Solutions for HW7

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Contents

1 Problem 1

Consider a "segment" or a "rod" robot of length $\ell = 2$, which can rotate about its center to any orientation, but which can not translate in space.

1.1 a

1.1.1 problem formulation

Assume the ends of this robot are distinguishable (red and blue ends). What is the configuration space of the robot and why?

1.1.2 solution

The rod-like robot has three degrees of freedom accounting for rotation for three each axis in $\mathbb{R}^3$, so the configuration space is $SO(3)$, which is a group of rotations like a 3D sphere $\mathbb{S}^3$.

1.2 b

1.2.1 problem formulation

Suppose you would like to do motion planning for the robot with distinguishable ends, taking the red end from $|(0,0,1)$ in $\mathbb{R}^3$ to (0,0,-1) in $\mathbb{R}^3$ and passing through the point $W = (-1,0,0)$ However, there are two obstacles present:

$$O_1 = \left\{(x,y,z) \in \mathbb{R}^3 | -a \leq x \leq -b, c \leq z, \text{ and } x^2+y^2+z^2 \leq 1\right\}$$
$$O_2 = \left\{(x,y,z) \in \mathbb{R}^3 | -a \leq x \leq -b, z \leq -c, \text{ and } x^2+y^2+z^2 \leq 1\right\}$$

for some positive constants $a, b, c > 0$. To find a path, make use of the stereographic charts, ϕ_N restricted over $\phi_N^{-1}(\mathbb{D})$ and ϕ_S restricted over $\phi_S^{-1}(\mathbb{D})$, where $\mathbb{D} = \left\{(x,y) \in \mathbb{R}^2 | x^2+y^2 \leq 1\right\}$ is the unit disk. The projection of O_2 on the disk using ϕ_N looks as in Figure [2] (left), while the projection of O_1 on the disk using ϕ_S looks as in Figure [2] (right). Using the graph defined over the $\mathbb{D}$ (Figure $\nVdash$ (right)), construct a graph on the sphere that you can use to solve a motion planning problem (give the list of vertices and enumerate the edges, you can also provide a diagram for it). This graph has to be conflict free, that is, its edges or vertices can not intersect with obstacles.

1.2.2 solution

As stated in the problem formulation, we are provided are two charts which are able to parameterize the whole space M and it is enough to use these two charts to solve a motion planning problem. Then, the first step is to set equivalences among these two charts. Since we are using two stereographic charts, based on its geometric projection property, we know that all the points x_k on the circle(when $z = 0$) are equivalent in each chart. Hence, the connection between these two charts are build naturally. We could search a feasible path in these two charts with the given connection points.

We are required to take the red end from (0,0,1) to (0,0,-1) and pass through the point $W = (-1,0,0)$. We construct graphs respectively in each chart and connect them as a whole path via equivalent points.

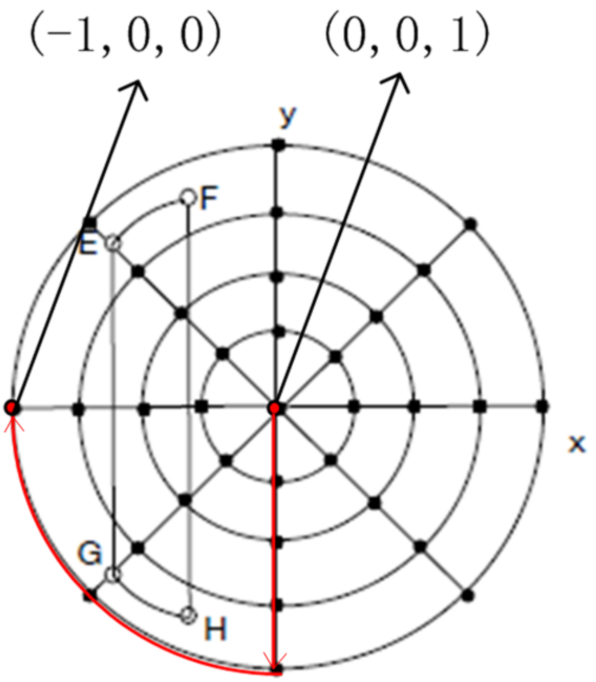

Figure 1: The path on the chart ϕ_S

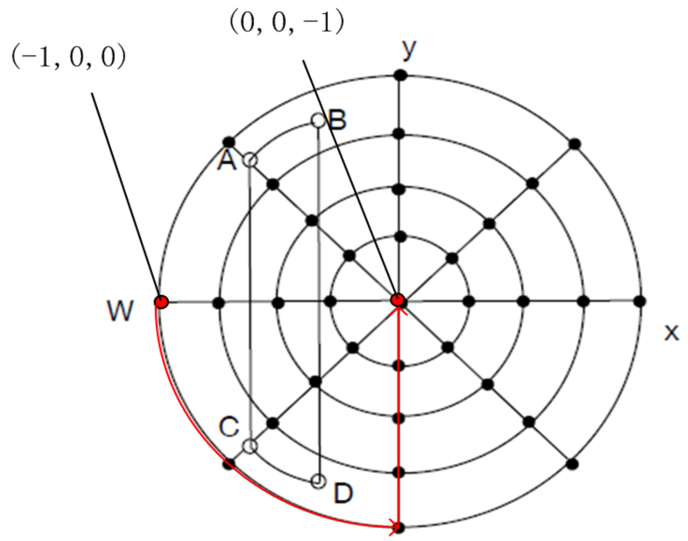

Figure 2: The path on the chart ϕ_N

From the above two figures, you can see that the point (-1,0,0) is selected as equivalent point on the two charts and it is used to build the connection between two charts. By following this path, the rod-like robot could bypass all the obstacles and move from (0,0,1) to (0,0,-1).

