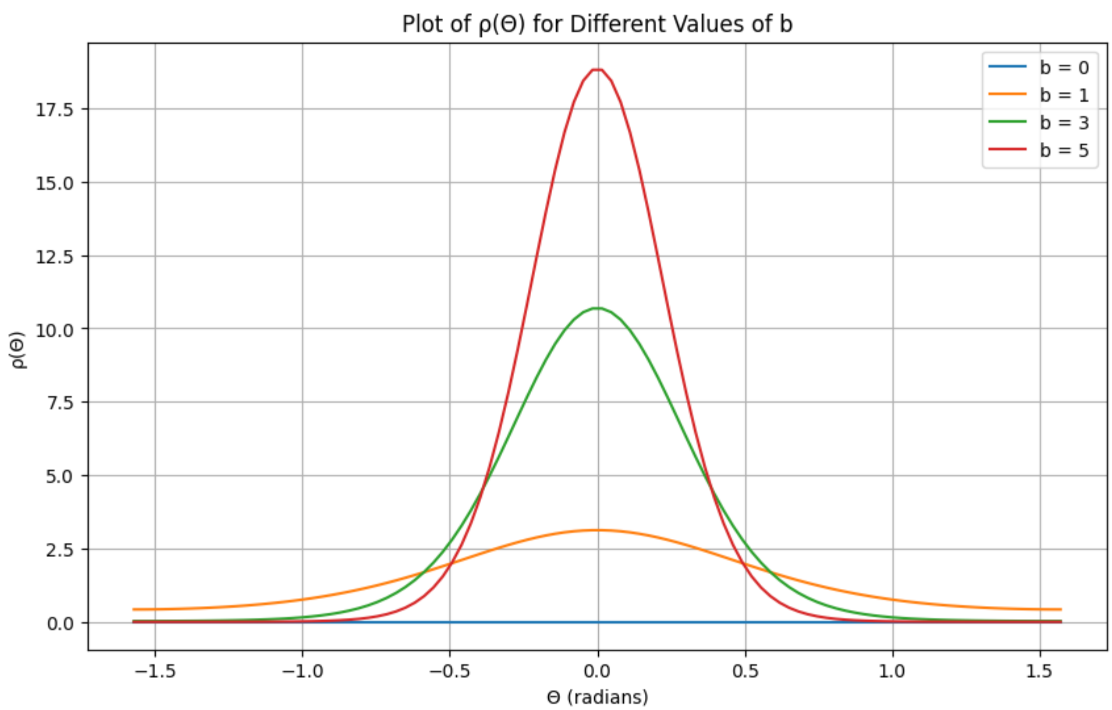
Appendix

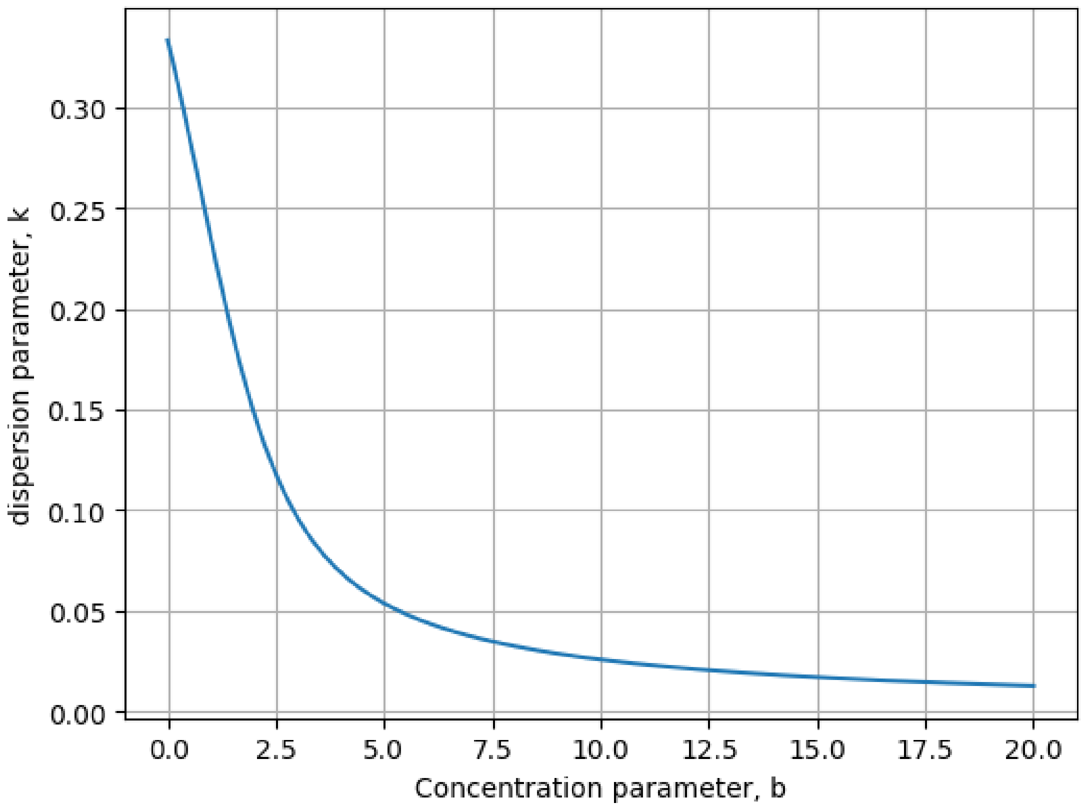
January 12, 2024

[ ]:
```python
import numpy as np
import matplotlib.pyplot as plt
```

[ ]:
```python
from scipy.special import erfi

# Define the function rho(Theta) based on the provided equations
def rho(theta, b):

    numerator = 4 * np.sqrt(b / (2 * np.pi)) * np.exp(b * (np.cos(2 * theta) + 1))
    if b!=0:
        denominator = erfi(np.sqrt(2 * b))
    else:
        denominator=1
    return numerator / denominator

# Define a range of theta values from 0 to 2
theta = np.linspace(-np.pi/2, np.pi/2, 100)

# Define different values of b to plot
b_values = [0, 1, 3,5]

# Plot rho(Theta) for different values of b
plt.figure(figsize=(10, 6))
for b in b_values:
    plt.plot(theta, rho(theta, b), label=f'b = {b}')

plt.title('Plot of  (Θ) for Different Values of b')
plt.xlabel('Θ (radians)')
plt.ylabel(' (Θ)')
plt.legend()
plt.grid(True)
plt.savefig('./pics/b_theta.pdf')
plt.show()
```



