MAE 145: Intro Robot Planning and Estimation
Homework #8

Assigned March 5. Due on March 13, Friday, by 10:30am

Notice:

Please upload 2 files for this homework. One to include the written part of the homework (problem 1, and the extra credit problem 3), and one python files for the second exercise. In your assignment include the names of the other students you have collaborated with to do the homework problems. The Collaboration Policy for this course is detailed in the syllabus.

Homework exercises:

1. Assume that you have a robot equipped with a sensor capable of measuring the distance and bearing to point landmarks. The sensor furthermore provides you with the identity of the observed landmarks. A sensor measurement $\mathbf{z} = (z_r, z_\theta)^\top$ is composed of the measured distance z_r and the measured bearing z_θ to the landmark l. Both the range and the bearing measurements are subject to zero-mean Gaussian noise with variances σ_r^2, and σ_θ^2, respectively. The range and the bearing measurements are independent of each other. A sensor model models the probability of a measurement $\mathbf{z}$ of landmark l observed by the robot from pose $\mathbf{x}$.

   (a) Design a sensor model $p(\mathbf{z}|\mathbf{x}, l)$ for this type of sensor. Furthermore, explain your sensor model.

   (b) Assume that it is known that your sensor can only provide measurements with a maximum range value of $z_{\max}$. Modify the sensor model in part (a) appropriately.

2. A particle filter consists of three steps, which you will implement in the following. The steps are listed in the following:

   (a) Sample new particle poses using the motion model.

   (b) Compute weights for the new particles using the sensor model.

   (c) Compute the new belief by sampling particles proportional to their weight with replacement.

   Consider a robot moving close to a wall (of length $[-50, 50]$) and with 3 doors at locations $[0.5, 1]$, $[2, 2.5]$, and $[22.5, 23]$. Do the following:

   (a) Implement a probabilistic motion model function of the robot $p(\mathbf{x}_t|\mathbf{x}_{t-1}, u_t)$ given as a *truncated* Gaussian random variable of mean $\mathbf{x}_{t-1} + u_t$ and variance $\sigma_u^2 = 0.5$, for $u_t \neq 0$. When $u_t = 0$, then the motion is deterministic, $\mathbf{x}_{t+1} = \mathbf{x}_t$. The truncated Gaussian should give values between $[-50, 50]$.

   (b) Implement the sensor likelihood model function:

$$
\begin{aligned}
&P(z = 1|0.5 \leq x \leq 1, \text{ or } 2 \leq x \leq 2.5 \text{ or } 22.5 \leq x \leq 23) = 0.7\\
&P(z = 0|x \text{ at door locations}) = 0.3\\
&P(z = 0|x \text{ not at door locations}) = 0.85\\
&P(z = 1|x \text{ not at door locations}) = 0.15
\end{aligned}
$$

(c) Consider the control data $u_t = 1$ for $1 \leq t \leq 14$ time steps, $u_t = 0$ for 2 steps, $15 \leq t \leq 16$, and $u_t = -1$, for $17 \leq t \leq 38$ steps. The sensor sequence is $\{z_t\} = \{0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0\}$ at each of the 38 steps. Implement a particle filter by using the most basic resampling method discussed in class. This filter should be a function of the number of particles. re-draw a new particle until you get something within $[-50, 50]$. *Note:* Particle deprivation effects could be apparent after a small number of iterations t with a small number of particles. Find out at what iteration and what number of M you may observe such problems. Visualize the particle locations at different times t by means of plots of vertical lines at the locations $\mathbf{x}_t^{[m]}$ and with height 1.

3. (Extra credit problem) Many early robots navigated using artificial landmarks in the environment that were easy to recognize. A good place to mount the markers is a ceiling, and a classical example is a visual marker. Suppose that you have the marker of Figure 1 attached to the ceiling.

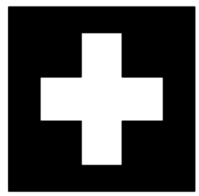

Figure 1: Marker for Exercise 3

Let the world coordinates be x_m y_m and its orientation relative to the global coordinate system θ_m. We will denote the robot's pose by x_r, y_r, and θ_r.

Now assume that we are given a routine that can detect the marker in the image plane of a perspective camera at the robot. Let x_i, y_i denote the coordinates of the marker on the image plane, and θ_i its angular orientation. The camera has focal length of f. From projective geometry, we know that each displacement d in the $x - y$ space gets projected to a proportional distance $d \cdot \frac{f}{h}$ in the image plane, for some constant h. (You have to make some choices of your coordinate systems; make these choices explicit.) Do the following:

(a) Describe mathematically where to expect the marker (in global coordinates x_m, y_m, θ_m) when its image coordinates are x_i, y_i, θ_i and the robot is at x_r, y_r, θ_r.

(b) Provide a mathematical equation for computing the image coordinates x_i, y_i, θ_i from the robot pose x_r, y_r, θ_r and the marker coordinates x_m, y_m, θ_m.

(c) Now give a mathematical equation for determining the robot coordinates x_r, y_r, θ_r assuming we know the true marker coordinates x_m, y_m, θ_m and the image coordinates x_i, y_i, θ_i.

(d) So far, we have assumed there is a single marker. Now suppose that there are multiple (indistinguishable) markers of the type shown above. How many such markers must a robot be able to see to uniquely indentify its pose? Draw such configuration, and argue why it is sufficient.