

Adaptive Estimation and Control of Models for Battery Electrochemistry

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UC San Diego

ME Seminar
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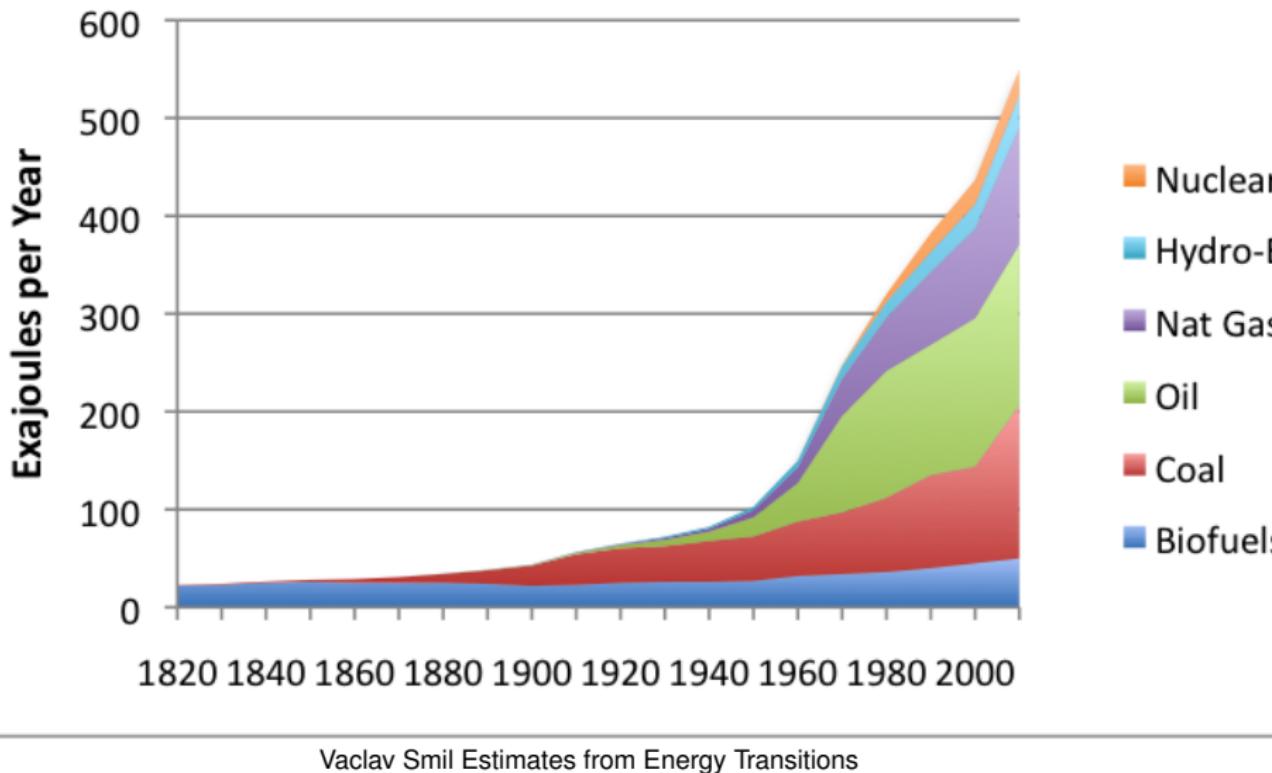
Core Philosophy:

(Dynamical models of physical phenomena)

+ (novel control paradigms)

= (transformative advancements)

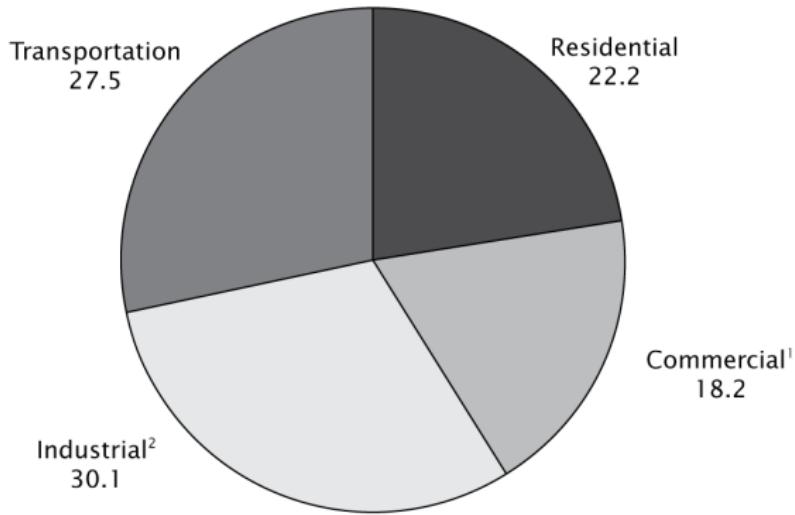
World Energy Consumption



Energy Consumption by End-Use Sector: 2010

(Quadrillion Btu)

Total Consumption=98.0



Source: Chart prepared by U.S. Census Bureau.

Energy Initiatives



- Denmark 50% wind penetration by 2025
- China leads manufacturing of renewable tech
- Brazil uses 86% renewables
- EV Everywhere
- SunShot
- Green Button
- Zero emissions vehicle (ZEV)
- 33% renewables by 2020
- Go Solar California

Energy Crisis Solutions

Energy storage (e.g., batteries)	Demand-side management (e.g., smart grids)
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Outline

1

Batteries

- [Electrochemical Modeling] Incorporating Physics
- [SOC/SOH Estimation] Looking Inside w/ Models, Meas., and Math
- [Constrained Control] Operate at the Limits, Safely
- [PHEV Power Management] Max eMPG and Max Batt Life

2

Demand Response in Smart Grids

3

Future

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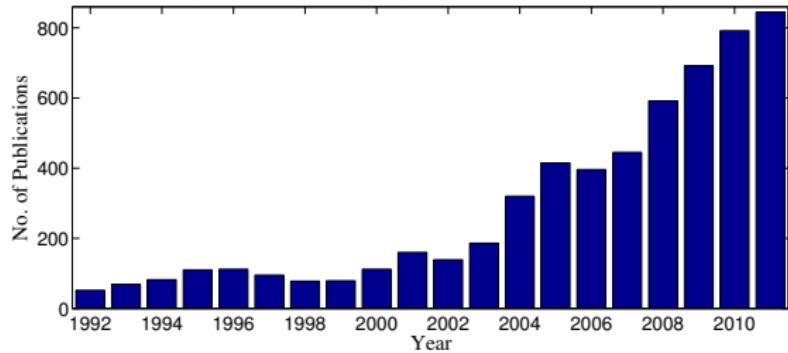
A Golden Era



A Golden Era



Keyword: "Battery Systems and Control"



The Battery Problem

Needs: Cheap, high energy, high power, long life

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Today's reality: Expensive, conservatively design/operated, die too quickly

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Some Motivating Facts

EV Batts

\$800 / kWh now (2010)

\$125 / kWh for parity to IC engine

Only 75% of available capacity is used

Range anxiety inhibits adoption

Lifetime risks caused by fast charging

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Two Solutions

Design better batteries
(materials science & chemistry)

Make current batteries better
(estimation and control)

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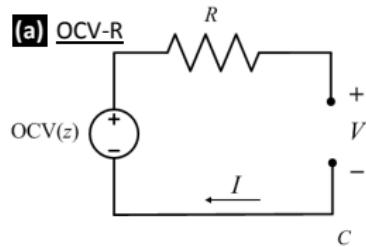
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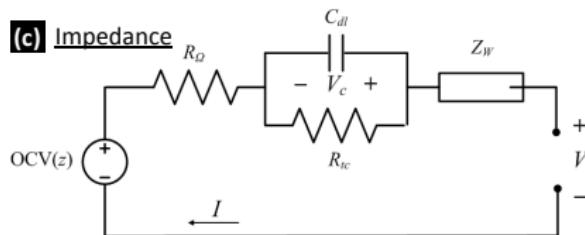
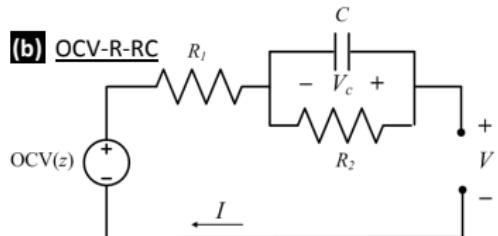
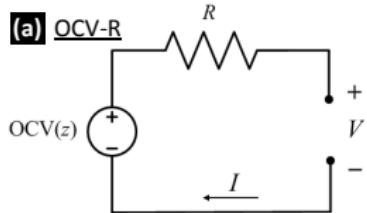
Battery Models

Equivalent Circuit Model



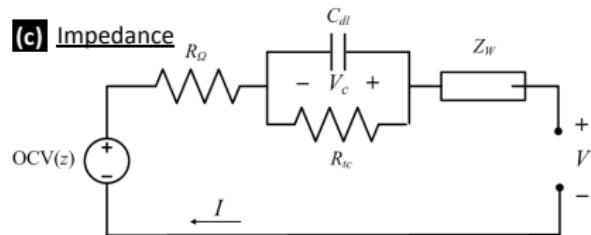
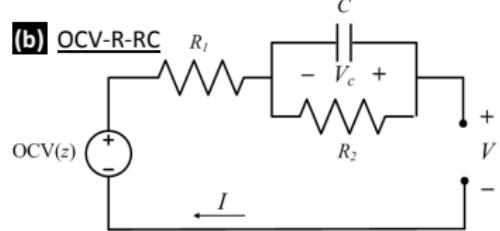
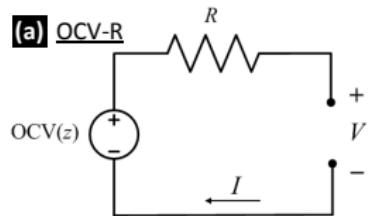
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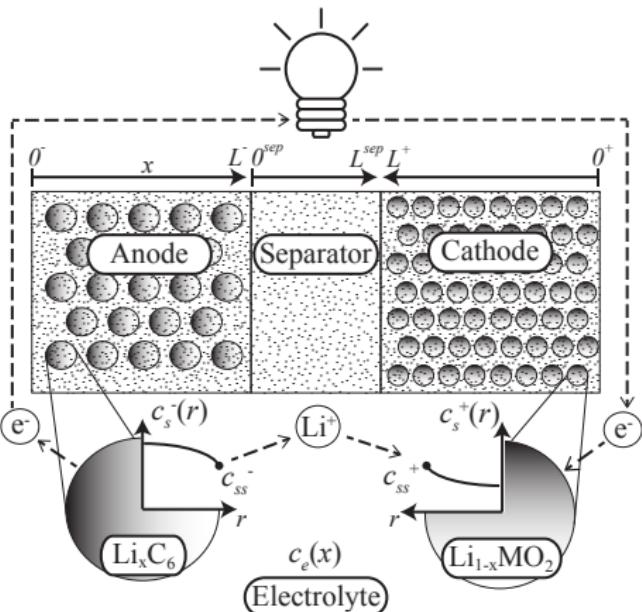