[ ]:
```python
from scipy.integrate import quad

# Function to be integrated for kappa
def integrand(theta, b):
    return rho(theta, b) * np.sin(theta)**3

# Calculate kappa for different values of b
b_values = np.linspace(0.000000001,20,1000)
kappa_values = []

for b in b_values:
    # Perform numerical integration
    kappa, _ = quad(integrand, 0, np.pi, args=(b,))
    kappa = kappa / 4  # scaling factor as per the equation
    kappa_values.append(kappa)

plt.plot(b_values,kappa_values)
plt.xlabel('Concentration parameter, b')
plt.grid(True)
plt.ylabel('dispersion parameter, k')
plt.savefig('./pics/kappa_b.pdf')
```



[ ]:
```
import pandas as pd
```

[ ]:
```
DataFrame=pd.read_csv('./data.csv')
DataFrame
```

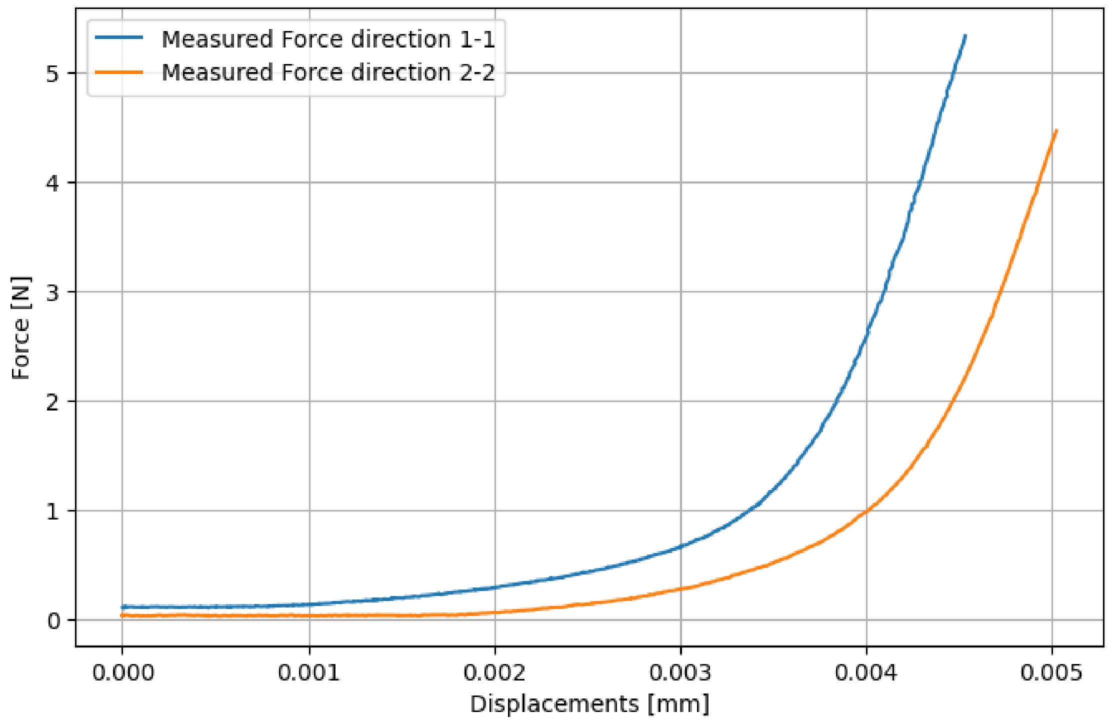
[ ]:
```
           DIC              DIC2D            DIC2D.1            DIC2D.2  \
0     Frame Id  Location u [pix]   Location u [pix]   Location u [pix]
1          NaN                s0                 s1                 s2
2                            NaN                NaN                NaN
3            1        1559.585813        1553.741541        1412.644109
4            2        1559.708498        1553.877931        1412.831641
..         …                  …                  …                  …
552        550        1584.570361        1553.220319        1394.087741
553        551        1584.513656        1553.239681        1394.081244
554        552         1584.86932        1553.649015        1394.361475
555        553        1584.605407        1553.764137        1394.326758
556        554         1584.61713        1553.747644         1394.29233

              DIC2D.3            DIC2D.4            DIC2D.5  \
0    Location u [pix]   Location v [pix]   Location v [pix]
1                  s3                 s0                 s1
```

```
2              NaN             NaN             NaN
3      1402.625357     1294.164038     1478.706625
4      1402.705853     1294.056155     1478.661087
..             …               …               …
552    1366.487721     1341.932798     1600.490575
553    1366.475637     1342.060295     1600.714695
554    1366.706303      1341.87463     1600.674964
555    1366.624568     1341.788098     1600.489383
556    1366.546468     1341.858183     1600.383823

               DIC2D.6             DIC2D.7  Log(Force 1)  Log(Force 2)  \
0     Location v [pix]   Location v [pix]        Force 1       Force 2
1                   s2                  s3             N             N
2                  NaN                 NaN           NaN           NaN
3          1450.315457         1291.798107       0.10061       0.02826
4          1450.034011          1291.78907       0.10389       0.03155
..                 …                   …             …           …
552        1530.933444         1332.198172       5.18464       4.29675
553        1531.085507         1332.212609       5.24877       4.37403
554        1531.077444         1332.205085       5.26028       4.39211
555        1530.941468         1331.921434       5.32605       4.45953
556        1531.025627         1331.974316           NaN           NaN

     Log(Force 3)  Log(Force 4)  Log(Displacement 1)  Log(Displacement 2)  \
0         Force 3       Force 4       Displacement 1       Displacement 2
1               N             N                   mm                   mm
2             NaN           NaN                  NaN                  NaN
3         0.00524       0.05785              0.00368              0.00401
4         0.00688       0.05621              0.00039              0.00434
..            …             …                    …                 …
552       5.10901       4.52365              4.50928              4.99269
553       5.16985         4.596              4.52309               5.0065
554       5.18958       4.61902              4.52539              5.01144
555       5.25206       4.68643              4.53559              5.02624
556           NaN           NaN                  NaN                  NaN

     Log(Displacement 3)  Log(Displacement 4)
0         Displacement 3       Displacement 4
1                     mm                   mm
2                    NaN                  NaN  NaN
3                0.00499               0.0142
4                0.00434              0.00828
..                   …                  …  …
552              4.49547              4.99861
553              4.50928              5.01275
554              4.50796              5.01736
555               4.5221              5.03183
```