1.3 c

1.3.1 problem formulation

Draw a path from start to the goal, passing through W, over the graph you obtained using the stereographic coordinates. Plot and trace out the path and the graph in $\mathbb{R}^3$. Note: An approximate drawing by hand is sufficient (make a big clear drawing), and, some extra credit will be given to those who provide a plot for it to visualize in 3 D. Recall that a parameterization of a segment between two vectors $\mathbf{u}, \mathbf{v}$ is given by $\gamma(t) = t\mathbf{u} + (1-t)\mathbf{v}, t \in [0, 1]$ (similar arc parameterizations can be found). You can use this to lift any segment in the plane to $\mathbb{R}^3$ using the stereographic charts.

1.3.2 solution

The path I obtained using stereographic coordinated is shown in part (b). Then, I have to plot the path in $\mathbb{R}^3$.

The hand-written path in $\mathbb{R}^3$ is shown as follow:

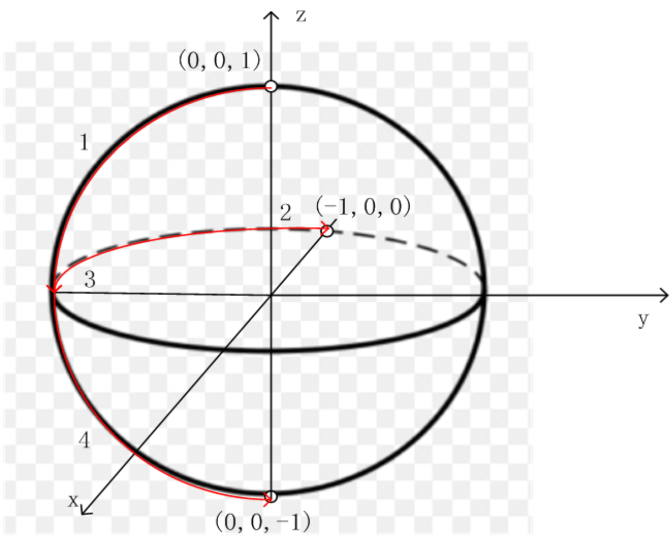

Figure 3: Hand-written path

There are four main arcs of the path and each of them is assigned with a number nearby in sequence. Then, I plot it in 3D using python.

