Motions of Baseballs and Footballs

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Professor O'Reilly
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Introduction

The motion of baseballs and footballs is an age-old problem, contemplated by the scholars of today and yesterday. In 17th century England, Sir Issac Newton investigated the curved motion of tennis balls. Two hundred years later, Lord Rayleigh discussed the motions of spinning balls while P.G. Tait extensively analyzed the paths of golf balls.[1] Today, Pedro Martinez of the Boston Redsox and David Beckham of the English national football team utilize this phenomenon on a daily basis in their efforts to help their teams win a world championship.

Magnus Force

Consider a sphere traveling at a velocity *V* through an oncoming flow as shown in Fig 1a. If the fluid is viscous, the sphere will experience a drag force. Next consider a counter-clockwise spinning sphere traveling at a velocity *V* through an oncoming flow as shown in Fig 1b. Once again, the sphere will experience a drag force if the fluid is viscous, however a pressure difference will emerge as well, resulting in a force normal to the velocity of the sphere. This force is the cause for the curved motion of a baseball or football and is known as the Magnus force.[2]

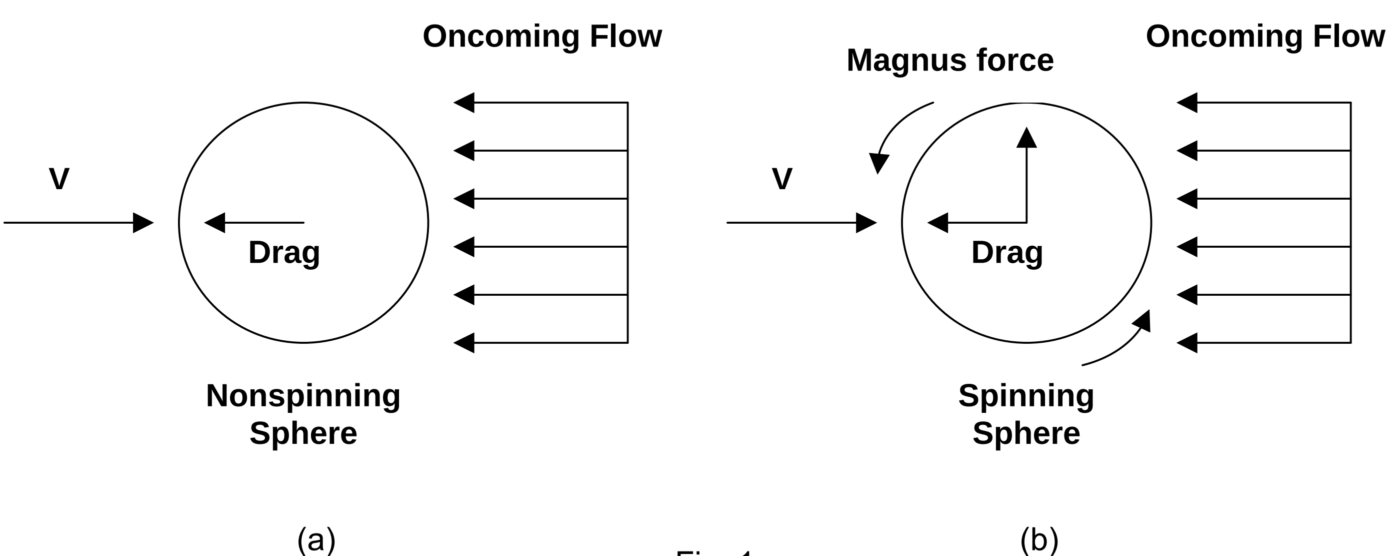

Fig. 1

[1] Adair, Robert K. The Physics of Baseball. New York: Collins Publishers Inc., 2002. pp 12-13.
[2] Credited to the German scientist Heinrich Gustav Magnus (1802 – 1870)

Fluid Dynamics – Magnus Force

Why does a pressure difference emerge when the sphere spins? A short digression into fluid dynamics will help answer this question. Again, consider a sphere with no spin through an oncoming flow of air in Fig. 2a. The 'no-slip condition' requires that the velocity of the flow at the surface of an object is equal to the velocity of that point on the object.[3] Due to the symmetry of the flow, the sphere experiences equal velocities and therefore equal pressures above and below the sphere. The result is no net force. Now consider the counter-clockwise spinning sphere in the oncoming flow in Fig. 2b. Above, the sphere is spinning in the direction of the flow, adding to generate a higher velocity due to the 'no-slip condition'. Below, the sphere is opposing the direction of the flow to generate a lower velocity. According to Bernoulli's principle of energy conservation, a higher velocity results in a lower pressure and a lower velocity results in a higher pressure.[4] The result is a net pressure difference across the sphere, resulting in a force normal to the direction of the sphere's velocity.

(a)

(b)

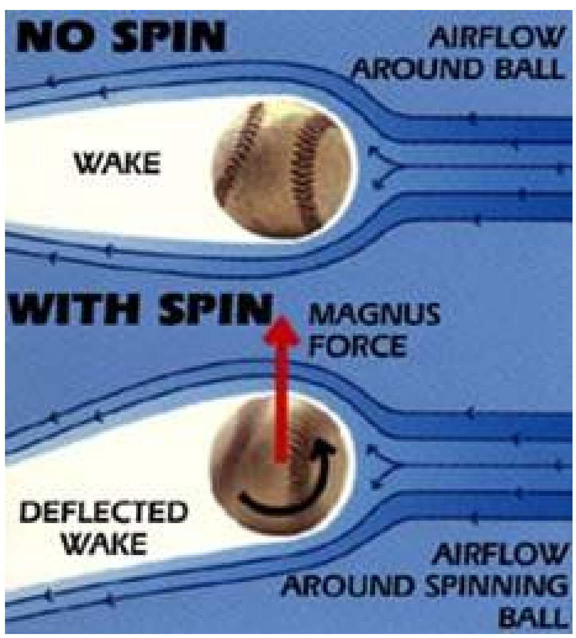

Fig. 2[5]

[3] Munson, Bruce R., Donald F. Young, and Theodore H. Okiishi. Fundamentals of Fluid Dynamics. 4th Edition. Hoboken: John Wiley & Son, 2002. pp 16.
[4] Briggs, Lyman J. "Effect of Spin and Speed on the Lateral Deflection (Curve) of a Baseball; and the Magnus Effect for Smooth Spheres". Armenti Jr., Angleo. The Physics of Sports:.Volume One. New York: American Institute of Physics, 1992.
[5] Peter J. Brancazio. "The Mechanics Of A Breaking Pitch." Popular Mechanics. Dec 7, 2004. <http://popularmechanics.com/science/sports/1997/4/breaking_pitch/index4.phtml>.

To analytically show this, we shall consider a ball of radius R spinning counter-clockwise with angular velocity ω through an oncoming flow of speed V as shown in Fig. 3.

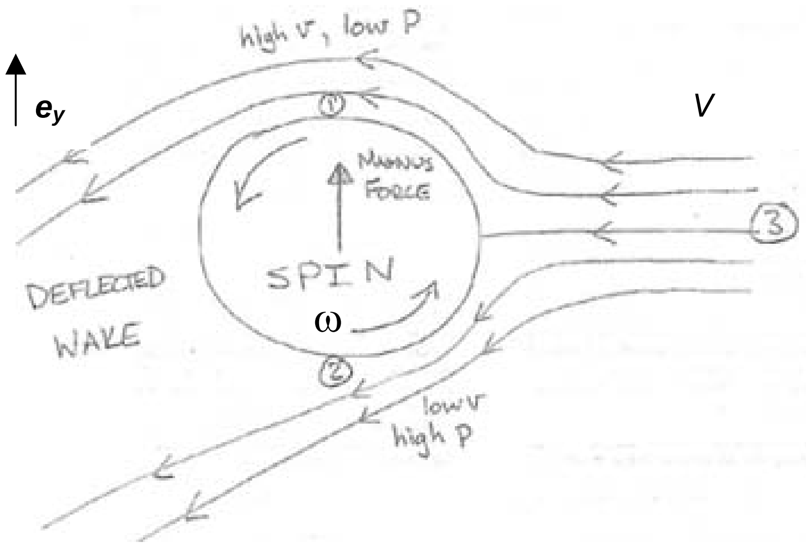

Fig. 3

The velocities at points 1 and 2 are given by:

$$v_1 = V + \omega R \qquad v_2 = V - \omega R \tag{1}$$

We can then apply Bernoulli's Equation to points 1 and 2 and equate them to point 3, since the streamlines are parallel

$$\frac{P_1}{\rho} + \frac{{v_1}^2}{2} + gz_1 = \frac{P_3}{\rho} + \frac{{v_3}^2}{2} + gz_3 = \frac{P_2}{\rho} + \frac{{v_2}^2}{2} + gz_2 \tag{2}$$

Neglecting potential energies and solving for the pressure difference we get

$$P_2 - P_1 = \rho\left(\frac{{v_1}^2 - {v_2}^2}{2}\right) \tag{3}$$

Then inserting (1) into (3) we obtain

$$P_2 - P_1 = 2\rho V \omega R \tag{4}$$

To calculate the force each pressure creates, we can integrate over the top and bottom hemispheres and dot with $\boldsymbol{e_y}$:

$$F = \int_A P\,\boldsymbol{dA} \cdot e_y = 2\pi R^2 (P_2 - P_1) \tag{5}$$

Inserting equation (4) into (5) gives and expression for the Magnus force, F_M

$$F_M = 4\pi\rho V \omega R^3 \tag{6}$$

Fluid Dynamics – Air Drag

The viscosity of the air also impedes the motion of the baseball and is known as air drag. For this project, we will approximate the baseball as a particle with a single force acting through the center of mass, in the direction opposing the motion of the ball. As a result, air drag should slow down the velocity of the center of mass, but not the angular velocity of the ball. This idealization is not the case in real life, where viscous effects slow the velocity of the center of mass and the angular velocity of the ball. According to O'Reilly, drag force has the representation,[6]

$$\boldsymbol{F}_d = -\frac{1}{2}\rho_f A C_d\left(\overline{\boldsymbol{v}}\cdot\overline{\boldsymbol{v}}\right)\frac{\overline{\boldsymbol{v}}}{\left\|\overline{\boldsymbol{v}}\right\|} \tag{7}$$

where ρ_f is the density of air, A is the cross-sectional area of the ball, and C_d is the drag coefficient. By noting that a vector doted with itself is the square of its magnitude, this equation simplifies to:

$$\boldsymbol{F}_d = -\frac{1}{2}\rho_f A C_d\left\|\overline{\boldsymbol{v}}\right\|\boldsymbol{v} \tag{8}$$

The drag coefficient, C_d, is a function, of a dimensionless quantity called the Reynolds number, Re.[7] In turn, the Reynold's number is a function of the velocity of the ball,

$$Re = \frac{\rho \mathrm{v} \mathrm{D}}{\mu} \tag{9}$$

where ρ is the density of the air, v is the magnitude of the velocity of the center of mass, D is the diameter of the baseball, and μ is the viscocity of the air. When the Reynolds number is sufficiently small, the air flow above and below the ball is laminar and the drag coefficient takes on an approximately constant value. However, above a critical Reynold's number value that depends on the shape of the object, the flow becomes turbulent and the drag coefficient drops. This fact comes from the fact that the ball slips more easily in a turbulent flow. These results are shown in Fig. 4 for a rough projectile and a smooth projectile.