Battery Models

Equivalent Circuit Model

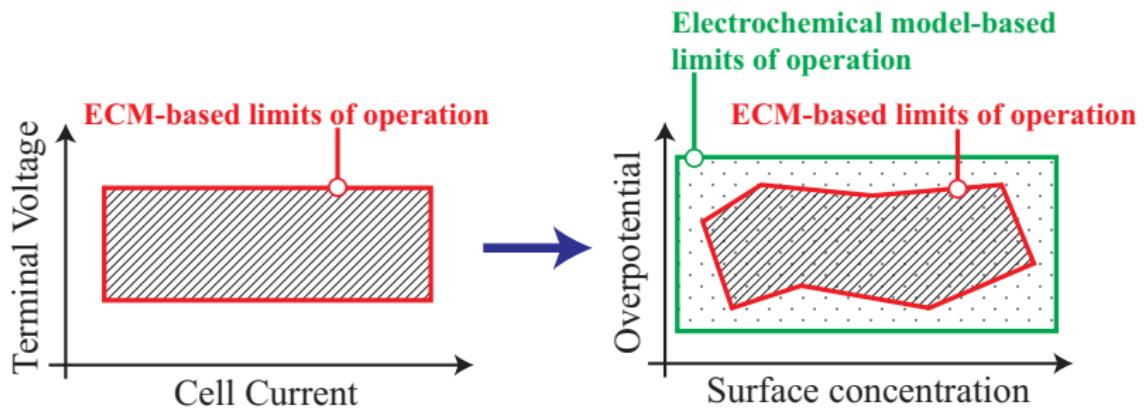


Electrochemical Model





Operate Batteries at their Physical Limits



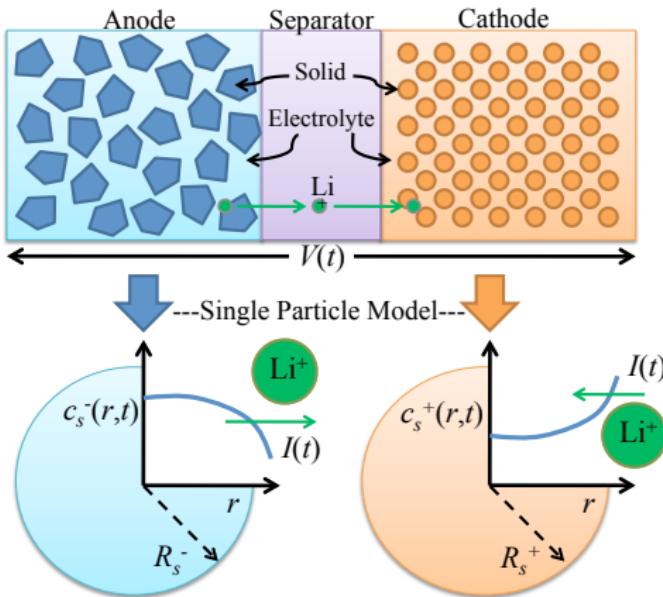
Electrochemical Model Equations

well, some of them

Description	Equation
Solid phase Li concentration	$\frac{\partial c_s^\pm}{\partial t}(x, r, t) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[D_s^\pm r^2 \frac{\partial c_s^\pm}{\partial r}(x, r, t) \right]$
Electrolyte Li concentration	$\varepsilon_e \frac{\partial c_e}{\partial t}(x, t) = \frac{\partial}{\partial x} \left[\varepsilon_e D_e \frac{\partial c_e}{\partial x}(x, t) + \frac{1-t_c^0}{F} i_e^\pm(x, t) \right]$
Solid potential	$\frac{\partial \phi_s^\pm}{\partial x}(x, t) = \frac{i_e^\pm(x, t) - I(t)}{\sigma^\pm}$
Electrolyte potential	$\frac{\partial \phi_e}{\partial x}(x, t) = -\frac{i_e^\pm(x, t)}{\kappa} + \frac{2RT}{F} (1 - t_c^0) \left(1 + \frac{d \ln f_{c/a}}{d \ln c_e}(x, t) \right) \frac{\partial \ln c_e}{\partial x}(x, t)$
Electrolyte ionic current	$\frac{\partial i_e^\pm}{\partial x}(x, t) = a_s F j_n^\pm(x, t)$
Molar flux btw phases	$j_n^\pm(x, t) = \frac{1}{F} i_0^\pm(x, t) \left[e^{\frac{\alpha_a F}{RT} \eta^\pm(x, t)} - e^{-\frac{\alpha_c F}{RT} \eta^\pm(x, t)} \right]$
Temperature	$\rho c_P \frac{dT}{dt}(t) = h [T^0(t) - T(t)] + I(t)V(t) - \int_{0^-}^{0^+} a_s F j_n(x, t) \Delta T(x, t) dx$

Animation of Li Ion Evolution

Single Particle Model (SPM)

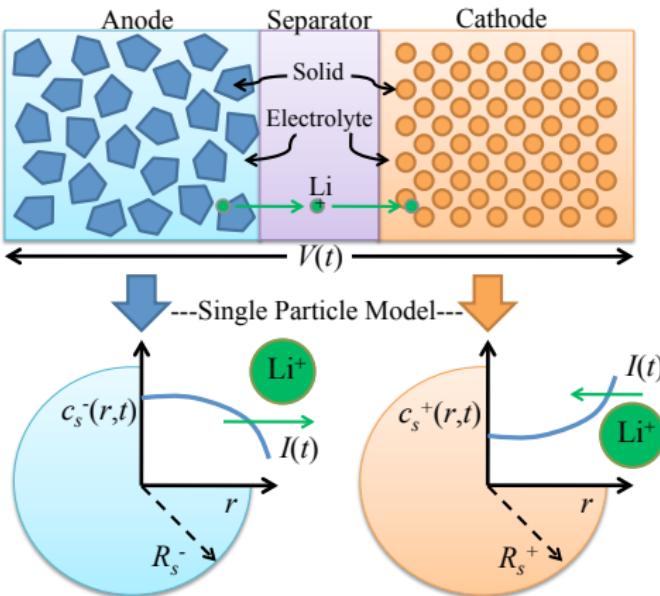


Single Particle Model (SPM)

Diffusion of Li in solid phase:

$$\frac{\partial c_s^-}{\partial t}(r, t) = \frac{D_s^-}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial^2 c_s^-}{\partial r^2}(r, t) \right]$$

$$\frac{\partial c_s^+}{\partial t}(r, t) = \frac{D_s^+}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial^2 c_s^+}{\partial r^2}(r, t) \right]$$



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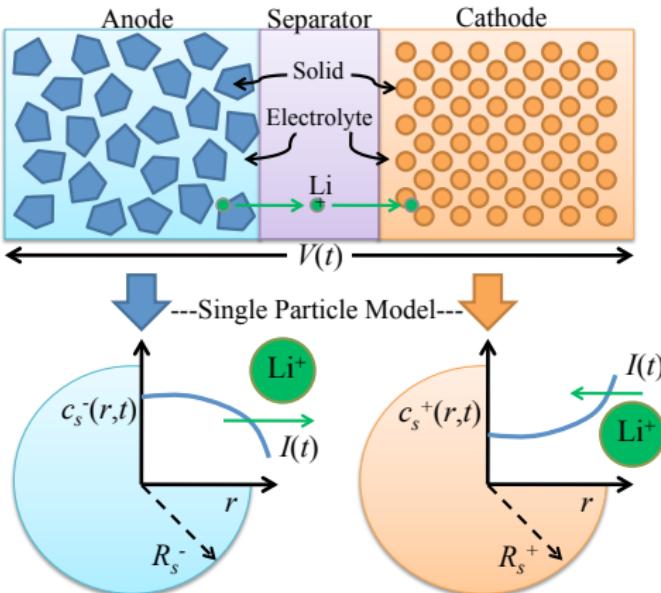
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Boundary conditions:

$$\frac{\partial c_s^-}{\partial r}(R_s^-, t) = -\rho^+ I(t)$$

$$\frac{\partial c_s^+}{\partial r}(R_s^+, t) = \rho^- I(t)$$



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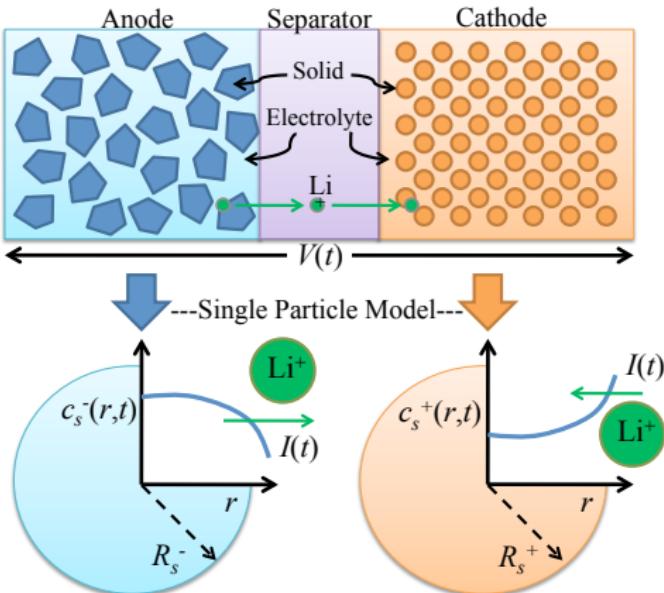
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Voltage Output Function:

$$V(t) = h(c_{ss}^-(t), c_{ss}^+(t), I(t); \theta)$$



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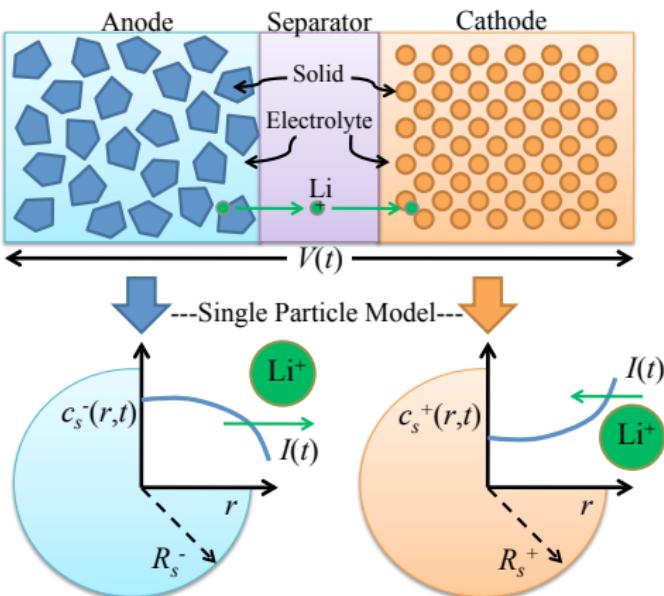
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Definitions

- SOC: Bulk concentration SOC_{bulk} , Surface concentration $c_{ss}^-(t)$
- SOH: Physical parameters, e.g. ε , q , n_{Li} , R_f



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The **SOC** Estimation Problem

Problem Statement

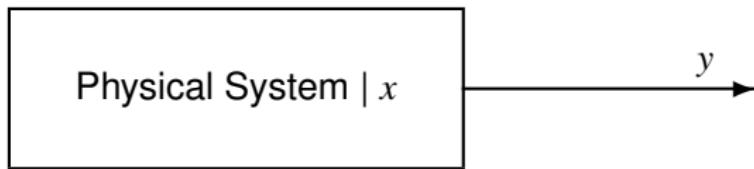
Estimate states $c_s^-(r, t), c_s^+(r, t)$ from measurements $I(t), V(t)$ and SPM

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Intro to Estimation

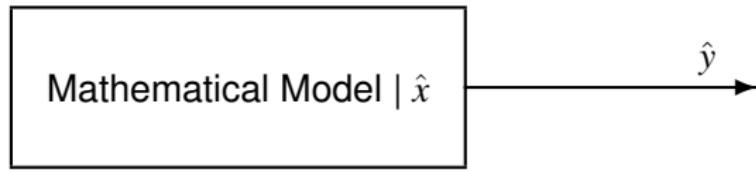
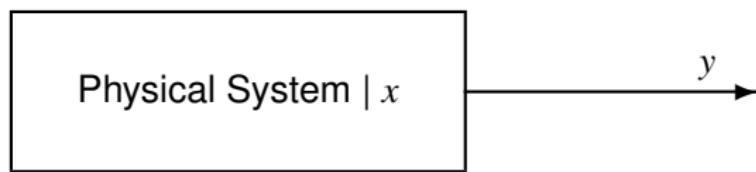


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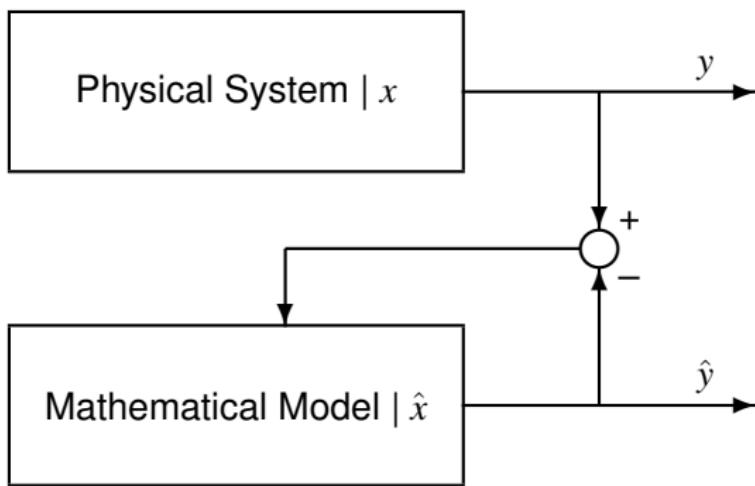


