DeepEMO: A Multi-Indicator Convolutional Neural Network-based Evolutionary Multi-Objective Algorithm

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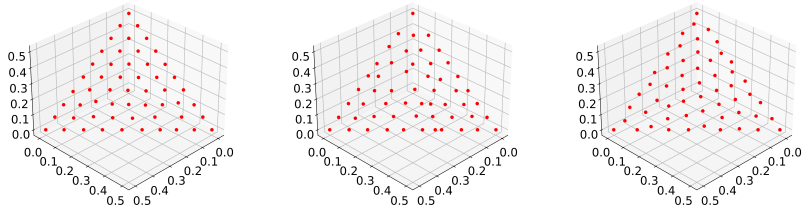
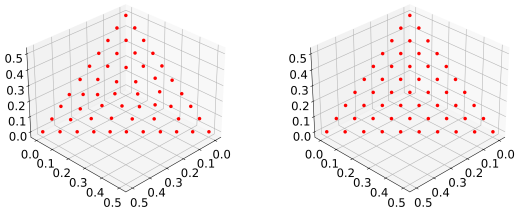
1 Supplementary Material

This section will include supplementary material regarding our experiments, such as tables and figures. In Figure 1, we show a graphic comparison of the final approximation sets obtained by each EMOA for the DTLZ1 test problem, which features a simplex-like linear geometry. Figure 2 shows a graphic comparison of the final approximation sets obtained by each EMOA for the three-objective DTLZ2^{-1} problem, which features a convex geometry. In Figure 3, we show a graphic comparison of the final approximation sets obtained by each EMOA for the IMOP5 test problem, which features a disconnected geometry. This problem is difficult for EMOAs as its Pareto front comprises eight circular regions.

In Tables 1 to 7, we present the complete numerical results of all the EMOAs regarding $R2$, E_s, IGD, IGD$^+$, Δ_p, ϵ^+, and SPD. In all tables, the two best values are shown in grayscale, with the darker tone being the best. A one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$ was used to check if the outperforming EMOA performed significantly better than the other EMOAs, with a # symbol being placed to indicate this. Table 1 shows the mean performance and standard deviation (in parentheses) for each problem as measured by the $R2$ indicator. Table 2 shows the mean performance and the standard deviation (in parentheses) for each problem as measured by the E_s indicator. Table 3 features the mean performance and standard deviation (in parentheses) for each MOP as measured by the IGD indicator. Table 4 showcases the mean performance and standard deviation (in parentheses) as measured by the IGD$^+$. Table 5 shows the mean performance and standard deviation (in parentheses) as measured by the Δ_p indicator. Table 6 includes the mean performance and standard deviation (in parentheses) as measured by the ϵ^+ indicator. Table 7 shows the mean performance and standard deviation (in parentheses) as measured by the SPD indicator.

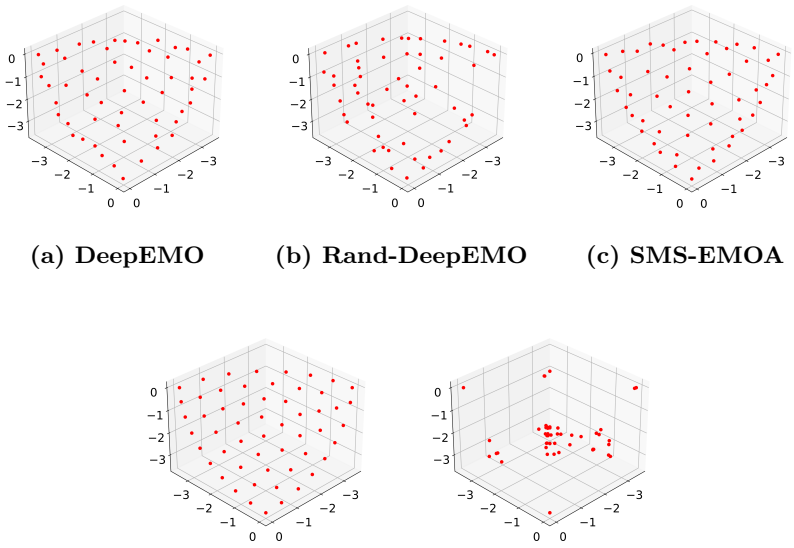


(a) DeepEMO **(b) Rand-DeepEMO** **(c) SMS-EMOA**



(d) E_s-EMOA **(e) $R2$-EMOA**

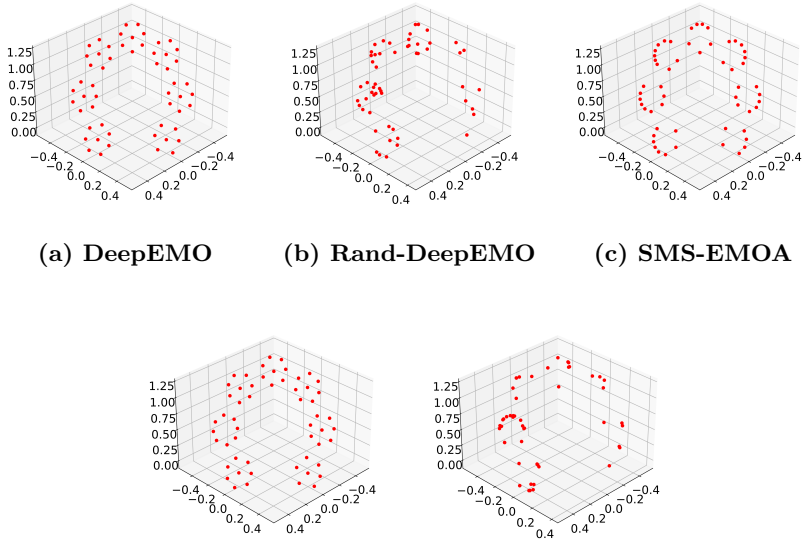
Fig. 1. Graphic comparison of performance between **(a)** DeepEMO, **(b)** Rand-DeepEMO, **(c)** SMS-EMOA, **(d)** E_s-EMOA, and **(e)** $R2$-EMOA in the DTLZ1 problem.