[6] Oliver M. O'Reilly. Lecture Notes on the Dynamics of Particles and Rigid Bodies. Berkeley: Copy Central, 2004. pp. 176 Eq. (8.53)

[7] Attributed to the Irish engineer Osborne Reynolds.

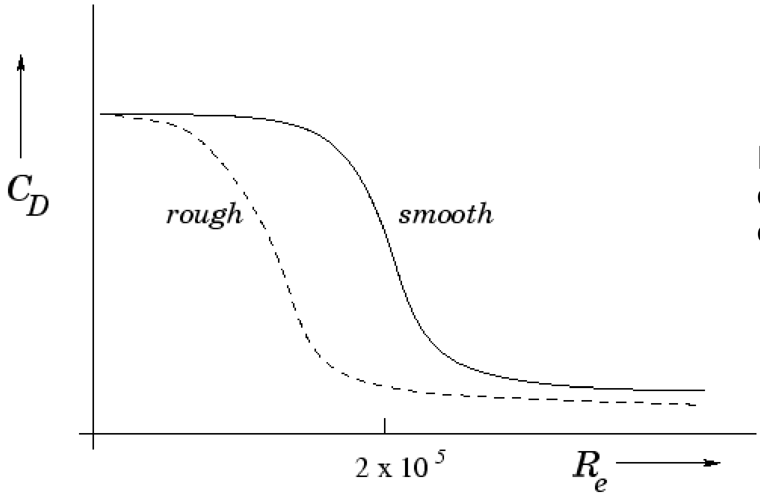

Fig 4: Typical dependence of the drag coefficient, C_d, on the Reynolds number, *Re.*[8]

Ironically, a rough object moves with less resistance at high speeds than a smooth one. As stated before, the critical Reynolds number depends on the roughness of the projectile. It turns out that a smooth baseball will have a transition speed well above the speed of the fastest pitches. Conversely, the transition speed for a rough ball occurs at very slow speeds relative to a typical baseball pitch. Therefore, raised stiches are put on an otherwise smooth ball in order to place the transition speed right in the center of a pitcher's speed range. These results are shown in Fig. 5.

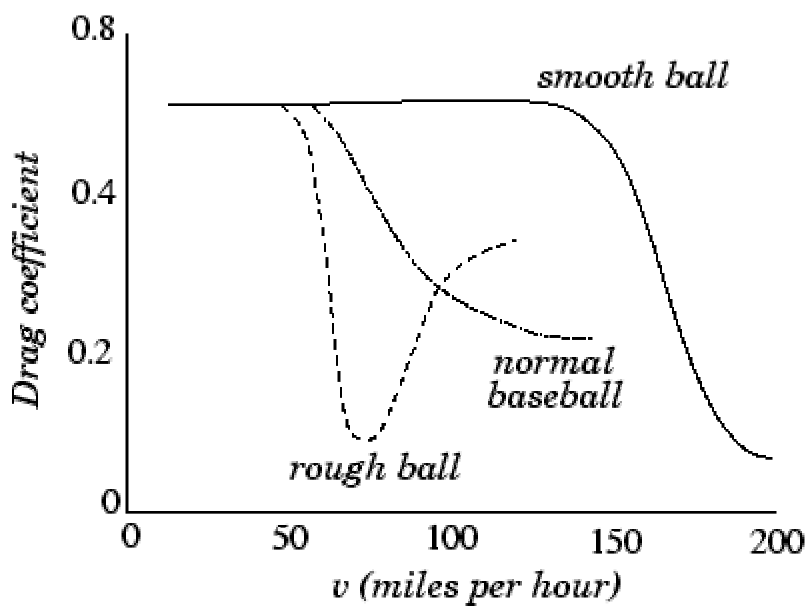

Fig. 5: Variation of the drag coefficient, C_d, with speed *v*, for normal, rough, and smooth baseballs.[9]

[8] Air Drag. Richard Fitzpatrick. Aug 25, 2003. University of Texas. Dec 8, 2004. <http://farside.ph.utexas.edu/teaching/329/lectures/node70.html>.
[9] R.K. Adair. The Physics of Baseball. New York: Harper & Row, 1990.

Since there is no closed form representation for the drag coefficient as a function of the Reynolds number for a baseball, we can approximate it using three piecewise linear functions based off of Fig. 5.

$$
\begin{aligned}
&C_d = 0.5 && Re \le 1.35 \times 10^5 \\
&C_d = 0.5 - \left(\frac{0.5-0.2}{2.8\times 10^5 - 1.35\times 10^5}\right)\left(Re - 1.35\times 10^5\right) && 1.35\times 10^5 < Re < 2.8\times 10^5 \\
&C_d = 0.2 && Re \ge 2.8\times 10^5
\end{aligned}
\tag{10}
$$

This approximation is shown graphically in Fig. 6.

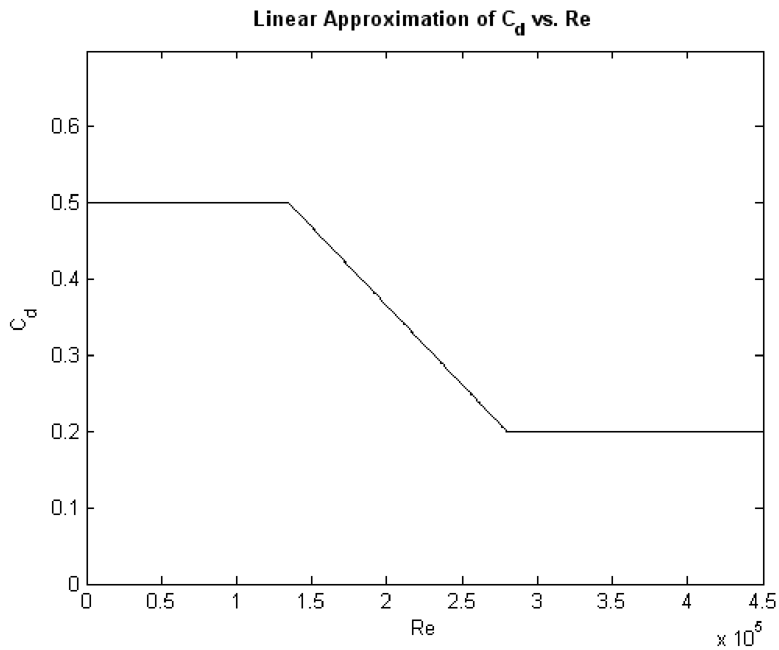

Fig. 6

EOM's

Now that we can calculate both the Magnus force and the drag force, the equations of motion can be formulated. Let's consider a free body diagram of the baseball, as shown in Fig. 7.

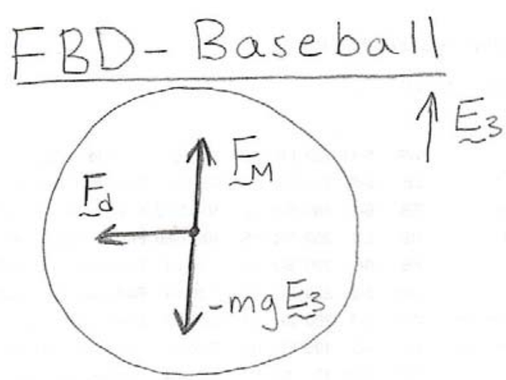

Fig. 7

The magnitude of the magnus has the representation given by Eq. (6). From O'Reilly's notes, the vector $\boldsymbol{F_M}$ is given by:

$$\boldsymbol{F}_M = mB\boldsymbol{\omega} \times \overline{\boldsymbol{v}} \tag{11}$$

Let's assume that the angular velocity is always orthogonal to the velocity of the center of mass, such that

$$||\boldsymbol{F}_M|| = mB||\boldsymbol{\omega}||\left\|\overline{\boldsymbol{v}}\right\| \tag{12}$$

Therefore Eq. (6) can be used to find mB and thus an expression for $\boldsymbol{F_M}$

$$mB = 4\pi\rho R^3 \Rightarrow \boldsymbol{F}_M = 4\pi\rho R^3 \boldsymbol{\omega} \times \overline{\boldsymbol{v}} \tag{13}$$

Applying Newton's second law:

$$m\dot{\overline{\boldsymbol{v}}} = \boldsymbol{F}_d + \boldsymbol{F}_M - mg\boldsymbol{E}_3 \tag{14}$$

The equations of motion are:

$$\begin{aligned}
m\ddot{x}_1 &= -\tfrac{1}{2}\rho A C_d \dot{x}_1\sqrt{\dot{x}_1{}^2 + \dot{x}_2{}^2 + \dot{x}_3{}^2} + 4\pi\rho R^3(\Omega_2\dot{x}_3 - \Omega_3\dot{x}_2) \\
m\ddot{x}_2 &= -\tfrac{1}{2}\rho A C_d \dot{x}_2\sqrt{\dot{x}_1{}^2 + \dot{x}_2{}^2 + \dot{x}_3{}^2} + 4\pi\rho R^3(\Omega_3\dot{x}_1 - \Omega_1\dot{x}_3) \\
m\ddot{x}_3 &= -\tfrac{1}{2}\rho A C_d \dot{x}_3\sqrt{\dot{x}_1{}^2 + \dot{x}_2{}^2 + \dot{x}_3{}^2} + 4\pi\rho R^3(\Omega_1\dot{x}_2 - \Omega_2\dot{x}_1) - mg
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
\dot{\overline{\boldsymbol{v}}} &= \dot{x}_1\boldsymbol{E}_1 + \dot{x}_2\boldsymbol{E}_2 + \dot{x}_3\boldsymbol{E}_3 \\
\boldsymbol{\omega} &= \Omega_1\boldsymbol{E}_1 + \Omega_2\boldsymbol{E}_2 + \Omega_3\boldsymbol{E}_3
\end{aligned} \tag{16}$$