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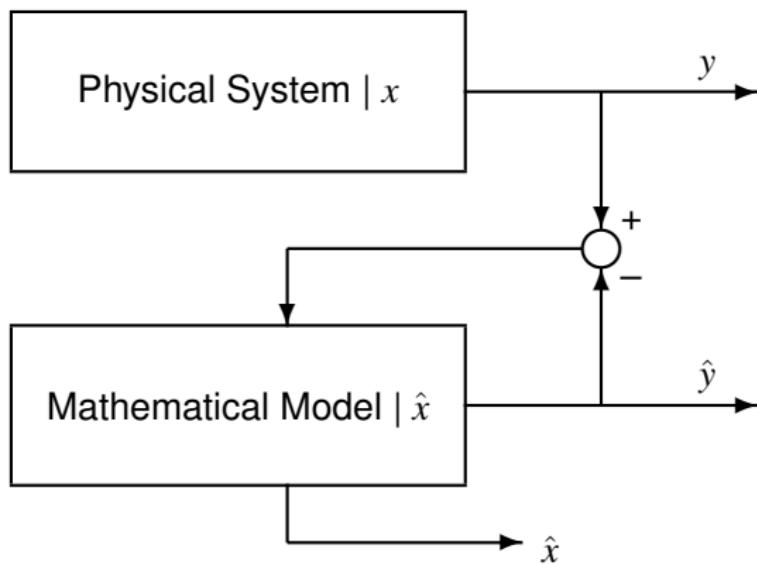


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Intro to Estimation



Backstepping PDE Estimator

Simplify the Math

- Model reduction to achieve observability
- Normalize time and space
- Scale spatial dimension

Backstepping PDE Estimator

Model Eqns. for Observer Design: $c(r, t)$

$$\begin{aligned}\frac{\partial c}{\partial t}(r, t) &= \varepsilon \frac{\partial^2 c}{\partial r^2}(r, t) && \text{Heat PDE} \\ c(0, t) &= 0\end{aligned}$$

$$\frac{\partial c}{\partial r}(1, t) - c(1, t) = -q\rho I(t)$$

$$\text{Measurement} = c(1, t) = \check{c}_{ss}^-(t)$$

Backstepping PDE Estimator

Estimator: $\hat{c}(r, t)$

$$\frac{\partial \hat{c}}{\partial t}(r, t) = \varepsilon \frac{\partial^2 \hat{c}}{\partial r^2}(r, t) + p_1(r) [c(1, t) - \hat{c}(1, t)]$$

$$\hat{c}(0, t) = 0$$

$$\frac{\partial \hat{c}}{\partial r}(1, t) - \hat{c}(1, t) = -q\rho I(t) + p_{10} [c(1, t) - \hat{c}(1, t)]$$

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$$\frac{\partial \hat{c}}{\partial r}(1, t) - \hat{c}(1, t) = -q\rho I(t) + p_{10} [c(1, t) - \hat{c}(1, t)]$$

Estimation Error Dynamics: $\tilde{c}(r, t) = c(r, t) - \hat{c}(r, t)$

$$\begin{aligned}\frac{\partial \tilde{c}}{\partial t}(r, t) &= \varepsilon \frac{\partial^2 \tilde{c}}{\partial r^2}(r, t) - p_1(r) \tilde{c}(1, t) \\ \tilde{c}(0, t) &= 0\end{aligned}$$

$$\frac{\partial \tilde{c}}{\partial r}(1, t) - \tilde{c}(1, t) = -p_{10} \tilde{c}(1, t)$$

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$$\frac{\partial \hat{c}}{\partial r}(1, t) - \hat{c}(1, t) = -q\rho I(t) + p_{10} [c(1, t) - \hat{c}(1, t)]$$

The Concept

$$\tilde{c}(r, t) = \tilde{w}(r, t) - \int_r^1 p(r, s) \tilde{w}(s) ds \quad \text{Backstepping Transformation}$$

$$\frac{\partial \tilde{w}}{\partial t}(r, t) = \varepsilon \frac{\partial^2 \tilde{w}}{\partial r^2}(r, t) + \lambda \tilde{w}(r, t) \quad \text{Exp. Stable Target System}$$

$$\tilde{w}(0, t) = 0$$

$$W(t) = \frac{1}{2} \int_0^1 \tilde{w}^2(x, t) dx$$

$$\frac{\partial \tilde{w}}{\partial r}(1, t) = \frac{1}{2} \tilde{w}(1, t)$$

$$\dot{W}(t) \leq -\gamma W(t)$$

Backstepping PDE Estimator

Estimator: $\hat{c}(r, t)$

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Kernel PDE

$$p(r, s) : \mathcal{D} \rightarrow \mathbb{R}, \quad \mathcal{D} = \{(r, s) | 0 \leq r \leq s \leq 1\}$$

$$\begin{aligned}p_{rr}(r, s) - p_{ss}(r, s) &= \frac{\lambda}{\varepsilon} p(r, s) & p_1(r) &= -p_s(r, 1) - \frac{1}{2} p(r, 1) \\ p(0, s) &= 0 & p_{10} &= \frac{3 - \lambda/\varepsilon}{2} \\ p(r, r) &= \frac{\lambda}{2\varepsilon} r\end{aligned}$$

Backstepping PDE Estimator

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Explicit Solution to Estimator Gains

$$p_1(r) = \frac{-\lambda r}{2\varepsilon z} \left[I_1(z) - \frac{2\lambda}{\varepsilon z} I_2(z) \right] \quad \text{where } z = \sqrt{\frac{\lambda}{\varepsilon}(r^2 - 1)}$$
$$p_{10} = \frac{1}{2} \left(3 - \frac{\lambda}{\varepsilon} \right)$$

The SOH Estimation Problem

Problem Statement

Estimate physical parameters from measurements $I(t)$, $V(t)$ and SPM

The SOH Estimation Problem

Problem Statement

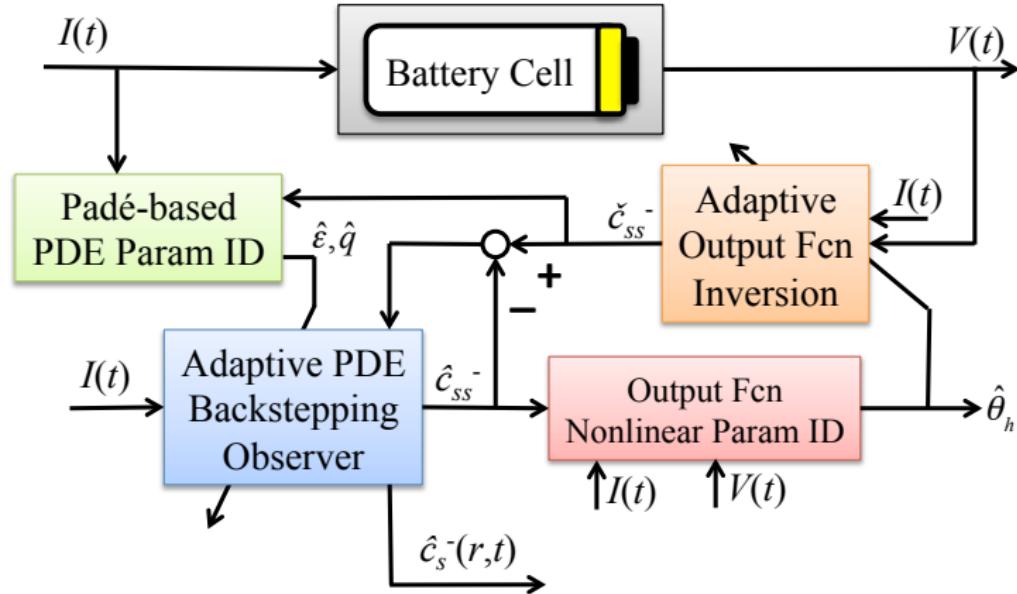
Estimate physical parameters from measurements $I(t)$, $V(t)$ and SPM

Relate uncertain parameters to SOH-related concepts

- Capacity fade
- Power fade

Adaptive Observer

Combined State & Parameter Estimation



Output Function Nonlinear Parameter ID

Nonlinearly Parameterized Output

$$V(t) = h(c_{ss}^-(t), I(t); \theta)$$

- θ contains many parameters
- Linear dependence b/w parameters?

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Identifiability Analysis

- Linearly independent parameter subset : $\theta_h = [n_{Li}, R_f]^T$
 - n_{Li} : Moles of cyclable Li (Capacity Fade)
 - R_f : Resistance of current collectors, electrolyte, etc. (Power Fade)

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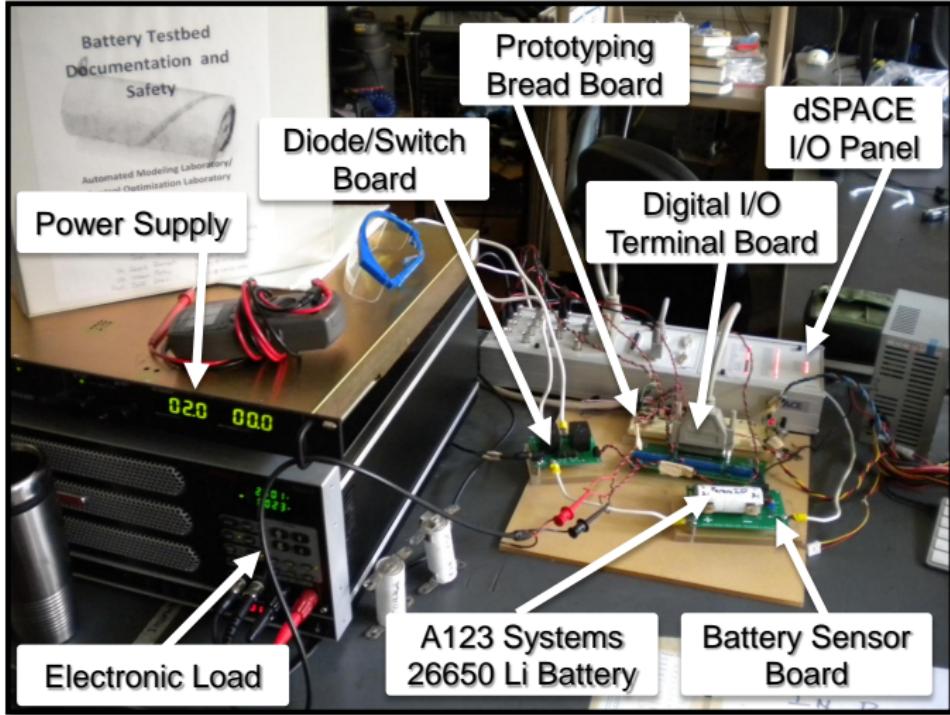
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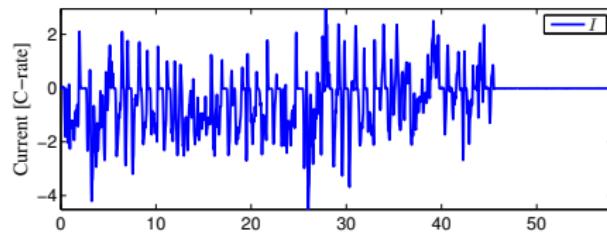
Apply nonlinear recursive least squares to θ_h

