

Computer Problem Solution

- a) Using the training data in **TrainingSamplesDCT8.mat**, what are reasonable estimates for the prior probabilities?

Solution:

Two priors probabilities, $P_Y(cheetah)$ and $P_Y(grass)$, could be estimated based on the number of vectors in the training set. The estimation of $P_Y(cheetah)$ and $P_Y(grass)$ are:

$$P_Y(cheetah) = N_{FG}/(N_{FG} + N_{BG}) = 0.1919 \quad (1)$$

$$P_Y(grass) = N_{BG}/(N_{FG} + N_{BG}) = 0.8081 \quad (2)$$

where

N_{BG} is the number of vectors in matrix **TrainsampleDCT_BG**

N_{FG} is the number of vectors in matrix **TrainsampleDCT_FG**

- b) Using the training data in **TrainingSamplesDCT8.mat**, compute and plot the index histograms $P_{X|Y}(x|cheetah)$ and $P_{X|Y}(x|grass)$.

Solution:

According to training data, the frequency histograms is the following picture:

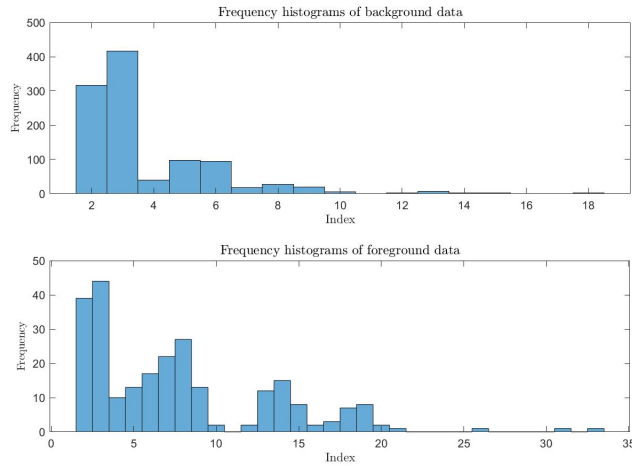


Figure 1: Frequency histograms

The index histograms of $P_{X|Y}(x|cheetah)$ and $P_{X|Y}(x|grass)$ is showed as following:

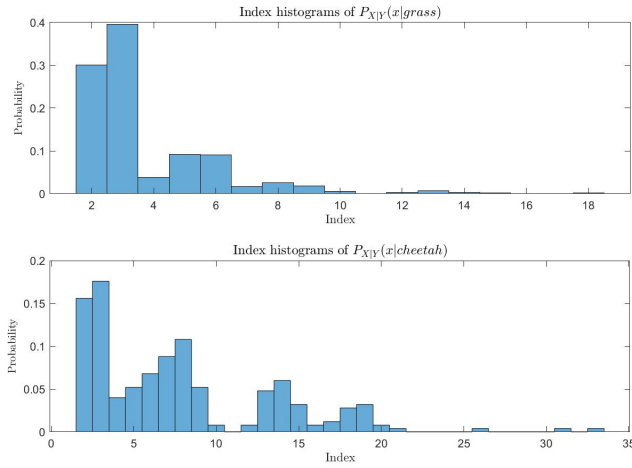


Figure 2: Index histograms

- c) For each block in the image **cheetah.bmp**, compute the feature X (index of the DCT coefficient with 2^{nd} greatest energy). Compute the state variable Y using the minimum probability of error rule based on the probabilities obtained in a) and b). Store the state in an array A. Using the commands `imagesc` and `colormap(gray(255))` create a picture of that array.

Solution:

Given a 8×8 block from the image **cheetah.bmp**, we can easily compute an array of 8×8 frequency coefficients by using function **dct2** on Matlab. Feature X would be index of the 2^{nd} greatest DCT coefficient. Given $X = x$ in one block, according to the minimum probability of error rule, we can pick state of cheetah if:

$$\frac{P_{X|Y}(x|cheetah)}{P_{X|Y}(x|grass)} > T = \frac{P_Y(grass)}{P_Y(cheetah)} \quad (3)$$

where

$P_{X|Y}(x|cheetah)$ and $P_{X|Y}(x|grass)$ are the estimation we get from training data.

