

# Modeling and Estimation of Demand Responsive Loads

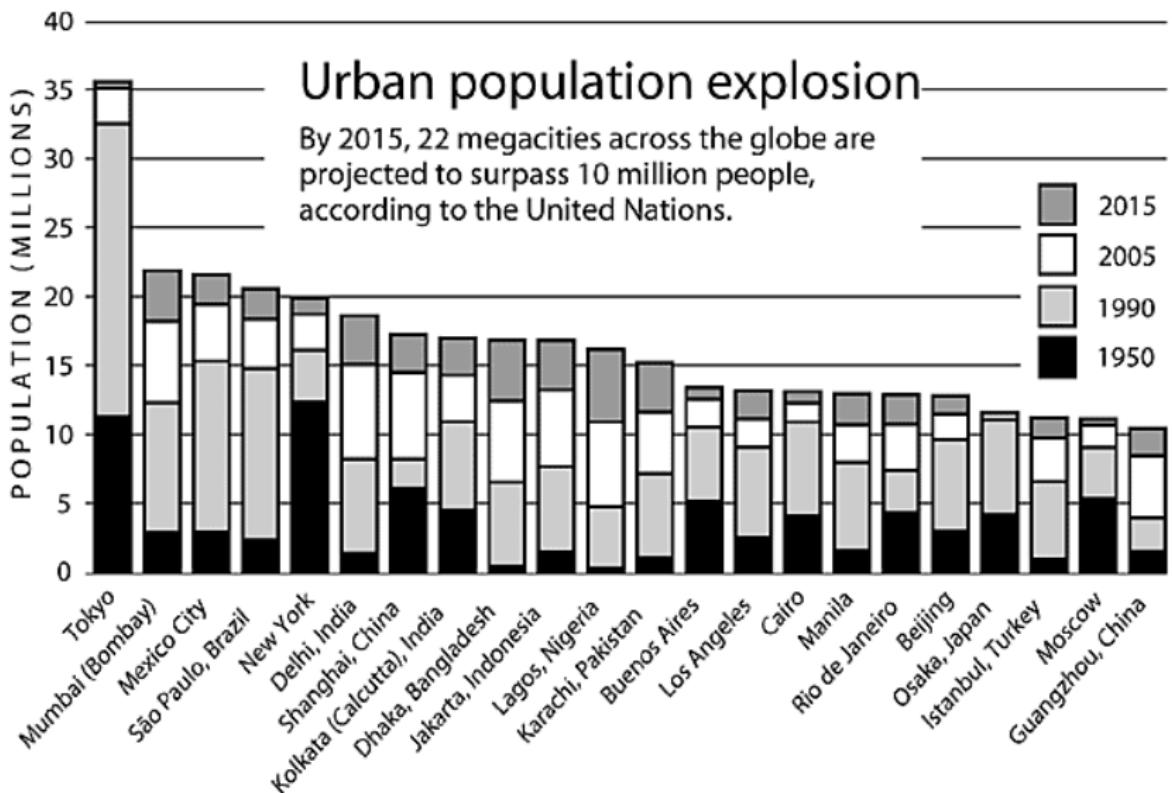
Scott Moura

Assistant Professor  
Civil & Environmental Engineering  
University of California, Berkeley

International Workshop on Smart City:  
Control and Automation Perspectives

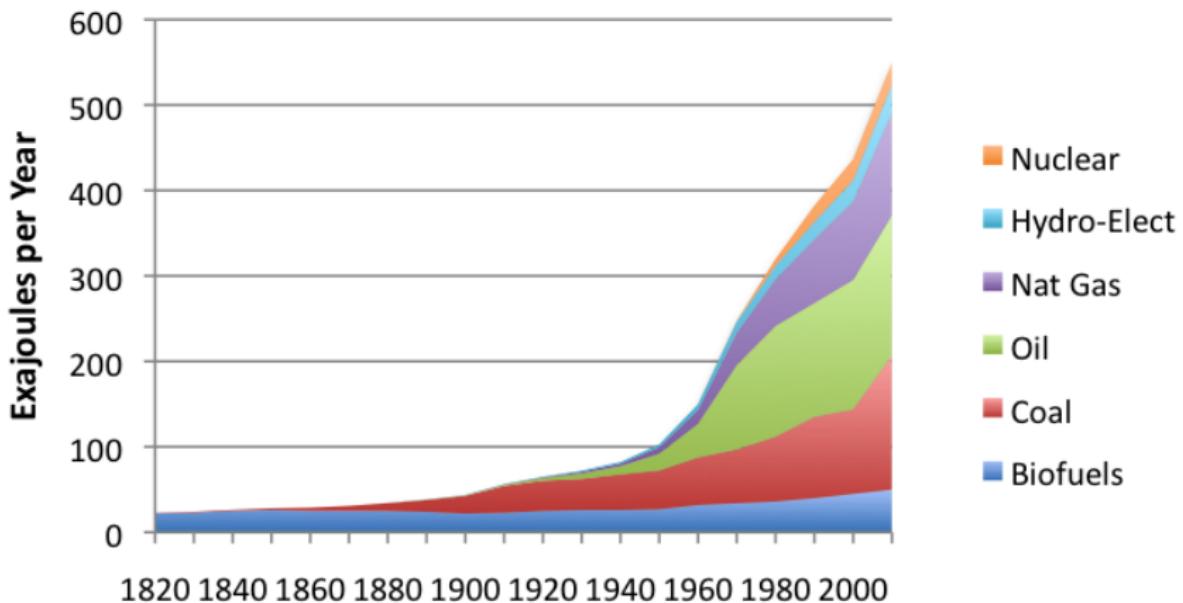
Hangzhou, China  
August 28, 2013





Source: United Nations, DESA, Population Division. World Urbanization Prospects.

## World Energy Consumption



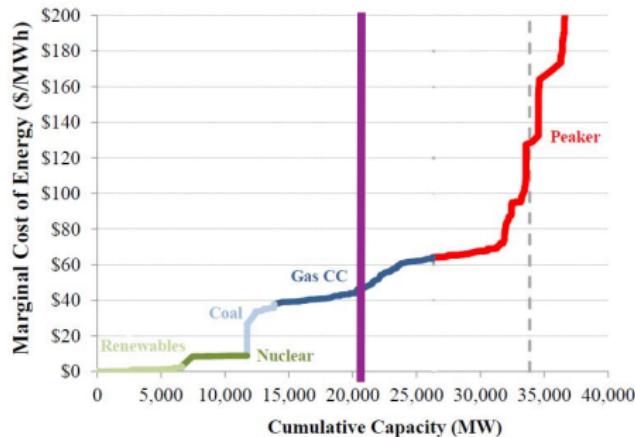
Source: Vaclav Smil Estimates from Energy Transitions

# Integrating Renewables

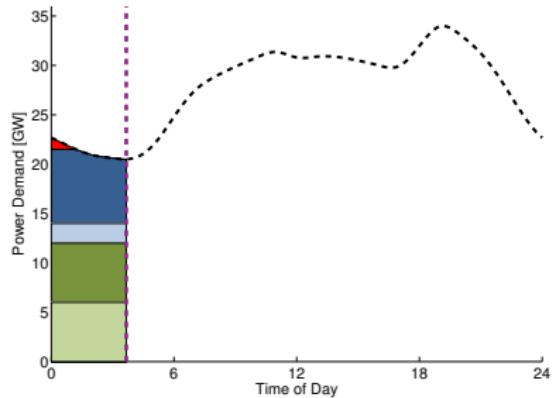
Ambitious targets:

- CA: 33% renewables by 2020
- US: 20% wind penetration by 2030
- Denmark: 50% wind penetration by 2025
- China: 15% renewables by 2020

# Power Market Supply/Demand

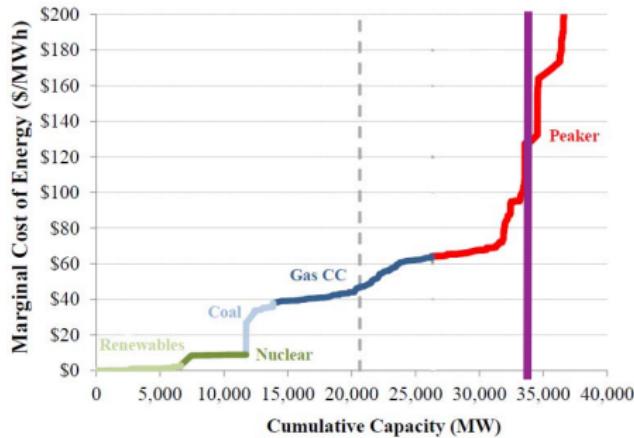


Supply

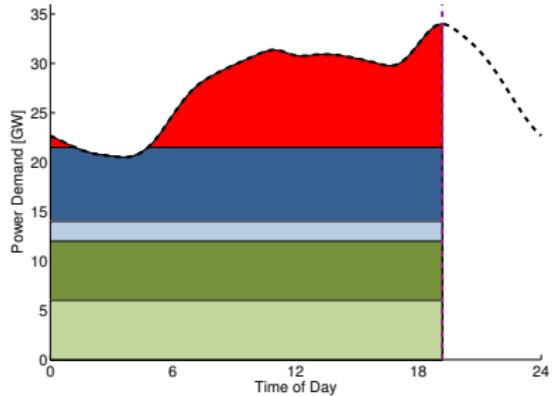


Demand

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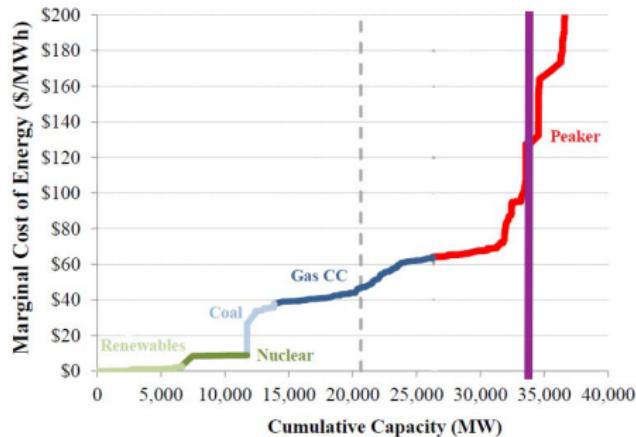
Supply



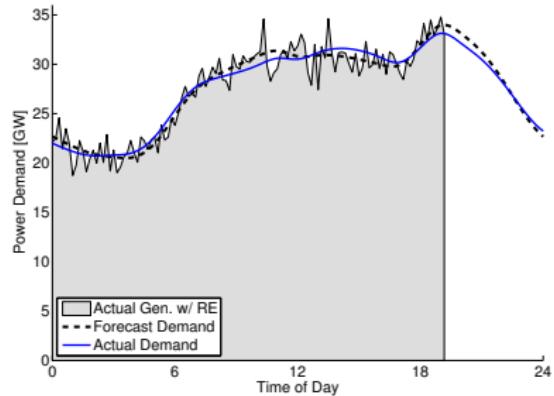
Demand

# Power Market Supply/Demand

Reality: Renewables are variable!



Supply



Demand

# The Cost of Renewables w/o doing something “SMART”

Increased variability is THE problem!

Assuming 33% RE penetration, and current practices,  
the following reserve capacity must be added [Helman 2010]:

- Load following: 2.3 GW → 4.4 GW
- Regulation: 227 MW → 1.4 GW

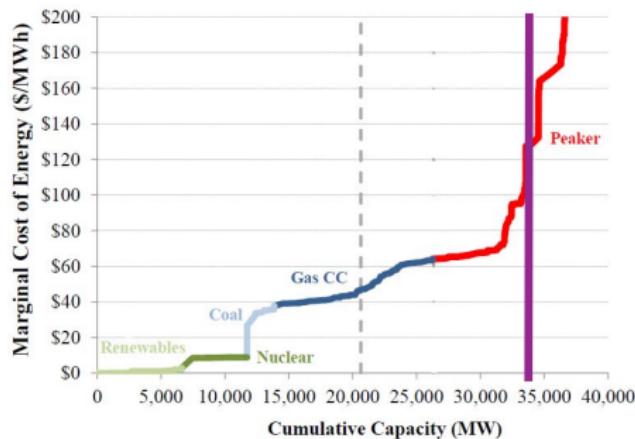
Added costs due to reserves at 15% renewable penetration

- \$2.50 - \$5 per MW of renewable generation [EWITS study, NREL, 2010]

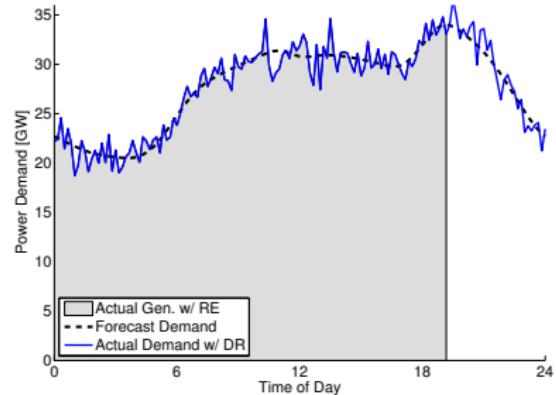
Additional dirty and expensive reserves defeats benefits!

# Power Market Supply/SMART Demand

**Goal:** Actual Demand with Demand Response (DR)



Supply



Demand

# Outline

## 1 Modeling and Estimation for DR Loads

- Modeling Aggregations via PDEs
- State Estimation
- Parameter Identification
- UC Campus Implementation

## 2 PEV-Grid Integration

## 3 Summary

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**Needs:** Monitor & actuate (very) large populations of flexible loads

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Systems controlled by on-off actuation, e.g. HVAC, water heaters, freezers

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## Some Interesting Facts

Thermostatically  
Controlled Loads  
(TCLs)

50% of U.S. electricity consumption is TCLs  
11% of thermostats are programmed  
Comfort is loosely coupled with control