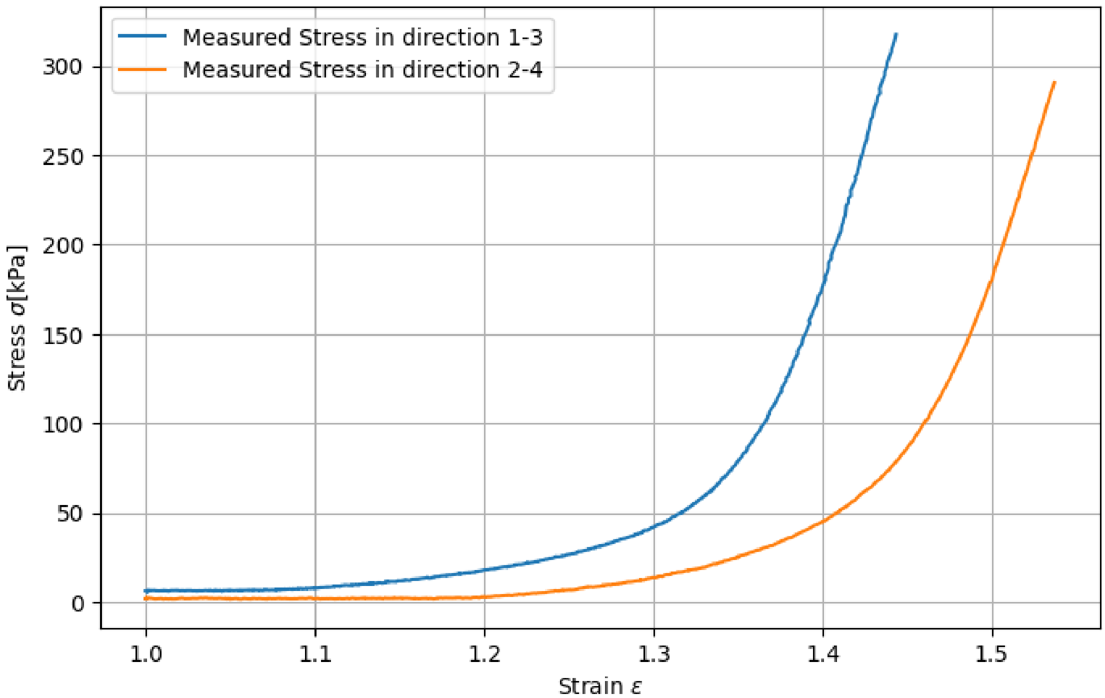
```
556                  NaN                  NaN

[557 rows x 18 columns]
```

[ ]:
```
dis_1=pd.to_numeric(DataFrame[' Log(Displacement 1)'], errors='coerce')/1000
f_1=pd.to_numeric(DataFrame[' Log(Force 1)'], errors='coerce')

dis_2=pd.to_numeric(DataFrame[' Log(Displacement 2)'], errors='coerce')/1000
f_2=pd.to_numeric(DataFrame[' Log(Force 2)'], errors='coerce')

dis_3=pd.to_numeric(DataFrame[' Log(Displacement 3)'], errors='coerce')/1000
f_3=pd.to_numeric(DataFrame[' Log(Force 3)'], errors='coerce')

dis_4=pd.to_numeric(DataFrame[' Log(Displacement 4)'], errors='coerce')/1000
f_4=pd.to_numeric(DataFrame[' Log(Force 4)'], errors='coerce')
```

[ ]:
```
plt.figure(figsize=(8,5))
plt.plot(dis_1,f_1,label='Measured Force direction 1-1')
plt.plot(dis_2,f_2,label='Measured Force direction 2-2')


plt.xlabel('Displacements [mm]')
plt.ylabel('Force [N]')

plt.legend()

plt.grid(True)
plt.savefig('./pics/force.pdf')
```



[ ]:
```
dis_13=10.23*10**(-3)    #10.23mm
dis_24=9.36*10**(-3)      #9.36mm
thickness=1.64*10**(-3)
A_13=thickness*dis_13*10**3
A_24=thickness*dis_24*10**3


plt.figure(figsize=(8,5))
plt.plot(1+(dis_1)/dis_13,f_1/A_13,label='Measured Stress in direction 1-3')
plt.plot(1+dis_2/dis_24,f_2/A_24,label='Measured Stress in direction 2-4')
# plt.plot(dis_3/dis_13,f_3/A_13,label='3')
# plt.plot(dis_4/dis_24,f_4/A_24,label='4')

plt.xlabel('Strain '+r'$\varepsilon$')
plt.ylabel('Stress '+r'$\sigma$'+'[kPa]')

plt.legend()

plt.grid(True)
plt.savefig('./pics/stress.pdf')
```



[ ]:
```
dis_1=dis_1.dropna()
dis_2=dis_2.dropna()
```

[ ]:
```
dis=[1+dis_1/dis_13,1+dis_2/dis_24]

dis2=dis[1]

f_1=f_1.dropna()
f_2=f_2.dropna()
```

[ ]:
```
def sigmaB(dis1,c,k1,k2,dis2,args):

    la1=np.array(dis1)

    la2=np.array(dis2)

    theta,k,i,j=args

    I=[[1,0,0],[0,1,0],[0,0,1]]

    I1=la1**2+la2**2+1/(la1*la2)

    I41=(np.cos(theta))**2*la1**2+(np.sin(theta))**2*la2**2
```

```
    m=[la1*np.cos(theta),la2*np.sin(theta),0]
    m_=[la1*np.cos(theta),-la2*np.sin(theta),0]

    b=[[la1**2,0,0],[0,la2**2,0],[0,0,1/(la1*la2)]]

    I14=k1*(np.exp(k2*(k*I1+(1-3*k)*I41-1)**2))*(k*I1+(1-3*k)*I41-1)*k

    psi4b=I14*(1-3*k)

    psi1b=c+4*I14*k

    pb=-2*(psi1b*b[2][2])


    sigmaB_11=2*(psi1b*b[i][j]+psi4b*m[i]*m[j]+psi4b*m_[i]*m_[j])+pb*I[i][j]


    return sigmaB_11
```

