ECE 269 Discussion Session

Nov. 22. 2019

Face Recognition Using Principal Component Analysis

1. **Computing PCs**

   Given a set of M training images (all of size $L-by-N$), we can represent them by a set of $LN-by-1$ vectors: $\Gamma_1, \Gamma_2, \cdots, \Gamma_M$. The computation of the PCs (eignefaces) is done by the following:

   First compute average face $\Psi = \frac{1}{M}\sum_{i=1}^{M}\Gamma_i$. Then construct covariance matrix C:

   $$C = AA^T$$

   where $A = [\Gamma_1 - \Psi, \Gamma_2 - \Psi, \cdots, \Gamma_M - \Psi]$. Your paper used the notation $\Phi_i := \Gamma_i - \Psi$. The eigenfaces $u_i, i = 1, 2, \cdots, M$ is just a set of M orthonormal eigenvectors of C associated with non-zero eigenvalues.

2. **Reconstructing an image using eigenfaces**

   Given any image Γ and a set of eigenfaces $u_1, u_2, \cdots, u_{M'}, M' \leq M$. We can reconstruct this image as:

   $$\hat{\Gamma} = \sum_{i=1}^{M'} w_i u_i + \Psi$$

   where $w_i = u_i^T(\Gamma - \Psi)$.

3. **Performance of reconstruction using MSE**

   The MSE (mean-square-error) between the reconstructed image and original image is computed by

   $$MSE = \frac{1}{NL}\sum_{i=1}^{L}\sum_{j=1}^{N}\left[\hat{\Gamma}(i,j) - \Gamma(i,j)\right]^2,$$

   where the images are assumed to be of size $L-by-N$.

   From the definition of MSE, we can see that if two images are identical, MSE=0, and smaller MSE values correspond to more accurate representation. Also, for 8-bit images (which is what we have for the training set), the MSE should not be larger than $(255)^2$.

   Furthermore, as we allow the use of more PCs to reconstruct an image, we would expect the MSE to be non-increasing. If the image we want to reconstruct can be represented by the set of eigenfaces exactly, the MSE should be very close to zero.

4. **Efficient calculation of eigenfaces**

   Note that the size of C is $NL-by-NL$. For our set of 190 training images, this matrix will be of size 31266-by-31266. But we only care about at most $M = 190$ eigenvectors of this matrix.

   If one tries to compute the eigenvalues and eigenvectors of C directly, one will end up with 31266 eigenvalues, and at most 190 of those will be nonzero values (or not small values such as 1.32×10^{-7}). And that is assuming your Matlab does not give you an “out of memory” error. So we seek an efficient way to compute the M eigenvectors we are interested in.

This is done by the following: consider an eigenvector v_i and its corresponding eigenvalue μ_i for the matrix $A^T A$:

$$A^T A v_i = \mu_i v_i.$$

If we multiply the above equation on both side from the left by the matrix A we will get:

$$AA^T A v_i = A\mu_i v_i = \mu_i A v_i.$$

So the vector Av_i will be an eigenvector of the matrix $C = AA^T$ and its associated eigenvalue remains to be μ_i. We know that for $\mu_i \neq \mu_j$, Av_i and Av_j are orthogonal because for symmetric matrices, eigenvectors associated with different eigenvalues are orthogonal (but not necessarily of unit norm).

Note that the matrix $A^T A$ is of size M-by-M. For our training set, 190-by-190. So instead of performing eigenvalue decomposition of the matrix C (which is of the size 31266×31266) and throw away the zero eigenvalues, we could perform eigenvalue decomposition on the matrix $A^T A$ (size 190×190), obtain a set of 190 orthonormal eigenvectors $v_1, v_2, \cdots, v_{190}$, and translate them to eigenvectors for C by

$$u_i = \frac{Av_i}{||Av_i||_2}, i = 1, 2, \cdots M.$$

The normalization in the above equation is important because the eigenfaces as defined in the paper need to be orthonormal. If you do not normalize the $u_i's$, you will be in trouble when trying to reconstruct faces.

5. **Other questions**

   (a) Do we compute the PCs once or do we compute the PCs based on the image we want to reconstruct?

   A: You compute the PCs once using the 190 neutral expression images. This set of eigenfaces will be used for reconstructing images for the later parts.

   (b) How to reconstruct an image using the eigenfaces?

   A: See part 2.

   (c) How many PCs do I need to use for reconstruction? i.e. How to choose M' in part 2?

   A: We use the eigenvectors that correspond to M' largest eigenvalues as PCs for reconstruction. For part a, the choice of M' is up to you. You can keep all eigenvectors whose eigenvalues are within 90% of the maximum eigenvalue, or the 20 largest, or according to some other criteria. Remember you will need to justify why you want to use this number of PCs. For part b-f, you will need to vary the value of M' to create a plot with MSE as the y-axis and M' as the x-axis.

   (d) What library functions can I use?

   A: You can use library functions for obtaining eigenvalues/eigenvectors; singular values; MSE calculation; imread, imrotate, imresize etc.

   (e) What is required for the project report?

   A: Your code with comments. The plots we specifically asked for. Comments based on your plot. Give some intuition to your result. Is this what we expect to see? Why or why not.