(a) DeepEMO **(b) Rand-DeepEMO** **(c) SMS-EMOA**

(d) E_s-EMOA **(e) $R2$-EMOA**

Fig. 2. Graphic comparison of performance between **(a)** DeepEMO, **(b)** Rand-DeepEMO, **(c)** SMS-EMOA, **(d)** E_s-EMOA, and **(e)** $R2$-EMOA in the DTLZ2^{-1} problem.



(a) DeepEMO **(b) Rand-DeepEMO** **(c) SMS-EMOA**

(d) E_s-EMOA **(e) $R2$-EMOA**

Fig. 3. Graphic comparison of performance between **(a)** DeepEMO, **(b)** Rand-DeepEMO, **(c)** SMS-EMOA, **(d)** E_s-EMOA, and **(e)** $R2$-EMOA in the IMOP5 problem.

Table 1. Mean and standard deviation (in parentheses) of $R2$ results. A symbol # is placed when the outperforming EMOA performed significantly better than the other EMOAs based on a one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$. The two best values are shown in gray scale, where the darker tone corresponds to the best value.

MOP	Dim.	SMS-EMOA	$R2$-EMOA	E_s-EMOA	Rand-DeepEMO	DeepEMO
DTLZ1	3	6.815529e-01^{5}# (5.712299e-02)	3.689387e-01^{1} (3.994651e-04)	5.918704e-01^{4}# (1.014548e-01)	5.123234e-01^{2}# (1.147065e-01)	5.517063e-01^{3}# (1.234633e-01)
DTLZ1^{-1}	3	2.193736e+03^{3}# (4.279848e+02)	3.446509e+04^{5}# (1.545554e+04)	6.598808e+02^{1} (1.485682e+02)	5.644166e+03^{4}# (4.740002e+03)	6.744177e+02^{2} (1.620693e+02)
DTLZ2	3	7.156855e+00^{5}# (2.363708e-01)	1.119036e+00^{1} (2.033107e-01)	1.912819e+00^{3}# (2.553794e-01)	4.945274e+00^{4}# (2.491535e+00)	1.173508e+00^{2} (2.372220e-01)
DTLZ2^{-1}	3	6.448971e+00^{2}# (3.393293e-01)	6.090353e+02^{5}# (1.415175e+02)	6.128197e+00^{1} (6.346480e-01)	9.133029e+01^{4}# (7.471806e+01)	7.416565e+00^{3}# (5.866231e-01)
DTLZ7	3	2.692379e+03^{1} (6.666652e+02)	3.741646e+03^{5}# (7.247880e+02)	2.755525e+03^{3} (7.747503e+02)	2.859202e+03^{4}# (6.703520e+02)	2.745643e+03^{2}# (5.542140e+02)
DTLZ7^{-1}	3	2.797489e+04^{3}# (4.135886e-01)	2.797913e+04^{5}# (3.282683e+00)	2.796073e+04^{1} (2.680030e+00)	2.797824e+04^{4}# (2.575768e+00)	2.796973e+04^{2}# (5.894600e+00)
IMOP1	2	1.323267e-01^{1} (1.590965e-04)	7.788520e+02^{5}# (9.511550e+00)	1.981020e-01^{3}# (1.377545e-02)	9.272196e+01^{4} (1.125635e+02)	1.323860e-01^{2} (3.274378e-04)
IMOP2	2	4.026869e+01^{1} (2.860542e+01)	7.203953e+02^{5}# (1.070688e+01)	4.999264e+01^{2} (2.671132e+01)	4.719226e+02^{3}# (1.065543e+02)	7.172669e+02^{4}# (8.236408e+00)
IMOP3	2	2.419825e+00^{3} (4.803895e+00)	6.595992e+02^{5}# (5.981732e+01)	2.149384e+00^{2} (3.120995e+00)	4.873941e+02^{4}# (1.792201e+01)	1.620519e+00^{1} (1.976019e+00)
IMOP4	3	4.593245e+02^{2} (8.069629e+00)	1.315927e+03^{5}# (5.327980e+00)	4.585174e+02^{1} (4.045225e+00)	1.193923e+03^{4}# (1.800646e+01)	4.598390e+02^{3} (6.866369e+00)
IMOP5	3	1.880301e+02^{3}# (4.284865e+00)	2.855712e+02^{5}# (4.247623e+01)	1.848943e+02^{1} (3.847052e+00)	2.627559e+02^{4}# (1.357482e+01)	1.854980e+02^{2} (3.714512e+00)
IMOP6	3	1.574111e+02^{3}# (1.162030e+02)	8.069710e+02^{5}# (4.268351e+01)	1.312674e+02^{1} (5.902483e-01)	7.730191e+02^{4}# (1.478006e+02)	1.313697e+02^{2} (9.452300e-01)
IMOP7	3	7.271210e+02^{2}# (3.495057e+02)	9.119309e+02^{5}# (3.356095e+00)	4.588144e+01^{1} (1.000246e+00)	8.527087e+02^{3}# (1.860357e+02)	8.572841e+02^{4}# (1.794092e+02)
IMOP8	3	2.834786e+02^{3} (2.961041e+02)	1.084524e+03^{5}# (2.963933e+02)	2.269768e+02^{1} (1.219134e+01)	5.316769e+02^{4} (4.659727e+02)	2.325103e+02^{2} (1.984310e+01)
VIE1	3	2.235373e+03^{3}# (5.997280e+00)	2.247898e+03^{4}# (9.418676e+01)	2.050822e+03^{1} (1.869753e+01)	2.175530e+03^{2}# (3.984952e+01)	2.235373e+03^{3}# (5.997280e+00)
VIE2	3	2.007229e+04^{3}# (2.429878e+00)	2.021204e+04^{5}# (1.784344e+02)	2.006071e+04^{1} (1.906801e+00)	2.015485e+04^{4}# (1.044185e+02)	2.006133e+04^{2} (1.580238e+00)
VIE3	3	1.217050e+04^{1} (5.298401e-01)	1.321261e+04^{5}# (4.919373e+02)	1.218598e+04^{3}# (1.922241e+01)	1.258677e+04^{4}# (1.762456e+02)	1.218432e+04^{2}# (1.637350e+01)

