在 Overleaf 平台上使用 CTEX 完成作业：Final project

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Problem: Signal Enhancement

It is desired to design an adaptive Wiener filter to enhance a sinusoidal signal buried in noise. The noisy sinusoidal signal is given by

$$x_n = s_n + v_n\ , \qquad \text{where}\ \ s_n = \sin(\omega_0 n)$$

with $\omega_0 = 0.075\pi$. The noise v_n is related to the secondary signal y_n by

$$v_n = y_n + y_{n-1} + y_{n-2} + y_{n-3} + y_{n-4} + y_{n-5} + y_{n-6}$$

The signal y_n is assumed to be an order-4 AR process with reflection coefficients:

$$\left\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\right\} = \{0.5, -0.5, 0.5, -0.5\}$$

As in an earlier experiment, the variance σ^2 of the driving white noise of the model must be chosen in such a way as to make the variance σ_v^2 of the noise component v_n approximately *one*.

1 Theoretical solutions

For a Wiener filter of order $M = 6$, determine the theoretical direct-form Wiener filter coefficient vector:

$$\boldsymbol{h} = [h_0, h_1, \cdots, h_6]$$

for estimating y_n (or, rather v_n) from y_n. Determine also the theoretical lattice/ladder realization coefficients:

$$\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_6], \quad \boldsymbol{g} = [g_0, g_1, \cdots, g_6]$$

解:

注意：反射系数存在两种定义，这里采用现代谱估计的定义。$k_p = a_p(p)$

1. 先假设驱动白噪声的方差为 1，计算 $v(n)$ 的方差，再用其作为缩放因子调整驱动白噪声的方差。
2. 步降法得，AR 参数：$a = [1, -0.25, -0.1875, 0.5, -0.5]$，$\sigma_u^2 = 0.0534$
3. direct-form weights: $\boldsymbol{h} = [h_0, h_1, \cdots, h_6] = \begin{bmatrix}1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{bmatrix}$
4. ladder coefficients: $\boldsymbol{g} = [g_0, g_1, \cdots, g_6] = \begin{bmatrix}1.1328 & 1.5990 & 1.5104 & 0.4838 & 2.5386 & -1.0878 & 1\end{bmatrix}$
5. lattice reflection coefficient: $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_6] = \begin{bmatrix}0.5 & -0.5 & 0.5 & -0.5 & -1.6358 & -0.9292\end{bmatrix}$

2 LMS algorithm

Generate input pairs $\{x_n, y_n\}$ (making sure that the transients introduced by the modeling filter have died out), and filter them through the LMS algorithm to generate the filter output pairs $\{\hat{x}_n, y_n\}$. On the same graph, plot e_n together with the desired signal s_n.

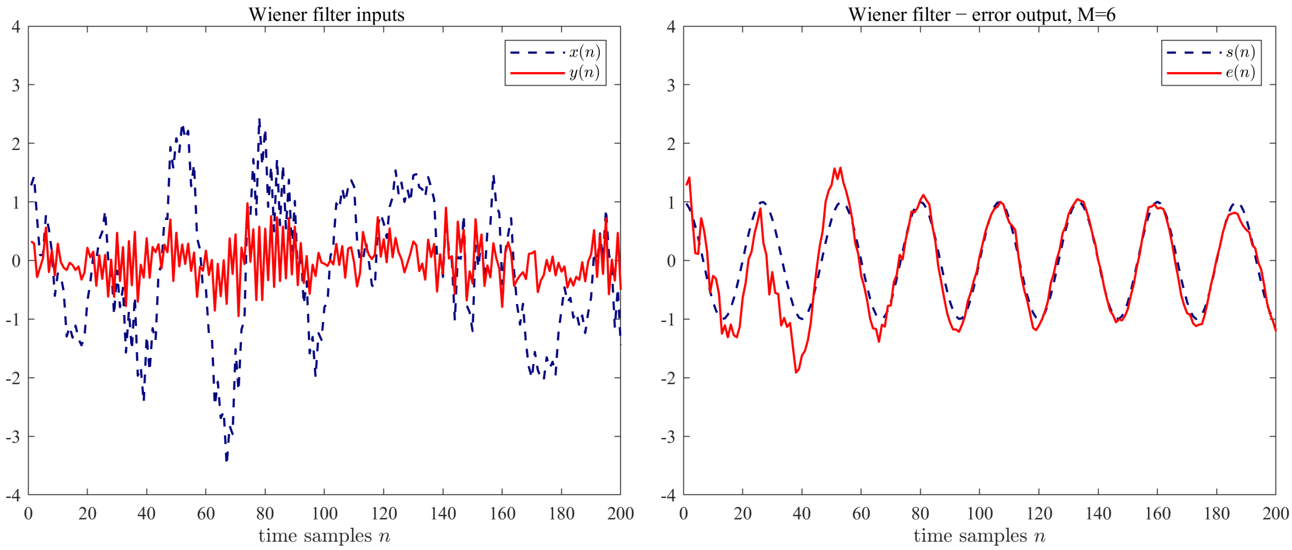
Plot also a few of the adaptive filter coefficients such as $h_4(n)$, $h_5(n)$, and $h_6(n)$. Observe their convergence to the theoretical Wiener solution.

