Solutions for HW6

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1 Homework exercise 1

In this exercise, we are required to compute the posterior probability of a sensor fault for $N = 1, 2, \cdots, 10$. The prior probability of sensor to be fault is given:$p_{\text{fault}} = 0.01$ and since the ranges are uniformly distributed in the interval, the prior Cumulative probability P of its distribution could be described as:

$$P_{z \leq Z} = \frac{z}{Z} \quad \forall z \in [0, Z]$$

In the above equation, z represents the range value read from the sensor during each scan and Z represents the maximum range value. In this case, $Z = 3$. Besides, it has another property:

$$p_{z_1 \leq z < z_2} = \frac{z_2 - z_1}{Z} \quad \forall z_1, z_2 \in [0, Z] \quad s.t. \quad z_1 < z_2$$

From the above equation, we could conclude that:

$$p = \begin{cases} \frac{1}{3} & z \in [0, 1) \\ \frac{1}{3} & z \in [1, 2) \\ \frac{1}{3} & z \in [2, 3] \end{cases}$$

From the title, we know that when the sensor is faulty, it would always output a range below 1m in regardless of the actual range. First, we focus on the $N = 1$. The task to find the posterior probability under the situation that the measurement is below 1m could be described as an formula using Bayer rules.

$$\begin{aligned} p(z_{\text{faulty}} | z_{z \leq 1}) &= \frac{p(z_{z \leq 1} | z_{\text{faulty}}) p(z_{\text{faulty}})}{p(z_{z \leq 1})} \\ &= \frac{1 * p(z_{\text{faulty}})}{p(z_{z \leq 1})} \\ &= 0.03 \end{aligned}$$

Since the measurement at every single time could be considered independent, which means the measurement at the previous time has no effect on it at the current time. In most cases, I think it is reasonable to introduce this assumption.

Then, the problem could be much easier because at each single time, the posterior probability for a sensor fault is independent. I define n represent a specific value in $N = 1, 2, \cdots, 10$, the posterior probability could be formulated as:

$$p_n = (0.03)^n (1 - 0.03)^{N-n}$$

2 Homework exercise 2

Assume there is a cleaning robot equipped with a cleaning unit to clean the floor. However, since neither the cleaning unit nor the sensor are perfect. Here, we are given the conditional probability of robot succeeding in cleaning a dirty floor as follow:

$$p(X_{t+1} = \text{clean} | x_t = \text{dirty}, u_{t+1} = \text{vacuum clean}) = 0.7$$

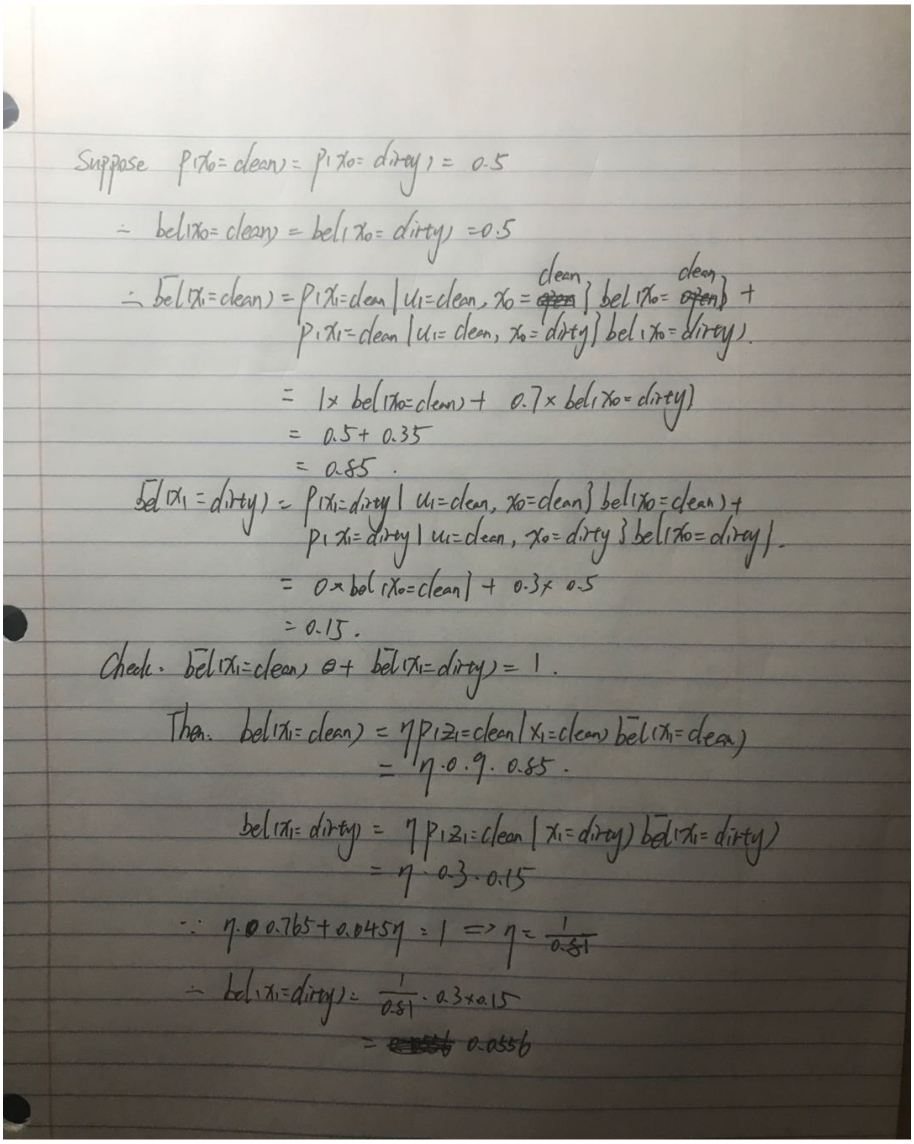
In the above equation, where x_{t+1} is the state of the floor after having vacuum-cleaned, u_{t+1} = vacuum clean is the control command, and x_t is the state of the floor before performing the action. And the probability of the sensor indicating the floor is clean although it is dirty is given by:

$$p(z_t = \text{clean} | x_t = \text{dirty}) = 0.3$$
$$p(z_t = \text{dirty} | x_t = \text{dirty}) = 0.7$$