Table 2. Mean and standard deviation (in parentheses) of E_s results. A symbol # is placed when the outperforming EMOA performed significantly better than the other EMOAs based on a one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$. The two best values are shown in gray scale, where the darker tone corresponds to the best value.

MOP	Dim.	SMS-EMOA	$R2$-EMOA	E_s-EMOA	Rand-DeepEMO	DeepEMO
DTLZ1	3	8.555404e+05^{4}# (1.096847e+04)	7.838071e+05^{1} (2.322225e+03)	7.881412e+05^{2}# (7.043412e+03)	1.796610e+06^{5}# (3.248982e+06)	7.892367e+05^{3}# (1.413359e+04)
DTLZ1^{-1}	3	9.250368e-04^{3}# (2.348007e-05)	4.000004e+12^{5}# (3.947013e+12)	6.132142e-04^{1} (2.054614e-05)	2.000000e+11^{4}# (6.155870e+11)	6.155464e-04^{2} (2.006527e-05)
DTLZ2	3	1.057069e+05^{2}# (9.514843e+03)	1.193565e+08^{4}# (5.109900e+08)	4.654907e+04^{1} (5.815003e+02)	3.199842e+10^{5}# (1.427364e+11)	5.358347e+07^{3}# (2.222651e+08)
DTLZ2^{-1}	3	1.794532e+03^{3}# (3.147280e+01)	2.580727e+51^{5}# (7.963084e+51)	1.086880e+03^{1} (1.337135e+01)	2.000037e+11^{4}# (6.155857e+11)	1.705908e+03^{2}# (7.696956e+01)
DTLZ7	3	2.794323e+05^{2}# (4.317235e+05)	1.382596e+23^{5}# (6.183155e+23)	9.604062e+05^{3} (3.399590e+06)	2.080375e+06^{4}# (4.081876e+06)	1.545118e+05^{1} (1.531743e+05)
DTLZ7^{-1}	3	5.671426e+05^{2}# (5.781104e+05)	1.051234e+18^{5}# (4.700993e+18)	2.425466e+05^{1} (1.811588e+04)	1.009549e+11^{3}# (4.492142e+11)	3.817727e+11^{4}# (9.814288e+11)
IMOP1	2	1.147259e+06^{3}# (5.335096e+04)	1.006376e+15^{5}# (3.514131e+15)	1.781171e+05^{1} (7.473682e+03)	2.096084e+08^{4}# (5.782098e+08)	1.145280e+06^{2}# (9.673941e+04)
IMOP2	2	8.309957e+05^{2}# (1.526420e+05)	1.391101e+14^{5}# (6.198192e+14)	3.983023e+05^{1} (4.562362e+04)	1.416161e+07^{3}# (1.091261e+07)	1.483422e+13^{4}# (3.722084e+13)
IMOP3	2	7.205966e+05^{3}# (3.147870e+05)	7.064968e+11^{5}# (2.122832e+12)	5.667994e+05^{2} (2.322607e+05)	3.182858e+07^{4}# (2.107698e+07)	5.059133e+05^{1} (2.046339e+05)
IMOP4	3	3.491875e+06^{3}# (5.221591e+05)	2.456896e+22^{5}# (6.232126e+22)	2.108116e+06^{1} (2.807477e+05)	1.134721e+11^{4}# (1.908632e+11)	2.134124e+06^{2} (3.428303e+05)
IMOP5	3	3.068360e+05^{3}# (1.602601e+04)	7.081429e+29^{5}# (3.166832e+30)	1.446778e+05^{2} (2.443692e+03)	1.028503e+11^{4}# (4.466868e+11)	1.443357e+05^{1} (2.425053e+03)
IMOP6	3	2.164288e+06^{3}# (9.031515e+06)	1.451166e+30^{5}# (6.482119e+30)	6.480664e+04^{2} (1.078281e+03)	9.105640e+07^{4}# (7.974157e+07)	6.474019e+04^{1} (7.823316e+02)
IMOP7	3	2.825797e+10^{2}# (1.473128e+10)	1.869560e+34^{4}# (7.856127e+34)	1.001604e+05^{1} (2.092979e+03)	1.874938e+26^{3}# (8.384973e+26)	2.470831e+38^{5}# (1.104989e+39)
IMOP8	3	9.683622e+05^{3}# (2.956174e+06)	7.807935e+38^{5}# (3.491815e+39)	2.059517e+04^{1} (1.058066e+03)	3.461083e+08^{4}# (7.961343e+08)	2.129625e+04^{2} (1.833898e+03)
VIE1	3	1.071705e+04^{2}# (4.091148e+02)	8.170045e+44^{4}# (3.653755e+45)	4.457618e+03^{1} (3.769419e+02)	2.000000e+11^{3}# (6.155870e+11)	1.071705e+04^{2}# (4.091148e+02)
VIE2	3	7.409281e+06^{3}# (4.571843e+05)	8.586644e+13^{5}# (3.069842e+14)	1.701927e+06^{2} (2.107662e+05)	1.310283e+12^{4}# (5.407335e+12)	1.655433e+06^{1} (1.522353e+05)
VIE3	3	2.762736e+05^{3}# (2.544827e+04)	1.324369e+45^{5}# (4.650078e+45)	2.664948e+02^{1} (1.059207e+02)	2.000002e+11^{4}# (6.155870e+11)	2.739037e+02^{2} (2.172698e+02)