You must generate enough input pairs in order to achieve convergence of the LMS algorithm and observe the steady-state converged output of the filter.

Experiment with the choice of the adaptation parameter μ. Start by determining $\lambda_{\max}$, $\lambda_{\min}$, the eigenvalue spread $\lambda_{\max}/\lambda_{\min}$ of $\mathbf{R}$ and the corresponding time constant.

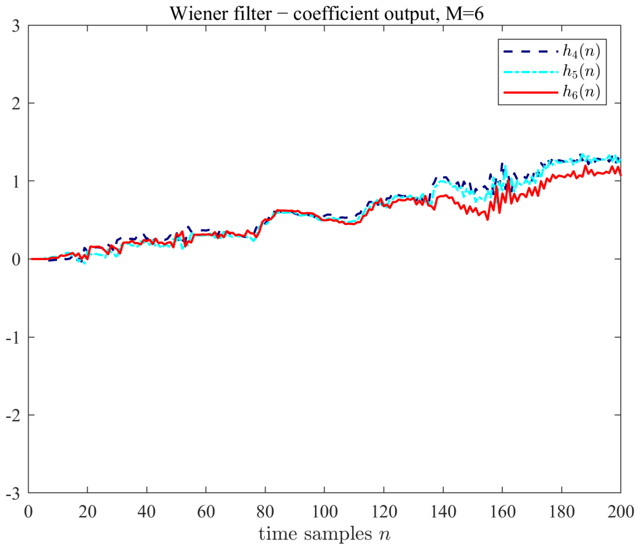
解:

1. 4 阶 AR 输入的自相关是 $r_{yy} = \begin{bmatrix} 0.1689 & -0.0844 & 0.1055 & -0.1161 & 0.1174 \end{bmatrix}$
2. 由于自适应的阶数是 6，所以将 r_{yy} 填充到 7 个元素。自相关矩阵非满秩。
   $r_{yy} = \begin{bmatrix} 0.1689 & -0.0844 & 0.1055 & -0.1161 & 0.1174 & 0 & 0 \end{bmatrix}$
3. $\lambda_{\max}$，$\lambda_{\min}$，$\lambda_{\max}/\lambda_{\min}$ 分别是 0.7062，-0.0577，-12.2457.
4. 上界 $\mu_{\max} = \frac{1}{2\max\lambda + \sum\lambda}$，取 $\mu = \frac{1}{5}\mu_{\max} = 0.0771$
5. *In generating y_n make sure that the transients introduced by the filter have died out.*

