1.

```
# 真實解
y_exact = exact_solution(t_values)

# 輸出結果
print("t\tEuler y\t\tTaylor2 y\t\tExact y\t\tEuler Error\tTaylor2 Error")
for t, ye, yt2, yt in zip(t_values, y_euler, y_taylor2, y_exact):
    print(f"{t:.1f}\t{ye:.6f}\t{yt2:.6f}\t{yt:.6f}\t{abs(ye - yt):.6f}\t{abs(yt2 - yt):.6f}")
```

input

t	Euler y	Taylor2 y	Exact y	Euler Error	Taylor2 Error
1.0	0.000000	0.000000	0.000000	0.000000	0.000000
1.1	0.100000	0.105000	0.105160	0.005160	0.000160
1.2	0.209917	0.220919	0.221243	0.011325	0.000324
1.3	0.330471	0.348612	0.349121	0.018651	0.000509
1.4	0.462354	0.488954	0.489682	0.027328	0.000728
1.5	0.606285	0.642883	0.643875	0.037590	0.000993
1.6	0.763041	0.811438	0.812753	0.049711	0.001315
1.7	0.933475	0.995703	0.997494	0.064019	0.001707
1.8	1.118537	1.197252	1.199439	0.080902	0.002187
1.9	1.319293	1.417344	1.420116	0.100823	0.002772
2.0	1.536943	1.657795	1.661282	0.124338	0.003487

2.

```
    max_u1_error = np.max(u1_error)
    max_u2_error = np.max(u2_error)

    print("t\tu1 (RK4)\tu1 (exact)\tu2 (RK4)\tu2 (exact)\tu1 error\tu2 error")
    for i in range(len(t_values)):
        t = t_values[i]
        u1_rk, u2_rk = u_values[i]
        u1_ex, u2_ex = u_exact[i]
        print(f"{t:.2f}\t{u1_rk:.6f}\t{u1_ex:.6f}\t{u2_rk:.6f}\t{u2_ex:.6f}\t{u1_error[i]:.2e}\t{u2_error[i]:.2e}")

    print(f"\n最大 u1 誤差 (h={h}): {max_u1_error:.2e}")
    print(f"最大 u2 誤差 (h={h}): {max_u2_error:.2e}")
```

input

```
=== h = 0.1 ===
t       u1 (RK4)        u1 (exact)      u2 (RK4)        u2 (exact)      u1 error        u2 error
0.00    1.333333        1.333333        0.666667        0.666667        0.00e+00        1.11e-16
0.10    -3.052437       1.793063        8.989305        -1.032002       4.85e+00        1.00e+01
0.20    -23.847795      1.423902        51.192704       -0.874681       2.53e+01        5.21e+01
0.30    -130.165202     1.131577        269.269193      -0.724999       1.31e+02        2.70e+02
0.40    -680.231485     0.909409        1399.368584     -0.608214       6.81e+02        1.40e+03
0.50    -3531.299585    0.738788        7258.241839     -0.515658       3.53e+03        7.26e+03
0.60    -18312.795052   0.605710        37634.965483    -0.440411       1.83e+04        3.76e+04
0.70    -94951.331907   0.499860        195131.871335   -0.377404       9.50e+04        1.95e+05
0.80    -492306.465639  0.413671        1011721.072078  -0.322954       4.92e+05        1.01e+06
0.90    -2552513.623867 0.341614        5245578.826590  -0.274409       2.55e+06        5.25e+06
1.00    -13234270.709168        0.279675        27197287.206587 -0.229888       1.32e+07        2.72e+07
```

```
    max_u1_error = np.max(u1_error)
    max_u2_error = np.max(u2_error)
```

input

```
=== h = 0.05 ===
t       u1 (RK4)        u1 (exact)      u2 (RK4)        u2 (exact)      u1 error        u2 error
0.00    1.333333        1.333333        0.666667        0.666667        0.00e+00        1.11e-16
0.05    1.721880        1.912059        -0.499599       -0.909077       1.90e-01        4.09e-01
0.10    1.726915        1.793063        -0.832598       -1.032002       6.61e-02        1.99e-01
0.15    1.617161        1.681967        -0.890373       -0.961459       1.52e-02        7.11e-02
0.20    1.481687        1.423902        -0.861042       -0.874681       5.78e-02        1.36e-02
0.25    1.348945        1.267646        -0.807505       -0.795221       8.13e-02        1.23e-02
0.30    1.227063        1.131577        -0.750341       -0.724999       9.55e-02        2.53e-02
0.35    1.117478        1.012999        -0.695886       -0.663060       1.04e-01        3.28e-02
0.40    1.019525        0.909409        -0.645732       -0.608214       1.10e-01        3.75e-02
0.45    0.931977        0.818630        -0.599934       -0.559389       1.13e-01        4.05e-02
0.50    0.853541        0.738788        -0.558092       -0.515658       1.15e-01        4.24e-02
0.55    0.783017        0.668275        -0.519706       -0.476225       1.15e-01        4.35e-02
0.60    0.719337        0.605710        -0.484290       -0.440411       1.14e-01        4.39e-02
0.65    0.661560        0.549809        -0.451407       -0.407635       1.12e-01        4.38e-02
0.70    0.608868        0.499860        -0.420673       -0.377404       1.09e-01        4.33e-02
0.75    0.560547        0.454695        -0.391754       -0.349296       1.06e-01        4.25e-02
0.80    0.515980        0.413671        -0.364365       -0.322954       1.02e-01        4.14e-02
0.85    0.474633        0.376158        -0.338259       -0.298076       9.85e-02        4.02e-02
0.90    0.436043        0.341614        -0.313226       -0.274409       9.44e-02        3.88e-02
0.95    0.399812        0.309583        -0.289089       -0.251739       9.02e-02        3.74e-02
1.00    0.365600        0.279675        -0.265698       -0.229888       8.59e-02        3.58e-02
```