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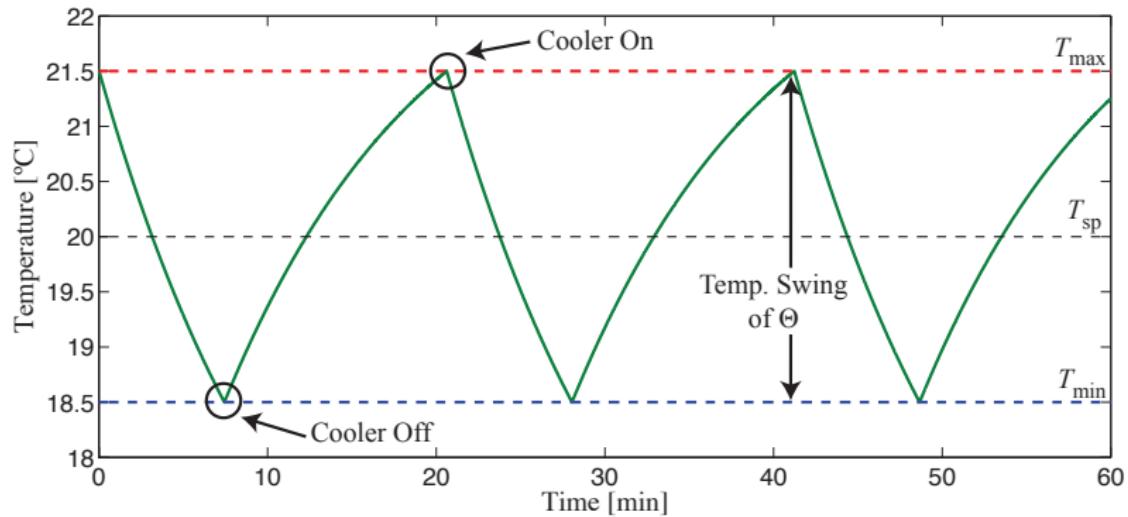
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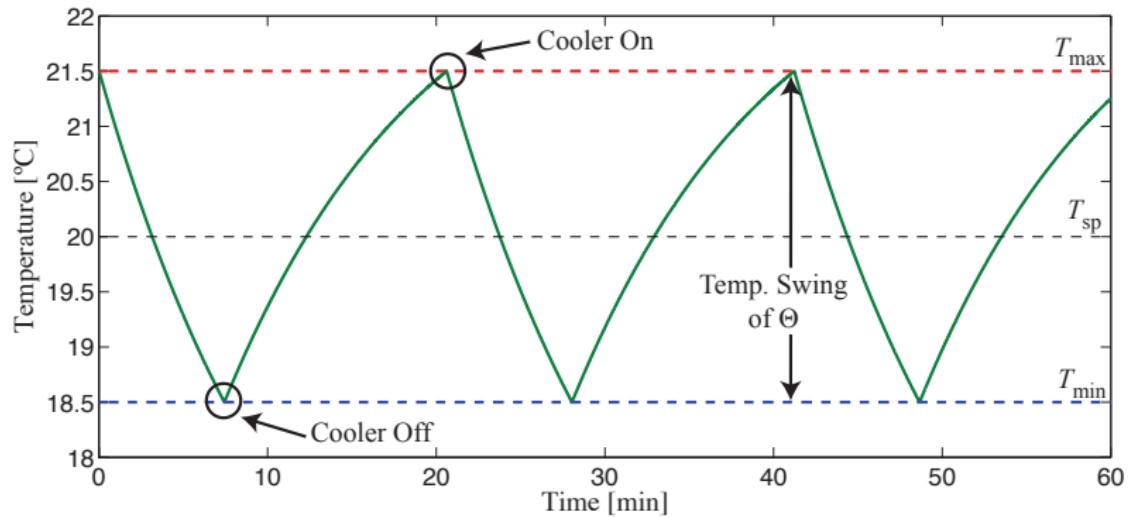
## The Punchline

Exploit flexibility of TCLs to decrease energy waste

# Modeling TCLs



# Modeling TCLs



$$\dot{T}_i(t) = \frac{1}{R_i C_i} [T_\infty - T_i(t) - s_i(t) R_i P_i], \quad i = 1, 2, \dots, N$$
$$s_i \in \{0, 1\}$$

# Modeling Aggregated TCLs

**Main Idea:** Convert 1000+ ODEs into two coupled linear PDEs

# Modeling Aggregated TCLs

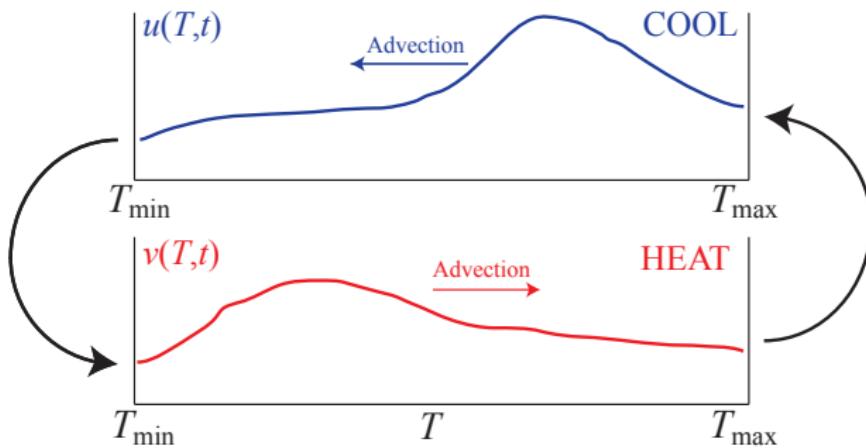
**Main Idea:** Convert 1000+ ODEs into two coupled linear PDEs

$$\begin{array}{l|l} u(T, t) & \# \text{TCLs / } ^\circ\text{C, in COOL state, @ temp } T, \text{ time } t \\ v(T, t) & \# \text{TCLs / } ^\circ\text{C, in HEAT state, @ temp } T, \text{ time } t \end{array}$$

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**Flux of TCLs in HEAT state:**

#TCLs / sec

$$\psi(T, t) = v(T, t) \frac{dT}{dt}(t) = \frac{1}{RC} [T_\infty - T(t)] v(T, t)$$

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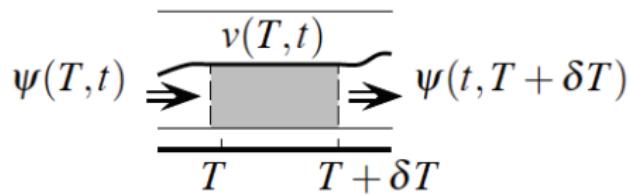
$u(T, t)$	# TCLs / °C, in COOL state, @ temp $T$ , time $t$
$v(T, t)$	# TCLs / °C, in HEAT state, @ temp $T$ , time $t$

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**Control volume:**



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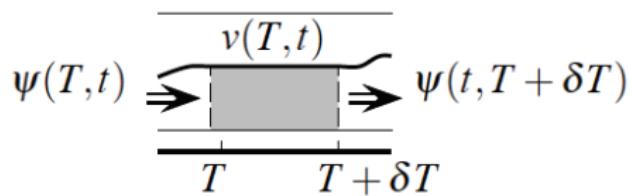
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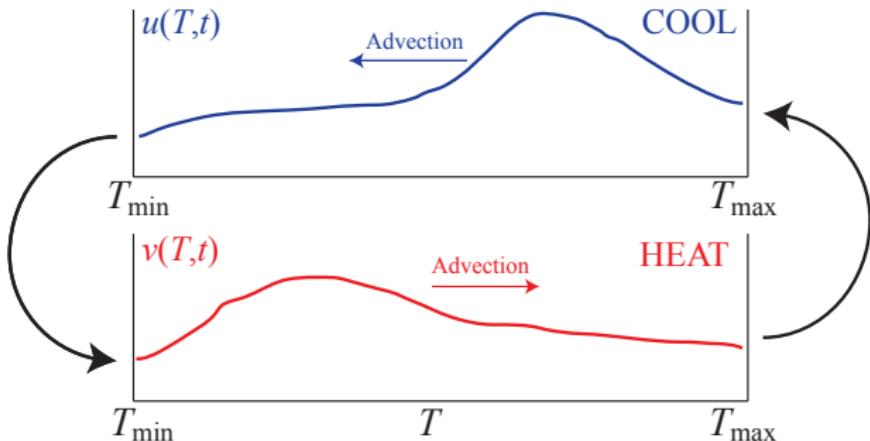
$$\psi(T, t) = v(T, t) \frac{dT}{dt}(t) = \frac{1}{RC} [T_\infty - T(t)] v(T, t)$$