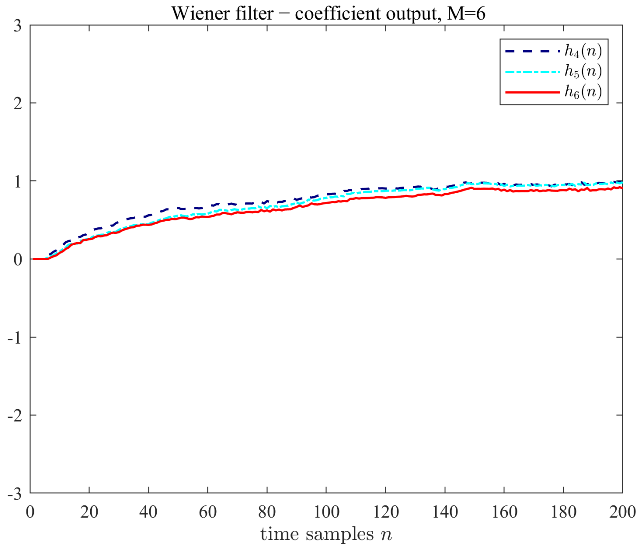


(a) (b)

Figure 1 Wiener 滤波器的输入与 LMS 的误差输出。(a) 主输入与次输入，(b) LMS 算法的增强-误差输出。



(a)



(b)

Figure 2 The direct-form coefficients of the Wiener filter。(a) 单次，(b) 20 次平均。

3 Tracking changes

Next, we change this experiment into a non-stationary one. Suppose the total number of input pairs that you used in parts (2) is N. And suppose that at time $n = N$, the input statistics changes suddenly so that the primary signal is given now by the model:

$$x_n = s_n + v_n \,, \quad \text{where} \quad v_n = y_n + y_{n-1} + y_{n-2} + y_{n-3}$$

and y_n changes from a fourth-order AR model to a second-order model with reflection coefficients (use the same σ_v^2 as before):

$$\{\gamma_1, \gamma_2\} = \{0.5, -0.5\}$$

Repeat parts (1) and (2), keeping the filter order the same, $M = 6$. Use $2N$ input pairs, such that the first N follow the original statistics and the second N follow the changed statistics. Compare the capability of the LMS and lattice adaptive filters in tracking such changes.

Here, the values of μ for the LMS case and λ for the lattice case, will make more of a difference in balancing the requirements of learning speed and quality of estimates.

解:

1. 先假设驱动白噪声的方差为 1，计算 $v(n)$ 的方差，再用其作为缩放因子调整驱动白噪声的方差。
2. 步降法得，AR 参数：$a = [1, 0.25, -0.5]$，$\sigma_u^2 = 0.1607$
3. direct-form weights: $\boldsymbol{h} = [h_0, h_1, \cdots, h_6] = \begin{bmatrix}1 & 1 & 1 & 1 & 1 & 0 & 0 & 0\end{bmatrix}$
4. ladder coefficients: $\boldsymbol{g} = [g_0, g_1, \cdots, g_6] = \begin{bmatrix}1.1250 & 1.5833 & 0.3889 & 1 & 0 & 0 & 0\end{bmatrix}$
5. lattice reflection coefficient: $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \cdots, \gamma_6] = \begin{bmatrix}0.5 & -0.5 & -0.7222 & 0.2387 & 1.8898 & -0.4253\end{bmatrix}$
6. 2 阶 AR 输入的自相关是 $r_{yy} = \begin{bmatrix}0.2875 & -0.2143 & 0.1607\end{bmatrix}$
7. 由于自适应的阶数是 6，所以将 r_{yy} 填充到 7 个元素。自相关矩阵非满秩。
   $r_{yy} = \begin{bmatrix}0.2875 & -0.2143 & 0.1607 & 0 & 0 & 0 & 0\end{bmatrix}$
8. $\lambda_{\max}$，$\lambda_{\min}$，$\lambda_{\max}/\lambda_{\min}$ 分别是 0.8190，−0.0258，−31.786.
9. 上界 $\mu_{\max} = \frac{1}{2\max\lambda + \sum\lambda}$，取 $\mu = \frac{1}{5}\mu_{\max} = 0.0550$