Euler Angles

For this problem, we chose to parameterize the rotation tensor $\mathrm{Q} = \sum_{\mathrm{k}=1}^{3} \mathrm{e}_\mathrm{k} \otimes \mathrm{E}_\mathrm{k}$ by a set of 3-2-1 Euler angles. The angular velocity has the representation:

$$\boldsymbol{\omega}_Q = \dot{\theta}\boldsymbol{E}_3 + \dot{\phi}\boldsymbol{t}_2 + \dot{\psi}\boldsymbol{e}_1 \tag{17}$$

The transformation matrix is:

$$\begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{e}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_1 \\ \boldsymbol{E}_2 \\ \boldsymbol{E}_3 \end{bmatrix} \quad (18)$$

The Euler basis can be derived by analyzing the individual rotations:

$$\begin{bmatrix} \boldsymbol{g}_1 \\ \boldsymbol{g}_2 \\ \boldsymbol{g}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_3 \\ \boldsymbol{t}_2 \\ \boldsymbol{e}_1 \end{bmatrix} = \begin{bmatrix} -\sin\phi & \sin\psi\cos\phi & \cos\psi\cos\phi \\ 0 & \cos\psi & -\sin\psi \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{e}_3 \end{bmatrix} \quad (19)$$

In addition, the dual Euler basis can be calculated through steps similar to those in O'Reilly's notes.[10]

$$\begin{bmatrix} \boldsymbol{g}^1 \\ \boldsymbol{g}^2 \\ \boldsymbol{g}^3 \end{bmatrix} = \begin{bmatrix} 0 & \sin\psi\sec\phi & \cos\psi\sec\phi \\ 0 & \cos\psi & -\sin\psi \\ 1 & \sin\psi\tan\phi & \cos\psi\tan\phi \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_1 \\ \boldsymbol{e}_2 \\ \boldsymbol{e}_3 \end{bmatrix} \quad (20)$$

The dual Euler basis can be use to calculate the Euler angles by $\omega \cdot g^i = \dot{\gamma}^i$ where γ^i represents the Euler angles. Following this result, a system first-order differential equations can be written for the 3-2-1 Euler angles:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \sin\psi\sec\phi & \cos\psi\sec\phi \\ 0 & \cos\psi & -\sin\psi \\ 1 & \sin\psi\tan\phi & \cos\psi\tan\phi \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \quad (21)$$

Also, the baseball experiences moment-free motion, so the angular velocity can be solved from the following first-order differential equtions, modified slightly from O'Reilly's notes:[11]

$$\begin{aligned} \lambda_1 \dot{\omega}_1 &= (\lambda_2 - \lambda_3)\omega_2\omega_3 \\ \lambda_2 \dot{\omega}_2 &= (\lambda_3 - \lambda_1)\omega_1\omega_3 \\ \lambda_3 \dot{\omega}_3 &= (\lambda_1 - \lambda_2)\omega_1\omega_2 \end{aligned} \quad (22)$$

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[10] Oliver M. O'Reilly. Lecture Notes on the Dynamics of Particles and Rigid Bodies. Berkeley: Copy Central, 2004. pp. 103 Eq. (5.66 – 5.69).
[11] Oliver M. O'Reilly. Lecture Notes on the Dynamics of Particles and Rigid Bodies. Berkeley: Copy Central, 2004. pp. 168 Eq. (8.18).

Euler Basis Singularities

By examining both the Euler and the dual Euler basis, one will find that a singularity exists for the angle $\phi = \pm\pi / 2$. For the Euler basis, $\phi = \pm\pi / 2$ gives $g_1 = E_1 = \pm t_1 = \pm g_3$. For this angle, the Euler basis fails to span Euclidean three-space. In addition, the same phenomenon can be seen by setting $\phi = \pm\pi / 2$ for the dual Euler basis. In this case, the $\sec\phi$ and $\tan\phi$ terms are undefined and the dual Euler basis fails to span Euclidean three-space as well. To avoid this singularity, it is necessary to restrict the second Euler angle, $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$. However, this problem is avoided in our simulations, because θ never equals exactly $\frac{\pi}{2}$.

Inertial Tensor

Since we modeled the baseball as a sphere of radius *R*, the sphere is symmetric about all axes of rotation hence, $\lambda_1 = \lambda_2 = \lambda_3$. As a result,

$$\boldsymbol{J}_0 = \frac{2}{5} m R^2 (\boldsymbol{E}_1 \otimes \boldsymbol{E}_1 + \boldsymbol{E}_2 \otimes \boldsymbol{E}_2 + \boldsymbol{E}_3 \otimes \boldsymbol{E}_3)$$

$$\boldsymbol{J}_{\square} = \frac{2}{5} m R^2 (\boldsymbol{e}_1 \otimes \boldsymbol{e}_1 + \boldsymbol{e}_2 \otimes \boldsymbol{e}_2 + \boldsymbol{e}_3 \otimes \boldsymbol{e}_3) \tag{21}$$

Linear Momentum

For our baseball simulation the linear momentum is given by Eq. (6.26) in O'Reilly's notes[12]:

$$\boldsymbol{G} = m \bar{\boldsymbol{v}} \tag{22}$$

The magnitude of linear momentum decreases over time, as a result of the drag force.

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[12] Oliver M. O'Reilly. Lecture Notes on the Dynamics of Particles and Rigid Bodies. Berkeley: Copy Central, 2004. pp. 121 Eq. (6.26)

Conservation of Energy

In total, three force act on the body: Magnus force, drag force, and gravitational force. Of these forces, only the gravitational force is conservative. Moreover, the ball goes through moment-free motion, hence there is no energy due to rotation and the work-energy theorm simplifies to

$$\begin{aligned} \dot{T} &= \sum_k \boldsymbol{F}_k \cdot \overline{\boldsymbol{v}} \\ &= -\dot{U} + (\boldsymbol{F}_M + \boldsymbol{F}_d) \cdot \overline{\boldsymbol{v}} \end{aligned} \tag{23}$$

where $\boldsymbol{F}_M$ is the magnus force, $\boldsymbol{F}_d$ is the drag force, and $-\dot{U}$ is the conservative force due to gravity. Continuing further,

$$\begin{aligned} \dot{T} + \dot{U} &= \boldsymbol{F}_M \cdot \overline{\boldsymbol{v}} + \boldsymbol{F}_d \cdot \overline{\boldsymbol{v}} \\ &= mB\left(\omega \times \overline{\boldsymbol{v}}\right) \cdot \overline{\boldsymbol{v}} - \frac{1}{2} \rho_f A C_d \left\| \overline{\boldsymbol{v}} \right\| \overline{\boldsymbol{v}} \cdot \overline{\boldsymbol{v}} \\ &= 0 - \frac{1}{2} \rho_f A C_d \left\| \overline{\boldsymbol{v}} \right\|^3 \\ \dot{E} &= -\frac{1}{2} \rho_f A C_d \left\| \overline{\boldsymbol{v}} \right\|^3 \end{aligned} \tag{24}$$

This result shows that the total energy of the baseball is decreasing due to the drag force only.

Constants

In order to simulate a baseball pitch, we must first know something about the parameters of a standard MLB baseball. According to the Official Rules: 1.09 of Major League Baseball:

> ***1.09***
>
> *The ball shall be a sphere formed by yarn wound around a small core of cork, rubber or similar material, covered with two stripes of white horsehide or cowhide, tightly stitched together. It shall weigh not less than five nor more than 5 1/4 ounces avoirdupois and measure not less than nine nor more than 9 1/4 inches in circumference.*[13]

[13] Official Rules. Major League Baseball.
<http://mlb.mlb.com/NASApp/mlb/mlb/official_info/official_rules/objectives_1.jsp>

For this problem, we assume the baseball has a mass of 5 ounces and a 9 inch circumference. Also, we shall consider the motion of the ball just as it passes over home plate. We assume a pitcher releases the ball from height of 1.7m off the ground and the distance from the pitchers mound is found to be 60.5 feet, as defined by MLB Official Rules: 1.07:

> ***1.07***
>
> *The pitcher's plate shall be a rectangular slab of whitened rubber, 24 inches by 6 inches. It shall be set in the ground as shown in Diagrams 1 and 2, so that the distance between the pitcher's plate and home base (the rear point of home plate) shall be 60 feet, 6 inches.*[14]

Finally, the density and viscosity of the air through which the ball travels needs to be known for our calculations. We can assume the air is at standard temperature and pressure. The density is ρ = 1.23 kg/m^3 and the viscosity is μ = 1.79x10^{-5} kg/m·s.

Simulations – Initial Conditions

For all of our simulations, we defined our initial conditions by the descriptions given in Adair, shown in Fig. 8:[15]

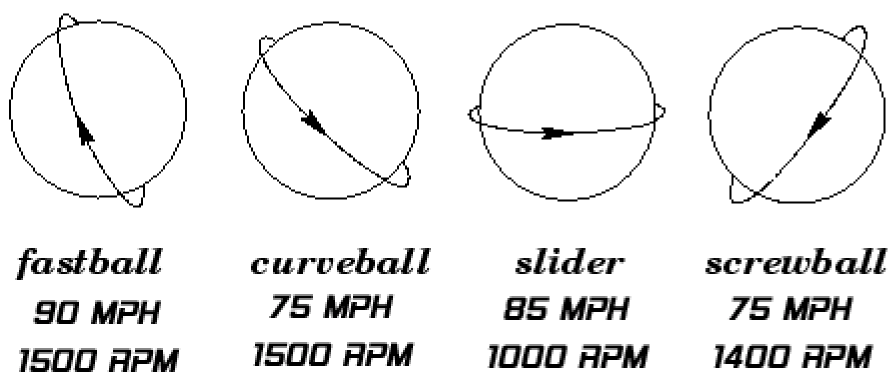

Fig. 8: Ball rotation directions for pitches thrown almost straight overhand by a right-handed pitcher.