**Control volume:**

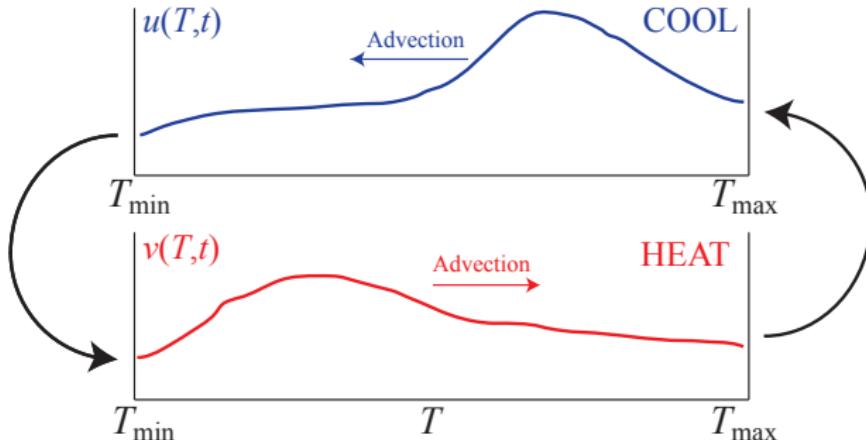


$$\begin{aligned} \frac{\partial v}{\partial t}(T, t) &= \lim_{\delta T \rightarrow 0} \left[ \frac{\psi(T + \delta T, t) - \psi(T, t)}{\delta T} \right] \\ &= \frac{\partial \psi}{\partial T}(T, t) \\ &= -\frac{1}{RC} [T_\infty - T(t)] \frac{\partial v}{\partial T}(T, t) + \frac{1}{RC} v(T, t) \end{aligned}$$

# PDE Model of Aggregated TCLs



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$$u_t(T, t) = \alpha \lambda(T) u_T(T, t) + \alpha u(T, t)$$

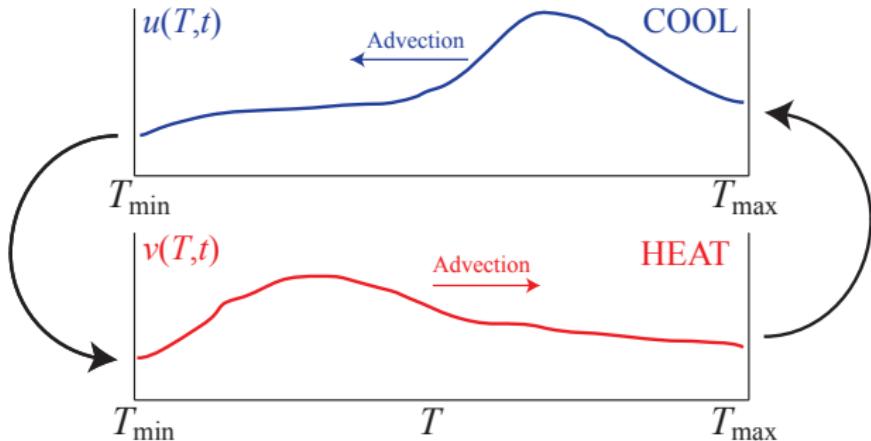
$$v_t(T, t) = -\alpha \mu(T) v_T(T, t) + \alpha v(T, t)$$

$$u(T_{\max}, t) = q_1 v(T_{\max}, t)$$

$$v(T_{\min}, t) = q_2 u(T_{\min}, t)$$

Video of 1,000 TCLs

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**Original Idea:** Malhame and Chong, Trans. on Automatic Control (1985)  
**Remark:** Assumes homogeneous populations

# Modeling Heterogeneous Aggregated TCLs

**Reality:** TCL populations are heterogeneous

e.g. variable heat capacity, power, deadband sizes

Video of 1,000 heterogeneous TCLs

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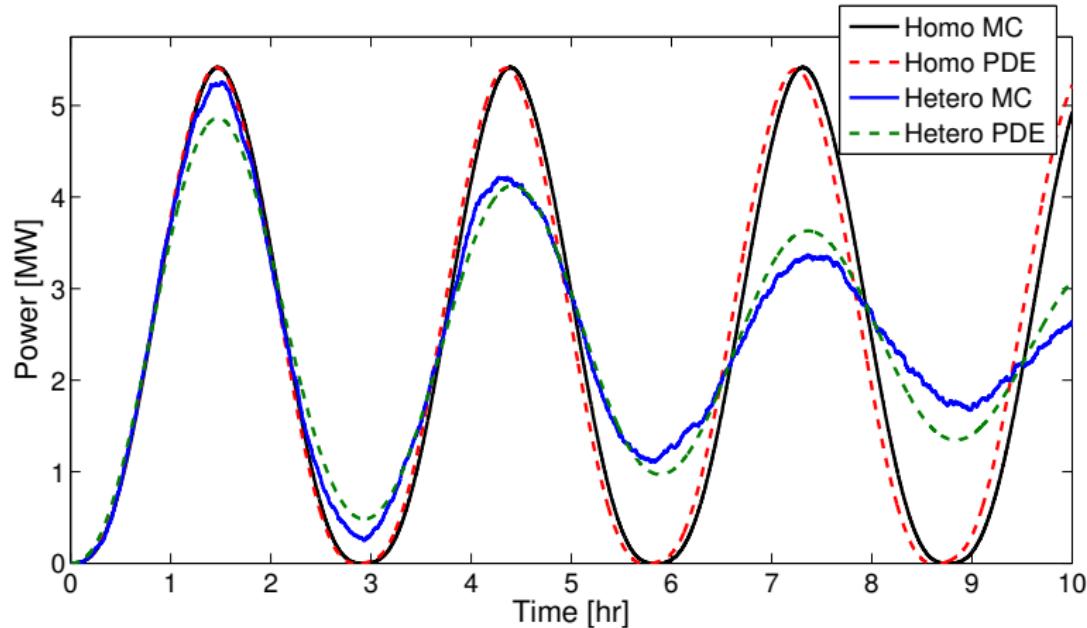
**Proposition:** The total number of TCLs is conserved over time.

$$Q(t) = \int_{T_{\min}}^{T_{\max}} u(T, t) dT + \int_{T_{\min}}^{T_{\max}} v(T, t) dT$$

$$\frac{dQ}{dt}(t) = 0, \quad \forall t$$

# Video Evolution of Heterogeneous PDE

# Model Comparison



# The State Estimation Problem

**Question:** Possible to monitor TCLs with minimal sensing infrastructure?

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# The State Estimation Problem

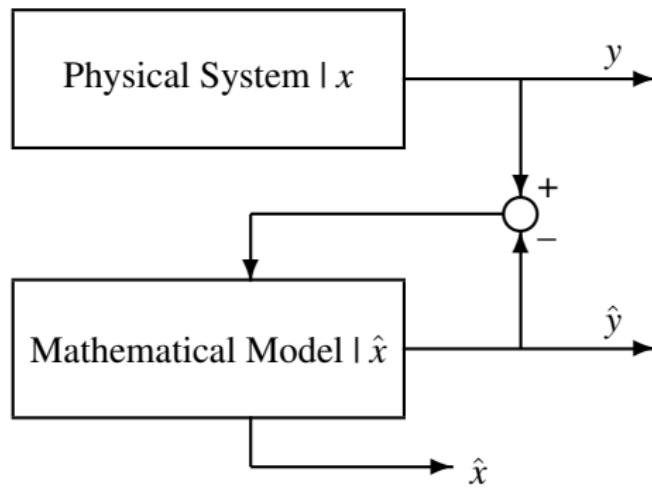
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## Problem Statement