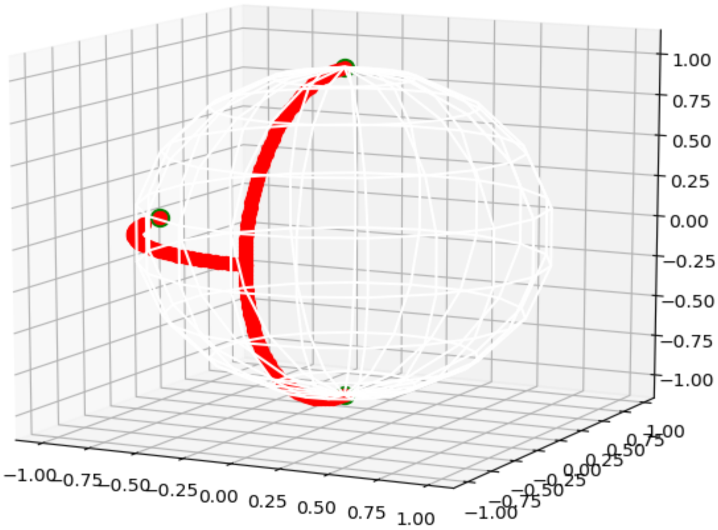

Figure 4: Visualization using python

I copy the codes here:

```
#A53307224
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D


def TD_point_south(x,y):
    z =  -(x**2 + y**2 - 1)/(x**2 + y**2 + 1)
    new_x = x*z + x
    new_y = y*z + y
    return new_x, new_y, z

def TD_point_north(x,y):
    z =  (x**2 + y**2 - 1)/(x**2 + y**2 + 1)
    new_x = -x*z + x
    new_y = -y*z + y
    return new_x, new_y, z

def main():
    start = [0,0,1]
    end = [0,0,-1]
    w = [-1,0,0]
    fig = plt.figure()
    ax = plt.axes(projection='3d')
    # Data for three-dimensional scattered points
    u, v = np.mgrid[0:2*np.pi:20j, 0:np.pi:10j]
    x = np.cos(u)*np.sin(v)
    y = np.sin(u)*np.sin(v)
    z = np.cos(v)
    ax.plot_wireframe(x, y, z, color="w")
    ax.scatter(start[0], start[1], start[2], color="g", s=100)
    ax.scatter(end[0], end[1], end[2], color="g", s=100)
```

```python
    ax.scatter(w[0], w[1], w[2], color="g", s=100)
    # Draw trajecroty
    x = np.linspace(0,0,50)
    y = np.linspace(0,-1,50)
    for i in zip(x,y):
        new_x, new_y, z = TD_point_south(i[0],i[1])
        ax.scatter(new_x, new_y, z, color="r", s=50)
    x = np.linspace(0,-1,50)
    y = -np.sqrt(1-x**2)
    for i in zip(x,y):
        new_x, new_y, z = TD_point_south(i[0],i[1])
        ax.scatter(new_x, new_y, z, color="r", s=50)
    x = np.linspace(-1,0,50)
    y = -np.sqrt(1-x**2)
    for i in zip(x,y):
        new_x, new_y, z = TD_point_north(i[0],i[1])
        ax.scatter(new_x, new_y, z, color="r", s=50)
    x = np.linspace(0,0,50)
    y = np.linspace(-1,0,50)
    for i in zip(x,y):
        new_x, new_y, z = TD_point_north(i[0],i[1])
        ax.scatter(new_x, new_y, z, color="r", s=50)
    plt.show()

if __name__ == '__main__':
    main()
```

1.4 d

1.4.1 problem formulation

(Extra credit) If the ends of the robot are indistinguishable, what is now the configuration space of the robot? what would be a valid path of the rod-robot from (0,0,1) to (0,0,-1) through W? Draw it by hand using the stereographic charts.

1.4.2 solution

If the ends of the robot are indistinguishable, we only need one chart to fully represent the whole space of M. In this case, both ϕ_S or ϕ_N is acceptable. Its configuration space is now like half a sphere because the two end points are always symmetric about the origin.

Hence, the valid path of the rod-robot from (0,0,1) to (0,0,-1) using ϕ_S could be:

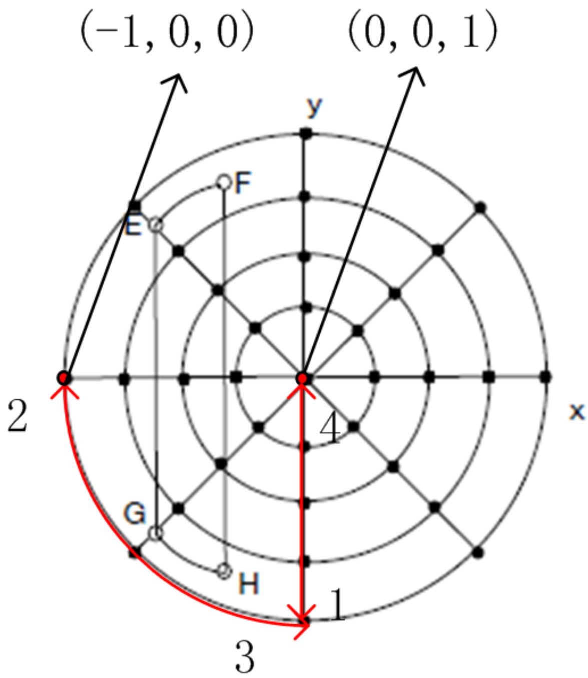

Figure 5: The path on the chart ϕ_S

Then, I draw it by hand in $\mathbb{R}^3$:

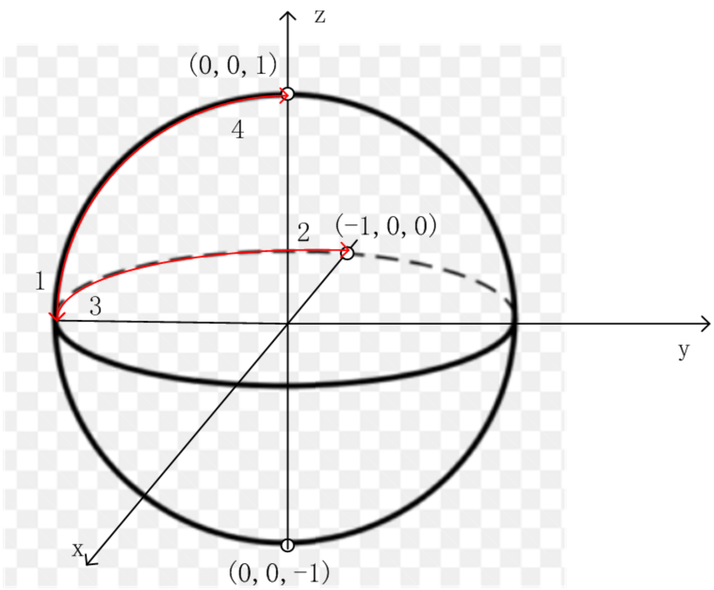

Figure 6: Hand-written path