10. 由于 μ 与改变前的 μ 近似，故继续采用前者值，而*rlsl* 的 λ 取 0.99 。

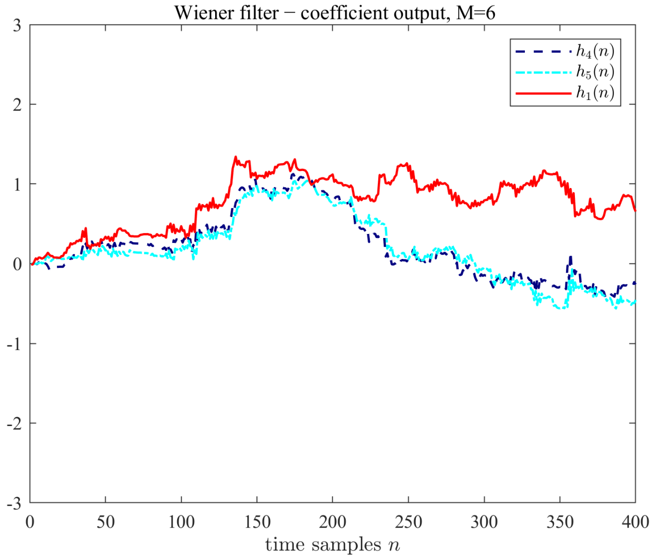


Figure 3 The direct-form coefficients for tracking using lms.

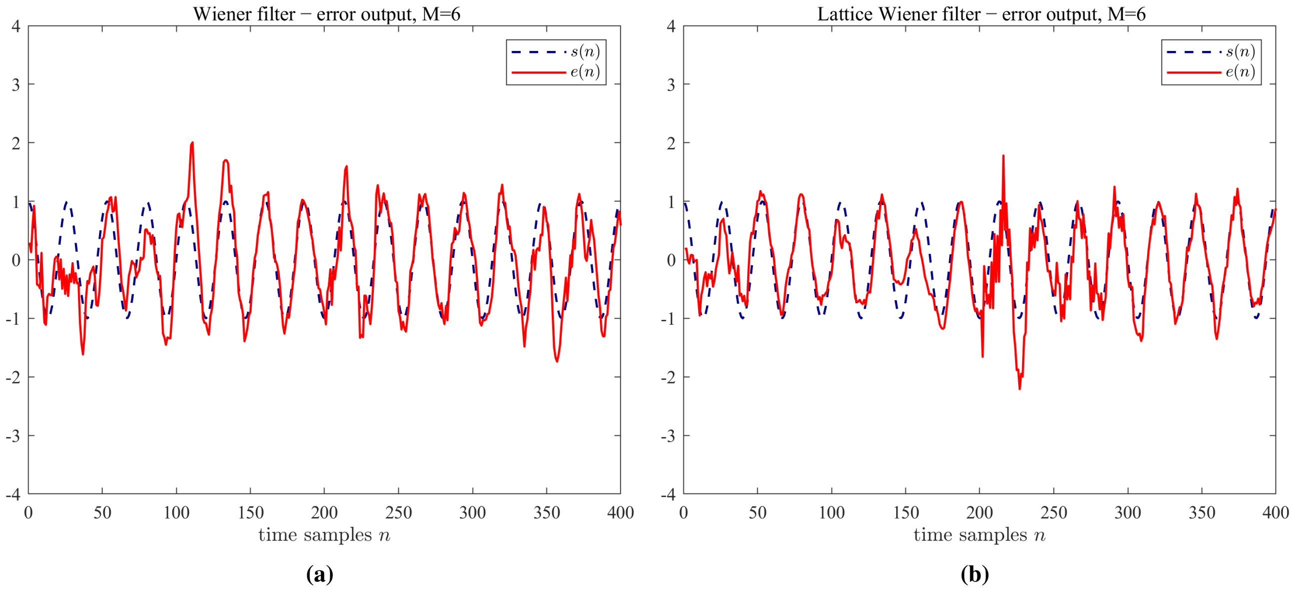
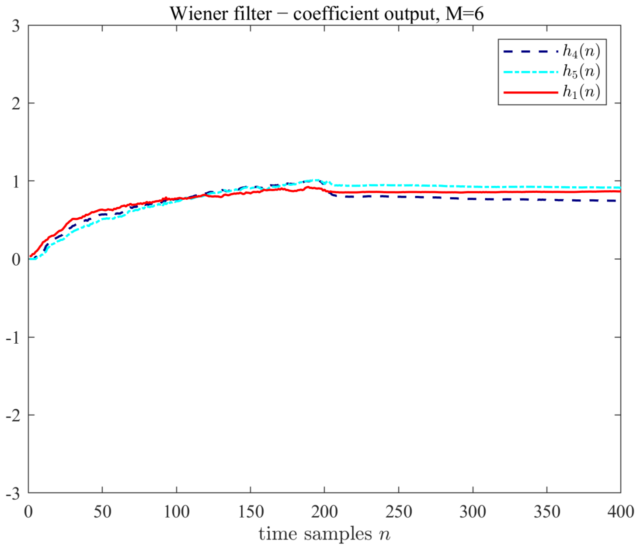
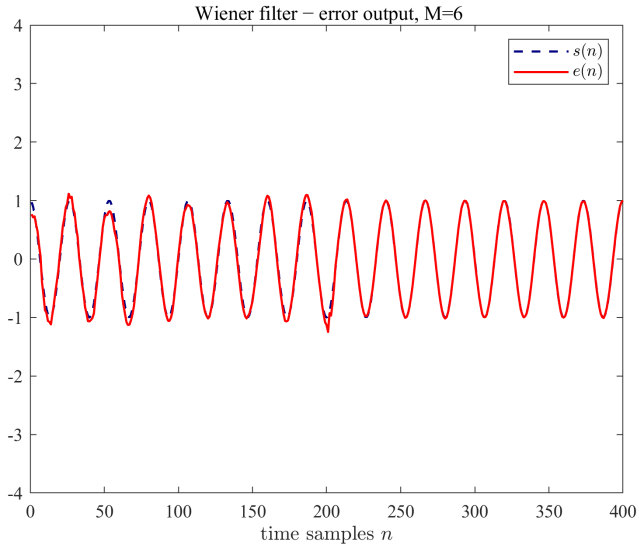