Estimate states  $u(T, t)$ ,  $v(T, t)$  from measurements of HVAC on/off signals

Finite-dimensional system:



# The State Estimation Problem

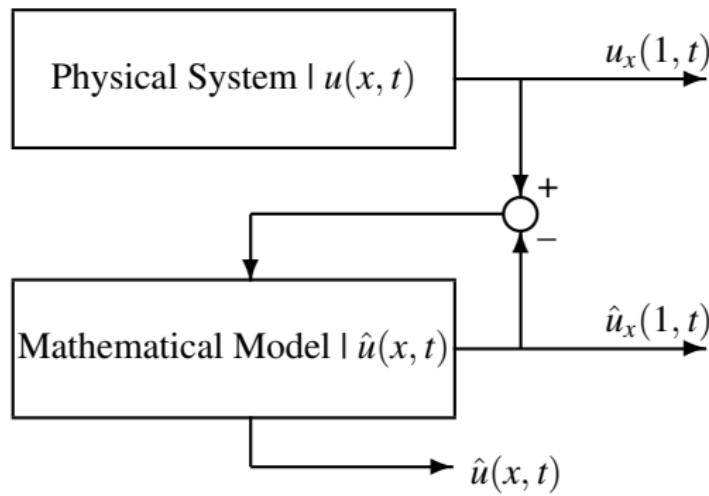
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# PDE State Estimator

## Heterogeneous PDE Model: $(u, v)$

$$u_t(x, t) = \alpha\lambda(x)u_x + \alpha u + \beta u_{xx}$$

$$u_x(0, t) = -v_x(0, t)$$

$$u(1, t) = q_1 v(1, t)$$

$$v_t(x, t) = -\alpha\mu(x)v_x + \alpha v + \beta v_{xx}$$

$$v(0, t) = q_2 u(0, t)$$

$$v_x(1, t) = -u_x(1, t)$$

## Measurements?

- $u(0, t), v(1, t)$
- $u_x(1, t), v_x(0, t)$

# PDE State Estimator

Estimator:  $(\hat{u}, \hat{v})$

$$\hat{u}_t(x, t) = \alpha\lambda(x)\hat{u}_x + \alpha\hat{u} + \beta\hat{u}_{xx} + p_1(x) [u(0, t) - \hat{u}(0, t)]$$

$$\hat{u}_x(0, t) = -v_x(0, t) + p_{10} [u(0, t) - \hat{u}(0, t)]$$

$$\hat{u}(1, t) = q_1 v(1, t)$$

$$\hat{v}_t(x, t) = -\alpha\mu(x)\hat{v}_x + \alpha\hat{v} + \beta\hat{v}_{xx} + p_2(x) [v(1, t) - \hat{v}(1, t)]$$

$$\hat{v}(0, t) = q_2 u(0, t)$$

$$\hat{v}_x(1, t) = -u_x(1, t) + p_{20} [v(1, t) - \hat{v}(1, t)]$$

# PDE State Estimator

Estimation Error Dynamics:  $(\tilde{u}, \tilde{v}) = (u - \hat{u}, v - \hat{v})$

$$\tilde{u}_t(x, t) = \alpha\lambda(x)\tilde{u}_x + \alpha\tilde{u} + \beta\tilde{u}_{xx} - \textcolor{magenta}{p_1}(x)\tilde{u}(0, t)$$

$$\tilde{u}_x(0, t) = -\textcolor{magenta}{p_{10}}\tilde{u}(0, t)$$

$$\tilde{u}(1, t) = 0$$

$$\tilde{v}_t(x, t) = -\alpha\mu(x)\tilde{v}_x + \alpha\tilde{v} + \beta\tilde{v}_{xx} - \textcolor{magenta}{p_2}(x)\tilde{v}(1, t)$$

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**Goal:** Design estimation gains:

- $p_1(x), p_2(x) : (0, 1) \rightarrow \mathbb{R}$
- $p_{10}, p_{20} \in \mathbb{R}$

such that  $(\tilde{u}, \tilde{v}) = (0, 0)$  is exponentially stable

# Backstepping Observer Design

Error Dynamics:  $\tilde{u}$

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Eliminate advection terms, “Gauge” Transformation

$$\xi(x, t) = \tilde{u}(x, t)e^{\frac{\alpha}{2\beta} \int_0^x \lambda(s)ds}$$

# Backstepping Observer Design

Transformed error state:  $\xi$

$$\begin{aligned}\xi_t(x, t) &= \beta \xi_{xx} + g(x)\xi - p_1^\xi(x)\xi(0, t) \\ \xi_x(0, t) &= p_{10}^\xi \xi(0, t) \\ \xi(1, t) &= 0\end{aligned}$$

$$\begin{aligned}g(x) &= \alpha \left[ 1 - \frac{\lambda'(x)}{2} \right] - \frac{\alpha^2 \lambda^2(x)}{4\beta} \\ p_1^\xi(x) &= p_1(x) e^{\frac{\alpha}{2\beta} \int_0^x \lambda(s) ds} \\ p_{10}^\xi &= \frac{\alpha}{2\beta} \lambda(0) - p_{10}\end{aligned}$$

# Backstepping Observer Design

Transformed error state:  $\xi$

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Backstepping transformation

$$\xi(x, t) = w_1(x, t) - \int_0^x p(x, y)w_1(y, t)dy$$

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Backstepping transformation

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Target system:  $w_1$ , exp. stable in  $\mathcal{L}^2$ -norm

$$\begin{aligned}w_{1t}(x, t) &= \beta w_{1xx}(x, t) - c_1 w_1(x, t), \quad c_1 \geq 0 \\ w_{1x}(0, t) &= w_1(0, t) \\ w_1(1, t) &= 0\end{aligned}$$

# Backstepping Observer Design

Kernel PDE:  $p(x, y)$

$$\begin{aligned}\beta p_{xx}(x, y) - \beta p_{yy}(x, y) &= -[c_1 + g(x)] p(x, y) \\ p(x, x) &= -\frac{1}{2\beta} \int_x^1 [c_1 + g(s)] ds \\ p(1, y) &= 0\end{aligned}$$

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Estimation gains

$$\begin{aligned}p_1^\xi(x) &= -\beta [p(x, 0) + p_y(x, 0)] \\ p_{10}^\xi &= 1 - p(0, 0) \\ p_1(x) &= p_1^\xi(x) e^{-\frac{\alpha}{2\beta} \int_0^x \lambda(s) ds} \\ p_{10} &= \frac{\alpha}{2\beta} \lambda(0) - p_{10}^\xi\end{aligned}$$

# Video Evolution of PDE estimator

**Key point:** Converges to true distribution, using only HVAC on/off signals.

# Parameter Identification

$$u_t(x, t) = \alpha\lambda(x)u_x + \alpha u + \beta u_{xx}$$

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## Algorithm #1: Power-based Identification [DSCC13]

- Assumes measurements of aggregate power consumption & B.C.'s

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## Algorithm #1: Power-based Identification [DSCC13]

