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Assignment #1

Problem 1

Answer to the problem goes here. Use a line per sentence. Leave a blank space to start a new paragraph. Next, an example typesetting mathematics in LaTeX.
Showing that equation $a + b = \frac{c}{d}$ in evidence:

$$a + b = \frac{c}{d} \tag{1}$$

Note that equation **??** was automatically numbered. If you prefer not numbered equations, see the next example.

a) "There is a student in Gryffindor who has taken all elective classes."

Example Problem 2

Showing that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

$$\begin{aligned}
\neg(p \to q) &\equiv \neg(\neg p \vee q) \\
&\equiv \neg(\neg p \vee q) \\
&\equiv \neg(\neg p) \wedge \neg(q) \\
&\equiv p \wedge \neg q
\end{aligned}$$

Note that & is where the equations align.

Example Problem 3

Constructing the *Truth Table* of $(p \to q) \wedge (\neg p \leftrightarrow q)$ in Table **??**:

Table 1: Caption here. Leave it blank if you will not refer it.

p	q	$\neg p$	$p \to q$	$\neg p \leftrightarrow q$	$(p \to q) \wedge (\neg p \leftrightarrow q)$
T	T	F	T	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

Example Problem 4

a) "There is a student in Gryffindor who has taken all elective classes."
Solution:
$$\exists x \forall y \forall z (H(x, \text{Gryffindor}) \wedge P(x, y)) \tag{2}$$
where

$H(x, z)$ is "x is of z house"
$P(x, y)$ is "x has taken y,"
the domain for x consists of all students in Hogwarts
the domain for y consists of all elective classes,
and the domain for z consists of all Hogwarts houses.

b) Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd."
Solution:

1. $$\forall n (P(n) \rightarrow Q(n)), \tag{3}$$
   where
   $P(n)$ is "n is an odd integer" and
   $Q(n)$ is "n^2 is odd."
2. Assume $P(n)$ is true.
3. By definition, an odd integer is $n = 2k + 1$, where k is some integer.
4. $$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \end{aligned}$$
5. $\therefore n^2$ is an odd integer. $\square$

c) Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, \{1, 2, 3\}\}$:
Then, $A \in B$ and $A \subseteq B$.

d) Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3, \}$, and universe $U = \{1, 2, 3, 4, 5\}$:
$$\begin{aligned} A \cup B &= \{1, 2, 3, 5\}, \\ A \cap B &= \{1, 3\}, \\ A - B &= \{5\}, \\ \bar{A} &= \{2, 4\}, \\ A - A &= \emptyset. \end{aligned}$$