

Hw8

1.

```
man.py
# (a) 二次多项式 least squares approximation
coeffs_quad = np.polyfit(x, y, 2)
y_quad = np.polyval(coeffs_quad, x)
error_quad = np.sum((y - y_quad) ** 2)
print("-- (a) 二次多项式 least squares --")
print(f"方程式: y = {coeffs_quad[0]:.4f} x^2 + {coeffs_quad[1]:.4f} x + {coeffs_quad[2]:.4f}")
print(f"误差 SSE = {error_quad:.4f}\n")

# (b) 指数型 least squares approximation
ln_y = np.log(y)
coeffs_exp = np.polyfit(x, ln_y, 1)
a = coeffs_exp[0]
ln_b = coeffs_exp[1]
b = np.exp(ln_b)
y_exp = b * np.exp(a * x)

-- (a) 二次多项式 least squares --
方程式: y = 6.6912 x^2 + -1.8837 x + 3.0864
误差 SSE = 0.0052

-- (b) 指数型 least squares --
方程式: y = 21.4445 * e^{0.3905 * x}
误差 SSE = 94.9630

-- (c) 幂次型 least squares --
方程式: y = 6.2390 * x^2.0196
误差 SSE = 0.0117
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2.

```
man.py
A33, _ = quad(lambda x: 1, a_int, b_int)

# 組 A 矩陣 (對稱矩陣)
A = np.array([
    [A11, A12, A13],
    [A12, A22, A23],
    [A13, A23, A33]
])

# 計算 B 向量元素 (積分值)
B1, _ = quad(lambda x: x**2 * f(x), a_int, b_int)
B2, _ = quad(lambda x: x * f(x), a_int, b_int)
B3, _ = quad(lambda x: f(x), a_int, b_int)

B = np.array([B1, B2, B3])

# 解聯立方程組
coeffs = np.linalg.solve(A, B)
a, b, c = coeffs

print("-- 答案 --")
print(f"P(x) = {a:.6f} x^2 + {b:.6f} x + {c:.6f}")

-- 答案 --
P(x) = -0.232631 x^2 + 0.326540 x + 0.498279
```

3.

```
man.py
29 # (c) 計算真實積分  $\int_{-0.1}^1 x^2 \sin(x) dx$ 
30 true_integral, _ = quad(lambda x: x**2 * np.sin(x), 0, 1)
31
32 # (d) 計算誤差  $E(S_4) = \int [f(x_j) - S_4(x_j)]^2$ 
33 S4_vals = S4(xj)
34 error = np.sum((fj - S4_vals)**2)
35
36 # 輸出結果
37 print("(a) Fourier coefficients:")
38 print(f"  a0 = {a0}")
39 print(f"  ak = {ak}")
40 print(f"  bk = {bk}")
41 print()
42 print(f"(b)  $\int S_4(x) dx = {integral\_S4:.6f}$ ")
43 print(f"(c)  $\int x^2 \sin(x) dx = {true\_integral:.6f}$ ")
44 print(f"(d) Error  $E(S_4) = {error:.6e}$ ")
45
46
```

input:

```
(a) Fourier coefficients:
a0 = 0.1976721964763756
ak = [ 0.07282666 -0.02226183 -0.03839017 -0.04386479]
bk = [-0.23724918 -0.123859 -0.07780929 -0.05222269]

(b)  $\int S_4(x) dx = 0.19767220$ 
(c)  $\int x^2 \sin(x) dx = 0.22324428$ 
(d) Error  $E(S_4) = 7.74015759e-02$ 
```