$P_Y(cheetah)$ and $P_Y(grass)$ are the estimation we get from training data.

T is the threshold.

Then we mask the top left corner of the 8×8 block as 1, regarding this pixel belongs to cheetah. Otherwise, we mask 0. By using a sliding window that moves by one pixel at each step, finally we get a array A containing the mask indicates which blocks contain grass and which contain the cheetah.

- d) The array A contains a mask that indicates which blocks contain grass and which contain the cheetah. Compare it with the ground truth provided in image **cheetah mask.bmp** (shown below on the right) and compute the probability of error of your algorithm.

Solution:

The comparison between ground truth and picture generated from array A is showed as following:

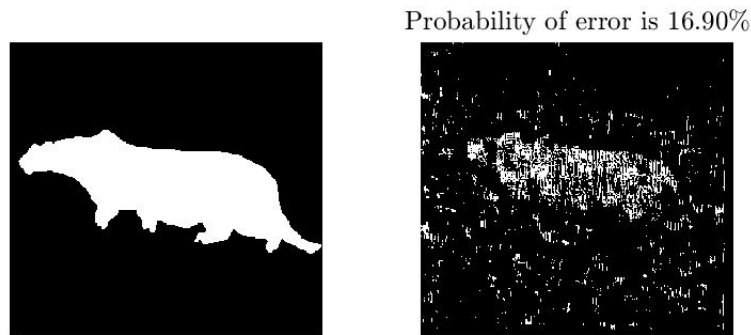


Figure 3: Comparision

The probabilities of error is 16.90%, as showed in the figure above.

Appendix

The following is the Matlab code.

```
1 clear all
2 %%
3 %Training
4 %Read the TrainingSamplesDCT_8.mat file
5 load( 'dataset/TrainingSamplesDCT_8.mat' );
6 %Save TrainsampleDCT_BG and TrainsampleDCT_FG in temporary value
7 train_BG = TrainsampleDCT_BG;
8 train_FG = TrainsampleDCT_FG;
9
10 %Find the second largest value in each row of matrix train_BG
11 [MBG,NBG] = max(train_BG,[],2);
```

```

12 train_BG(bsxfun(@eq, train_BG, MBG)) = -1; % Set the largest
    value in each row to -inf
13 [MBG,N_BG] = max(train_BG,[],2);
14
15 %Find the second largest value in each row of matrix train_FG
16 [MFG,N_FG] = max(train_FG,[],2);
17 train_FG(bsxfun(@eq, train_FG, MFG)) = -1; % Set the largest
    value in each row to -inf
18 [MFG,N_FG] = max(train_FG,[],2);
19
20 %Plot the frequency histogram
21 subplot(2,1,1);
22 h1 = histogram(N_BG);
23 ylim([0, 500]);
24 ylabel('Frequency', 'interpreter', 'latex', 'FontSize', 10);
25 xlabel('Index', 'interpreter', 'latex');
26 title({'Frequency histograms of background data'}, 'FontSize', 12,
    'interpreter', 'latex');
27 subplot(2,1,2);
28 h2 = histogram(N_FG);
29 ylim([0, 50]);
30 ylabel('Frequency', 'interpreter', 'latex', 'FontSize', 10);
31 xlabel('Index', 'interpreter', 'latex');
32 title({'Frequency histograms of foreground data'}, 'FontSize', 12,
    'interpreter', 'latex');
33 %Save the statistic data
34 F_x_BG = zeros(1,64);
35 F_x_BG(min(N_BG):max(N_BG)) = h1.Values;
36 F_x_FG = zeros(1,64);
37 F_x_FG(min(N_FG):max(N_FG)) = h2.Values;
38 %Save the histogram figure
39 set(gcf, 'Position', [400,100,900,600]);
40 saveas(gcf, ['Images/histograms1.jpg']);
41 close(gcf);
42
43 %Calculate the estimation of class-conditionals for two classes
    and priors probabilities
44 P_x_BG = F_x_BG ./ sum(F_x_BG);
45 P_x_FG = F_x_FG ./ sum(F_x_FG);
46 P_BG = size(train_BG,1) / (size(train_BG,1) + size(train_FG,1));
47 P_FG = size(train_FG,1) / (size(train_BG,1) + size(train_FG,1));
48