Table 3. Mean and standard deviation (in parentheses) of IGD results. A symbol # is placed when the outperforming EMOA performed significantly better than the other EMOAs based on a one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$. The two best values are shown in gray scale, where the darker tone corresponds to the best value.

MOP	Dim.	SMS-EMOA	$R2$-EMOA	E_s-EMOA	Rand-DeepEMO	DeepEMO
DTLZ1	3	2.861410e-02^{2}# (3.162496e-04)	2.827091e-02^{1} (2.939249e-05)	2.892274e-02^{4}# (2.866761e-04)	2.988609e-02^{5}# (8.128918e-04)	2.880920e-02^{3}# (2.233730e-04)
DTLZ1^{-1}	3	3.667983e+02^{1} (3.274004e-02)	3.723776e+02^{5}# (1.143572e+00)	3.669734e+02^{3}# (6.065991e-02)	3.701679e+02^{4}# (7.080741e-01)	3.669690e+02^{2}# (4.952593e-02)
DTLZ2	3	1.045549e-01^{5}# (2.150346e-03)	7.813285e-02^{2}# (1.592205e-03)	7.661003e-02^{1} (7.005726e-04)	9.636619e-02^{4}# (3.013346e-03)	7.876167e-02^{3}# (1.834633e-03)
DTLZ2^{-1}	3	2.756250e-01^{2}# (1.872538e-03)	6.316006e-01^{5}# (3.930287e-02)	2.635957e-01^{1} (1.734480e-03)	3.878818e-01^{4}# (3.884384e-02)	2.770555e-01^{3}# (2.960541e-03)
DTLZ7	3	3.262437e-01^{3}# (2.533027e-01)	7.428838e-01^{5}# (3.327275e-01)	2.945584e-01^{2} (3.450529e-01)	3.495229e-01^{4}# (2.790459e-01)	2.767777e-01^{1} (2.236599e-01)
DTLZ7^{-1}	3	1.203165e-01^{4}# (1.446368e-02)	1.250064e-01^{5}# (2.222334e-02)	5.341874e-02^{1} (1.483899e-03)	1.016095e-01^{3}# (1.893063e-02)	6.670036e-02^{2} (2.450917e-02)
IMOP1	2	5.125835e-02^{2}# (3.776483e-03)	8.976944e-01^{5}# (9.408380e-03)	2.051430e-02^{1} (3.488812e-03)	2.777318e-01^{4}# (1.706220e-01)	5.280787e-02^{3}# (6.685649e-03)
IMOP2	2	1.079952e-01^{1} (1.904404e-02)	8.075596e-01^{5}# (1.361505e-02)	1.119683e-01^{2} (1.714082e-02)	4.823508e-01^{3}# (1.089253e-01)	8.036038e-01^{4}# (1.050013e-02)
IMOP3	2	8.405944e-02^{1} (2.557894e-02)	8.269868e-01^{5}# (5.680047e-02)	8.862462e-02^{3} (2.518382e-02)	6.788577e-01^{4}# (4.240582e-02)	8.691152e-02^{2} (2.043327e-02)
IMOP4	3	4.865782e-02^{2} (1.213043e-02)	9.872588e-01^{5}# (4.277788e-03)	4.737670e-02^{1} (9.966395e-03)	8.944643e-01^{4}# (1.691783e-02)	4.904950e-02^{3} (1.245756e-02)
IMOP5	3	9.385251e-02^{3}# (1.243771e-03)	2.211618e-01^{5}# (2.245907e-01)	8.606775e-02^{1} (5.635506e-04)	1.235335e-01^{4}# (7.597249e-03)	8.649406e-02^{2} (8.851700e-04)
IMOP6	3	1.555263e-01^{3}# (1.164452e-01)	6.549822e-01^{5}# (4.267674e-04)	1.250250e-01^{1} (4.934768e-04)	6.100868e-01^{4}# (1.132456e-01)	1.251928e-01^{2} (5.610394e-04)
IMOP7	3	7.921229e-01^{2}# (3.698789e-01)	1.006864e+00^{5}# (6.923686e-03)	6.415742e-02^{1} (1.201222e-03)	9.211792e-01^{3}# (2.024124e-01)	9.298298e-01^{4}# (2.026094e-01)
IMOP8	3	1.785156e-01^{3} (1.793178e-01)	9.180657e-01^{5}# (1.197965e-01)	1.290660e-01^{2} (4.528589e-03)	3.887299e-01^{4}# (3.477471e-01)	1.290447e-01^{1} (5.670750e-03)
VIE1	3	1.750223e-01^{1} (3.132802e-03)	3.036876e-01^{4}# (5.727569e-02)	2.123987e-01^{2}# (6.836933e-03)	2.393316e-01^{3}# (5.007878e-02)	1.750223e-01^{1} (3.132802e-03)
VIE2	3	3.037353e-02^{3}# (1.406457e-03)	1.150057e-01^{5}# (3.214621e-02)	2.133348e-02^{2} (5.992631e-04)	7.256103e-02^{4}# (1.793425e-02)	2.125693e-02^{1} (5.876940e-04)
VIE3	3	6.734126e-01^{1} (1.698856e-03)	1.777128e+00^{5}# (7.591623e-01)	6.951721e-01^{3}# (2.019560e-02)	1.232485e+00^{4}# (3.355433e-01)	6.923582e-01^{2}# (3.133966e-02)