- Assumes measurements of aggregate power consumption & B.C.'s

## Algorithm #2: Passive Identifier [IJC]

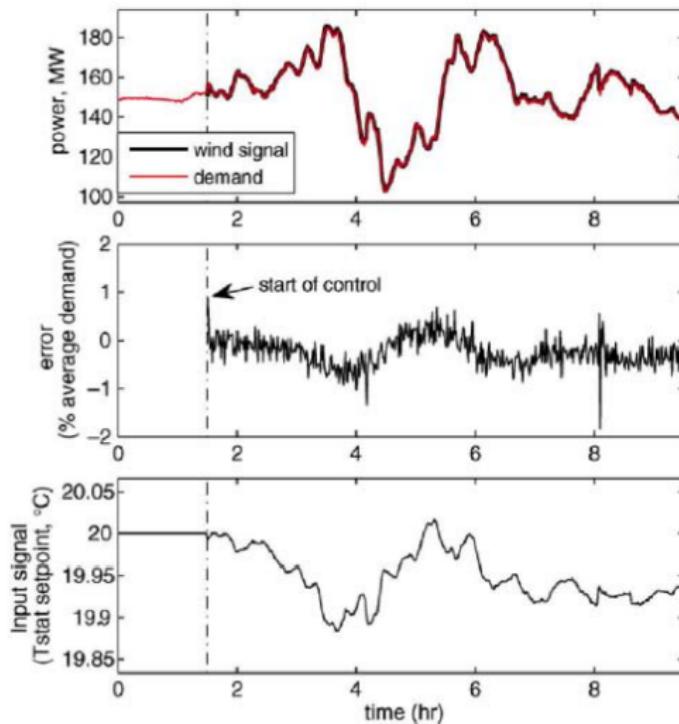
- Assumes full state measurements

## Algorithm #3: Swapping Identifier [IJC]

- Assumes full state measurements

# Output Regulation Problem

[Callaway, 2009]



- $P(t) \approx w(t)$
- Tracking error  $\approx 1\%$
- Setpoint changes  $\approx 0.1^\circ C$

# UC San Diego Campus: A Living Laboratory



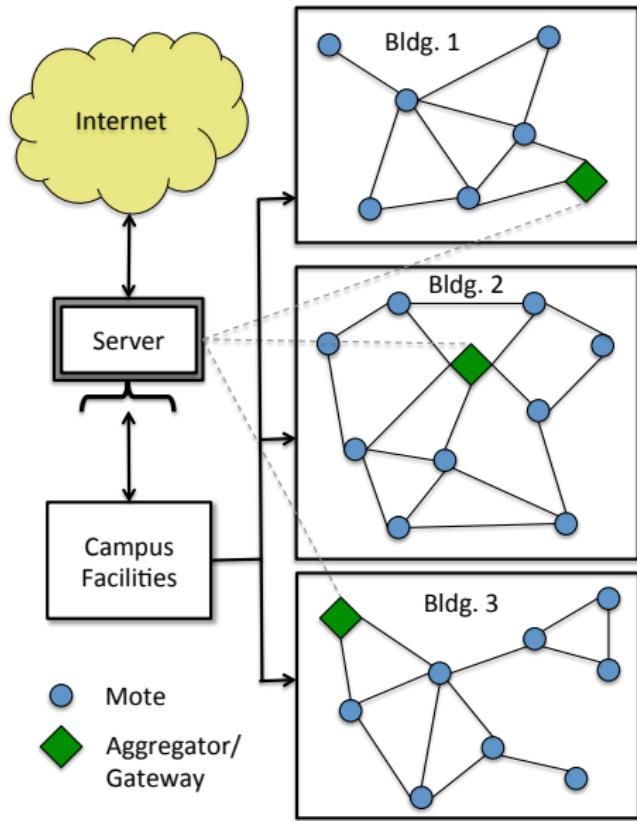
# UC San Diego Campus: A Living Laboratory

**Goal:** Exploit Flexible Loads in Bldgs

- ① Deploy wireless sensor network
- ② Model/estimator verification
- ③ Control design
- ④ Campus implementation

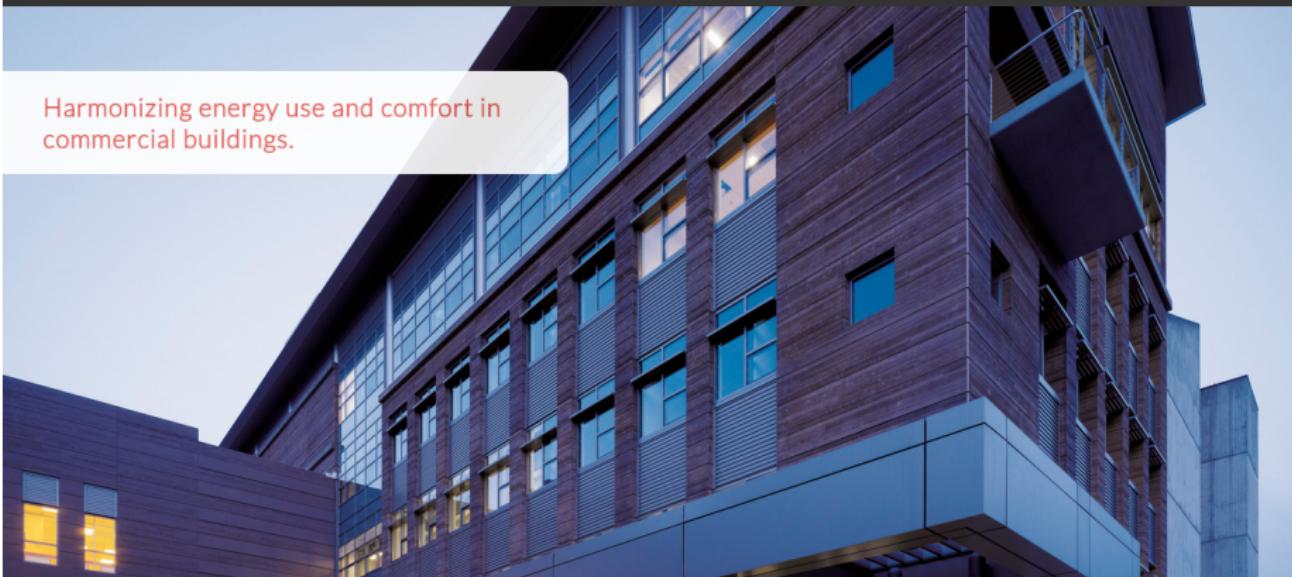


Libelium Waspmotes and Meshlium Gateway



# BUILDING ROBOTICS

Harmonizing energy use and comfort in commercial buildings.



Software platform for collecting building energy data  
Prof. David Culler's Group | EECS @ UC Berkeley

## Campus Map

Click a highlighted building to view energy usage information.

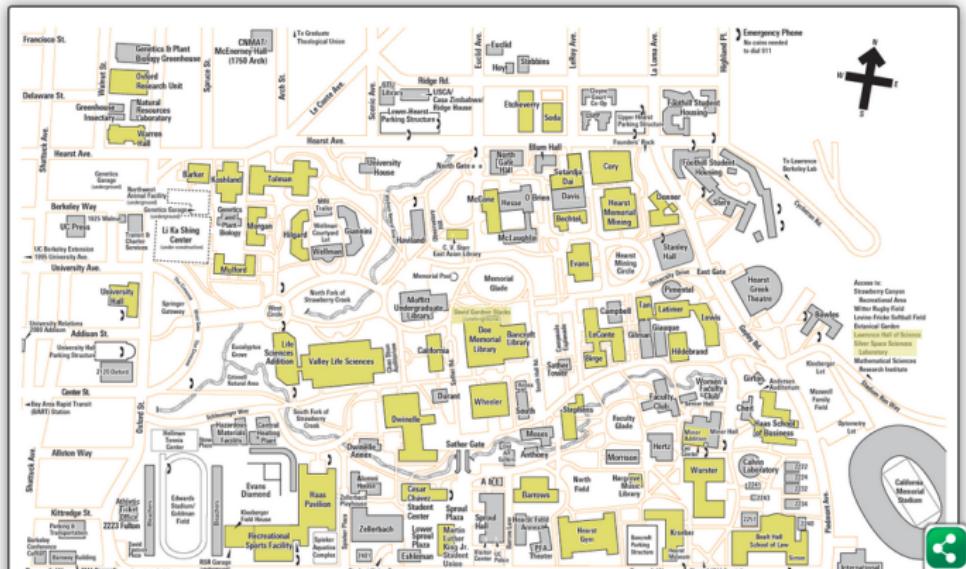
**Zoom In**

[Zoom out](#)

