

# sbm\_hls\_temp

temporary repository for ACA2021 final project

## QUBO2Ising explanation

### QUBO formulation of currency exchange problem

$$x = \operatorname{argmax}_x \left[ \sum_{(i,j) \in E} x_{ij} \log c_{ij} - M_1 \sum_{i \in V} \left( \sum_{j, (i,j) \in E} x_{ij} - \sum_{j, (j,i) \in E} x_{ji} \right)^2 - M_2 \sum_{i \in V} \sum_{j, (i,j) \in E} x_{ij} \left( \sum_{j, (i,j) \in E} x_{ij} - 1 \right) \right]$$

is equivalent to

$$x = \operatorname{argmin}_x \left\{ \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (-\log_2(c_{ij})) x_{ij} + M_1 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} x_{ij} - \sum_{j=0}^{n-1} x_{ji} \right)^2 + M_2 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} x_{ij} \right) \left( \sum_{j=0}^{n-1} x_{ij} - 1 \right) \right\}$$

Expand the 1st term:

$$\begin{aligned} & \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (-\log_2(c_{ij})) x_{ij} \\ &= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (-\log_2(c_{ij})) x_{ij} x_{ij} \end{aligned}$$

Expand the 2nd term:

$$\begin{aligned} & M_1 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} x_{ij} - \sum_{j=0}^{n-1} x_{ji} \right)^2 \\ &= M_1 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} (x_{ij} - x_{ji}) \right)^2 \\ &= M_1 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} (x_{ij} - x_{ji}) \right) \left( \sum_{k=0}^{n-1} (x_{ik} - x_{ki}) \right) \\ &= M_1 \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} (x_{ij} - x_{ji})(x_{ik} - x_{ki}) \\ &= M_1 \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} (x_{ij}x_{ik} - x_{ij}x_{ki} - x_{ji}x_{ik} + x_{ji}x_{ki}) \end{aligned}$$

Expand the 3rd term:

$$\begin{aligned} & M_2 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} x_{ij} \right) \left( \sum_{j=0}^{n-1} x_{ij} - 1 \right) \\ &= M_2 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} x_{ij} \right) \left( \sum_{k=0}^{n-1} x_{ik} - 1 \right) \\ &= M_2 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} x_{ij}x_{ik} - \sum_{j=0}^{n-1} x_{ij} \right) \\ &= M_2 \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} x_{ij}x_{ik} - \sum_{j=0}^{n-1} x_{ij}x_{ij} \right) \end{aligned}$$

Change subscripts:

$$x_{ab} \rightarrow x_{a \times n + b}$$

## Another formulation in "A Currency Arbitrage Machine based on the Simulated Bifurcation Algorithm for Ultrafast Detection of Optimal Opportunity," 2020

$$x = \underset{x}{\operatorname{argmin}} \left\{ m_c \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (-\log_2(c_{ij})) x_{ij} + \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} x_{ij} - \sum_{j=0}^{n-1} x_{ji} \right)^2 + \sum_{i=0}^{n-1} \sum_{j \neq j'}^{n-1} (x_{ij} x_{ij'} + x_{ji} x_{j'i}) + \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} x_{ij} x_{ji} \right\}$$

Expand the 1st term:

$$\begin{aligned} & \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (-\log_2(c_{ij})) x_{ij} \\ &= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} (-\log_2(c_{ij})) x_{ij} x_{ij} \end{aligned}$$

Expand the 2nd term:

$$\begin{aligned} & \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} x_{ij} - \sum_{j=0}^{n-1} x_{ji} \right)^2 \\ &= \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} (x_{ij} - x_{ji}) \right)^2 \\ &= \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} (x_{ij} - x_{ji}) \right) \left( \sum_{k=0}^{n-1} (x_{ik} - x_{ki}) \right) \\ &= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} (x_{ij} - x_{ji})(x_{ik} - x_{ki}) \\ &= \sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} (x_{ij} x_{ik} - x_{ij} x_{ki} - x_{ji} x_{ik} + x_{ji} x_{ki}) \end{aligned}$$

Expand the 3rd term:

$$\begin{aligned} & \sum_{i=0}^{n-1} \sum_{j \neq j'}^{n-1} (x_{ij} x_{ij'} + x_{ji} x_{j'i}) \\ &= \frac{1}{2} \sum_{i=0}^{n-1} \left( \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} (x_{ij} x_{ik} + x_{ji} x_{ki}) - \sum_{j=0}^{n-1} (x_{ij} x_{ij} + x_{ji} x_{ji}) \right) \end{aligned}$$

Expand the 4th term: already in expanded form

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} x_{ij} x_{ji}$$

Change subscripts:

$$x_{ab} \rightarrow x_{a \times n + b}$$

## QUBO

$$Q \in \mathbb{R}^{N \times N}, Q = Q^T, x_i \in \{0, 1\}$$

$$f_Q(Q, x) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} Q_{ij} x_i x_j$$

$$x_{opt} = \underset{x}{\operatorname{argmin}} f_Q(Q, x)$$

## Ising model

$$J \in \mathbb{R}^{N \times N}, J = J^T, h \in \mathbb{R}^N, s_i \in \{-1, 1\}$$

$$H(J, h, s) = -\frac{1}{2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} J_{ij} s_i s_j + \sum_{i=0}^{N-1} h_i s_i$$

$$s_{opt} = \underset{s}{\operatorname{argmin}} H(J, h, s)$$

## Convert QUBO to Ising model

$$\text{Let } x_i = \frac{s_i + 1}{2}$$

$$f_Q(Q, x)$$

$$= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} Q_{ij} x_i x_j$$

$$= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} Q_{ij} \frac{s_i + 1}{2} \frac{s_j + 1}{2}$$

$$= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4} (s_i s_j + s_i + s_j + 1)$$

$$= \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4} s_i s_j + \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4} s_i + \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4} s_j + \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4}$$

$$= -\frac{1}{2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{-Q_{ij}}{2} s_i s_j + \sum_{i=0}^{N-1} s_i \left( \sum_{j=0}^{N-1} \frac{Q_{ij}}{4} \right) + \sum_{j=0}^{N-1} s_j \left( \sum_{i=0}^{N-1} \frac{Q_{ij}}{4} \right) + \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4}$$

$$= -\frac{1}{2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{-Q_{ij}}{2} s_i s_j + \sum_{i=0}^{N-1} \left( \sum_{j=0}^{N-1} \frac{Q_{ij}}{4} + \sum_{j=0}^{N-1} \frac{Q_{ji}}{4} \right) s_i + \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4}$$

$$\text{Let } J_{ij} = \frac{-Q_{ij}}{2}, h_i = \left( \sum_{j=0}^{N-1} \frac{Q_{ij}}{4} + \sum_{j=0}^{N-1} \frac{Q_{ji}}{4} \right), C = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \frac{Q_{ij}}{4}$$

$$f_Q(Q, x) = -\frac{1}{2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} J_{ij} s_i s_j + \sum_{i=0}^{N-1} h_i s_i + C = H(J, h, s) + C$$