[ ]:
```
def sigmaA(dis1,c,k1,k2,dis2,args):

    psi1=c/2

    la1=np.array(dis1)

    la2=np.array(dis2)

    theta,i,j=args

    I=[[1,0,0],[0,1,0],[0,0,1]]

    m=[la1*np.cos(theta),la2*np.sin(theta),0]
    m_=[la1*np.cos(theta),-la2*np.sin(theta),0]

    b=[[la1**2,0,0],[0,la2**2,0],[0,0,1/(la1*la2)]]

    pa=-2*psi1*b[2][2]

    I41=(np.cos(theta))**2*la1**2+(np.sin(theta))**2*la2**2

    psi4A=(k1)*(I41-1)*np.exp(k2*(I41-1)**2)

    sigmaA=2*(psi1*b[i][j]+psi4A*m[i]*m[j]+psi4A*m_[i]*m_[j])+pa*I[i][j]

    return sigmaA
```

[ ]:
```
kappa=0.26
theta=np.pi/6
```

[ ]:
```
from scipy.optimize import curve_fit

def fitting_func(dis, c, k1, k2, args=[theta,kappa,0,0]):
    return np.array(sigmaB(dis, c, k1, k2, dis2, args))

initial_guess=[7.64, 100, 1.6]

params, _ = curve_fit(fitting_func, dis[0], f_1/A_13, p0=initial_guess)

# Extracting fitted parameters
c_fit, k1_fit, k2_fit = params
print("Fitted Parameters: c =", c_fit, "k1 =", k1_fit, "k2 =", k2_fit)
```

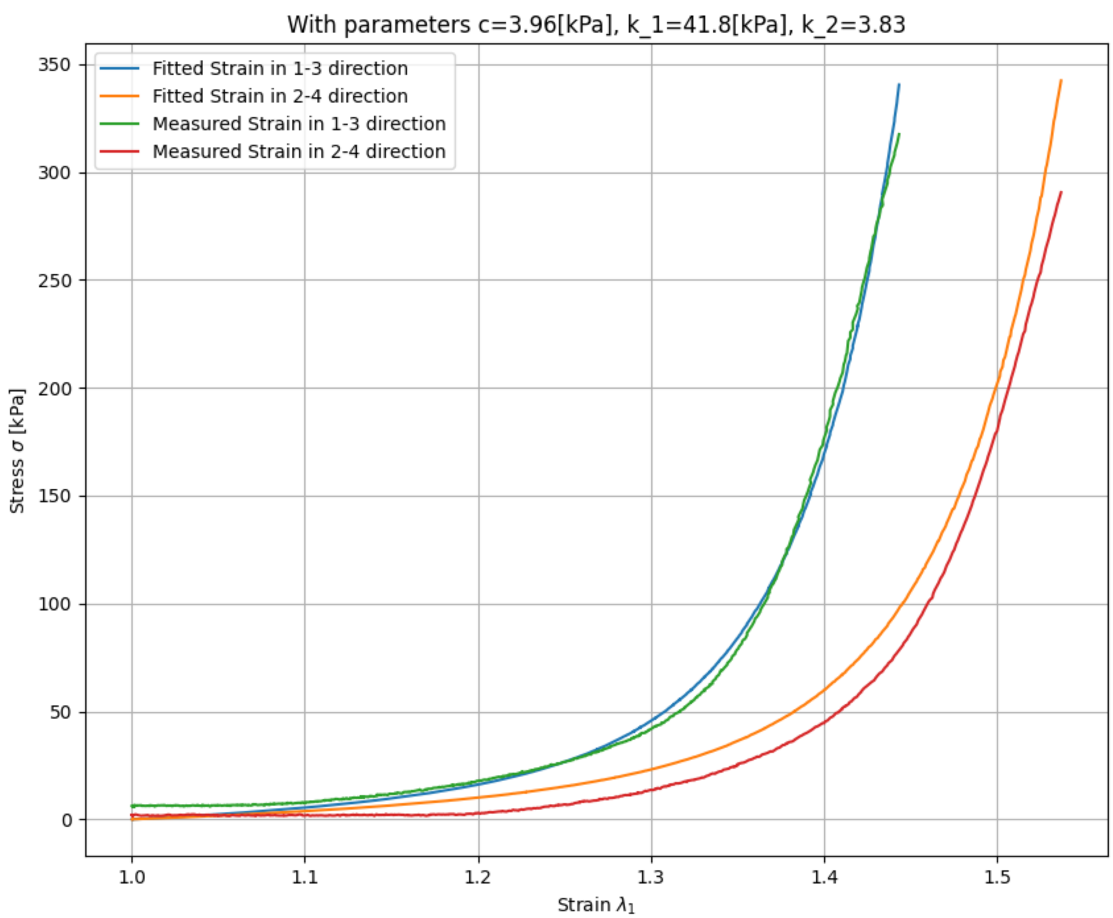
```
Fitted Parameters: c = 3.9611183100445424 k1 = 41.84508106862842 k2 =
3.8269215239602725
```

[ ]:
```
def fitting_funca(dis, c, k1, k2, args=[theta,0,0]):
    return np.array(sigmaA(dis, c, k1, k2, dis2, args))

initial_guess=[c_fit,k1_fit,k2_fit]

paramsa, _ = curve_fit(fitting_funca, dis[0], f_1/A_13, p0=initial_guess)

# Extracting fitted parameters
c_fita, k1_fita, k2_fita = paramsa
print("Fitted Parameters: c =", c_fita, "k1 =", k1_fita, "k2 =", k2_fita)
```

```
Fitted Parameters: c = 5.671223770799812 k1 = 3.896399913822921 k2 =
1.853643776965365
```