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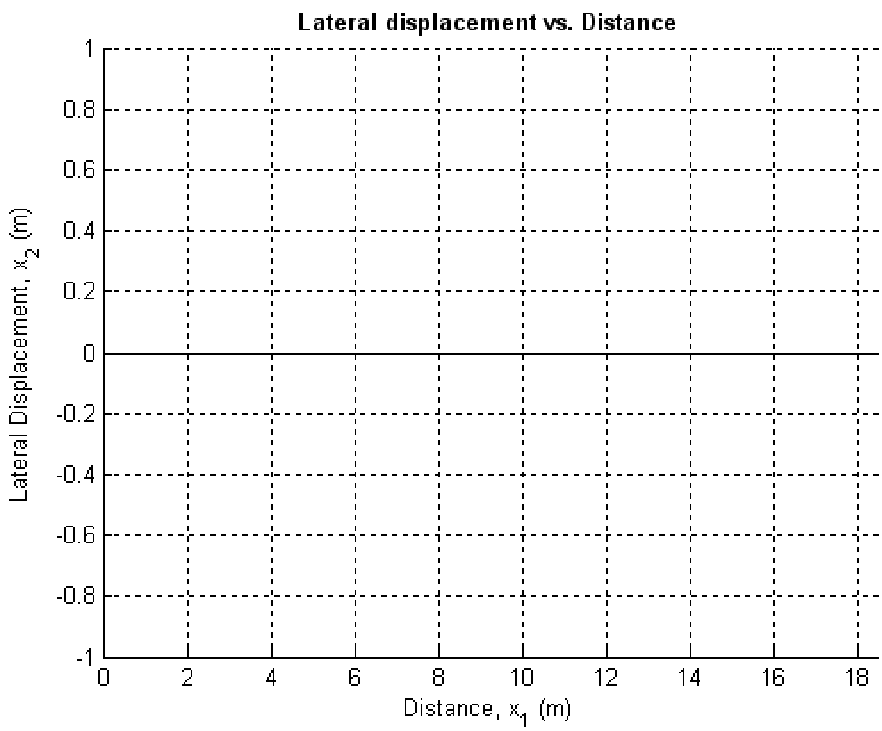
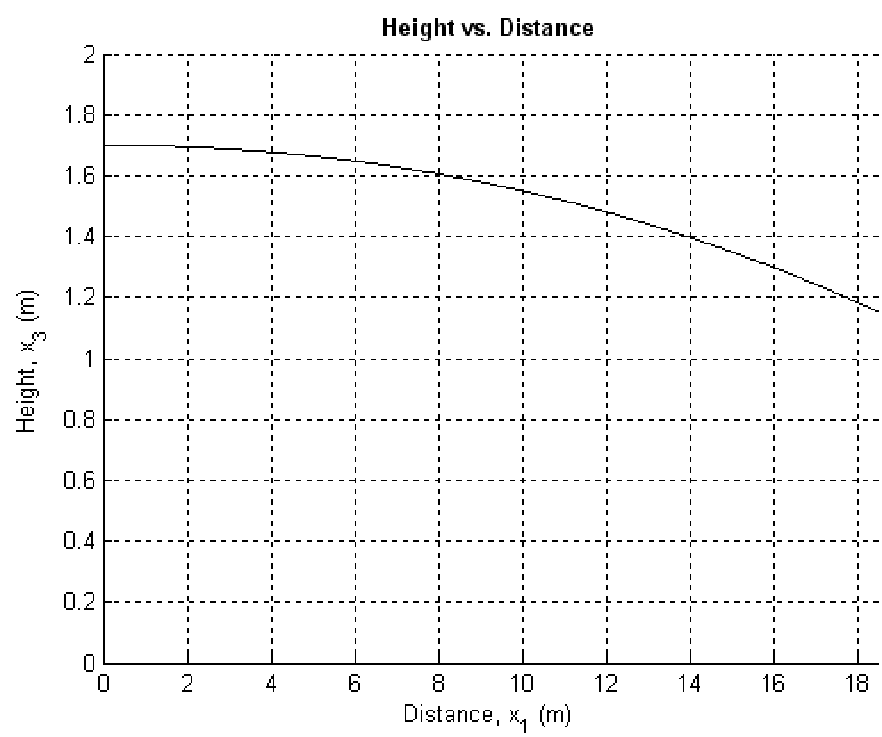
[14] Official Rules. Major League Baseball. <http://mlb.mlb.com/NASApp/mlb/mlb/official_info/official_rules/objectives_1.jsp>

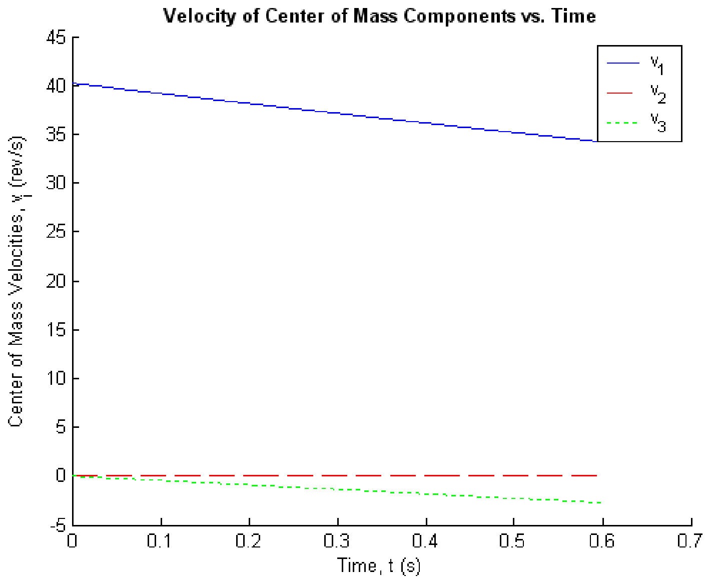
[15] Adair, Robert K. The Physics of Baseball. New York: Collins Publishers Inc., 2002.

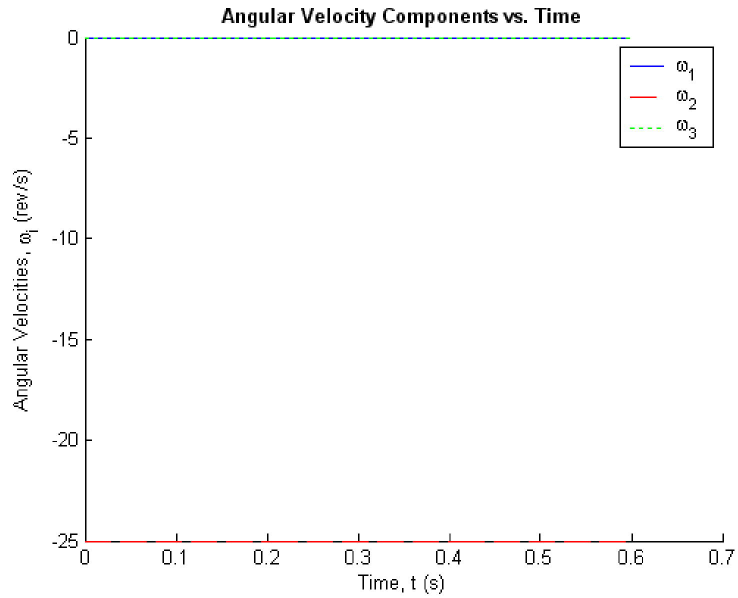
Simulations – Fastball

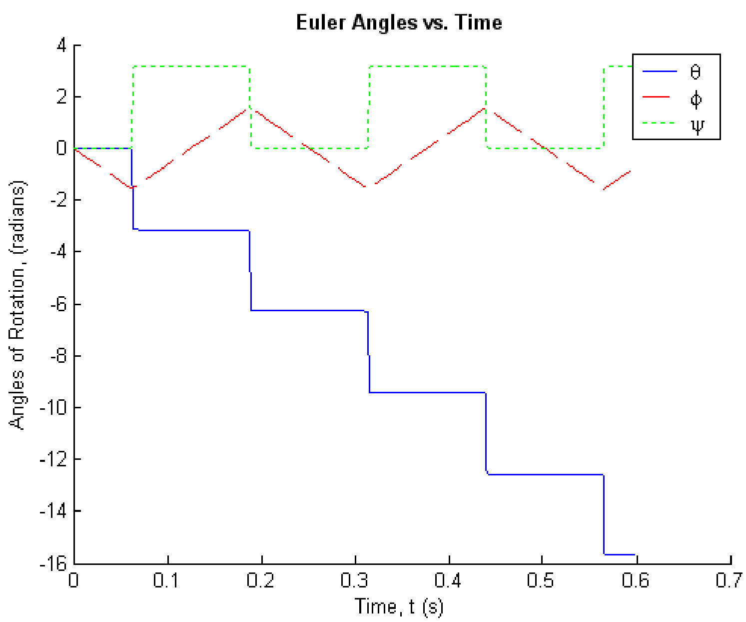
Velocity Components	Initial Condition (MPH)	Rotation Components	Initial Condition (RPM)
v_{10}	90	Ω_{10}	0
v_{20}	0	Ω_{20}	-1500
v_{30}	0	Ω_{30}	0

