

Problem 1: Convolution as Linea Map

Solution

- (a) Obviously the size of matrix T must be $(N + 1) \times (N + 1)$. Let $T_{i1}, T_{i2}, \dots, T_{i(N+1)}$ be the i -th row of matrix T . Since $y = Tx$, we can get

$$v(i - 1) = \sum_{j=1}^{N+1} T_{ij}u(j - 1) = T_{i1}u(0) + T_{i2}u(1) + \dots + T_{i(N+1)}u(N) \quad (1)$$

Accoring to the convolution function,

$$v(i - 1) = \sum_{k=-\infty}^{+\infty} h(k)u(i - 1 - k) = \sum_{k=(i-1-N)}^{i-1} h(k)u(i - 1 - k) \quad (2)$$

Comparing function (1) and function (2), it is easy to conclude that

$$T_{ij} = h(i - j), \text{ where } 1 \leq i \leq (N + 1), 1 \leq j \leq (N + 1)$$

So the matrix is a matrix where each element could be represented as $T_{ij} = h(i - j)$, where $1 \leq i \leq (N + 1), 1 \leq j \leq (N + 1)$.

- (b) The structure of matrix T could be described as follow:

$$T_{i,j} = T_{i+1,j+1}$$

Problem 2: Adjacency graph

Solution

Problem 3: Vector Spaces of Polynomials

Solution

Problem 4: Symmetric and Hermitian matrices

Solution

Problem 5: Properties of Vector Spaces

Solution

Problem 6: Linear Independence

Solution

Problem 7: Finding Basis

Solution