HW 4: Optimal Economic Dispatch of Power Systems

Due: Friday April 1 at 5:00pm PT

This assignment will provide hands-on practice for optimization with application to the economic dispatch problem in power systems. The assignment is organized in a tutorial fashion, thereby allowing you to practice optimization theory on a relevant real-world energy system example.

Reading

Read Sections 6.1 and 6.2 of Kirschen and Strbac's textbook "Fundamentals of Power System Economics," found in the Readings section of bCourses.

[Optional] Read "Grid of the Future: Quantification of Benefits from Flexible Energy Resources in Scenarios With Extra-High Penetration of Renewable Energy", posted in bCourses.

Background

Consider a power system with several generators supplying the required system load. Economic dispatch (ED) is the process of allocating generation levels to the individual generating units, such that the system load may be supplied entirely and most economically, subject to generator operating conditions and power system network constraints. This optimization task is a fundamental and daily operation of power systems. In this assignment, you will formulate and solve progressively more complex and realistic versions of the ED problem.

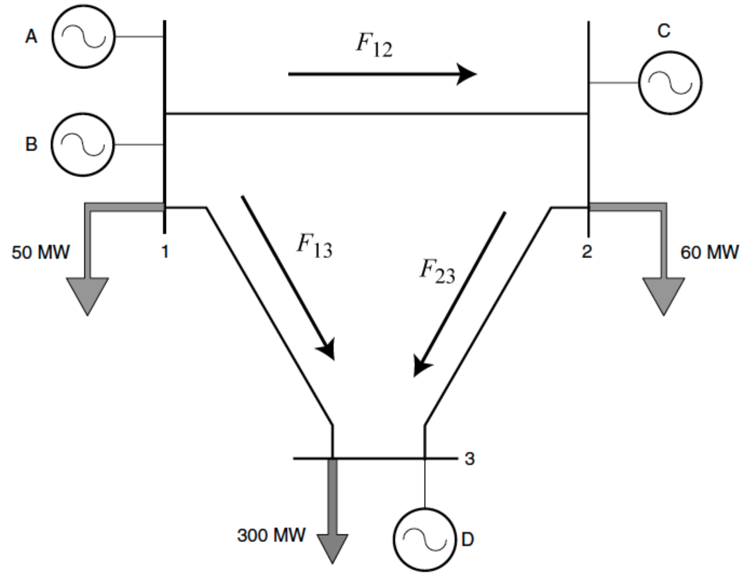

Figure 1: Three-bus network.

Consider the three-bus network power system in Fig. 1, which contains four generators, denoted A, B, C, D, and loads at each bus denoted by the downward arrows. Each branch has transmission capacity limits, provided in Table 1. Each generator has power capacity limits and a "marginal generation cost" provided in Table 2. "Marginal cost" implies the generation cost as a function of power is linear, and the marginal cost indicates the slope.

Table 1: Branch Data for Three-Bus System

Branch	Reactance [p.u.]	Capacity [MW]
1-2	$x_{12} = 0.2$	126
1-3	$x_{13} = 0.2$	250
2-3	$x_{23} = 0.1$	90

Table 2: Generator Data for Three-Bus System

Generator	Capacity [MW]	Marginal Cost [USD/MWh]
A	140	7.5
B	285	6
C	90	14
D	85	10

Problem 1: Intuition
Before formulating anything mathematically, use your intuition to determine how much each generator should supply to meet demand, while minimizing cost. Explain your solution, and the intuitive reasoning behind it. Do not spend too much time guessing. Your answer will undoubtedly be suboptimal.

Problem 2: Formulation
Formulate an optimization problem to minimize generating costs subject to supplying sufficient power to meet demand. Provide the following:

(a) Define variables for the reactance, branch capacity, generator capacity, marginal cost, generator power, and bus demand.

(b) What are the optimization variables?

(c) Write down the objective function, using your notation from (a).

(d) Write down ALL the constraints, using your notation from (a). Label the physical meaning of each constraint. For now, ignore the branch capacity constraint. This will be incorporated later.

(e) Is this a linear program (LP), quadratic program (QP), or nonlinear program (NLP)? Why?

Problem 3: LP Solution

(a) Encode this formulation into matrices c, A, b, A_{eq}, b_{eq} corresponding to the canonical LP problem $\min c^T x$, subject to $Ax \leq b$, $A_{eq}x = b_{eq}$. Write down matrices c, A, b, A_{eq}, b_{eq} in your report.

(b) Solve the LP that you have formulated using MATLAB function `linprog` or Python function `optimize.linprog` from the SciPy package. Provide the value of the objective function (total cost) and the value of the optimization variables (generator powers) at the optimum. Describe qualitatively the optimal solution and compare to your guess. Which constraints are active?