Table 4. Mean and standard deviation (in parentheses) of IGD^+ results. A symbol # is placed when the outperforming EMOA performed significantly better than the other EMOAs based on a one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$. The two best values are shown in gray scale, where the darker tone corresponds to the best value.

MOP	Dim.	SMS-EMOA	$R2$-EMOA	E_s-EMOA	Rand-DeepEMO	DeepEMO
DTLZ1	3	1.723662e-02^{1} (2.997624e-04)	1.750618e-02^{2}# (2.381428e-04)	1.775521e-02^{4}# (3.922386e-04)	1.820652e-02^{5}# (4.997667e-04)	1.770381e-02^{3}# (3.523215e-04)
DTLZ1^{-1}	3	3.634408e+02^{1} (3.606626e-02)	3.692607e+02^{5}# (1.191903e+00)	3.636146e+02^{3}# (6.732243e-02)	3.669071e+02^{4}# (7.276475e-01)	3.636082e+02^{2}# (5.437623e-02)
DTLZ2	3	2.519279e-02^{1} (3.222554e-04)	3.045805e-02^{3}# (1.288124e-04)	3.056343e-02^{5}# (8.038148e-04)	2.979966e-02^{2}# (1.163911e-03)	3.052701e-02^{4}# (1.438658e-04)
DTLZ2^{-1}	3	1.300036e-01^{2} (1.603028e-03)	3.506713e-01^{5}# (2.656736e-02)	1.294503e-01^{1} (1.863423e-03)	1.876921e-01^{4}# (1.878283e-02)	1.318378e-01^{3}# (3.019644e-03)
DTLZ7	3	1.174114e-01^{1} (1.768499e-01)	4.501602e-01^{5}# (2.465525e-01)	1.507089e-01^{3}# (2.246874e-01)	1.575071e-01^{4}# (1.760925e-01)	1.205011e-01^{2}# (1.059940e-01)
DTLZ7^{-1}	3	1.756750e-02^{2} (1.824919e-02)	4.849035e-02^{5}# (1.499030e-02)	1.653446e-02^{1} (5.428712e-04)	2.600850e-02^{4}# (6.447150e-03)	2.165257e-02^{3}# (9.545484e-03)
IMOP1	2	3.872137e-04^{1} (5.215087e-06)	7.227611e-01^{5}# (1.134124e-02)	7.289502e-04^{3}# (2.879971e-05)	6.599975e-02^{4}# (7.834779e-02)	3.903555e-04^{2} (1.555003e-05)
IMOP2	2	3.157071e-03^{1} (1.173276e-03)	3.039119e-01^{5}# (2.363738e-05)	3.859210e-03^{2}# (1.098733e-03)	2.610628e-01^{3}# (7.677068e-02)	3.039075e-01^{4}# (2.074418e-05)
IMOP3	2	4.036363e-02^{1} (1.011898e-02)	4.761706e-01^{5}# (1.088737e-02)	4.294459e-02^{2} (1.028424e-02)	3.641483e-01^{4}# (3.871938e-02)	4.424858e-02^{3} (7.594932e-03)
IMOP4	3	1.342977e-02^{2} (4.920049e-03)	6.962487e-01^{5}# (4.975672e-03)	1.305994e-02^{1} (4.235382e-03)	5.960938e-01^{4}# (1.542981e-02)	1.371423e-02^{3} (5.084249e-03)
IMOP5	3	4.903519e-02^{3}# (6.189105e-04)	1.256344e-01^{5}# (1.371114e-01)	4.254827e-02^{1} (3.172216e-04)	6.228372e-02^{4}# (3.283851e-03)	4.271106e-02^{2} (4.332631e-04)
IMOP6	3	6.340468e-02^{3}# (4.128570e-02)	4.829676e-01^{5}# (7.972308e-02)	5.209422e-02^{1} (7.982253e-04)	4.656887e-01^{4}# (9.674080e-02)	5.218971e-02^{2} (9.182813e-04)
IMOP7	3	4.153648e-01^{2}# (2.023482e-01)	5.182143e-01^{5}# (4.725092e-04)	2.335656e-02^{1} (5.070965e-04)	4.875721e-01^{3}# (1.093828e-01)	4.889347e-01^{4}# (1.089516e-01)
IMOP8	3	8.010665e-02^{3} (9.964600e-02)	6.434371e-01^{5}# (9.303049e-02)	7.279422e-02^{2} (4.199884e-03)	2.792545e-01^{4}# (2.844809e-01)	7.211751e-02^{1} (5.967181e-03)
VIE1	3	8.772351e-02^{1} (2.586308e-03)	1.085663e-01^{4}# (1.872102e-02)	9.604337e-02^{3}# (4.324776e-03)	9.530881e-02^{2}# (1.273319e-02)	8.772351e-02^{1} (2.586308e-03)
VIE2	3	4.456801e-03^{1} (9.748758e-05)	9.077799e-03^{5}# (1.809990e-03)	5.327477e-03^{2}# (3.357166e-04)	5.812080e-03^{4}# (6.742103e-04)	5.390655e-03^{3}# (3.163599e-04)
VIE3	3	1.983055e-01^{1} (7.654473e-05)	9.247887e-01^{5}# (5.928640e-01)	2.159523e-01^{2}# (8.240867e-03)	2.509725e-01^{4}# (4.329906e-02)	2.233116e-01^{3}# (2.886938e-02)