Figure 4 Estimation error while tracking. (a) lms, (b) rlsl。

11. 由图 3 和图 4 可以看出*rlsl* 算法的跟踪能力较*lms* 强，或者说对环境的改变更敏感。
12. 当 μ 取值较小时，收敛会变慢。time samples 需要更多。
13. 由图 5，当变化后的噪声 σ_v^2 为变化前的 1/100 时，将使得系统更完美，增强效果好，但 200 个点内 direct-form weights 跟踪失效。



(a)



(b)

Figure 5 Estimation error while tracking $\sigma_v^2/100$. 30 次平均 (a) coefficient with lms (b) estimation error with lms。

4 Free parameter tuning

Finally, feel free to "tweak" the statements of all of the above parts as well as the definition of the models in order to show more clearly and more dramatically the issues involved, namely, learning speed versus quality, and the effect of the adaptation parameters, eigenvalue spread, and time constants. One other thing to notice in this experiment is that, while the *adaptive weights tend to fluctuate a lot as they converge, the actual filter outputs* $\{\hat{x}_n, y_n\}$ *behave better and are closer to what one might expect*.

解:

1. 由图 5，当变化后的噪声 σ_v^2 为变化前的 1/100 时，将使得系统更完美，增强效果好，但 direct-form weights 跟踪失效。事实上，由图 6，$h_4(n)$、$h_5(n)$ 和 $h_6(n)$ 的值随变化后的噪声方差下降跟踪能力渐弱。或者说，μ 的值需要对应放大，得到更快的学习速率。