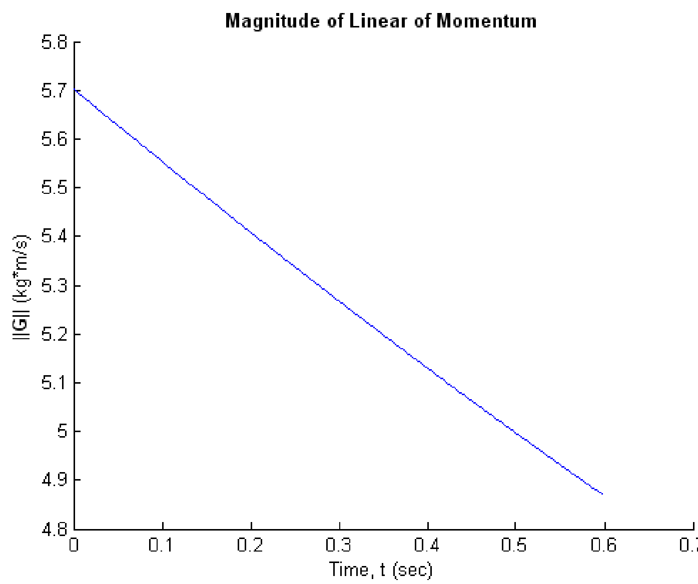


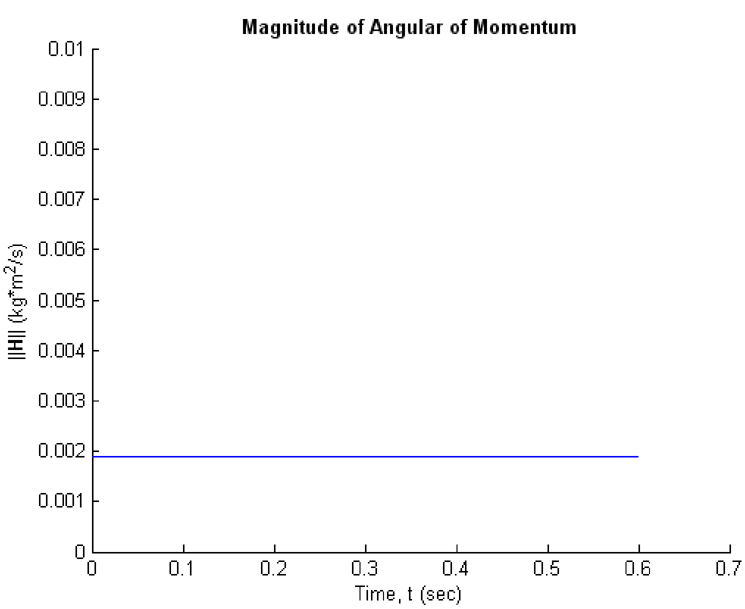


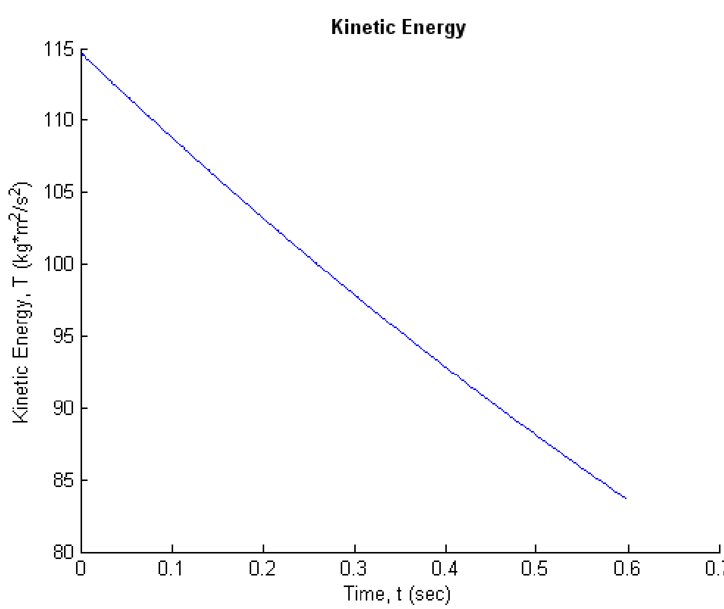

Velocity of Center of Mass Components vs. Time
Center of Mass Velocities, v_i (rev/s)
Time, t (s)
v_1
v_2
v_3
45
40
35
30
25
20
15
10
5
0
-5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7


Angular Velocity Components vs. Time
Angular Velocities, ω_i (rev/s)
Time, t (s)
ω_1
ω_2
ω_3
0
-5
-10
-15
-20
-25
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7


Euler Angles vs. Time
Angles of Rotation, (radians)
Time, t (s)
θ
ϕ
ψ
4
2
0
-2
-4
-6
-8
-10
-12
-14
-16
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

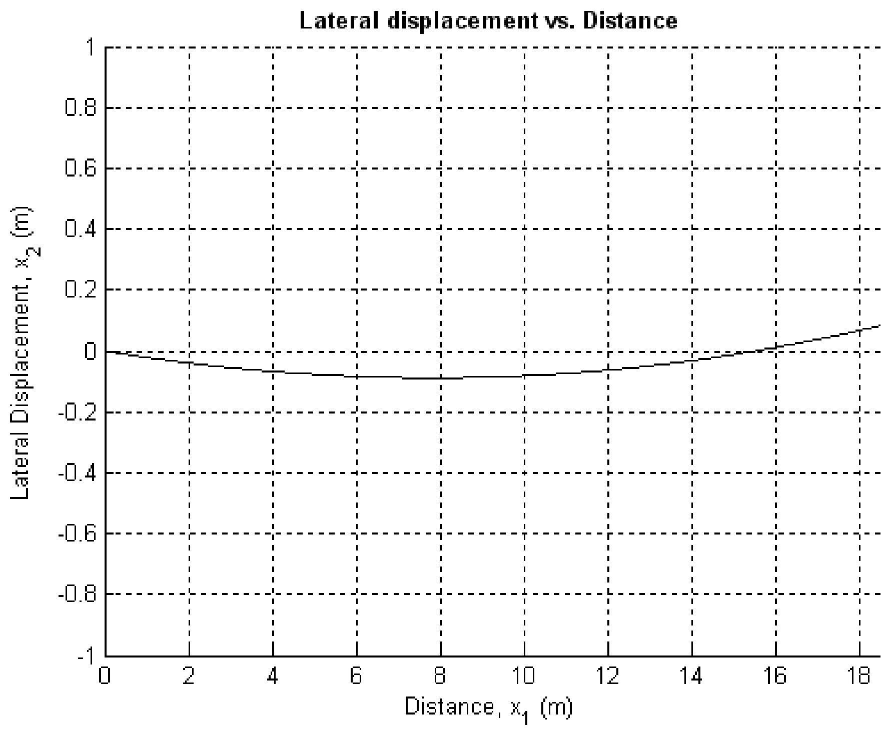

Magnitude of Linear of Momentum
||G|| (kg*m/s)
Time, t (sec)
5.8
5.7
5.6
5.5
5.4
5.3
5.2
5.1
5
4.9
4.8
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

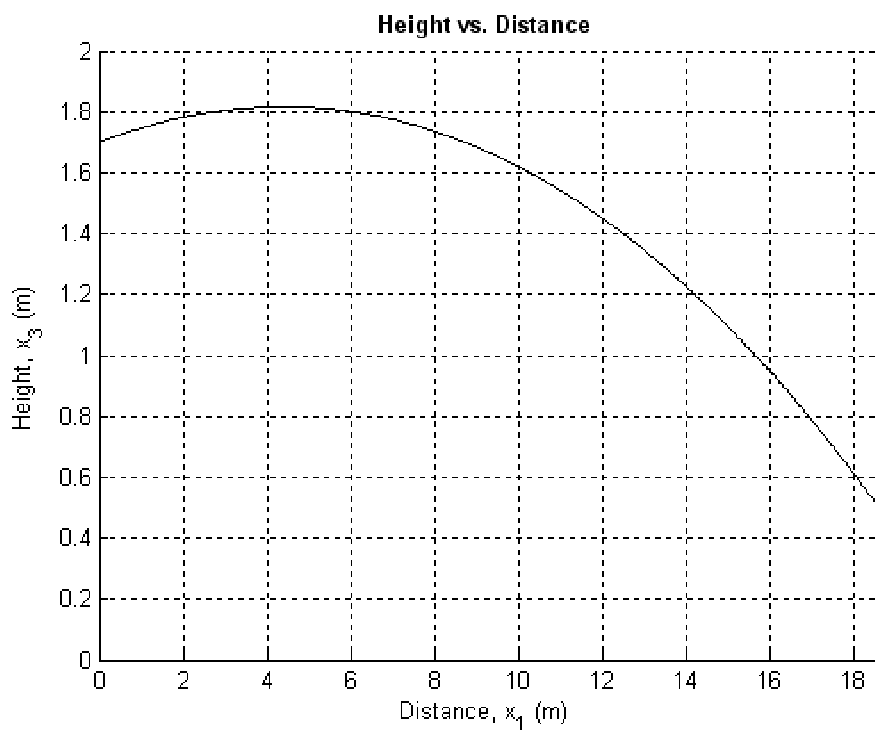

Magnitude of Angular of Momentum
||H|| (kg*m^2/s)
Time, t (sec)
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

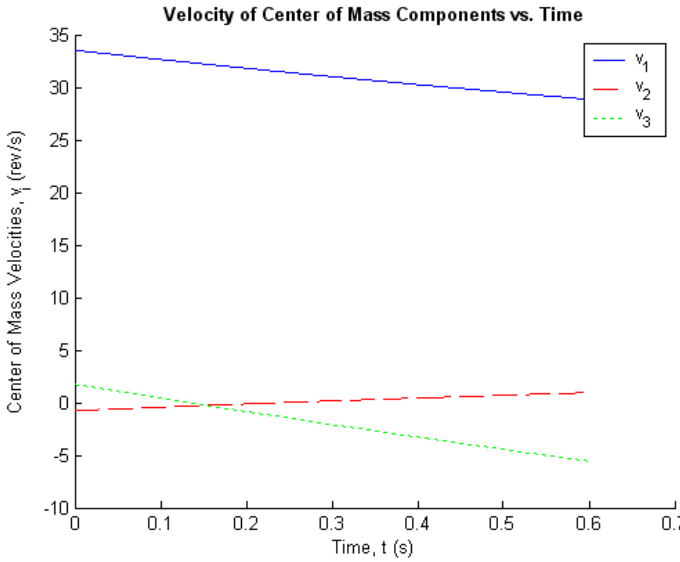

Kinetic Energy
Kinetic Energy, T (kg*m^2/s^2)
Time, t (sec)
115
110
105
100
95
90
85
80
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