Table 5. Mean and standard deviation (in parentheses) of Δ_p results. A symbol # is placed when the outperforming EMOA performed significantly better than the other EMOAs based on a one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$. The two best values are shown in gray scale, where the darker tone corresponds to the best value.

MOP	Dim.	SMS-EMOA	$R2$-EMOA	E_s-EMOA	Rand-DeepEMO	DeepEMO
DTLZ1	3	2.861410e-02[2]# (3.162496e-04)	2.827091e-02[1] (2.939249e-05)	2.892274e-02[3]# (2.866761e-04)	2.988609e-02[4]# (8.128918e-04)	1.685785e-01[5]# (6.251909e-01)
DTLZ1^{-1}	3	3.667983e+02[1] (3.274004e-02)	3.723776e+02[5]# (1.143572e+00)	3.669734e+02[3]# (6.065991e-02)	3.701679e+02[4]# (7.080741e-01)	3.669690e+02[2]# (4.952593e-02)
DTLZ2	3	1.045549e-01[5]# (2.150346e-03)	7.813285e-02[2]# (1.592205e-03)	7.661003e-02[1] (7.005726e-04)	9.636619e-02[4]# (3.013346e-03)	7.876167e-02[3]# (1.834633e-03)
DTLZ2^{-1}	3	2.756250e-01[2]# (1.872538e-03)	6.316006e-01[5]# (3.930287e-02)	2.635957e-01[1] (1.734480e-03)	3.878818e-01[4]# (3.884384e-02)	2.770555e-01[3]# (2.960541e-03)
DTLZ7	3	3.262437e-01[3]# (2.533027e-01)	7.428838e-01[5]# (3.327275e-01)	2.945584e-01[2] (3.450529e-01)	3.495229e-01[4]# (2.790459e-01)	2.767777e-01[1] (2.236599e-01)
DTLZ7^{-1}	3	1.203165e-01[4]# (1.446368e-02)	1.250064e-01[5]# (2.222334e-02)	5.341874e-02[1] (1.483899e-03)	1.016095e-01[3]# (1.893063e-02)	6.670036e-02[2] (2.450917e-02)
IMOP1	2	5.125835e-02[2]# (3.776483e-03)	8.976944e-01[5]# (9.408380e-03)	2.051430e-02[1] (3.488812e-03)	2.777318e-01[4]# (1.706220e-01)	5.280787e-02[3]# (6.685649e-03)
IMOP2	2	1.079952e-01[1] (1.904404e-02)	8.075596e-01[5]# (1.361505e-02)	1.119683e-01[2] (1.714082e-02)	4.823508e-01[3]# (1.089253e-01)	8.036038e-01[4]# (1.050013e-02)
IMOP3	2	8.405944e-02[1] (2.557894e-02)	8.269868e-01[5]# (5.680047e-02)	8.862462e-02[3] (2.518382e-02)	6.788577e-01[4]# (4.240582e-02)	8.691152e-02[2] (2.043327e-02)
IMOP4	3	4.865782e-02[2] (1.213043e-02)	9.872588e-01[5]# (4.277788e-03)	4.737670e-02[1] (9.966395e-03)	8.944643e-01[4]# (1.691783e-02)	4.904950e-02[3] (1.245756e-02)
IMOP5	3	9.385251e-02[3]# (1.243771e-03)	2.211618e-01[5]# (2.245907e-01)	8.606775e-02[1] (5.635506e-04)	1.235335e-01[4]# (7.597249e-03)	8.649406e-02[2] (8.851700e-04)
IMOP6	3	1.555263e-01[3]# (1.164452e-01)	6.549822e-01[5]# (4.267674e-04)	1.250250e-01[1] (4.934768e-04)	6.100868e-01[4]# (1.132456e-01)	1.251928e-01[2] (5.610394e-04)
IMOP7	3	7.921229e-01[2]# (3.698789e-01)	1.006864e+00[5]# (6.923686e-03)	6.415742e-02[1] (1.201222e-03)	9.211792e-01[3]# (2.024124e-01)	9.298298e-01[4]# (2.026094e-01)
IMOP8	3	1.785156e-01[3] (1.793178e-01)	9.180657e-01[5]# (1.197965e-01)	1.290660e-01[2] (4.528589e-03)	3.887299e-01[4]# (3.477471e-01)	1.290447e-01[1] (5.670750e-03)
VIE1	3	4.269261e-01[1] (6.285103e-03)	4.325627e-01[2] (8.710993e-02)	5.830041e-01[4]# (2.487121e-02)	4.666026e-01[3]# (3.965330e-02)	4.269261e-01[1] (6.285103e-03)
VIE2	3	3.037353e-02[3]# (1.406457e-03)	1.150057e-01[5]# (3.214621e-02)	2.214329e-02[2] (2.084490e-03)	7.256103e-02[4]# (1.793425e-02)	2.206675e-02[1] (2.112312e-03)
VIE3	3	1.276038e+00[1] (1.432809e-02)	2.994820e+00[3]# (2.278282e+00)	1.678026e+01[5]# (5.425940e-01)	2.925609e+00[2]# (1.356164e+00)	1.672819e+01[4]# (7.589820e-01)

Table 6. Mean and standard deviation (in parentheses) of ϵ^+ results. A symbol # is placed when the outperforming EMOA performed significantly better than the other EMOAs based on a one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$. The two best values are shown in gray scale, where the darker tone corresponds to the best value.