Custom-Built Battery-in-the-Loop Testbed



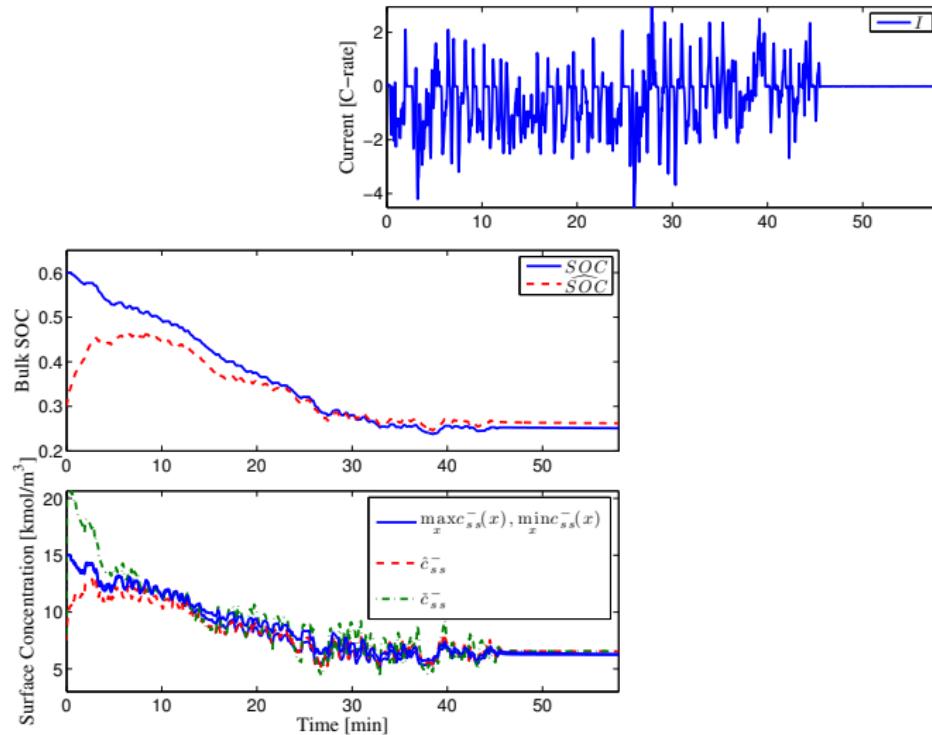
Results

UDDS Drive Cycle Input



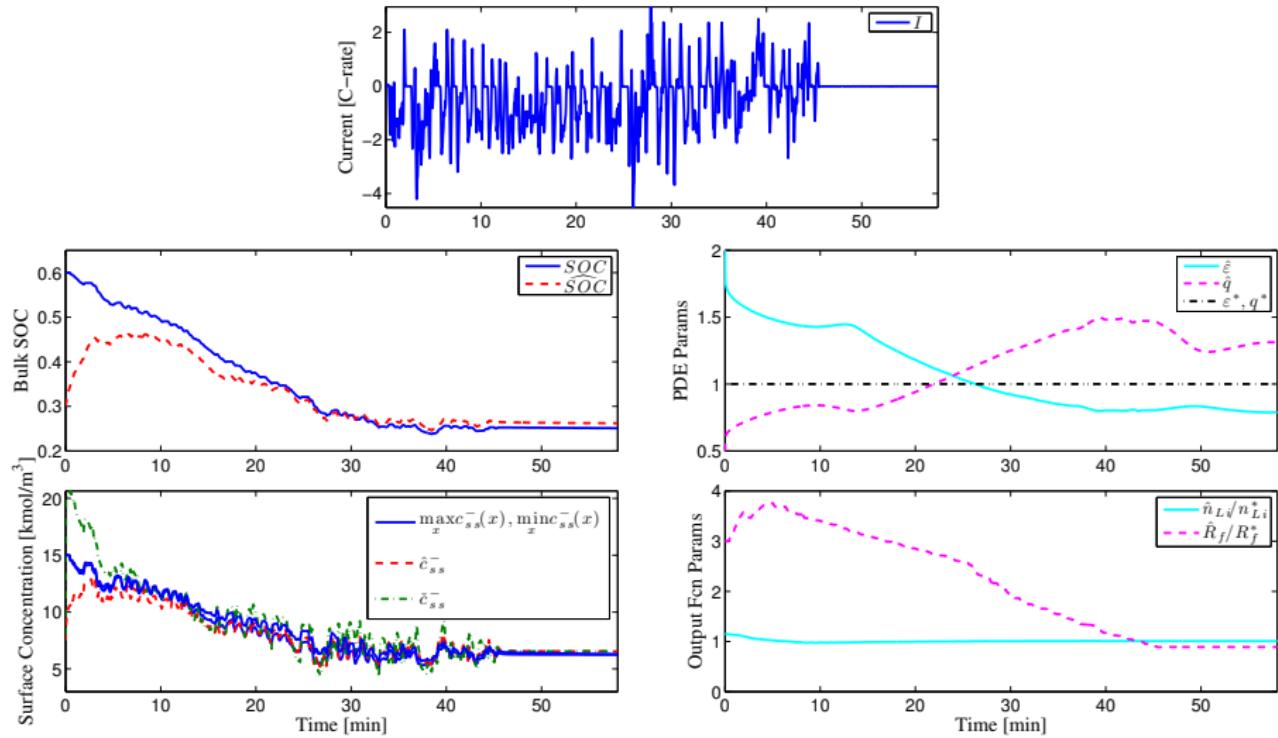
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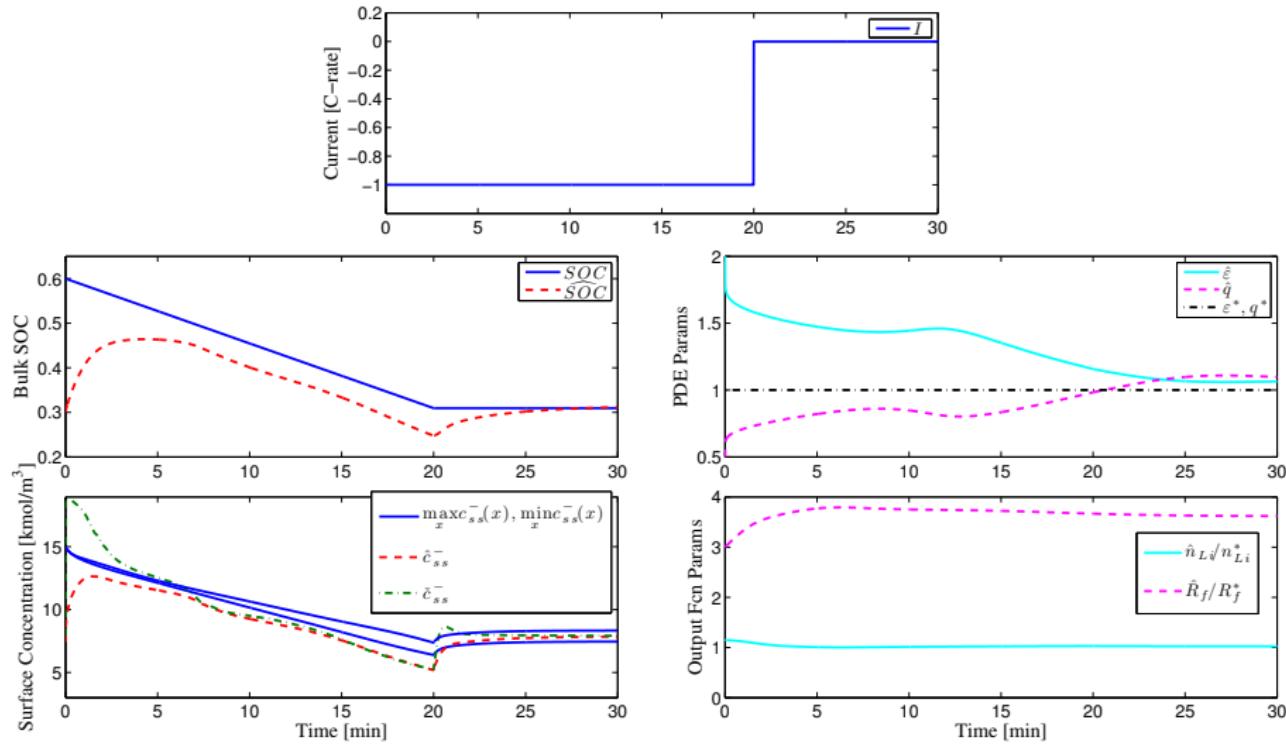
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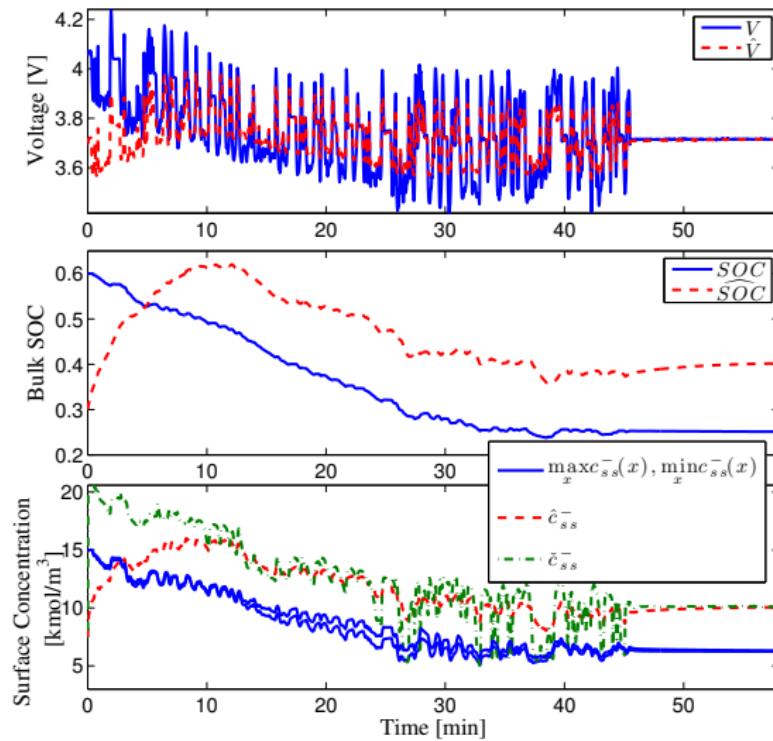
Results

Constant 1C Discharge

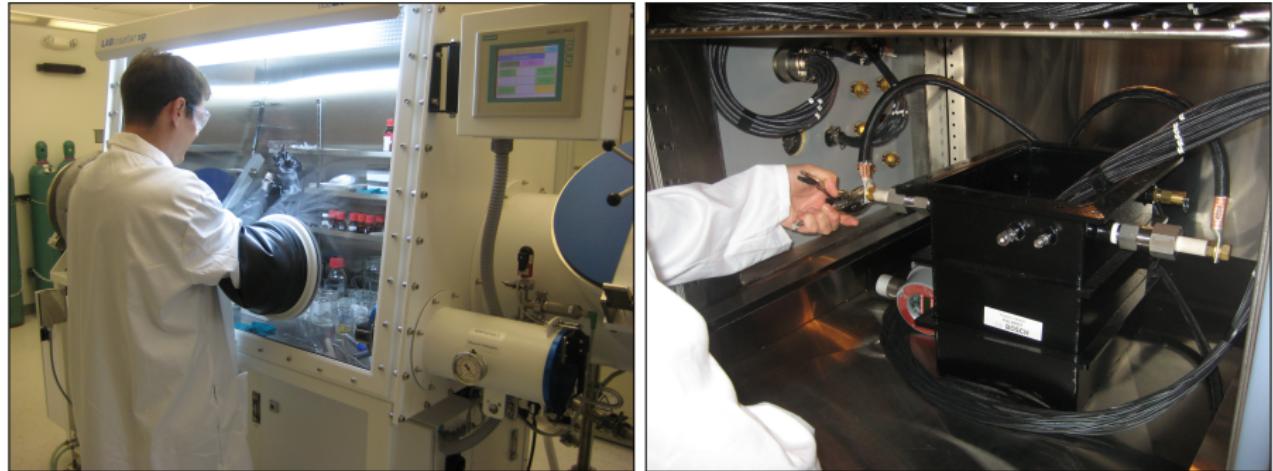


Results

No Parameter Adaption - Bias in State Estimates



Experimental Testing | ARPA-E AMPED Program



BOSCH



COBASYS

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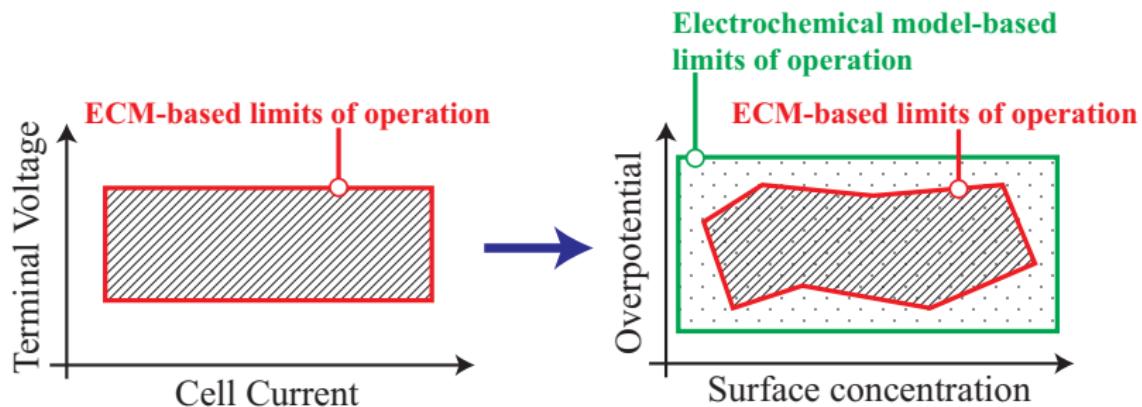
Operate Batteries at their Physical Limits



Operate Batteries at their Physical Limits

Problem Statement

Given accurate state estimates, govern the electric current such that safe operating constraints are satisfied.