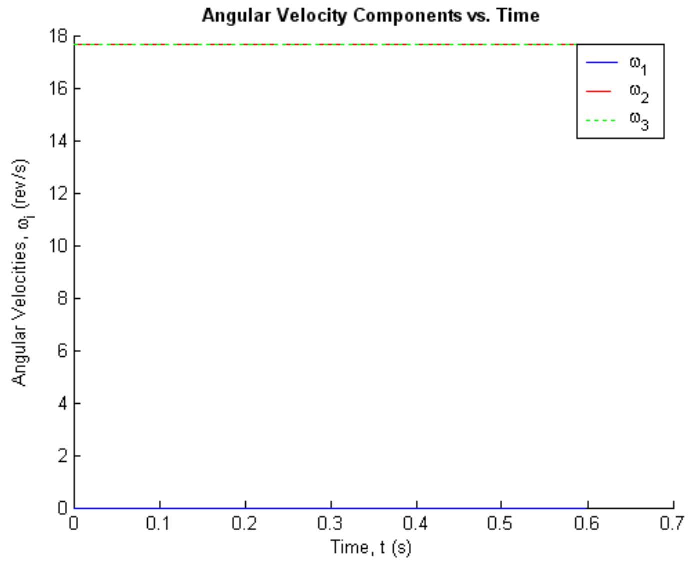
Simulations – Curveball

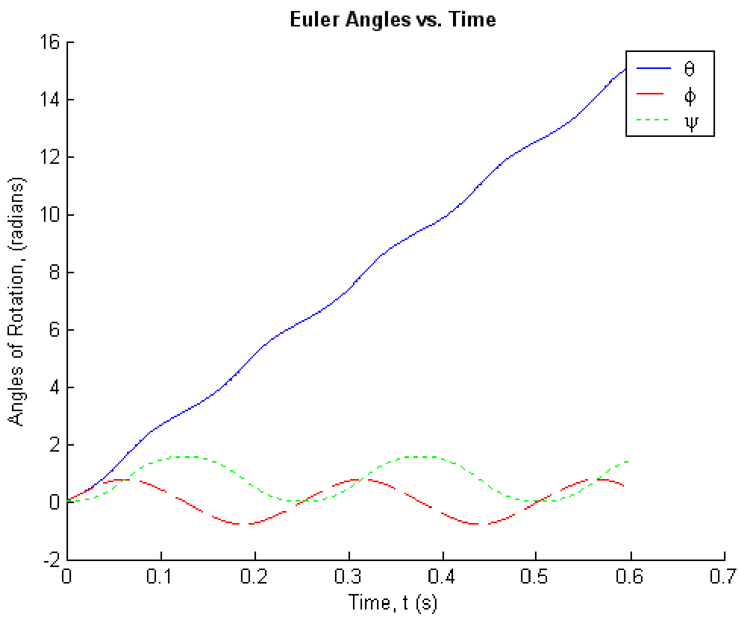
Velocity Components	Initial Condition (MPH)	Rotation Components	Initial Condition (RPM)
v_{10}	75	Ω_{10}	0
v_{20}	-0.75	Ω_{20}	1500 cos 45˚
v_{30}	1.75	Ω_{30}	1500 cos 45˚

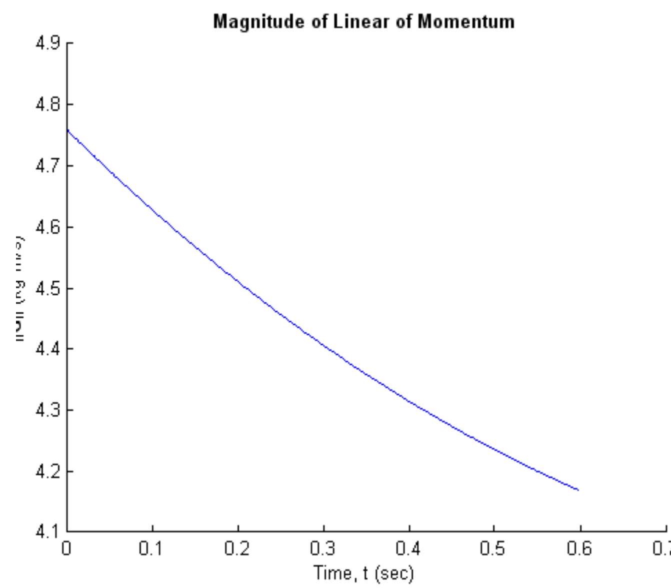


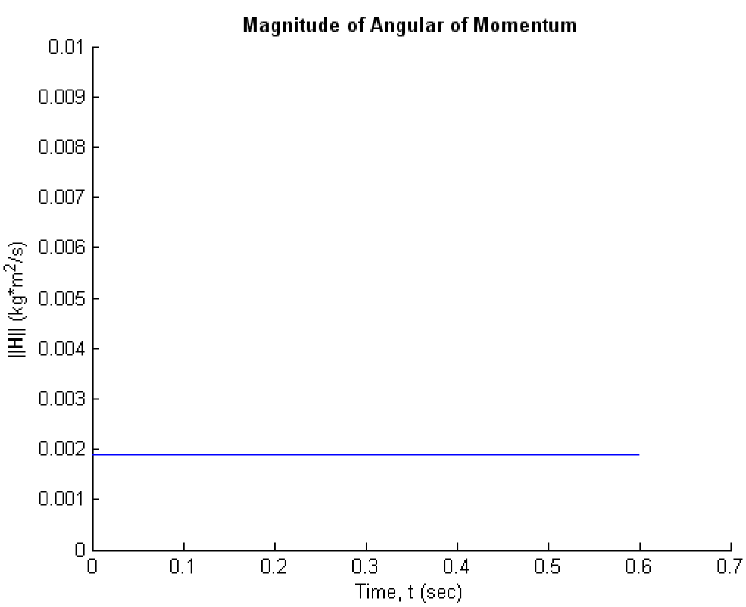


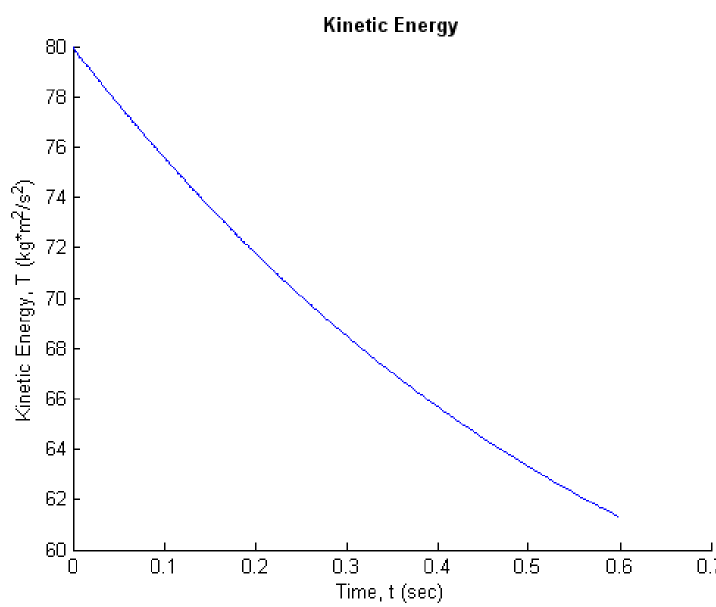

Velocity of Center of Mass Components vs. Time
Center of Mass Velocities, v_i (rev/s)
Time, t (s)
v_1
v_2
v_3


Angular Velocity Components vs. Time
Angular Velocities, ω_i (rev/s)
Time, t (s)
ω_1
ω_2
ω_3


Euler Angles vs. Time
Angles of Rotation, (radians)
Time, t (s)
θ
ϕ
ψ

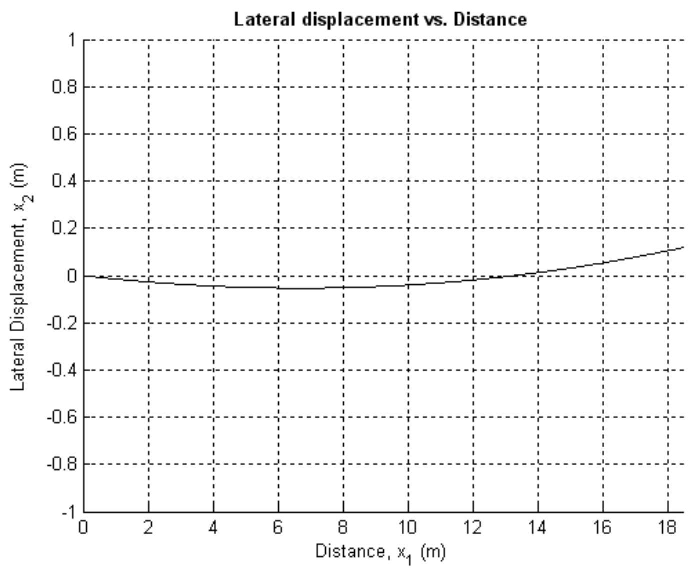

Magnitude of Linear of Momentum
Time, t (sec)

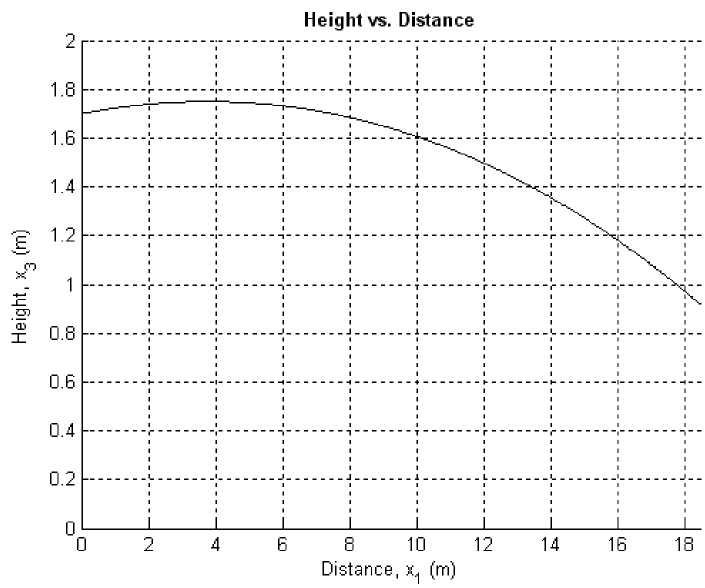

Magnitude of Angular of Momentum
||H|| (kg*m^2/s)
Time, t (sec)

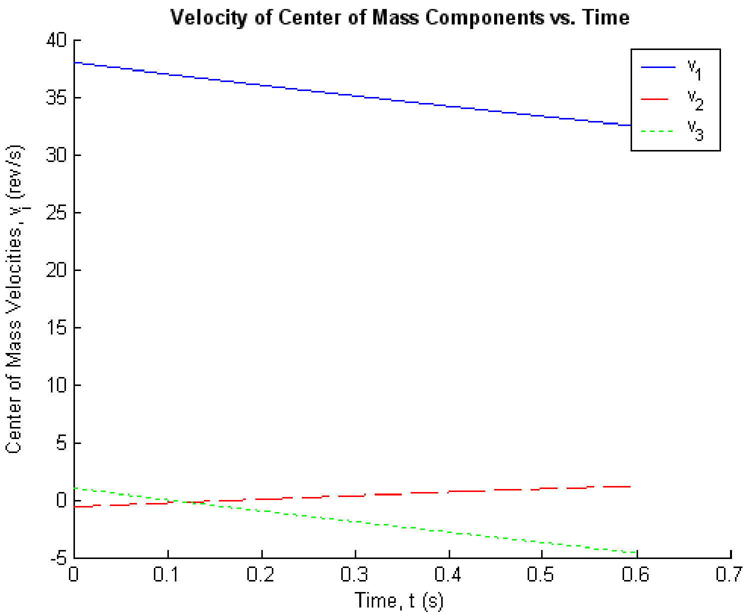

Kinetic Energy
Kinetic Energy, T (kg*m^2/s^2)
Time, t (sec)