Reset

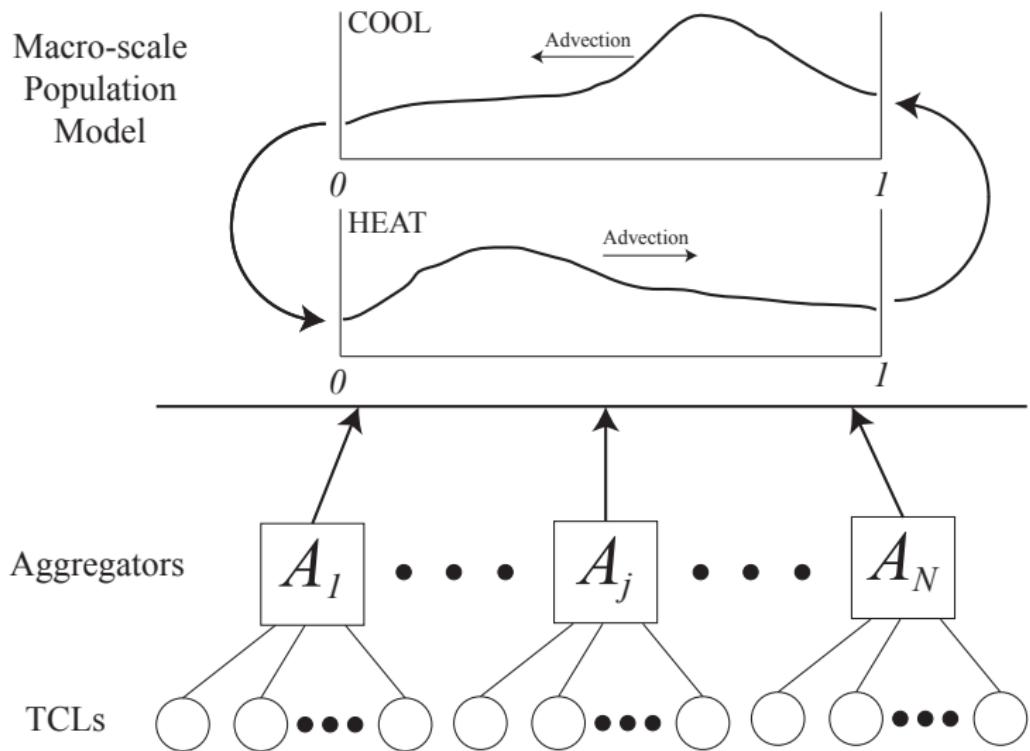
Popup Building List

[Close Building List](#)



Software platform for collecting building energy data  
Prof. David Culler's Group | EECS @ UC Berkeley

# Hierarchical Control/Monitoring Structure



# Outline

## 1 Modeling and Estimation for DR Loads

- Modeling Aggregations via PDEs
- State Estimation
- Parameter Identification
- UC Campus Implementation

## 2 PEV-Grid Integration

## 3 Summary

# The PEV-Grid Integration Problem

**Obstacle:** PEVs impose unprecedented constraints and demands on grid

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## Some Interesting Facts

Plug-in Electric  
Vehicles  
(PEVs)

Potentially dispatchable loads  
“carbitrage” opportunity  
Firm variable renewables

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## The Punchline

Exploit flexibility of PEV charging to enhance efficiency across infrastructures

# Modeling Aggregated PEVs

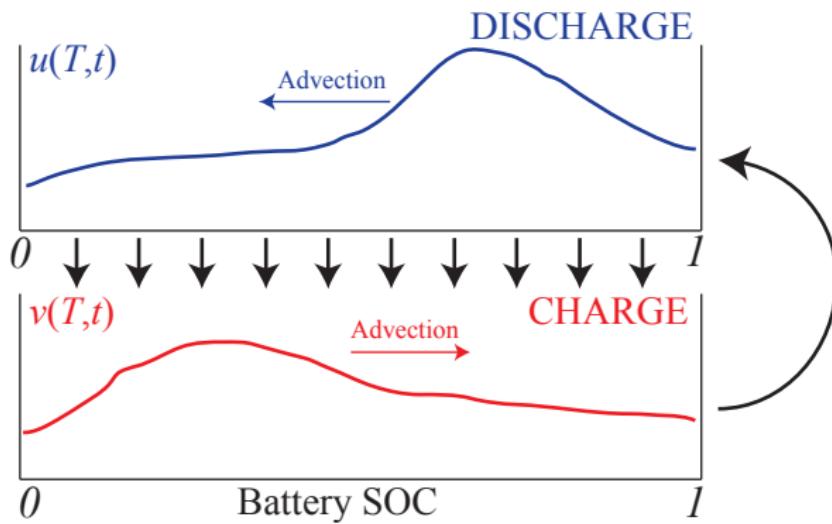
**Main Idea:** Mathematically model as coupled linear PDEs

$$\begin{array}{l|l} u(T, t) & \# \text{PEVs / SOC, in DISCHARGE state, @ SOC } x, \text{ time } t \\ v(T, t) & \# \text{PEVs / SOC, in CHARGE state, @ SOC } x, \text{ time } t \end{array}$$

# Modeling Aggregated PEVs

**Main Idea:** Mathematically model as coupled linear PDEs

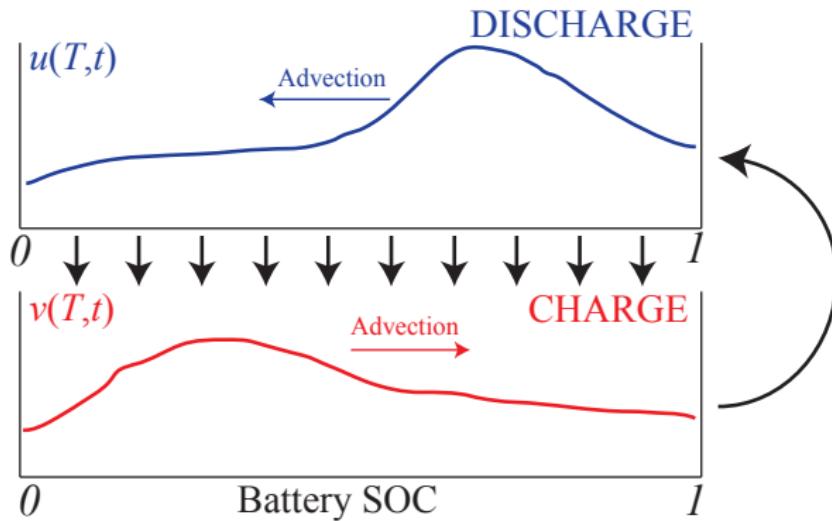
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**Open Questions:** Modeling, state estimation, control/optimization, implementation