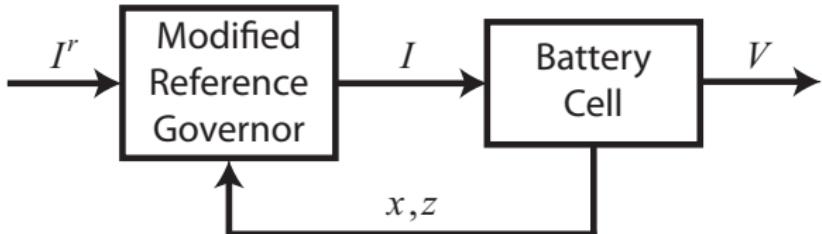


Constraints

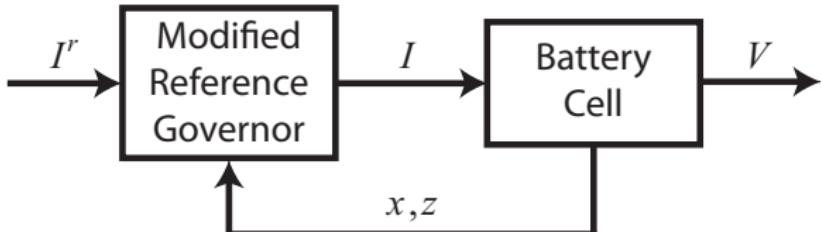
Variable	Definition	Constraint
$I(t)$	Current	Power electronics limits
$c_s^\pm(x, r, t)$	Li concentration in solid	Saturation/depletion
$\frac{\partial c_s^\pm}{\partial r}(x, r, t)$	Li concentration gradient	Diffusion-induced stress
$c_e(x, t)$	Li concentration in electrolyte	Saturation/depletion
$T(t)$	Temperature	High/low temps accel. aging
$\eta_s(x, t)$	Side-rxn overpotential	Li plating, dendrite formation

Each variable, y , must satisfy $y_{\min} \leq y \leq y_{\max}$.

The Algorithm: Modified Reference Governor (MRG)



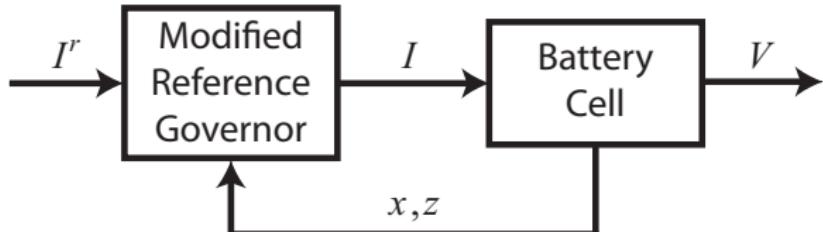
The Algorithm: Modified Reference Governor (MRG)



MRG Equations

$$I[k+1] = \beta^*[k]I^r[k], \quad \beta^* \in [0, 1],$$
$$\beta^*[k] = \max \{\beta \in [0, 1] : (x(t), z(t)) \in \mathcal{O}\}$$

The Algorithm: Modified Reference Governor (MRG)



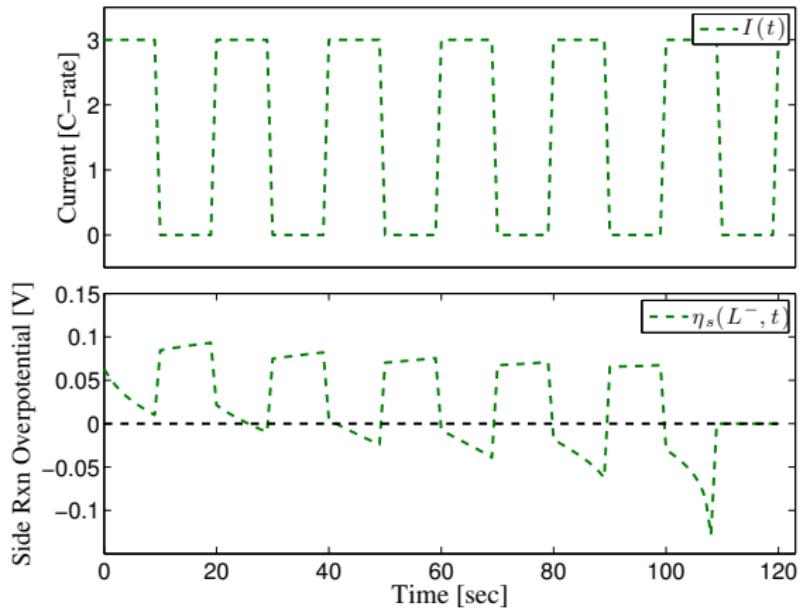
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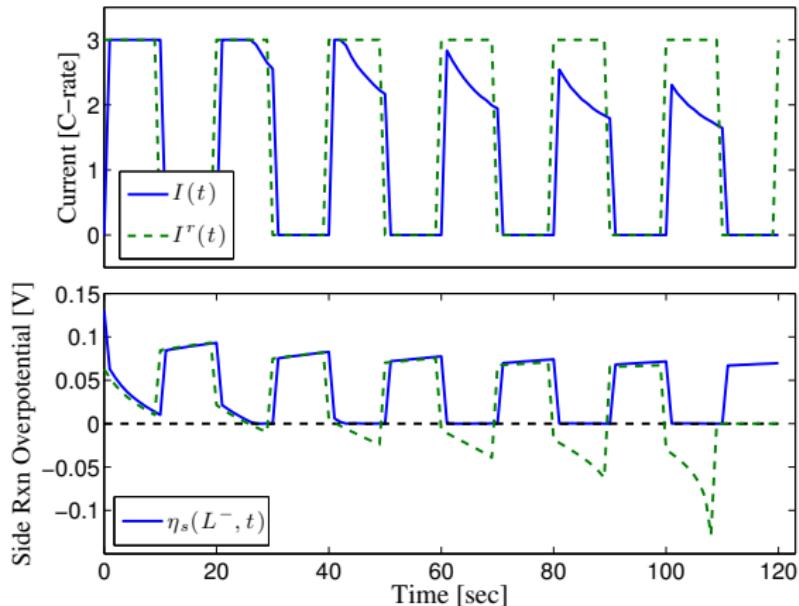
Def'n: Admissible Set \mathcal{O}

$$\mathcal{O} = \{(x(t), z(t)) : y(\tau) \in \mathcal{Y}, \forall \tau \in [t, t + T_s]\}$$
$$\dot{x}(t) = f(x(t), z(t), \beta I^r)$$
$$0 = g(x(t), z(t), \beta I^r)$$
$$y(t) = C_1 x(t) + C_2 z(t) + D \cdot \beta I^r + E$$