MOP	Dim.	SMS-EMOA	$R2$-EMOA	E_s-EMOA	Rand-DeepEMO	DeepEMO
DTLZ1	3	3.351217e-02[1] (1.337600e-03)	3.576627e-02[2]# (1.294748e-04)	3.644716e-02[3]# (1.543333e-03)	4.588518e-02[5]# (6.490948e-03)	3.693057e-02[4]# (2.911785e-03)
DTLZ1^{-1}	3	5.506488e+02[4]# (3.754296e-04)	5.506486e+02[3]# (4.838443e-04)	5.506496e+02[5]# (3.381233e-03)	5.506485e+02[2] (3.275105e-04)	5.506485e+02[1] (3.773523e-04)
DTLZ2	3	6.140099e-02[1] (3.119889e-03)	8.548580e-02[4]# (2.328459e-04)	8.008654e-02[2]# (4.564751e-03)	9.237722e-02[5]# (8.661539e-03)	8.542433e-02[3]# (1.781895e-04)
DTLZ2^{-1}	3	3.269054e-01[2]# (1.691369e-02)	8.201939e-01[5]# (5.216523e-02)	3.175923e-01[1] (1.375828e-02)	5.187449e-01[4]# (8.093046e-02)	3.319803e-01[3]# (2.096213e-02)
DTLZ7	3	5.060694e-01[1] (7.040513e-01)	1.653302e+00[5]# (7.022865e-01)	5.751527e-01[3] (8.215834e-01)	7.260028e-01[4]# (6.814760e-01)	5.658259e-01[2] (5.899179e-01)
DTLZ7^{-1}	3	6.539725e-02[2] (1.101246e-01)	2.641862e-01[5]# (7.826103e-02)	4.442333e-02[1] (5.352483e-03)	1.728907e-01[4]# (9.554187e-02)	8.954716e-02[3]# (8.187483e-02)
IMOP1	2	2.736021e-03[1] (8.688928e-05)	9.911009e-01[5]# (1.148696e-02)	1.076117e-02[3]# (5.544852e-04)	1.234198e-01[4]# (1.430895e-01)	2.791864e-03[2] (1.628172e-04)
IMOP2	2	2.939543e-02[1] (1.287751e-02)	9.999627e-01[5]# (2.543199e-05)	3.413629e-02[2] (1.180452e-02)	9.018331e-01[3]# (2.214916e-01)	9.999574e-01[4]# (2.180804e-05)
IMOP3	2	1.565163e-01[1] (4.179985e-02)	8.667794e-01[5]# (1.302946e-02)	1.650813e-01[2] (4.353941e-02)	7.305232e-01[4]# (4.100049e-02)	1.679943e-01[3] (3.359432e-02)
IMOP4	3	8.978590e-02[1] (3.085621e-02)	9.961003e-01[5]# (1.423072e-03)	8.981573e-02[2] (3.134513e-02)	9.570172e-01[4]# (1.241352e-02)	9.317336e-02[3] (3.495967e-02)
IMOP5	3	1.642659e-01[1] (2.419221e-06)	3.318207e-01[5]# (2.419582e-01)	1.704069e-01[2]# (4.284000e-03)	2.191348e-01[4]# (2.625320e-02)	1.708921e-01[3]# (5.688217e-03)
IMOP6	3	5.646745e-01[3]# (2.769343e-02)	9.651527e-01[5]# (9.957586e-02)	5.545996e-01[2] (1.331839e-02)	9.507222e-01[4]# (9.024993e-02)	5.508729e-01[1] (1.099010e-02)
IMOP7	3	8.084795e-01[2]# (3.827055e-01)	9.995894e-01[5]# (4.949771e-04)	7.598088e-02[1] (7.155305e-03)	9.468639e-01[3]# (2.036990e-01)	9.490800e-01[4]# (2.000066e-01)
IMOP8	3	3.014464e-01[3] (5.387722e-01)	1.633005e+00[5]# (5.402161e-01)	1.742035e-01[2] (2.952561e-02)	7.445496e-01[4]# (7.807219e-01)	1.731876e-01[1] (2.647645e-02)
VIE1	3	2.324179e-01[1] (1.326168e-02)	2.950161e-01[4]# (4.709265e-02)	2.354697e-01[2] (1.692497e-02)	2.581480e-01[3] (5.747417e-02)	2.324179e-01[1] (1.326168e-02)
VIE2	3	2.653366e-02[4]# (3.640883e-04)	3.341458e-02[5]# (8.476573e-03)	2.474499e-02[2]# (2.958495e-03)	2.270797e-02[1] (5.404677e-03)	2.556834e-02[3]# (2.668093e-03)
VIE3	3	1.141736e+00[1] (3.422487e-03)	2.691092e+00[5]# (1.350787e+00)	1.205296e+00[3]# (2.994757e-02)	1.157543e+00[2]# (2.011475e-02)	1.212537e+00[4]# (2.543573e-02)

Table 7. Mean and standard deviation (in parentheses) of SPD results. A symbol # is placed when the outperforming EMOA performed significantly better than the other EMOAs based on a one-tailed Wilcoxon test using a significance level of $\alpha = 0.05$. The two best values are shown in gray scale, where the darker tone corresponds to the best value.