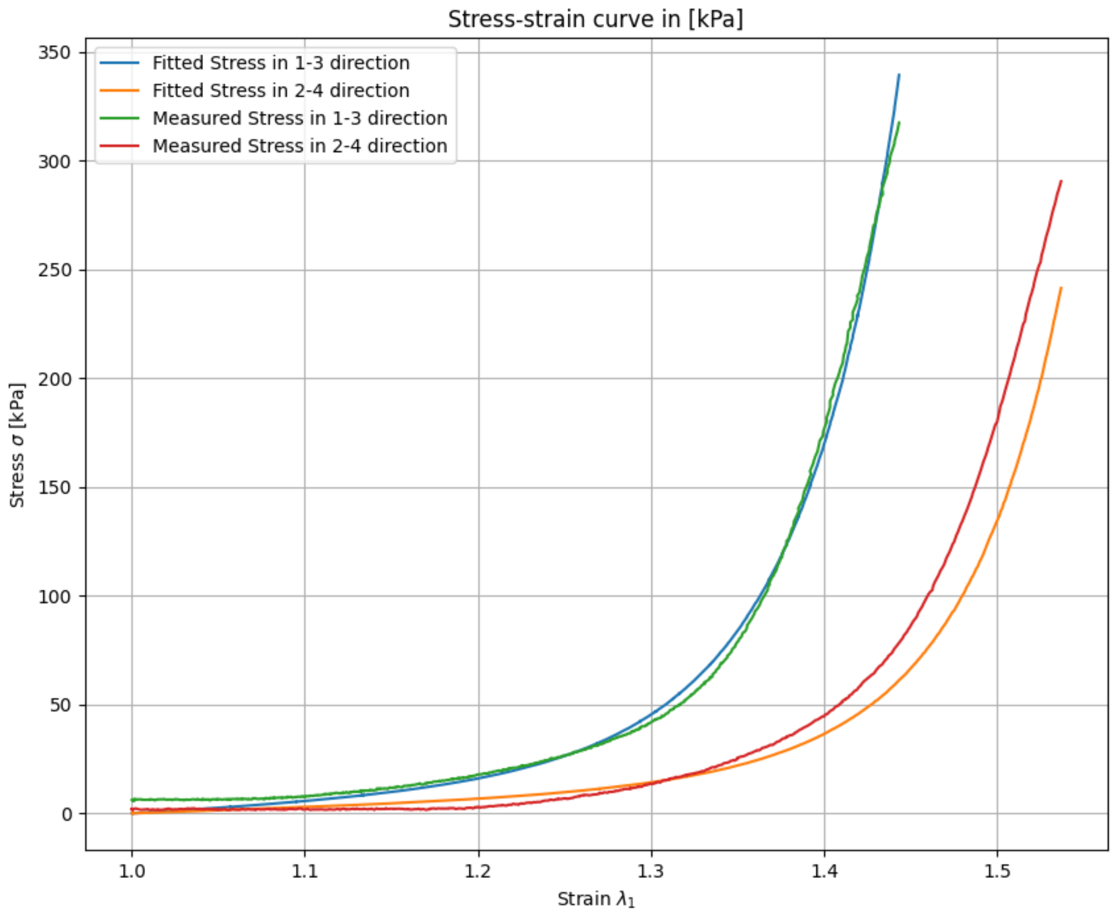
[ ]:
```
c=c_fit
k1=k1_fit
k2=k2_fit



args1=[theta,kappa,0,0]

testb=sigmaB(dis[0],c,k1,k2,dis[1],args1)

plt.figure(figsize=(10,8))
plt.title('With parameters c='+str(round(c_fit,2))+'[kPa],␣
↪k_1='+str(round(k1_fit,1))+'[kPa], k_2='+str(round(k2_fit,2)))

plt.plot(dis[0],testb,label='Fitted Strain in 1-3 direction')
```

```
args2=[theta,kappa,1,1]

test=sigmaB(dis[1],c,k1,k2,dis[0],args2)

plt.plot(dis[1],test,label='Fitted Strain in 2-4 direction')

plt.plot(dis[0],f_1/A_13,label='Measured Strain in 1-3 direction')
plt.plot(dis[1],f_2/A_24,label='Measured Strain in 2-4 direction')

plt.grid(True)
plt.xlabel('Strain '+r'$\lambda_1$')
plt.ylabel('Stress '+r'$\sigma$'+' [kPa]')
plt.legend()

plt.savefig('./pics/kappa.pdf')
```



[ ]:
```
c=c_fita
k1=k1_fita
```

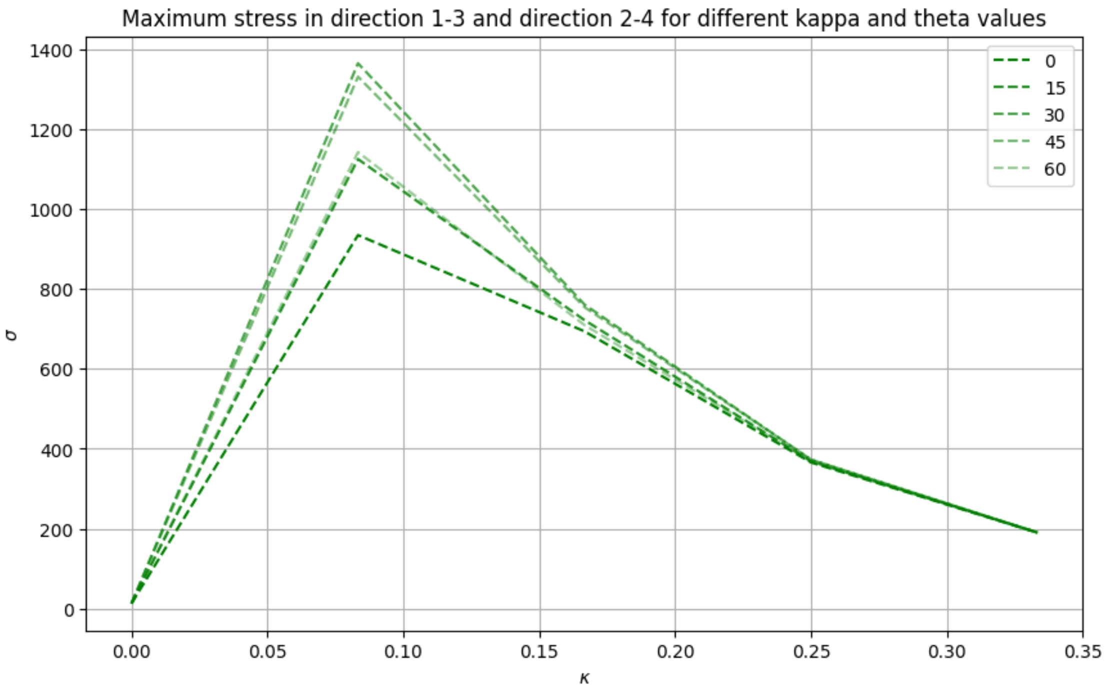
```
k2=k2_fita

args1=[theta,0,0]

test=sigmaA(dis[0],c,k1,k2,dis[1],args1)


plt.figure(figsize=(10,8))
plt.title('Stress-strain curve in [kPa]')

plt.plot(dis[0],test,label='Fitted Stress in 1-3 direction')


args2=[theta,1,1]

test=sigmaA(dis[1],c,k1,k2,dis[0],args2)

plt.plot(dis[1],test,label='Fitted Stress in 2-4 direction')

plt.plot(dis[0],f_1/A_13,label='Measured Stress in 1-3 direction')
plt.plot(dis[1],f_2/A_24,label='Measured Stress in 2-4 direction')

plt.grid(True)
plt.xlabel('Strain '+r'$\lambda_1$')
plt.ylabel('Stress '+r'$\sigma$'+' [kPa]')
plt.legend()

plt.savefig('./pics/no_kappa.pdf')
```