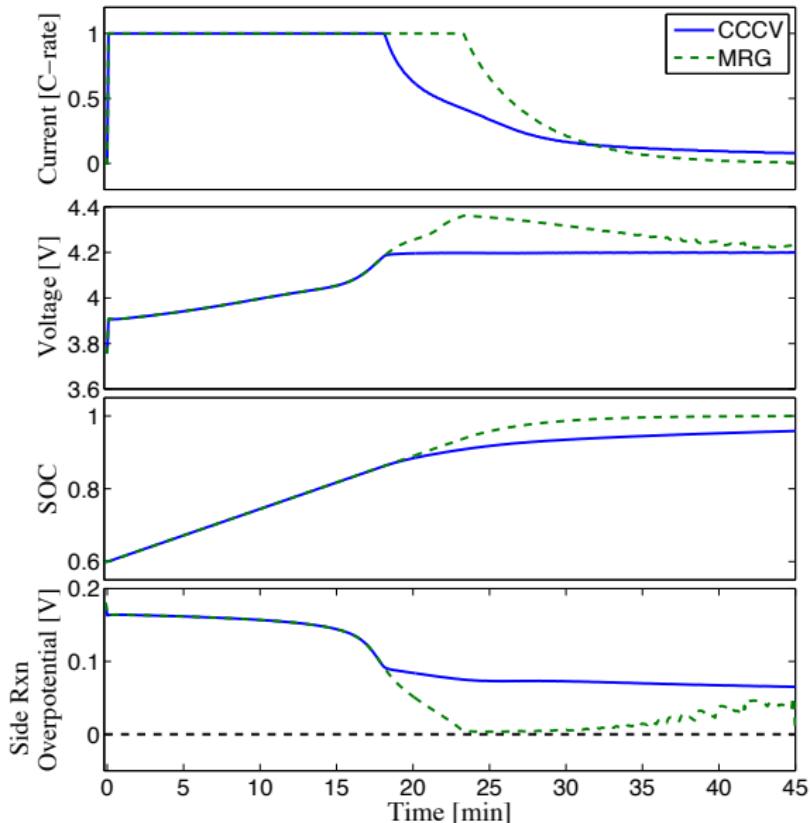
Constrained Control of EChem States



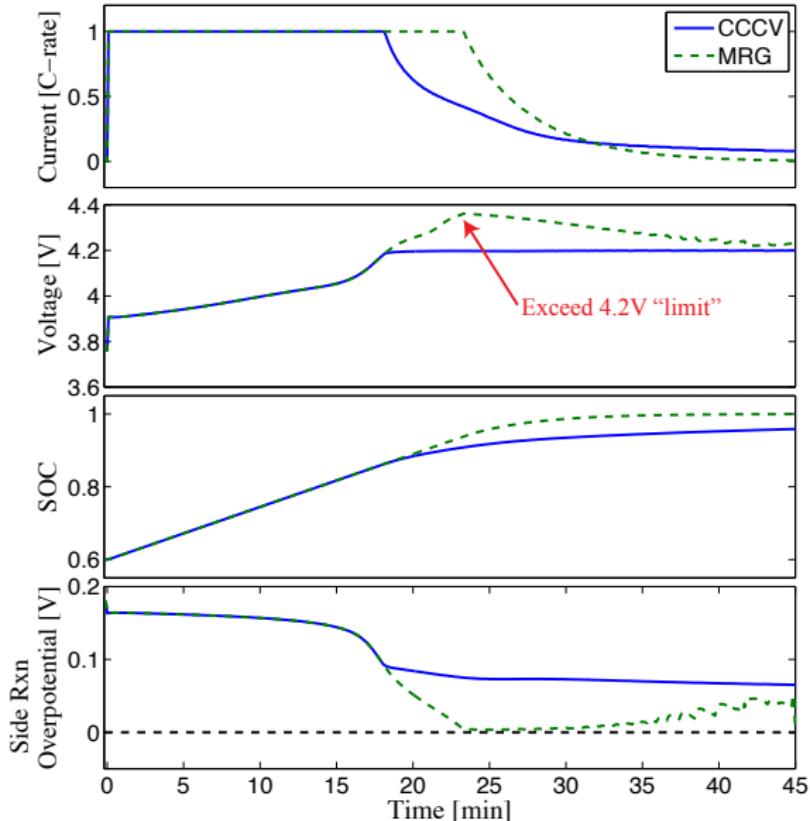
Constrained Control of EChem States



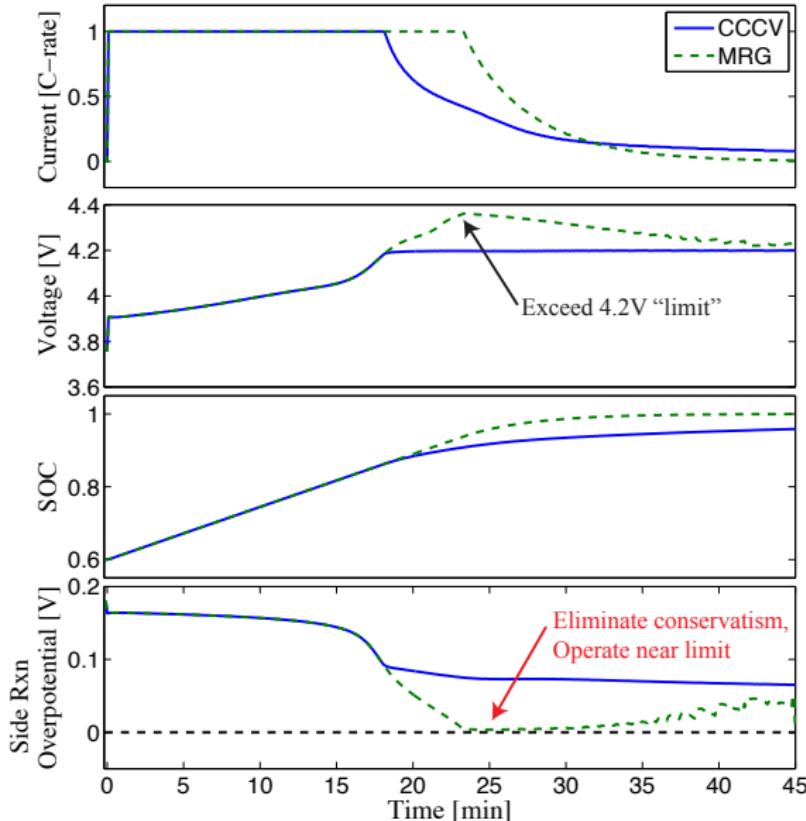
Application to Charging



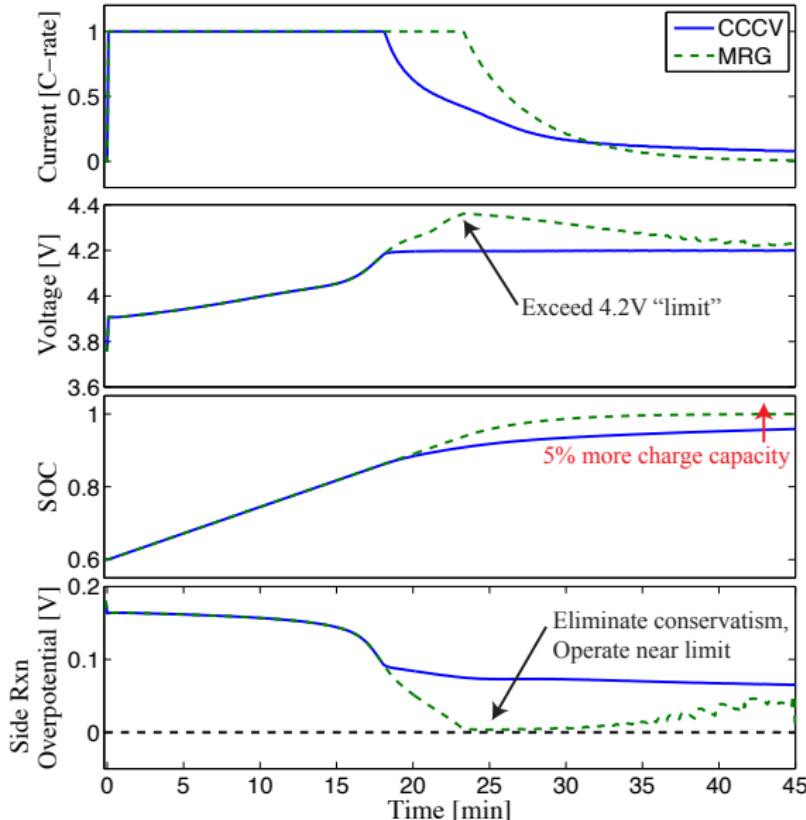
Application to Charging



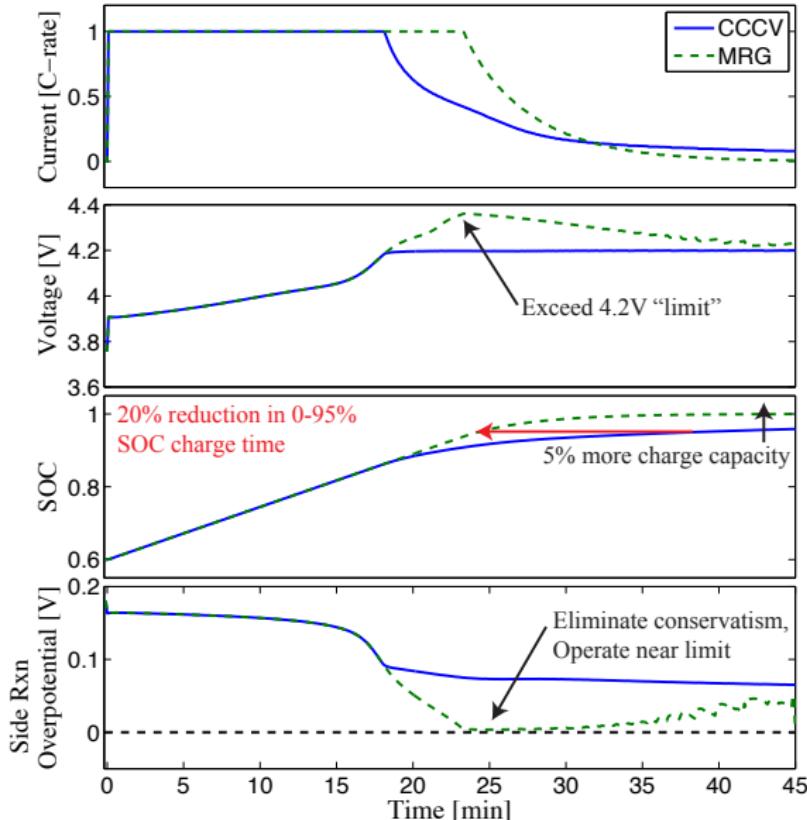
Application to Charging



Application to Charging



Application to Charging



Outline

1

Batteries

- [Electrochemical Modeling] Incorporating Physics
- [SOC/SOH Estimation] Looking Inside w/ Models, Meas., and Math
- [Constrained Control] Operate at the Limits, Safely
- [PHEV Power Management] Max eMPG and Max Batt Life

2

Demand Response in Smart Grids

3

Future

PHEV Power Management

Problem Statement

Design a supervisory control algorithm for plug-in hybrid electric vehicles (PHEVs) that splits **engine** and **battery** power **in some optimal sense**.



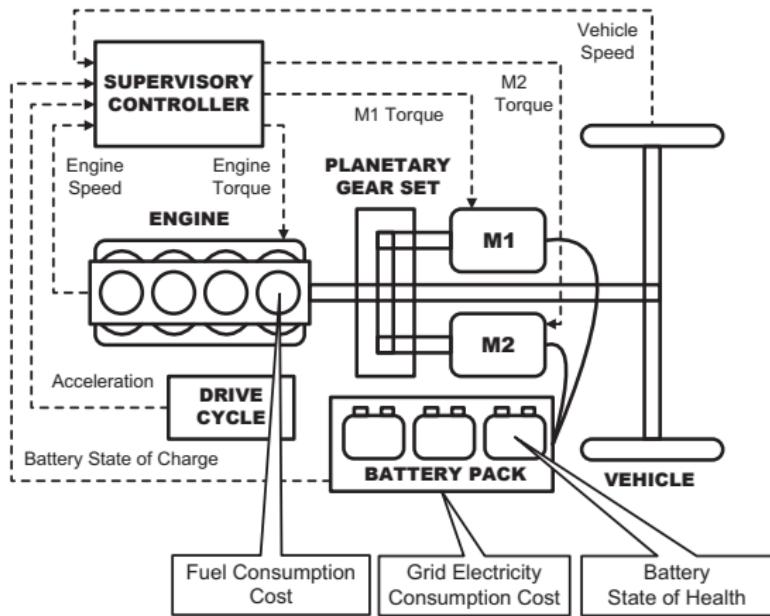
J. Voelcker, "Plugging Away in a Prius," *IEEE Spectrum*, vol. 45, pp. 30-48, 2008.



Power-Split PHEV Model

Ex: Toyota Prius, Ford Escape Hybrid

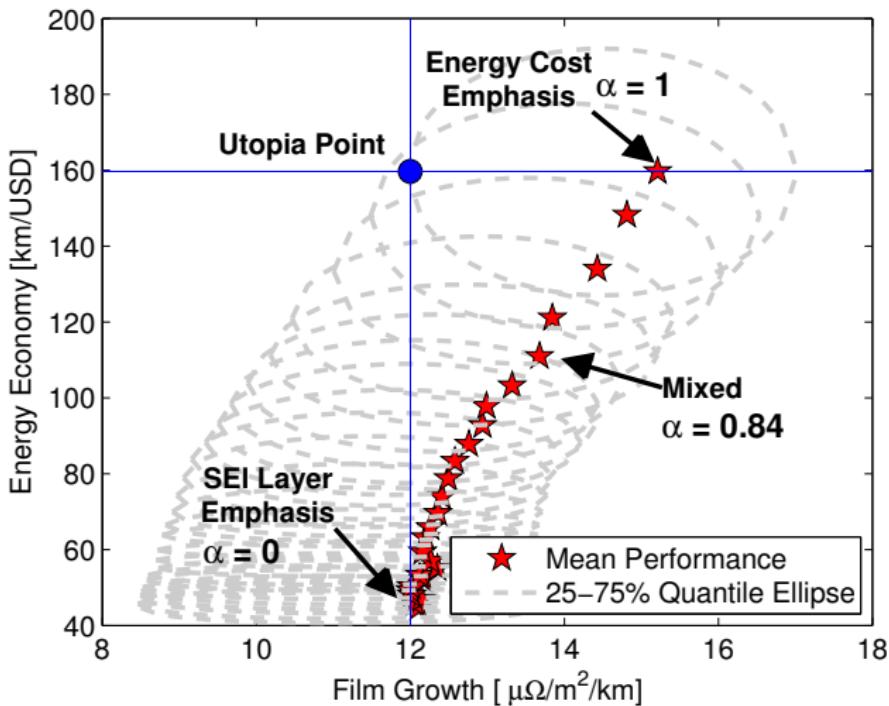
- Control Inputs
 - Engine Torque
 - M1 Torque
- State Variables
 - Engine speed
 - Vehicle speed
 - Battery SOC
 - Vehicle acceleration (Markov Chain)



Control Optimization: Minimize energy consumption cost AND battery aging

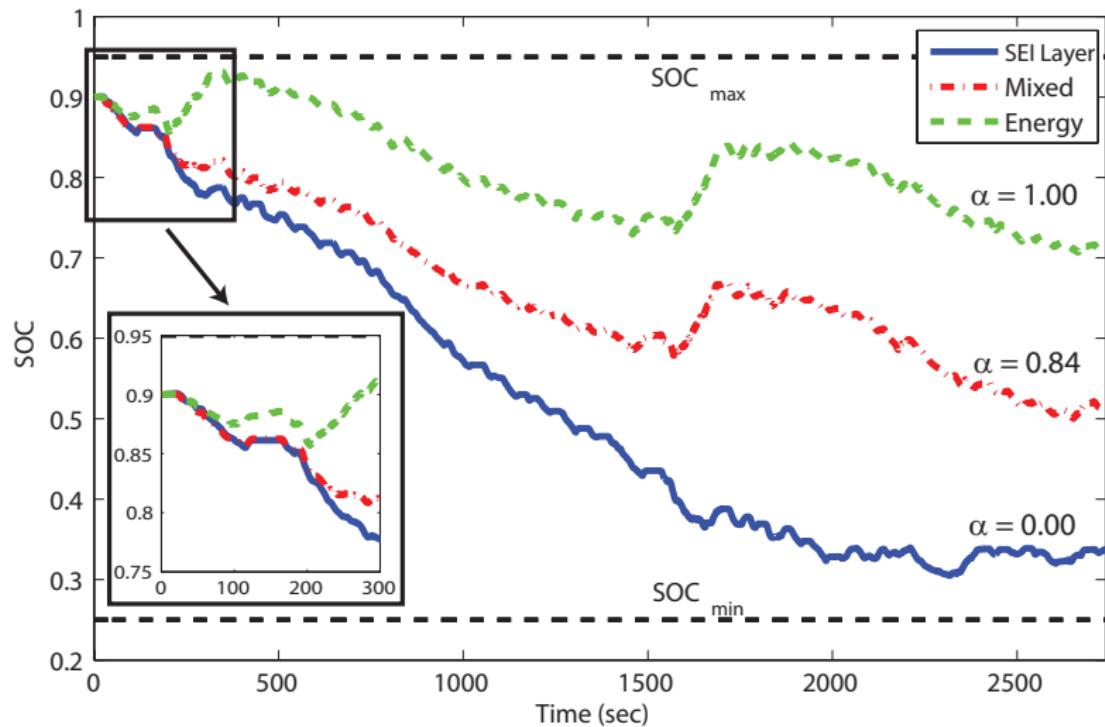
Pareto Set of Optimal Solutions

Anode-side SEI Layer Growth



SOC Trajectories

Anode-side SEI Layer Growth | UDDSx2



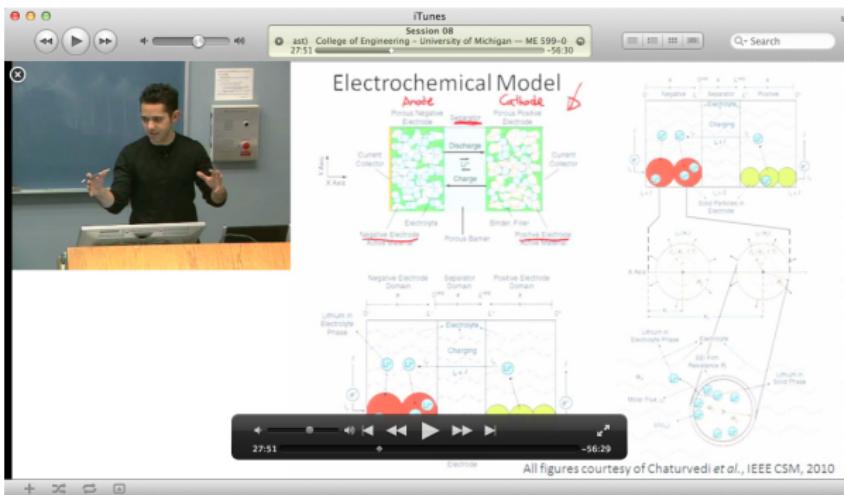
Battery Systems and Control Course

Funded by DOE-ARRA, University of Michigan

Enrollment

- Winter 2010: 59 + 5 distance
- Winter 2011: 50 + 26 distance
- ME, EE, ChemE, CS, Energy Systems, MatSci, Physics, Math

- Undergraduates
- Graduate students
- Professionals
 - Tesla Motors, General Motors, Roush, US Army



Summary of Contributions

Simultaneous SOC/SOH estimation
of physically meaningful variables via electrochemical models,
PDE estimation theory, and adaptive control.

