
ECE-269 Linear Algebra #Assignment1

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Due on Tuesday, October 15, 11:59pm

Problem 1: vector space other than \mathbb{R}^n .

Suppose $\vec{x}, \vec{y} \in \mathbb{R}^n$. Then

$$|\vec{x} \bullet \vec{y}| \leq \|\vec{x}\| \|\vec{y}\|. \quad (1)$$

Proof

First note that, if either \vec{x} or \vec{y} is the zero vector, then $\vec{x} \bullet \vec{y} = 0$ and $\|\vec{x}\| \|\vec{y}\| = 0$. In this case the theorem is trivially true because $|\vec{x} \bullet \vec{y}| = |0| = 0 = \|\vec{x}\| \|\vec{y}\|$.

Suppose, then, that neither \vec{x} nor \vec{y} is the zero vector. We will establish the truth of an inequality equivalent to (??), namely

$$-\|\vec{x}\| \|\vec{y}\| \leq \vec{x} \bullet \vec{y} \leq \|\vec{x}\| \|\vec{y}\|. \quad (2)$$

To do so we validate the right- and left-hand sides of inequality (2) separately.

Step 1 ($\vec{x} \bullet \vec{y} \leq \|\vec{x}\| \|\vec{y}\|$):

The stipulation that neither \vec{x} nor \vec{y} is the zero vector allows for the following definitions of (unit) vectors \vec{u} and \vec{w} :

$$\vec{u} = \frac{\vec{x}}{\|\vec{x}\|}, \quad \vec{w} = \frac{\vec{y}}{\|\vec{y}\|}. \quad (3)$$

Observe that

$$\begin{aligned} 0 &\leq \|\vec{u} - \vec{w}\|^2 \\ &= (\vec{u} - \vec{w}) \bullet (\vec{u} - \vec{w}) \\ &= \vec{u} \bullet \vec{u} + \vec{w} \bullet \vec{w} - 2(\vec{u} \bullet \vec{w}) \\ &= \|\vec{u}\|^2 + \|\vec{w}\|^2 - 2(\vec{u} \bullet \vec{w}) \\ &= 1 + 1 - 2(\vec{u} \bullet \vec{w}) \\ &= 2 - 2(\vec{u} \bullet \vec{w}) \end{aligned} \quad (4)$$

which implies that $2(\vec{u} \bullet \vec{w}) \leq 2$, or

$$\vec{u} \bullet \vec{w} \leq 1. \quad (5)$$

Substituting the values for \vec{u} and \vec{w} defined by (3) into inequality (5) yields $\frac{\vec{x}}{\|\vec{x}\|} \bullet \frac{\vec{y}}{\|\vec{y}\|} \leq 1$, which implies $\vec{x} \bullet \vec{y} \leq \|\vec{x}\| \|\vec{y}\|$.

Step 2 ($-\|\vec{x}\| \|\vec{y}\| \leq \vec{x} \bullet \vec{y}$):

Begin by defining the unit vector \vec{s} as

$$\vec{s} = \frac{-\vec{x}}{\|-\vec{x}\|} = \frac{-\vec{x}}{\|\vec{x}\|}.$$

Complete this step for extra credit. To do so, go through a similar argument represented by the expressions in (5), but with the vector $\vec{s} - \vec{w}$. Pay close attention to the minus signs as you proceed.

This is where you should begin Step 2.