MOP	Dim.	SMS-EMOA	$R2$-EMOA	E_s-EMOA	Rand-DeepEMO	DeepEMO
DTLZ1	3	9.055368e+00^{5}# (1.862275e-02)	9.148001e+00^{3}# (1.064257e-02)	9.170850e+00^{2} (2.016585e-02)	9.076815e+00^{4}# (4.053745e-02)	9.207686e+00^{1} (2.237013e-01)
DTLZ1^{-1}	3	5.500000e+01^{1} (0.000000e+00)	5.287191e+01^{3}# (1.601598e+00)	5.500000e+01^{1} (0.000000e+00)	5.489999e+01^{2} (3.078166e-01)	5.500000e+01^{1} (0.000000e+00)
DTLZ2	3	2.719427e+01^{5}# (1.924554e-01)	2.918328e+01^{2}# (1.302854e-01)	2.982793e+01^{1} (7.523631e-02)	2.762914e+01^{4}# (3.355055e-01)	2.911081e+01^{3}# (1.483171e-01)
DTLZ2^{-1}	3	5.383237e+01^{3}# (4.486853e-02)	3.169553e+01^{5}# (4.078808e+00)	5.475480e+01^{1} (8.152090e-03)	4.988262e+01^{4}# (1.492805e+00)	5.395493e+01^{2}# (8.854022e-02)
DTLZ7	3	2.673063e+01^{3}# (7.251372e+00)	1.258404e+01^{5}# (4.948288e+00)	3.093610e+01^{1} (1.057513e+01)	2.369720e+01^{4}# (7.000567e+00)	3.037893e+01^{2} (8.943325e+00)
DTLZ7^{-1}	3	1.932238e+01^{3}# (1.139173e+00)	1.524358e+01^{5}# (1.556741e+00)	2.261089e+01^{1} (2.605415e-01)	1.729186e+01^{4}# (1.385740e+00)	2.090379e+01^{2}# (2.375215e+00)
IMOP1	2	8.519319e+00^{2}# (1.905160e-01)	1.034160e+00^{5}# (4.591984e-02)	9.962349e+00^{1} (8.086014e-02)	5.627870e+00^{4}# (1.302105e+00)	8.488366e+00^{3}# (2.326329e-01)
IMOP2	2	7.312690e+00^{2}# (3.648345e-01)	1.532493e+00^{5}# (9.270027e-02)	7.559593e+00^{1} (3.197709e-01)	3.900257e+00^{3}# (8.939274e-01)	1.557683e+00^{4}# (7.262938e-02)
IMOP3	2	8.126736e+00^{3}# (6.662503e-01)	1.735499e+00^{5}# (4.025599e-01)	8.194289e+00^{2} (7.406894e-01)	2.855633e+00^{4}# (4.513087e-01)	8.400648e+00^{1} (6.811295e-01)
IMOP4	3	1.068482e+01^{3} (3.478179e-01)	1.087290e+00^{5}# (3.100570e-02)	1.072415e+01^{1} (2.837158e-01)	1.747847e+00^{4}# (1.083156e-01)	1.068777e+01^{2} (3.540219e-01)
IMOP5	3	2.097595e+01^{3}# (7.357660e-02)	1.407017e+01^{5}# (5.036230e+00)	2.105391e+01^{2} (5.378903e-02)	1.709453e+01^{4}# (4.161487e-01)	2.106648e+01^{1} (8.141869e-02)
IMOP6	3	2.449089e+01^{3}# (4.255569e+00)	6.396641e+00^{5}# (2.866519e-01)	2.643096e+01^{2} (9.714243e-02)	7.544883e+00^{4}# (3.899855e+00)	2.643856e+01^{1} (7.593774e-02)
IMOP7	3	5.882547e+00^{2}# (8.991557e+00)	1.127093e+00^{5}# (7.277314e-02)	2.406796e+01^{1} (1.297629e-01)	2.504921e+00^{3}# (4.208082e+00)	2.334991e+00^{4}# (3.852071e+00)
IMOP8	3	3.769519e+01^{3}# (8.892837e+00)	1.075654e+01^{5}# (3.935661e+00)	4.092913e+01^{1} (4.285666e-01)	2.856126e+01^{4}# (1.263666e+01)	4.063746e+01^{2} (8.245871e-01)
VIE1	3	4.644379e+01^{2}# (1.405075e-01)	3.480495e+01^{4}# (2.216949e+00)	5.152835e+01^{1} (2.675843e-01)	4.564080e+01^{3}# (1.240949e+00)	4.644379e+01^{2}# (1.405075e-01)
VIE2	3	1.078860e+01^{3}# (1.419263e-01)	8.049622e+00^{5}# (7.536916e-01)	1.209430e+01^{1} (1.459292e-01)	9.260691e+00^{4}# (5.977132e-01)	1.209226e+01^{2} (1.488862e-01)
VIE3	3	3.140308e+01^{5}# (4.239334e-01)	3.393923e+01^{4}# (7.311392e+00)	5.481397e+01^{2} (1.119817e-01)	4.787282e+01^{3}# (2.857204e+00)	5.483158e+01^{1} (1.141229e-01)