[ ]:
```
kappa_values=np.linspace(0,1/3,5)
theta_values=np.linspace(0,np.pi/3,5)
```

[ ]:
```
max_m_00=(f_1/A_13).max()
max_m_11=(f_1/A_24).max()

kappa_values=np.linspace(0.0,1/3,5)
theta_values=np.linspace(0,np.pi/3,5)

i=0
plt.figure(figsize=(10,6))
plt.title('Maximum stress in direction 1-3 and direction 2-4 for different␣
 ↪kappa and theta values')
for theta in theta_values:
    alpha_value=1-0.15*i
    i+=1
    sigma_00_values=[]
    sigma_11_values=[]
```

```
    for kappa in kappa_values:
        c = c_fit,
        k1 = k1_fit,
        k2 = k2_fit

        args1=[theta,kappa,0,0]
        sigma_b_00=sigmaB(dis[0],c,k1,k2,dis[1],args1)

        args2=[theta,kappa,1,1]
        sigma_b_11=sigmaB(dis[1],c,k1,k2,dis[0],args2)

        sigma_00_values.append(np.max(sigma_b_00))
        sigma_11_values.append(np.max(sigma_b_11))

        # plt.plot(kappa,np.max(sigma_b_00),'bo',alpha=alpha_value)
        # plt.plot(kappa,np.max(sigma_b_11),'go',alpha=alpha_value)


    # plt.
 ↪plot(kappa_values,sigma_00_values,'b-',alpha=alpha_value,label=str(round(theta*180/
 ↪np.pi)))
    plt.
 ↪plot(kappa_values,sigma_11_values,'g--',alpha=alpha_value,label=str(round(theta*180/
 ↪np.pi)))

# label='Theta='+str(round(theta*180/np.pi))

plt.xlabel(r'$\kappa$')
plt.ylabel(r'$\sigma$')
plt.grid(True)
plt.legend()
plt.savefig('./pics/theta_22_dir_green_1.pdf')
```




```
[ ]: import numpy as np
     import matplotlib.pyplot as plt

     def von_mises_failure(sigma1_list, sigma2_list, input_value):
         """
         Calculate the von Mises stress for lists of stress values and check against␣
      ↪a threshold.
         Stops calculation when von Mises stress surpasses the input value.

         :param sigma1_list: List of stress values in direction 1
         :param sigma2_list: List of stress values in direction 2
         :param input_value: Threshold value for von Mises stress
         :return: Index where failure occurs, or a message if no failure.
         """
         plt.figure()
         plt.title('Von Mises Stress Criterion')
         plt.xlabel('Stress in 1-3 direction [MPa]')
         plt.ylabel('Von Mises Stress [MPa]')
         plt.grid(True)

         for i, (sigma1, sigma2) in enumerate(zip(sigma1_list, sigma2_list)):
             von_mises_stress = np.sqrt(((sigma1 - sigma2)**2 + sigma1**2 +␣
      ↪sigma2**2) / 2)
             von_mises_plot=von_mises_stress/input_value
```

```
        if von_mises_stress > input_value:
            plt.plot(sigma1/10**6, von_mises_plot, 'ro', label=f'Stress at␣
↪failure: {round(sigma1/10**6, 3)} MPa')
            plt.legend()
            plt.savefig('./pics/von_mises_failure.pdf')
            return i  # Return the index of failure
        else:
            plt.plot(sigma1/10**6, von_mises_plot, 'bo')

    plt.show()
    return print('No failure detected')  # No failure detected
```

[ ]:
```
def tsai_hill_criterion(sigma_hh_list, sigma_zz_list, tau_hz_list, X, Y, S):
    """
    Calculate the modified Tsai-Hill failure criterion for lists of stress␣
↪values.

    Stops calculation when K >= 1 for any data point.

    :return: Index where failure occurs, or None if no failure.
    """
    plt.figure()
    plt.title('Tsai-Hill failure criteria')
    plt.xlabel('Stress in 1-3 direction [MPa]')
    plt.ylabel(r'$\Lambda$')
    plt.grid(True)

    for i, (sigma_hh, sigma_zz, tau_hz) in enumerate(zip(sigma_hh_list,␣
↪sigma_zz_list, tau_hz_list)):
        K = (sigma_hh / X)**2 + (sigma_zz / Y)**2 + (tau_hz / S)**2

        if K >= 1:
            plt.plot(sigma_hh/10**6,K,'ro',label=('Stress in fiber direction␣
↪'+r'$\sigma_{f}=$'+str(round(sigma_hh/10**6,3))))
            plt.legend()
            plt.savefig('./pics/failure.pdf')
            return i  # Return the index of failure
        else:
            plt.plot(sigma_hh/10**6,K,'mo')

    return print('No failure detected')  # No failure detected

def hashin_rotem_criterion(sigma_hh_list, sigma_zz_list, tau_hz_list, X, Y, S):
    """
    Calculate the Hashin-Rotem failure criterion for lists of stress values.

    Stops calculation when K >= 1 for any data point.
```

```
    :return: Index where failure occurs, or None if no failure.
    """

    plt.figure()
    plt.title('Hasin-Rotem failure criteria')
    plt.xlabel('Stress in 1-3 direction [MPa]')

    plt.ylabel(r'$\Lambda$')
    plt.grid(True)

    for i, (sigma_hh, sigma_zz, tau_hz) in enumerate(zip(sigma_hh_list,␣
 ↪sigma_zz_list, tau_hz_list)):
        Kf = sigma_hh / X
        Km = (sigma_zz / Y)**2 + (tau_hz / S)**2
        K = max(Kf, Km)
        if K >= 1:
            plt.plot(sigma_hh/10**6,Kf,'bo',label='Matrix')
            plt.plot(sigma_hh/10**6,Km,'go',label='Fiber')

            plt.plot(sigma_hh/10**6,K,'ro',label=('Stress in fiber direction␣
 ↪'+r'$\sigma_{f}=$'+str(round(sigma_hh/10**6,3))))
            plt.legend()
            plt.savefig('./pics/failure_hasin.pdf')
            return i  # Return the index of failure
        else:
            plt.plot(sigma_hh/10**6,Kf,'bo')
            plt.plot(sigma_hh/10**6,Km,'go')

    return print('No failure detected')  # No failure detected
```