Constrained control of batteries
via an electrochemical model
and reference governors.

PHEV power management
using electrochemical models
and stochastic optimal control.

Impact through education.

Energy Crisis Solutions

Energy storage (e.g., batteries)	Demand-side management (e.g., smart grids)
-------------------------------------	---

Energy Crisis Solutions

Energy storage
(e.g., batteries)

Demand-side management
(e.g., smart grids)

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3 Future

The Renewable Integration Problem

Needs: 33% renewables in CA by 2020

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Obstacle: Must install 4 GW reserve capacity to support variability

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Some Interesting Facts

Thermostatically
Controlled Loads
(TCLs)

50% of U.S. electricity consumption is TCLs
11% of thermostats are programmed
Comfort is loosely coupled with control

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The Punchline

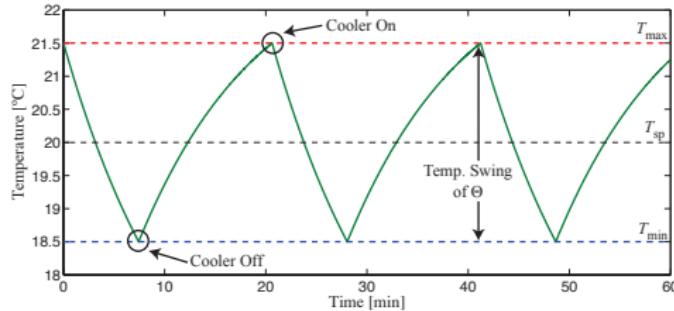
Flexible loads (e.g. TCLs) can absorb variability in renewable generation

Modeling Aggregations of TCLs

Individual TCL
models



(Tens of) Thousands
of hybrid ODEs

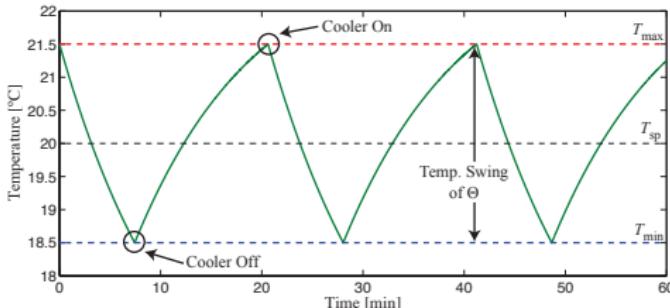


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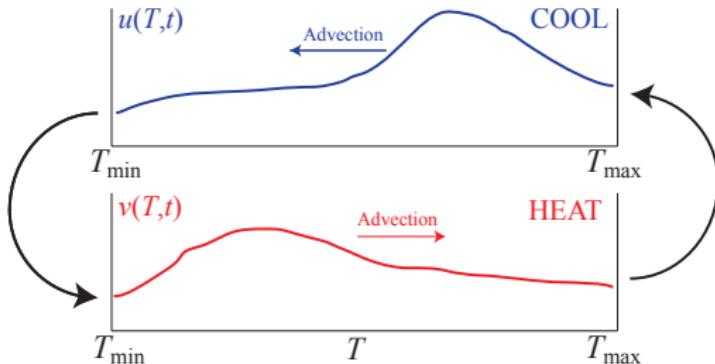
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Model aggregations
of TCLs



Two coupled linear
PDEs

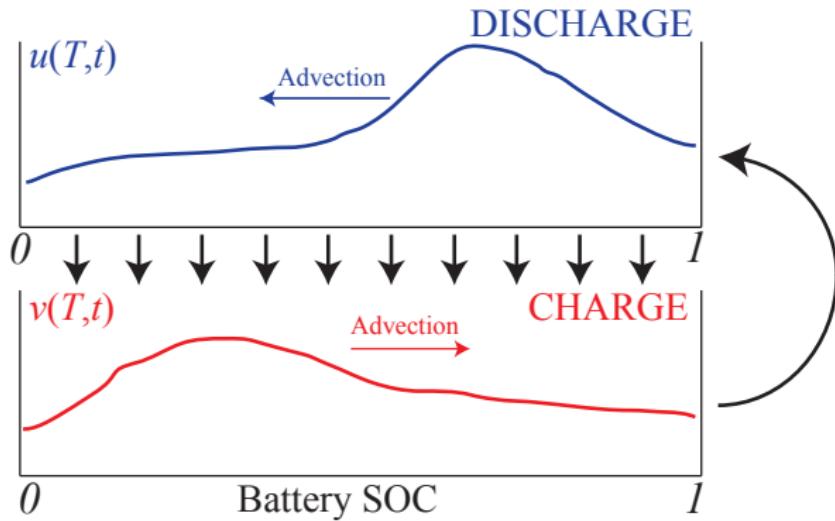


$u(T, t)$	# TCLs/°C, in COOL state, @ temp T , time t
$v(T, t)$	# TCLs/°C, in HEAT state, @ temp T , time t

Modeling Aggregated PEVs

Main Idea: Mathematically model as coupled linear PDEs

- | | |
|-----------|--|
| $u(T, t)$ | # PEVs / SOC, in DISCHARGE state , @ SOC x , time t |
| $v(T, t)$ | # PEVs / SOC, in CHARGE state , @ SOC x , time t |



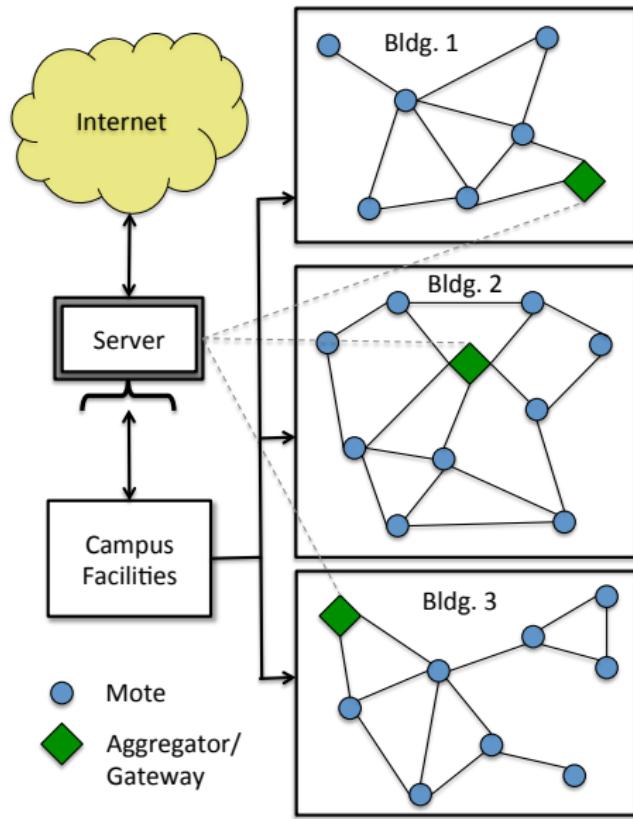
University Campus: A Living Laboratory

Goal: Smart Campus

- ① Deploy temp. wireless sensor network
- ② Verify models and estimation algorithms
- ③ Derive control algorithms
- ④ Implement via campus facilities & management



Libelium Waspmotes and Meshlium Gateway



Outline

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Future: EChem-based Battery Management Systems

Vision: Enhanced energy, power, chg times, life from existing batts.

Open technical problems for BMS:

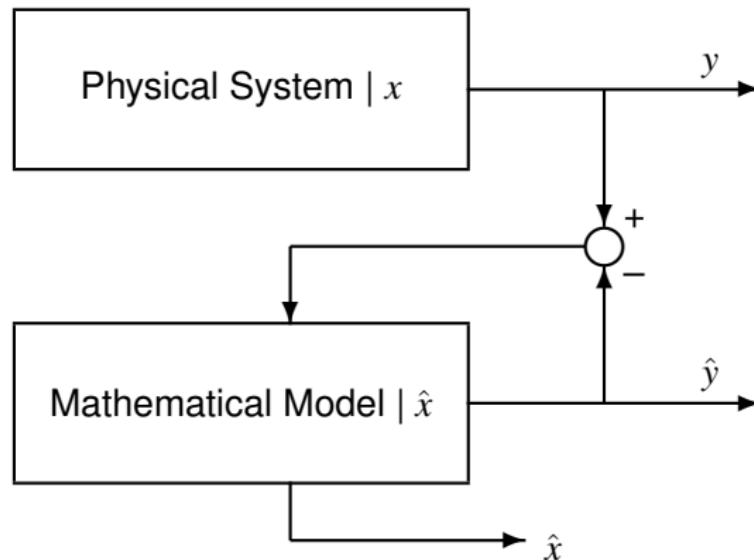
- (1) State observability

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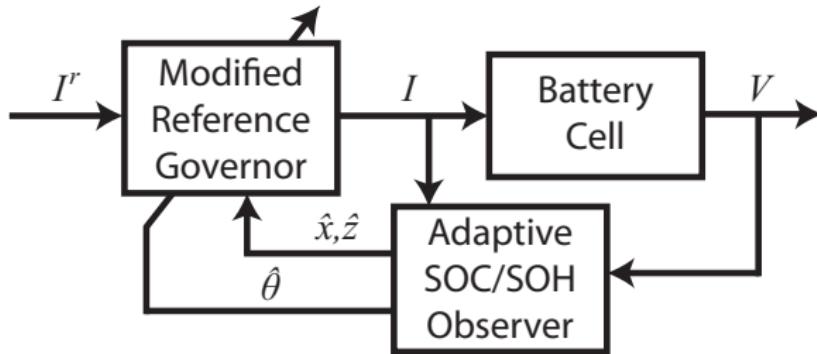


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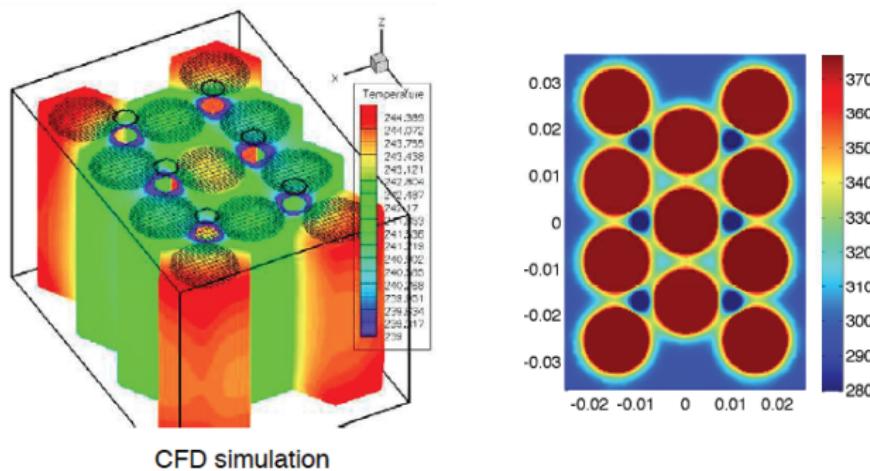


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Collaborators:

- UCSD, University of Michigan
- Bosch RTC, Ford Motor Company
- Auto Lab, UofM Energy Institute, US-China Clean Energy Research Center, Automotive Research Center, InterPro