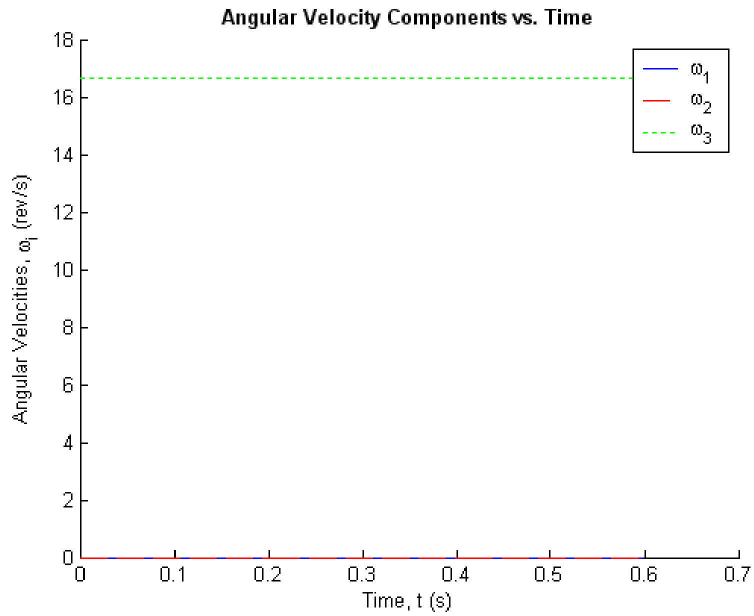
Simulations – Slider

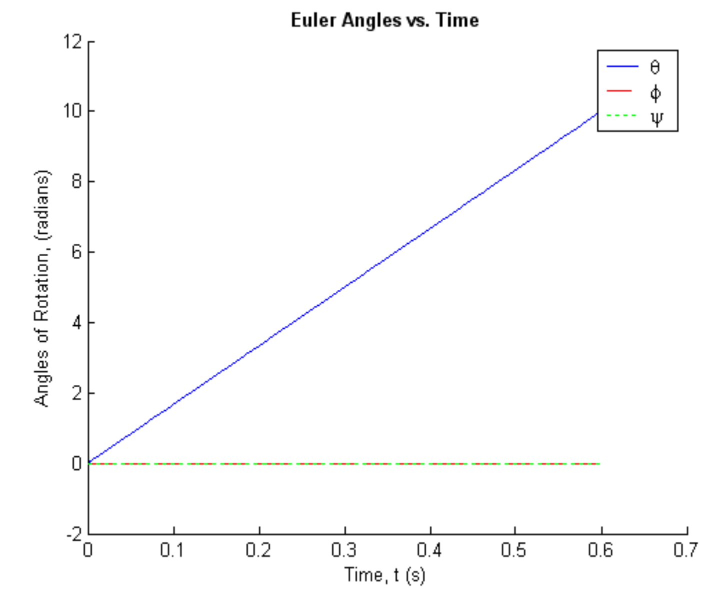
Velocity Components	Initial Condition (MPH)	Rotation Components	Initial Condition (RPM)
v_{10}	85	Ω_{10}	0
v_{20}	-0.6	Ω_{20}	0
v_{30}	1	Ω_{30}	1000