[ ]:

```
import numpy as np

def rotate_stress_tensor_lists(sigma_xx_list, sigma_xy_list, sigma_yx_list,␣
 ↪sigma_yy_list, theta):
    """
    Rotate multiple 2D stress states represented by separate lists for each␣
 ↪tensor component.

    :param sigma_xx_list: List of sigma_xx values.
    :param sigma_xy_list: List of sigma_xy values.
    :param sigma_yx_list: List of sigma_yx values.
    :param sigma_yy_list: List of sigma_yy values.
    :param theta: The rotation angle in radians.
    :return: Rotated stress components as four lists (sigma_xx', sigma_xy',␣
 ↪sigma_yx', sigma_yy').
    """
```

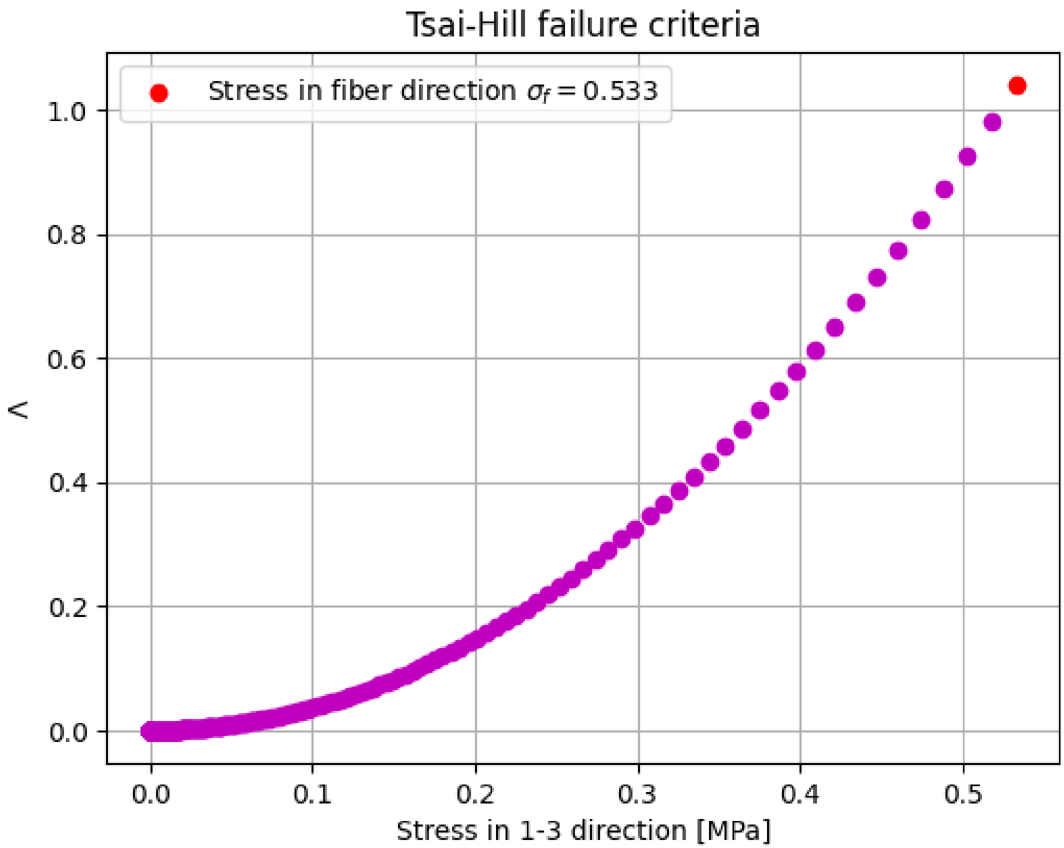
```
    cos_theta = np.cos(theta)
    sin_theta = np.sin(theta)

    # Rotation matrix for 2D stress
    R = np.array([[cos_theta, -sin_theta],
                  [sin_theta, cos_theta]])

    # Initialize lists for the rotated components
    rotated_sigma_xx = []
    rotated_sigma_xy = []
    rotated_sigma_yx = []
    rotated_sigma_yy = []

    for sigma_xx, sigma_xy, sigma_yx, sigma_yy in zip(sigma_xx_list,␣
 ↪sigma_xy_list, sigma_yx_list, sigma_yy_list):
        sigma = np.array([[sigma_xx, sigma_xy],
                          [sigma_yx, sigma_yy]])

        # Rotate the stress tensor
        rotated_sigma = R @ sigma @ R.T

        # Store the rotated components
        rotated_sigma_xx.append(rotated_sigma[0, 0])
        rotated_sigma_xy.append(rotated_sigma[0, 1])
        rotated_sigma_yx.append(rotated_sigma[1, 0])
        rotated_sigma_yy.append(rotated_sigma[1, 1])

    return rotated_sigma_xx, rotated_sigma_xy, rotated_sigma_yx,␣
 ↪rotated_sigma_yy
```