# When to charge PEVs?

**Research Question:**

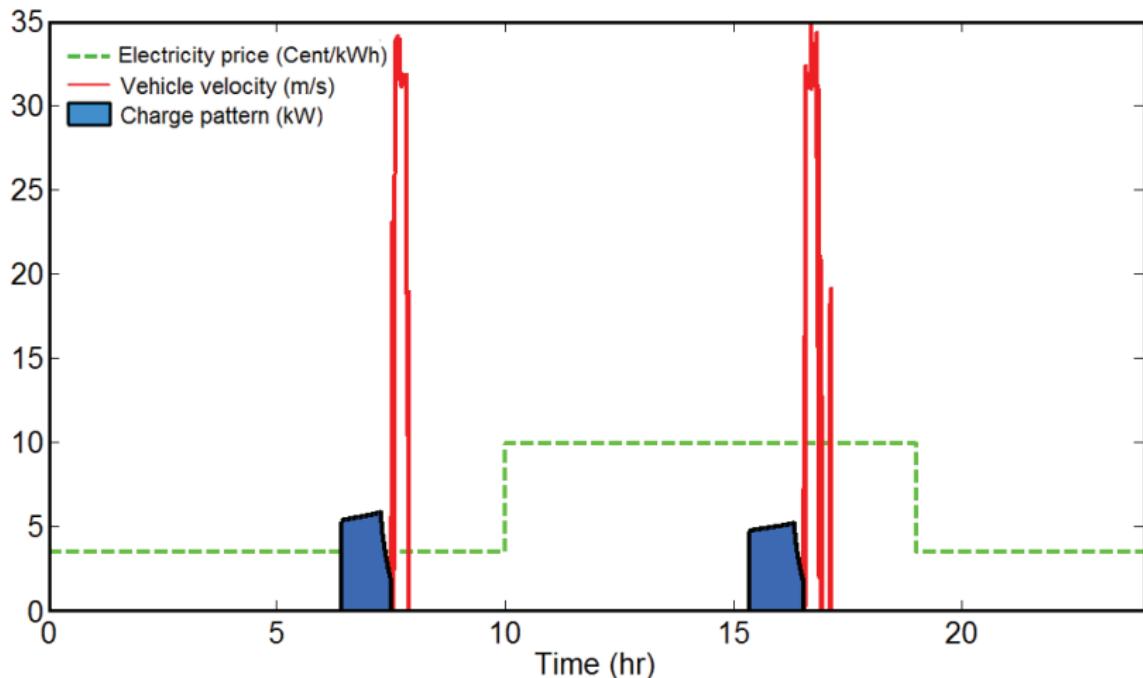
When to charge PEV's to maximize consumer-side benefits?

**Objectives:**

Fuel & electricity cost, battery health

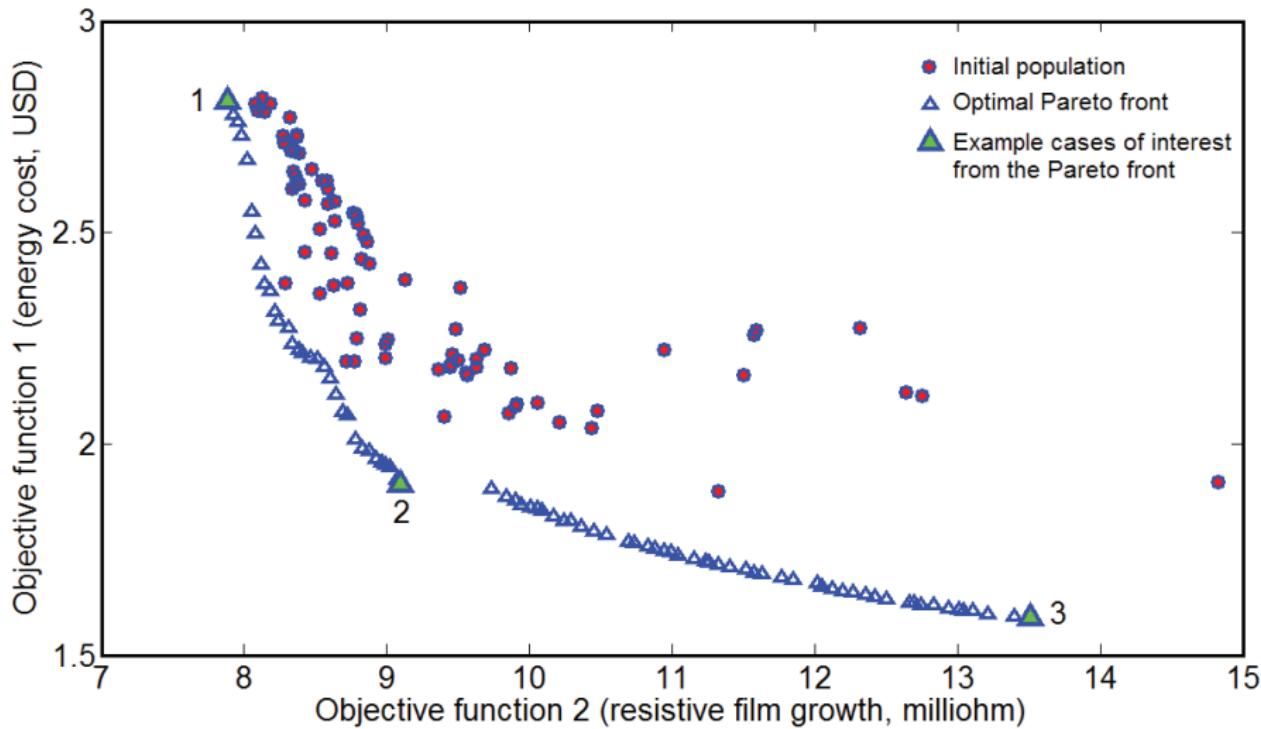
**Decision variables:**

When, how long, at what rate?



# Multiobjective Optimization

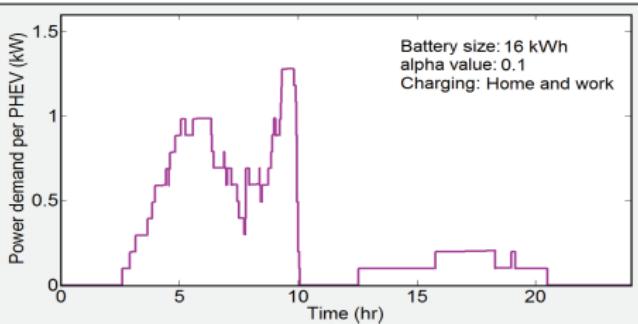
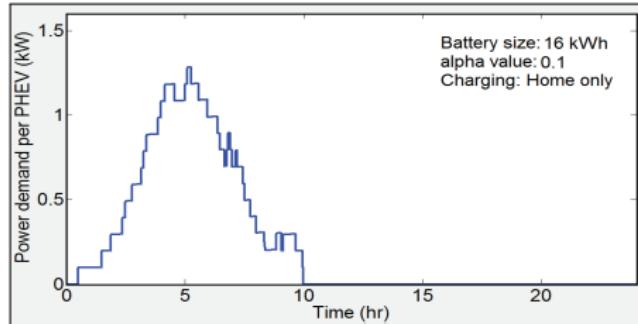
Energy Cost vs. Battery Health



# Infrastructure Req's?

**Research Question:**  
**Sensitivity to:**  
**Underlying Data:**

Does charging at work lower peak loads?  
Fuel/electricity price, battery size, health vs. energy  
NHTS Survey, UMTRI driving patterns



# Outline

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# Summary of TCL Modeling & Estimation

## ① Modeling

- Aggregate dynamics via PDEs
- Homogeneous vs. Heterogeneous

## ② Estimation

- State observer via PDE backstepping
- Parameter identification algorithms

## ③ UCSD Campus Deployment

## ④ PEV-Grid Integration

- Extension of aggregation approach

# Collaborators



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Xie Xie!

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