```

```

49 %Plot the index histogram
50 subplot(2,1,1);
51 h1 = histogram(NBG, 'Normalization', 'pdf');
52 ylim([0, 0.4]);
53 ylabel('Probability', 'interpreter', 'latex', 'FontSize', 10);
54 xlabel('Index', 'interpreter', 'latex');
55 title({'Index histograms of $$P_{X|Y}(x|grass)$$'}, 'FontSize'
    ,12, 'interpreter', 'latex');
56 subplot(2,1,2);
57 h2 = histogram(NFG, 'Normalization', 'pdf');
58 ylim([0, 0.2]);
59 ylabel('Probability', 'interpreter', 'latex', 'FontSize', 10);
60 xlabel('Index', 'interpreter', 'latex');
61 title({'Index histograms of $$P_{X|Y}(x|cheetah)$$'}, 'FontSize'
    ,12, 'interpreter', 'latex');
62 %Save the histogram figure
63 set(gcf, 'Position', [400,100,900,600]);
64 saveas(gcf, ['Images/histograms2.jpg']);
65 close(gcf);
66
67 %Read original image
68 I = imread('dataset/cheetah.bmp');
69 I = im2double(I);
70 %Define the loop numbers
71 loop_row = size(I,1) - 8 + 1;
72 loop_column = size(I,2) - 8 + 1;
73
74 mask = zeros(size(I));
75 position_ref = load('dataset/Zig-Zag Pattern.txt');
76 T = P_BG / P_FG; % Caculate the threshold
77
78 for i=1:1:loop_row
79     for j=1:1:loop_column
80         block = I(i:i+7,j:j+7);
81         DCT_block = dct2(block);
82         DCT_block = abs(DCT_block);
83         [x,y] = find(DCT_block==max(DCT_block(:))); % Find the
            largest coefficient
84         DCT_block(x,y) = -1; % Set the largest value as -1
85         [x,y] = find(DCT_block==max(DCT_block(:))); % Find the
            second largest coefficient and its position
86         feature = position_ref(x,y) + 1;

```

```
87         %Decide the binary mask
88     %         %Before decide the mask, we should caluate two class-
            conditionals
89     %         P_FG_Decision = P_x_FG(1,feature) * P_FG / (P_x_FG(1,
            feature)* P_FG + P_x_BG(1,feature) * P_BG);
90     %         P_BG_Decision = P_x_BG(1,feature) * P_BG / (P_x_FG(1,
            feature)* P_FG + P_x_BG(1,feature) * P_BG);
91         if P_x_FG(1,feature)/P_x_BG(1,feature) > T
92             mask(i,j) = 1;
93         end
94     end
95 end
96
97 subplot(1,2,1)
98 I = imread('dataset/cheetah_mask.bmp');
99 I = im2double(I);
100 imshow(I);
101 subplot(1,2,2)
102 imshow(mask);
103 %Calculate the probability of error
104 error = length(find((mask-I)~=0)) / (size(I,1) * size(I,2));
105 title(['Probability of error is ', num2str(error*100, '%.2f'), '%',
        ], 'FontSize', 12, 'interpreter', 'latex');
106 %set(gcf, 'Position', [900,600]);
107 saveas(gcf, ['Images/segmentation.jpg']);
108 close(gcf);
```