(c) Since the problem is four-dimensional, we cannot solve the problem graphically. However, we can project the problem onto two-dimensional planes to gain some insight. On the $P_A - P_B$ plane, sketch (by-hand) the projected iso-contours, constraints, and gradient at the optimum.

Problem 4: Branch Capacity Calculation Next we incorporate the branch capacity constraints. To do this, we must calculate the branch power flows F_{12}, F_{13}, F_{23} shown in Fig. 1. Section 6.2.2 of Kirschen provides guidelines for computing these power flows by hand, using the principle of superposition. However, to incorporate these constraints into an optimization problem, we must ultimately formulate matrix inequality constraints.

Given the assumed flow directions shown in Fig. 1, we can write the power balance equation at each bus as follows:

$$\begin{aligned} \text{Bus 1}: \quad F_{12} + F_{13} &= P_A + P_B - D_1 \qquad (1) \\ \text{Bus 2}: \quad -F_{12} + F_{23} &= P_C - D_2 \qquad (2) \\ \text{Bus 3}: \quad -F_{13} - F_{23} &= P_D - D_3 \qquad (3) \end{aligned}$$

where P_i represents the power generated from generator $i \in \{A, B, C, D\}$, and D_j represents the power demand at bus $j \in \{1, 2, 3\}$. In this case, we get three equations in three unknowns. However, these equations are linearly dependent since power balance (generation equals demand) holds for the entire system. For example, adding Equations (1) and (2) gives (3). Since one of these equations can be eliminated with no loss of information, we are left with two equations and three unknowns. This is hardly surprising because we have not taken into account the impedances (reactances in Table 1, in this case) of the branches. It turns out we can use Kirchoff's Voltage Law to create one more equation

Table 3: Coefficients for Quadratic Generator Cost

Generator, i	$c_{i,1}$	$c_{i,2}$
A	4.5	0.06
B	3	0.01
C	5	0.08
D	4	0.1

$$x_{12}F_{12} + x_{23}F_{23} - x_{13}F_{13} = 0 \qquad (4)$$

(a) Combine (4) with two equations from the set (1)-(3) and the branch capacity limits in Table 1 to formulate an additional linear matrix equality constraint.

(b) Augment your constraint matrices A, b, A_{eq}, b_{eq} from Problem 3 to include the branch capacity constraints. Solve the new LP in Matlab. Provide the value of the objective function (total cost) and the value of the decision variables (generator powers) at the optimum. Describe qualitatively the optimal solution and compare to your guess. Which constraints are active?

(c) On the $P_A - P_B$ plane, sketch by hand the projected iso-contours, constraints, and gradient at the optimum.

(d) Suppose the electric utility decides to invest into improving one of the three branches within the three-bus network. Which branch should they improve, and why? Conversely, which branch (or branches) would yield no reduction in economic dispatch costs if it were improved?

Problem 5: Quadratic Programming Formulation In reality, power generation costs are not linear. A quadratic cost function for generator $i = A, B, C, D$ of the following form is much more widely accepted

$$\textbf{(Power Generation Cost)}_i = c_{i,1}P_i + c_{i,2}P_i^2 \qquad (5)$$

(a) Reformulate your cost function into quadratic form $J = \frac{1}{2}P^T QP + R^T P$, using the coefficients listed in Table 3. Write down matrices Q, R in your report.

(b) Solve this new QP formulation using MATLAB function `quadprog`. Provide the value of the objective function (total cost) and the value of the decision variables (generator powers) at the optimum. Describe qualitatively the optimal solution. Which constraints are active?

(c) On the $P_A - P_B$ plane, sketch by hand the projected iso-contours, constraints, and gradient at the optimum.

Problem 6: Additional Analysis

(a) For this part, suppose the capacity of branch 2-3 is increased from 90 MW to 150 MW. Resolve the QP problem. Provide the new total cost and generator powers. Describe qualitatively the optimal solution and what has changed.

(b) In reality, the generator costs are typically piecewise. For this part, suppose that generator B has a cost function given by (6). Describe qualitatively how you would solve this ED problem. Be detailed.

$$\textbf{(Power Generation Cost)}_B = \begin{cases} 4P_B & \text{if } P_B \leq 50\text{MW} \\ 25 + 3P_B + 0.01P_B^2 & \text{if } P_B > 50\text{MW}. \end{cases} \tag{6}$$

Deliverables

Submit the following on bCourses. Be sure that the function files are named exactly as specified (including spelling and case), and make sure to zip all files.

`LASTNAME_FIRSTNAME_HW4.PDF`
`LASTNAME_FIRSTNAME_HW4.zip` which contains your respective Matlab or Python files.