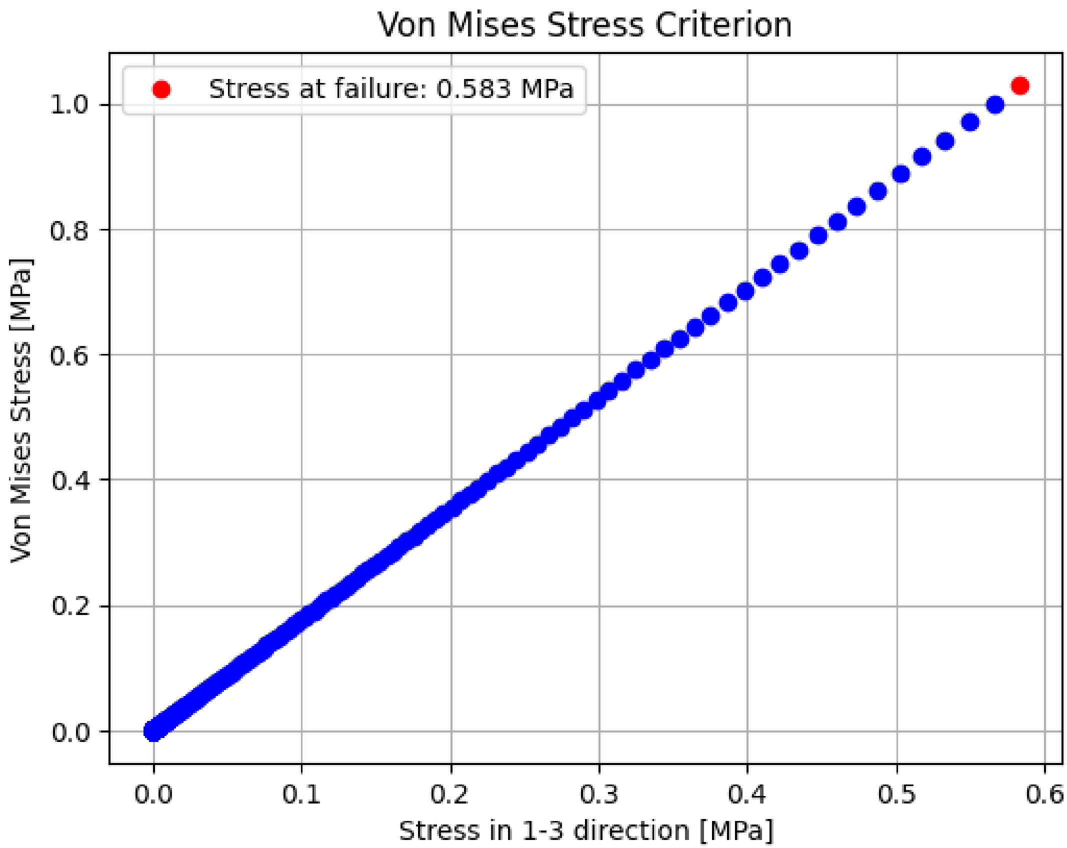
[ ]:
```
X,Y,S=1.68*10**6,0.55*10**6,0.42*10**6

sigma_matrix=[]

dis_1=np.linspace(1,2,1000)
dis_2=np.linspace(1,2,1000)

for i in range(2):
    for j in range(2):
        args=theta,kappa,i,j
        sigma_ij=sigmaB(dis_1,c_fit,k1_fit,k2_fit,dis_2,args)
        sigma_matrix.append(sigma_ij)

sigma_dir_13=rotate_stress_tensor_lists(sigma_matrix[0],sigma_matrix[1],sigma_matrix[2],sigma_

print(hashin_rotem_criterion(sigma_dir_13[0], sigma_dir_13[3], sigma_dir_13[1],␣
 ↪X, Y, S))
```

```
print(tsai_hill_criterion(sigma_dir_13[0], sigma_dir_13[3], sigma_dir_13[1], X,␣
↪Y, S))

von_mises_failure(sigma_dir_13[0], sigma_dir_13[3], 0.566*10**6)
```

880
878

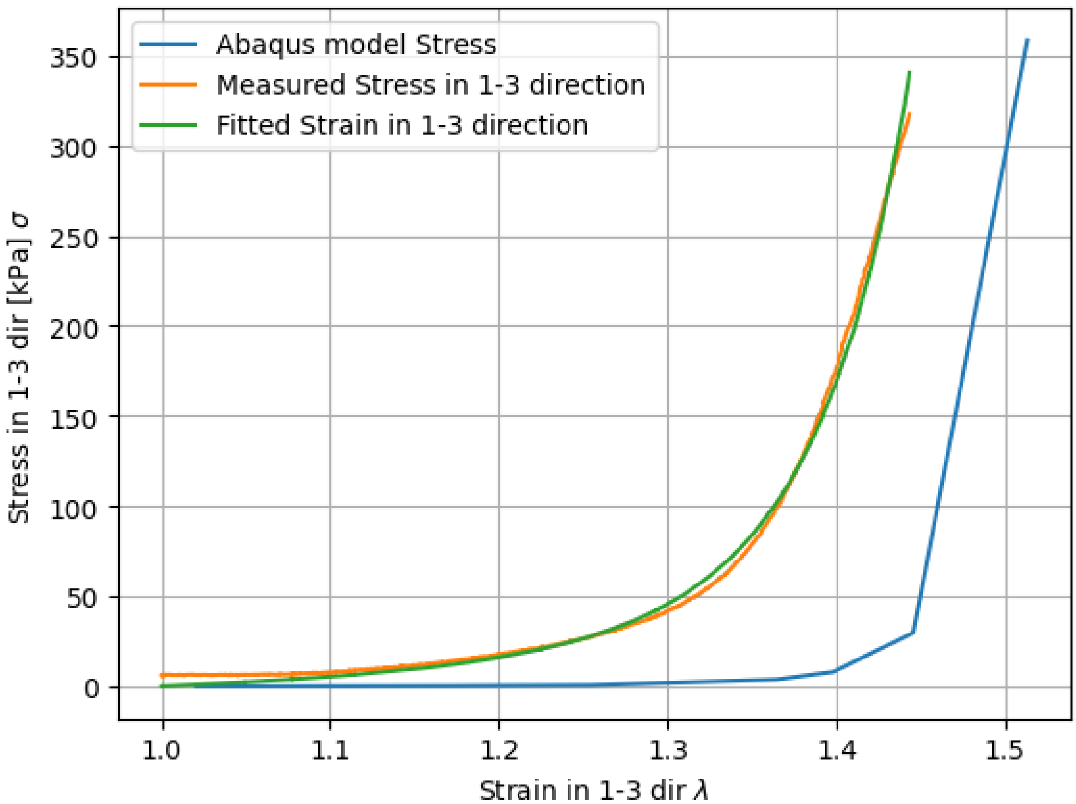
[ ]: 881


Tsai-Hill failure criteria
Stress in fiber direction $\sigma_f = 0.533$
1.0
0.8
0.6
0.4
0.2
0.0
Λ
0.0
0.1
0.2
0.3
0.4
0.5
Stress in 1-3 direction [MPa]



[ ]:
```
stress_strain_2=pd.read_csv('./Book1.csv')
stress_strain_2=stress_strain_2[:10]
```

[ ]:
```
plt.plot(1+stress_strain_2['0'],stress_strain_2['0.1']/100,label='Abaqus model␣
  ↪Stress')
plt.plot(dis[0],f_1/A_13,label='Measured Stress in 1-3 direction')
plt.plot(dis[0],testb,label='Fitted Strain in 1-3 direction')
plt.grid(True)
plt.xlabel('Strain in 1-3 dir'+r' $\lambda$')
plt.ylabel('Stress in 1-3 dir [kPa]'+r' $\sigma$')
plt.legend()
plt.savefig('./pics/abaqus_sim.pdf')
```



Abaqus model Stress
Measured Stress in 1-3 direction
Fitted Strain in 1-3 direction
Stress in 1-3 dir [kPa] σ
Strain in 1-3 dir λ
350
300
250
200
150
100
50
0
1.0
1.1
1.2
1.3
1.4
1.5