

## Problem 1: Convolution as Linea Map

### Solution

- (a) Obviously the size of matrice  $T$  must be  $(N + 1) \times (N + 1)$ . Let  $T_{i1}, T_{i2}, \dots, T_{i(N+1)}$  be the  $i$ -th row of matrice  $T$ . Since  $y = Tx$ , we can get

$$v(i - 1) = \sum_{j=1}^{N+1} T_{ij} u(j - 1) = T_{i1} u(0) + T_{i2} u(1) + \dots + T_{i(N+1)} u(N) \quad (1)$$

Accoring to the convolution function,

$$v(i - 1) = \sum_{k=-\infty}^{+\infty} h(k) u(i - 1 - k) = \sum_{k=(i-1-N)}^{i-1} h(k) u(i - 1 - k) \quad (2)$$

Comparing function (1) and function (2), it is easy to conclude that

$$T_{ij} = h(i - j), \text{ where } 1 \leq i \leq (N + 1), 1 \leq j \leq (N + 1)$$

So the matrice is a matrice where each element could be represented as  $T_{ij} = h(i - j)$ , where  $1 \leq i \leq (N + 1)$ ,  $1 \leq j \leq (N + 1)$ .

- (b) The structure of matrice  $T$  could be described as follow:

$$T_{i,j} = T_{i+1,j+1}$$

## Problem 2: Adjacency graph

### Solution

## Problem 3: Vector Spaces of Polynomials

### Solution

## Problem 4: Symmetric and Hermitian matrices

### Solution

## Problem 5: Properties of Vector Spaces

Solution

## Problem 6: Linear Independence

Solution

## Problem 7: Finding Basis

Solution