Also, the probability that the sensor correctly detects a clean floor is given by:

$$p(z_t = \text{clean} | x_t = \text{clean}) = 0.9$$
$$p(z_t = \text{dirty} | x_t = \text{clean}) = 0.1$$

Besides, since we have no knowledge of the state of the floor, it could be set as $p_{\text{dirty}} = p_{\text{clean}} = 0.5$ as the prior distribution on the state of the floor. Hence, the probability of the floor being still dirty after the robot has vacuum-cleaned it is computed:



Suppose $p(x_0 = clean) = p(x_0 = dirty) = 0.5$

$\therefore bel(x_0 = clean) = bel(x_0 = dirty) = 0.5$

$\therefore \overline{bel}(x_1 = clean) = p(x_1 = clean \mid u_1 = clean, x_0 = \cancel{open}\ clean)\, bel(x_0 = \cancel{open}\ clean) + p(x_1 = clean \mid u_1 = clean, x_0 = dirty)\, bel(x_0 = dirty).$

$= 1 \times bel(x_0 = clean) + 0.7 \times bel(x_0 = dirty)$

$= 0.5 + 0.35$

$= 0.85.$

$\overline{bel}(x_1 = dirty) = p(x_1 = dirty \mid u_1 = clean, x_0 = clean)\, bel(x_0 = clean) + p(x_1 = dirty \mid u_1 = clean, x_0 = dirty)\, bel(x_0 = dirty).$

$= 0 \times bel(x_0 = clean) + 0.3 \times 0.5$

$= 0.15.$

Check. $\overline{bel}(x_1 = clean) + \overline{bel}(x_1 = dirty) = 1.$

Then. $bel(x_1 = clean) = \eta\, p(z_1 = clean \mid x_1 = clean)\, \overline{bel}(x_1 = clean)$

$= \eta \cdot 0.9 \cdot 0.85.$

$bel(x_1 = dirty) = \eta\, p(z_1 = clean \mid x_1 = dirty)\, \overline{bel}(x_1 = dirty)$

$= \eta \cdot 0.3 \cdot 0.15$

$\therefore \eta \cdot 0.765 + 0.045\eta = 1 \Rightarrow \eta = \frac{1}{0.81}$

$\therefore bel(x_1 = dirty) = \frac{1}{0.81} \cdot 0.3 \times 0.15$

$= 0.0556$

Figure 1: The probability of the floor being dirty

3 Homework exercise 3

3.1 i

In this section, we are required to generate samples from a normal distribution $\mathcal{N}(\mu, \sigma^2)$. Using Box-Muller transform method, I could generate the random Gaussian distributed samples from uniform distribution. First, we generate two samples u, v from the uniform distribution $\in [0, 1]$, then, the gaussian normal distribution $\in \mathcal{N}(0, 1)$ value could be computed by:

$$z_1 = \sqrt{-2\text{log}u} \cos 2\pi v$$
$$z_2 = \sqrt{-2\text{log}u} \sin 2\pi v$$

Then, we multiple it by σ and add μ and obtain what we want. Finally, we compute the its probability from Guassian distribution:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

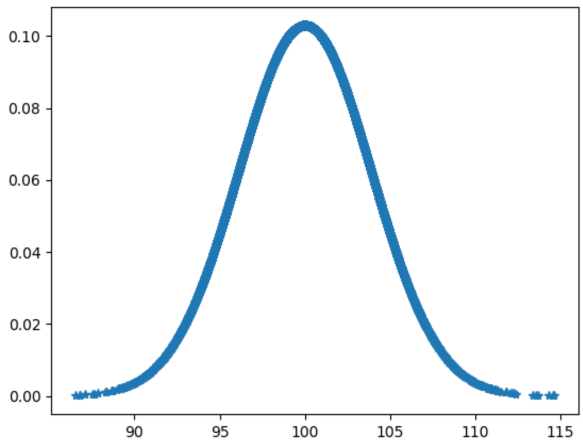


Figure 2: 10000 samples

3.2 ii

In this section, we are required to simulate the motion of a robot over a line. The motion model is very simple, which is given by:

$$x_t = x_{t-1} + u_t + w_t$$

Here, w_t represents the random noise generated from normal gaussian distribution. At each time we add the control input, we add the noise at the same time. After doing this for 10 times, we could obtain a sequence of x_t as each time t. I assume all the control inputs are ones, which means robot would always move forwards one more step and the initial state $x_0 = 0$.

```
 RESTART: C:\Users\Administrator\Desktop\winter quarter\MAE145\Homework\Homework
6\python_program.py
0.7583946847220544
0.37319676630081233
-0.6344774696177398
-1.660727952650923
0.4036757189532597
-0.2848849369780182
0.46428655510202277
1.2316371381849518
0.17498296949522843
0.8770614677667581
[0, 1.7583946847220544, 3.1315914510228664, 3.4971139814051266, 2.83638602875420
35, 4.240061747707463, 4.955176810729444, 6.419463365831467, 8.65110050401642, 9
.826083473511648, 11.703144941278406]
```

Figure 3: Results

In the above figure, the individual floats represent the noise generated at each time t and the list represents the sequence of steps of the robot.