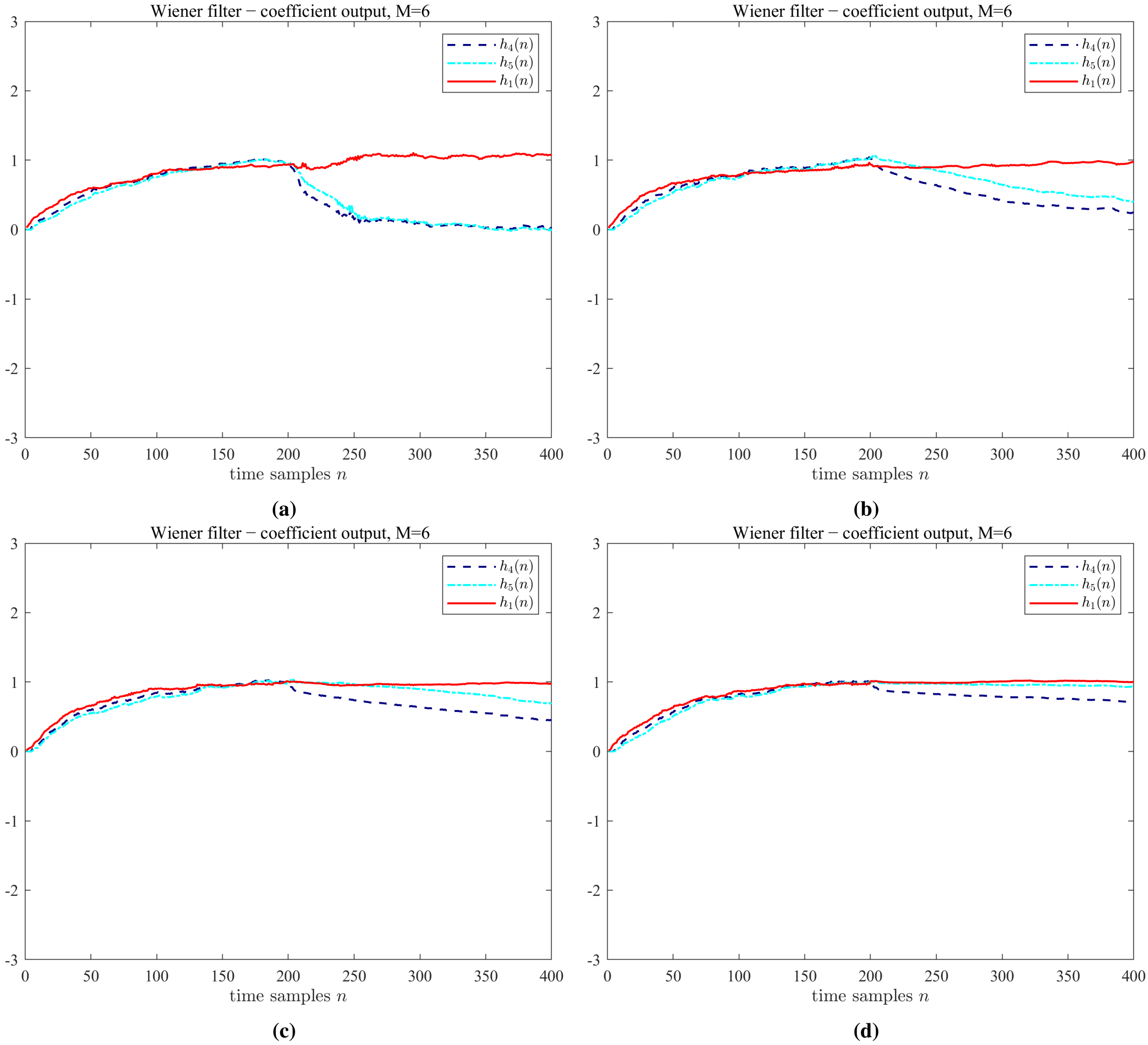


Figure 6 coefficients tracking while σ_v^2/b. 30 次平均 lms. (a) b=1, (b) b=4, (c) b=9, (d) b=25