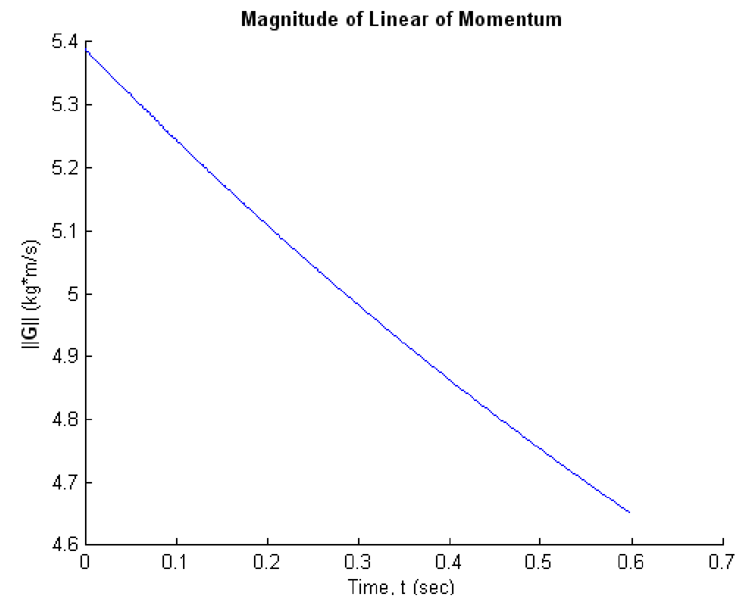


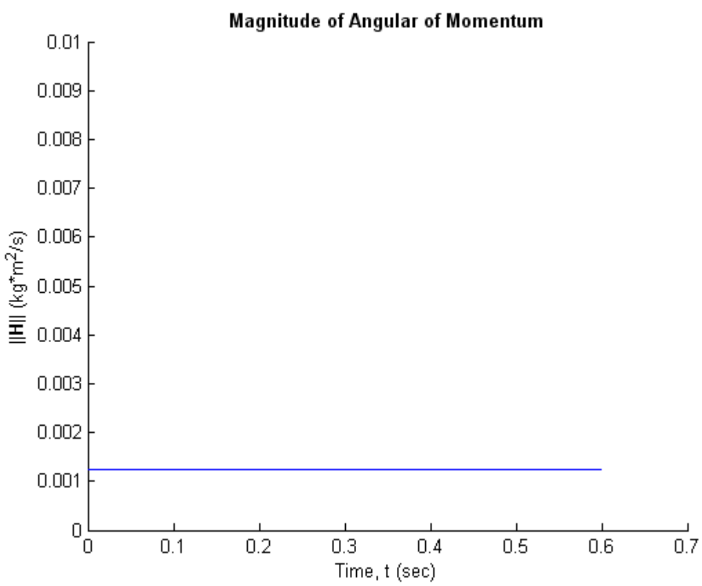


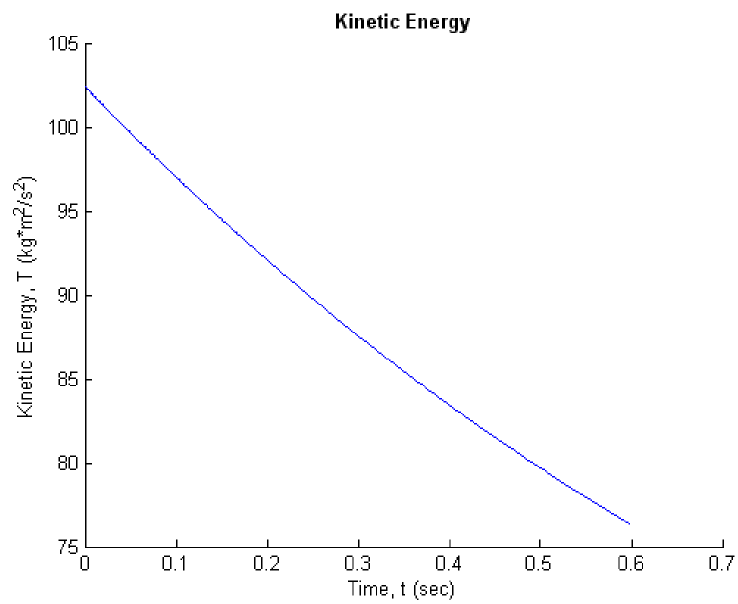






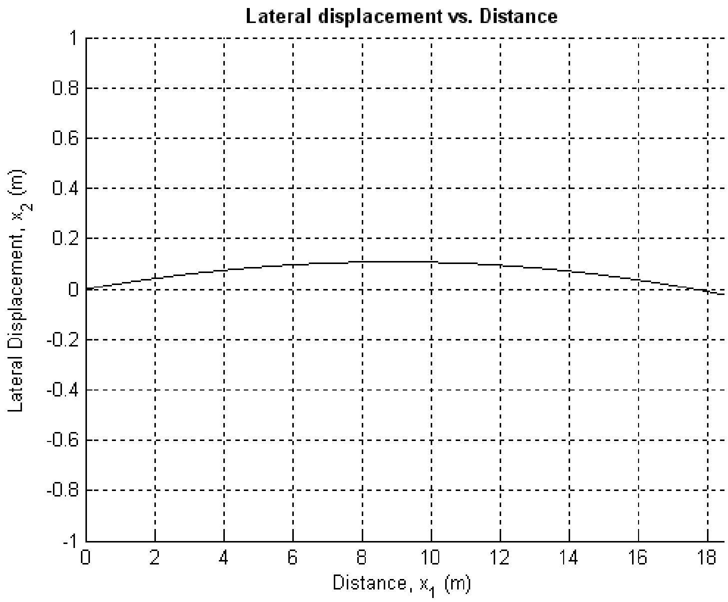


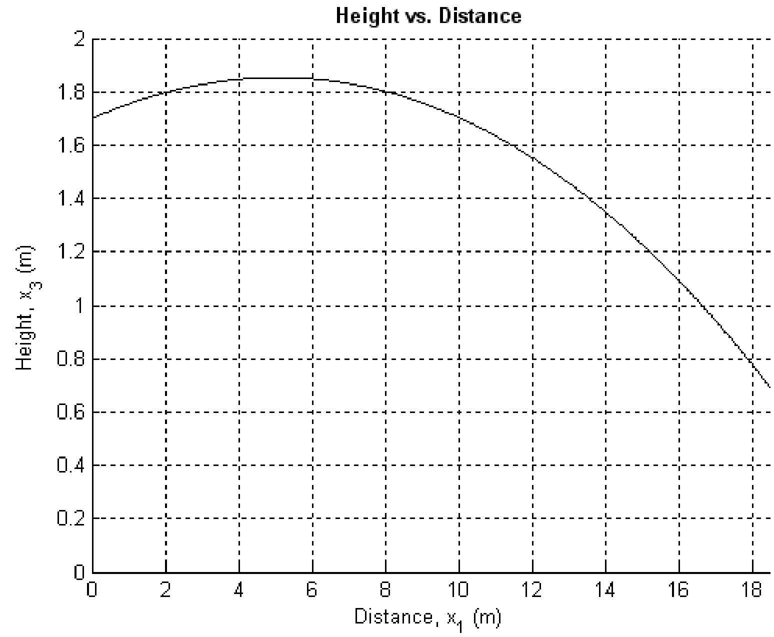


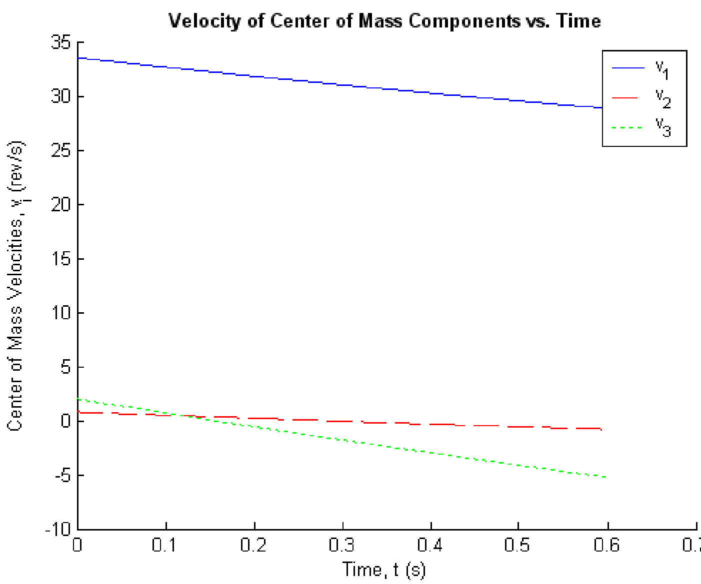


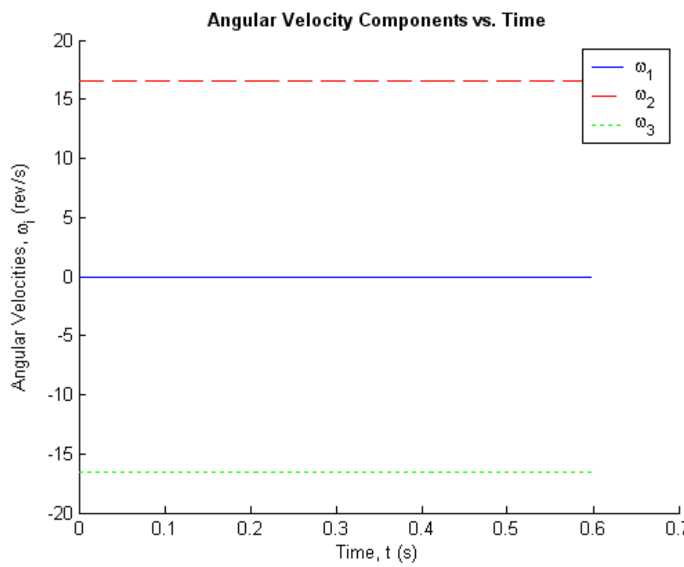
Simulations – Screwball

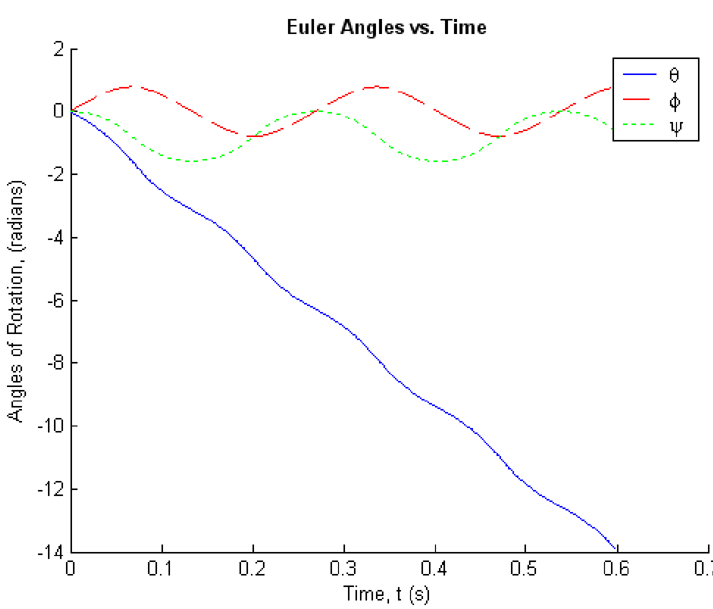
Velocity Components	Initial Condition (MPH)	Rotation Components	Initial Condition (RPM)
v_{10}	75	Ω_{10}	0
v_{20}	0.8	Ω_{20}	1400 cos 45˚
v_{30}	2	Ω_{30}	-1400 cos 45˚

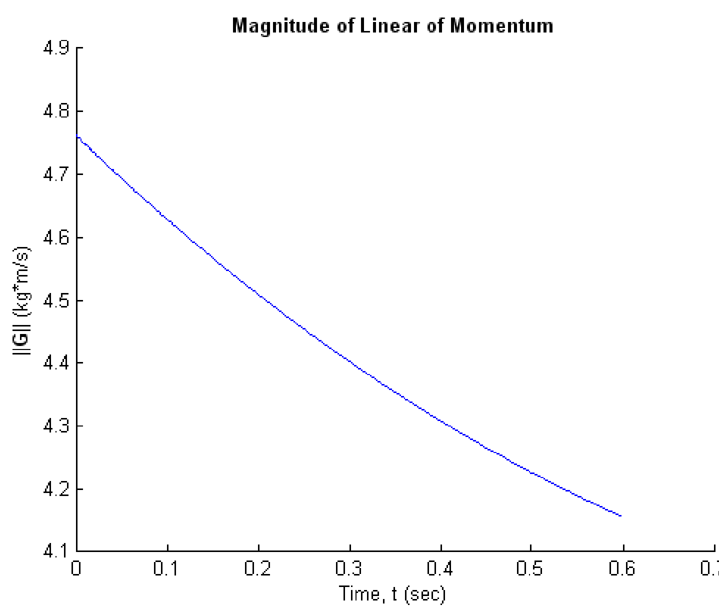

Lateral displacement vs. Distance
Lateral Displacement, x2 (m)
Distance, x1 (m)

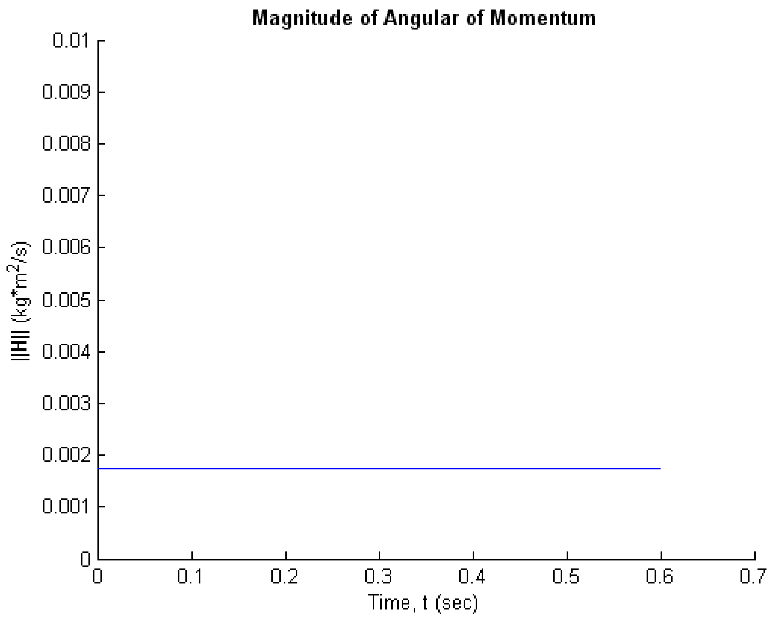

Height vs. Distance
Height, x3 (m)
Distance, x1 (m)

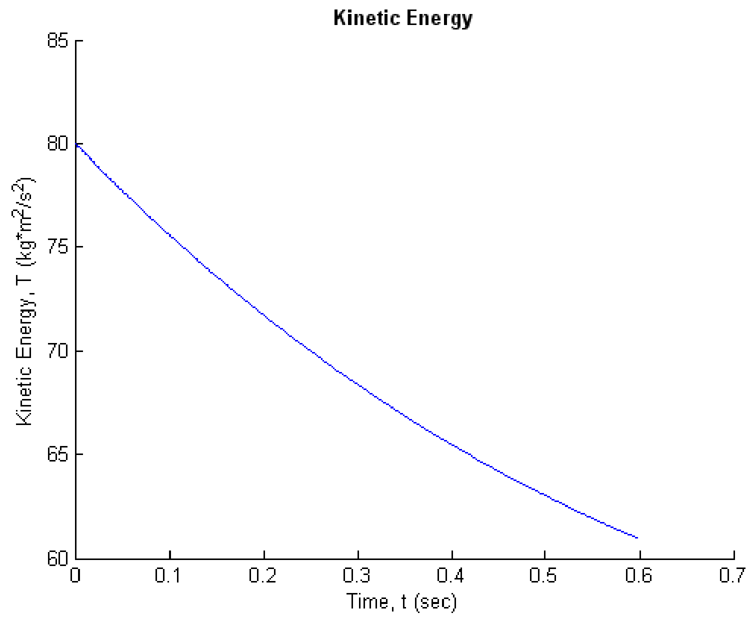

Velocity of Center of Mass Components vs. Time
Center of Mass Velocities, vi (rev/s)
Time, t (s)
v1
v2
v3


Angular Velocity Components vs. Time
Angular Velocities, ωi (rev/s)
Time, t (s)
ω1
ω2
ω3


Euler Angles vs. Time
Angles of Rotation, (radians)
Time, t (s)
θ
ϕ
ψ


Magnitude of Linear of Momentum
||G|| (kg*m/s)
Time, t (sec)





Conclusion

As can be seen from our simulations, the mechanics of pitching is based (no pun intended) on serveral interesting physical phenomena. The speed and rotational velocity at which the pitcher throws the ball can make it curve in ways that are difficult for the batter to hit. Thus, pitchers like Pedro Martinez are actually physicists without knowing it. They are masters of the Magnus, drag, and gravitational forces, able to manipulate them in ways to befuddle the opposition.