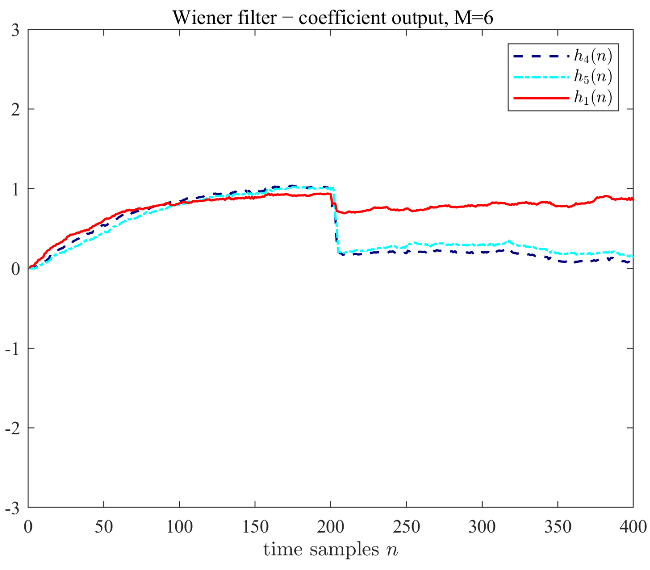
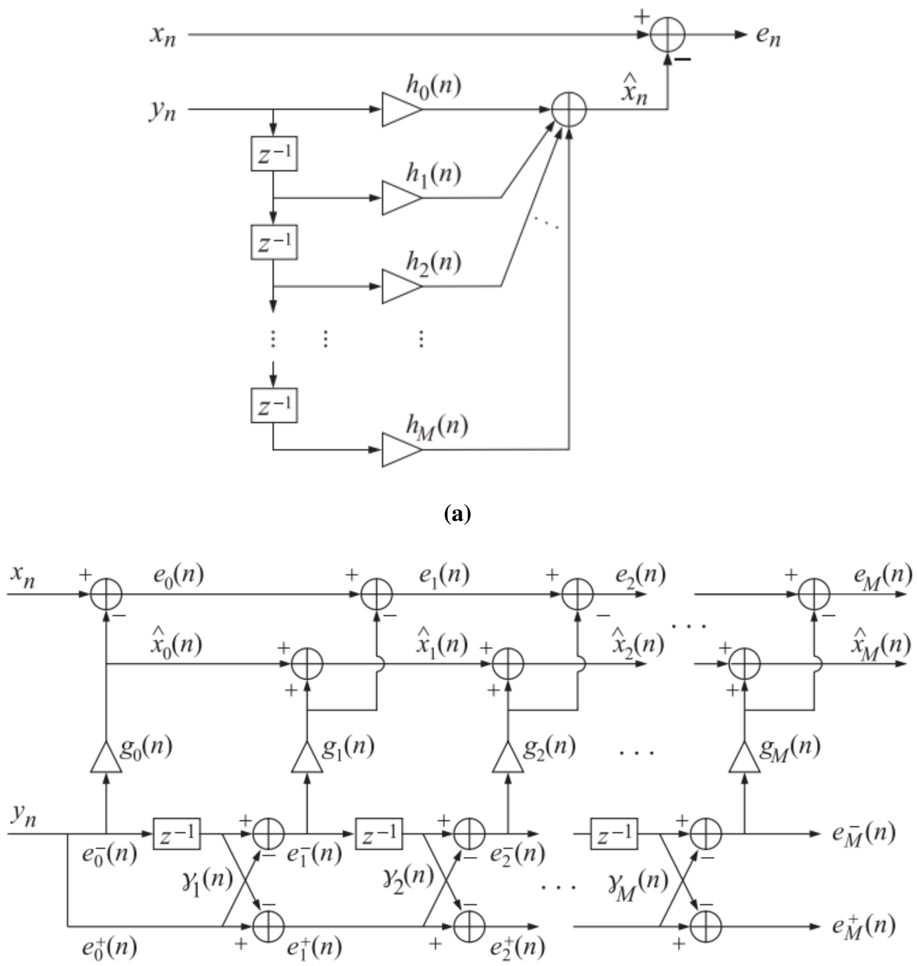


Figure 7 The direct-form coefficients for tracking using lms, $\sigma_v^2/100$, 10μ.



(a)

(b)

Figure 8 Adaptive filter. (a) Adaptive FIR Wiener filter. (b) Adaptive lattice Wiener filter.