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Funding:

- ARPA-E AMPED
- DoD HESM

Future: Solve EV Mobility - Modular Battery Packs

Goal: Solve range anxiety

Future: Solve EV Mobility - Modular Battery Packs

Fast Charging



Battery Swapping



Future: Solve EV Mobility - Modular Battery Packs

Modular User-Replaceable Batts

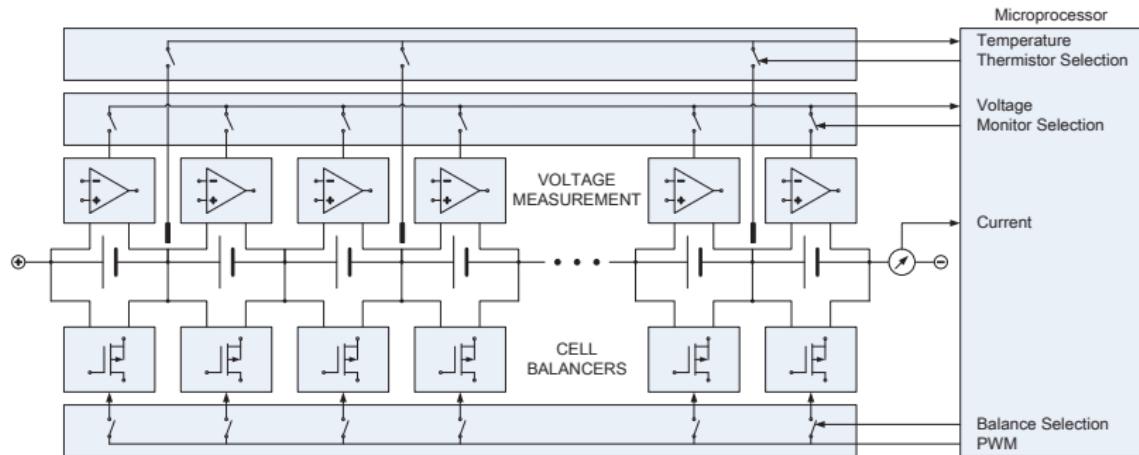


Scalable to EVs?

Future: Solve EV Mobility - Modular Battery Packs

Key components:

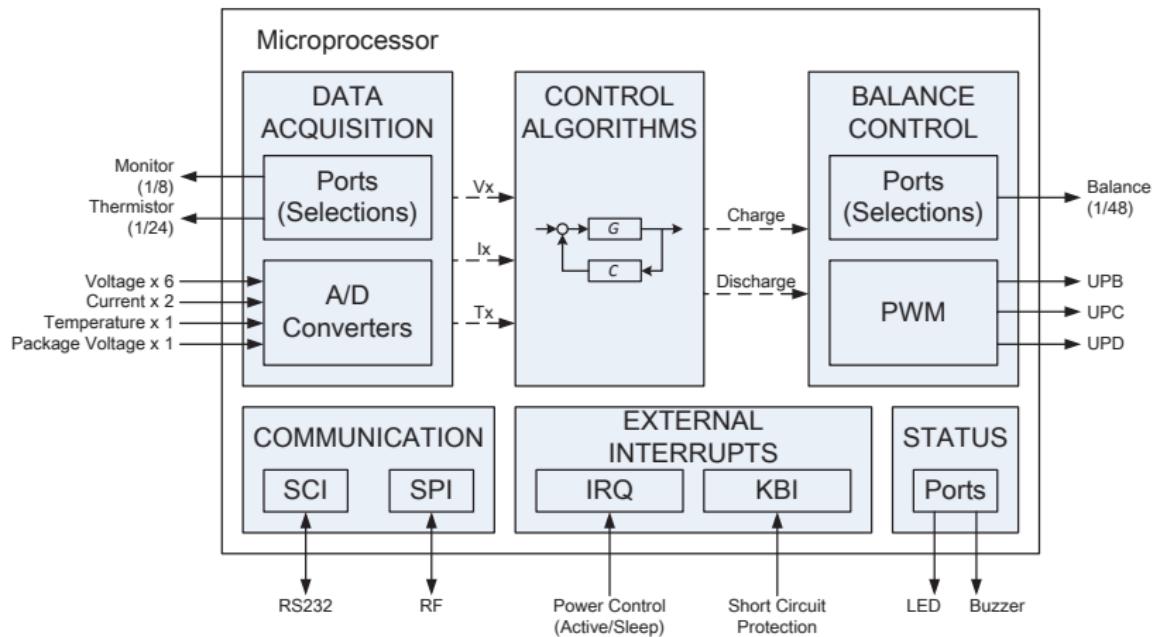
(1) Battery Autonomous Architecture (BAM)



Future: Solve EV Mobility - Modular Battery Packs

Key components:

- (1) Battery Autonomous Architecture (BAM)
- (2) Battery Management System (BMS)



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Key components:

- (1) Battery Autonomous Architecture (BAM)
- (2) Battery Management System (BMS)
- (3) Economics ↔ Design

- Financial model?
- Lease from third-party owner?
- Lower acquisition & operational costs?

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Funding:

- ARPA-E RANGE
- Automotive Industry

Future: Demand-side Management

Vision: A fundamental systems science for demand response.

Apps: Buildings & PEVs

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Open technical problems for Coupled PDEs:

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Open technical problems for Coupled PDEs:

(1) System ID & State Estimation

$$u_t(x, t) = \alpha\lambda(x)u_x + \alpha u + \beta u_{xx}$$

$$v_t(x, t) = -\alpha\mu(x)v_x + \alpha v + \beta v_{xx}$$

$$u(1, t) = q_1 v(1, t), \quad u_x(0, t) = -v_x(0, t)$$

$$v(0, t) = q_2 u(0, t), \quad v_x(1, t) = -u_x(1, t)$$

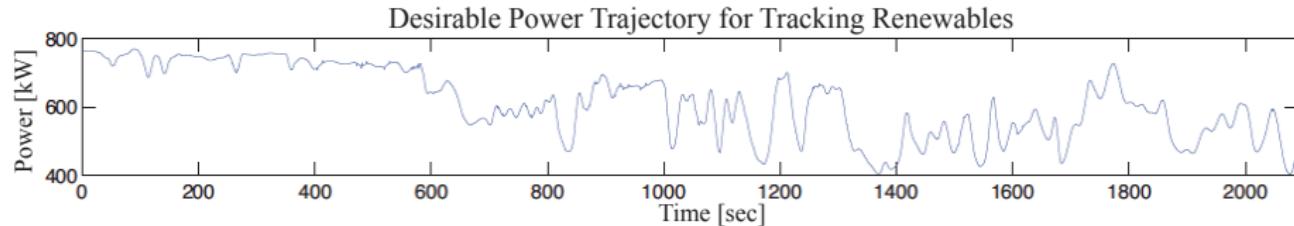
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Open technical problems for Coupled PDEs:

- (1) System ID & State Estimation
- (2) Reference Tracking Control via Temperature Setpoint Changes



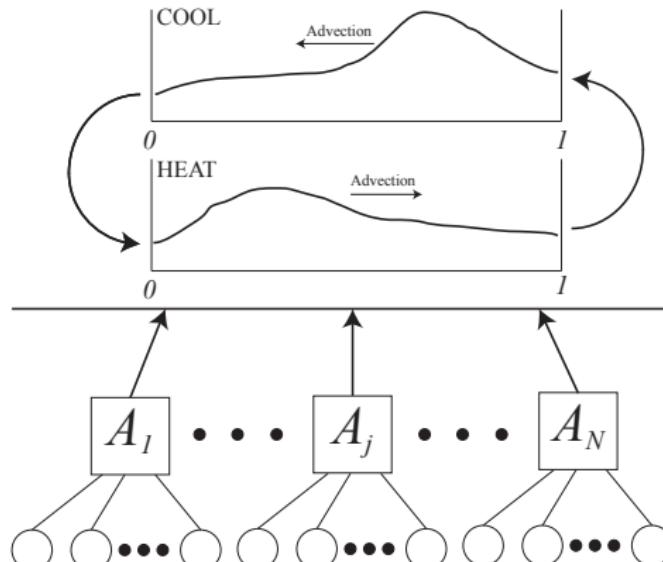
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- (3) Hierarchical Control Architecture



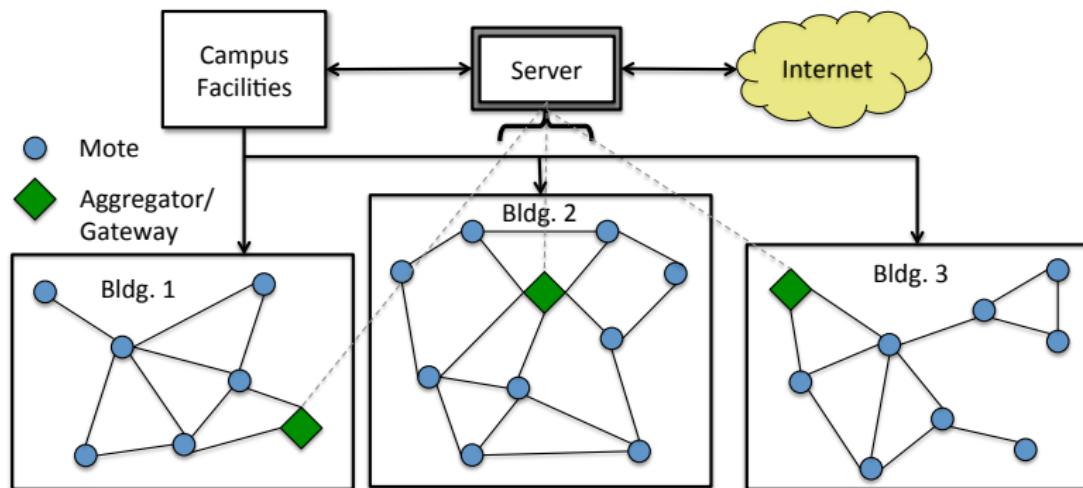
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Funding:

- DOE EERE
- NSF EPAS, CPS

Courses I could teach

Undergrad

ME 240 - Intro to Dynamic and Vibrations

ME 360 - Modeling, Analysis, and Control of Dynamic Systems

ME 433 - Advanced Energy Solutions

ME 461 - Automatic Control

Graduate

ME 502 - Methods of Differential Equations in Mechanics

ME 561 - Design of Digital Control Systems

ME 564 - Linear Systems Theory

ME 566 - Modeling, Analysis, and Control of HEVs

ME 569 - Control of Adv. Powertrain Systems

ME 662 - Advanced Nonlinear Control

Special Courses

ME 499/599 - Battery Systems and Control

NEW - Control of Distributed Parameter Systems

my lab...

Themes: systems & control | energy & transportation

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Key Tools:

- PDE modeling
- Lyapunov stability
- Adaptive control
- Optimization

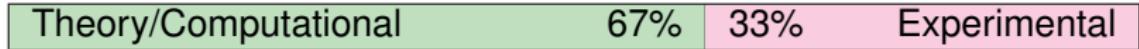
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Balance



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Outreach, Diversity, and Inclusion

- SHPE, NSBE, SWE
- SMES-G, Grad Admissions

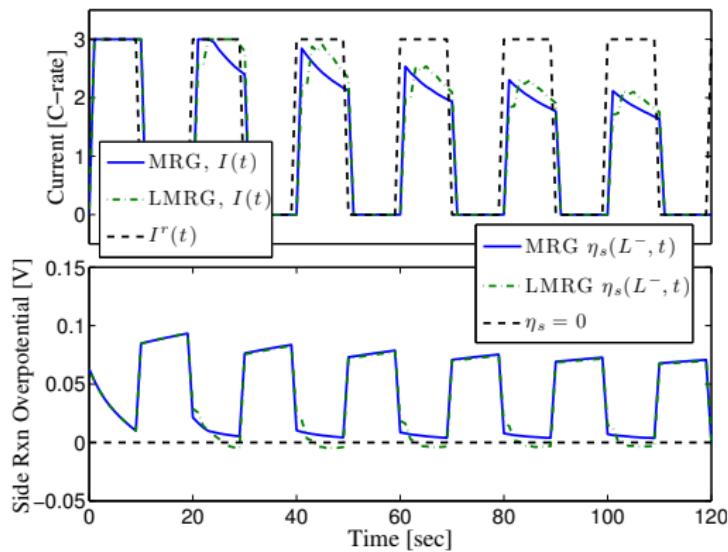
Publications available at <http://flyingv.ucsd.edu/smoura/>
smoura@ucsd.edu



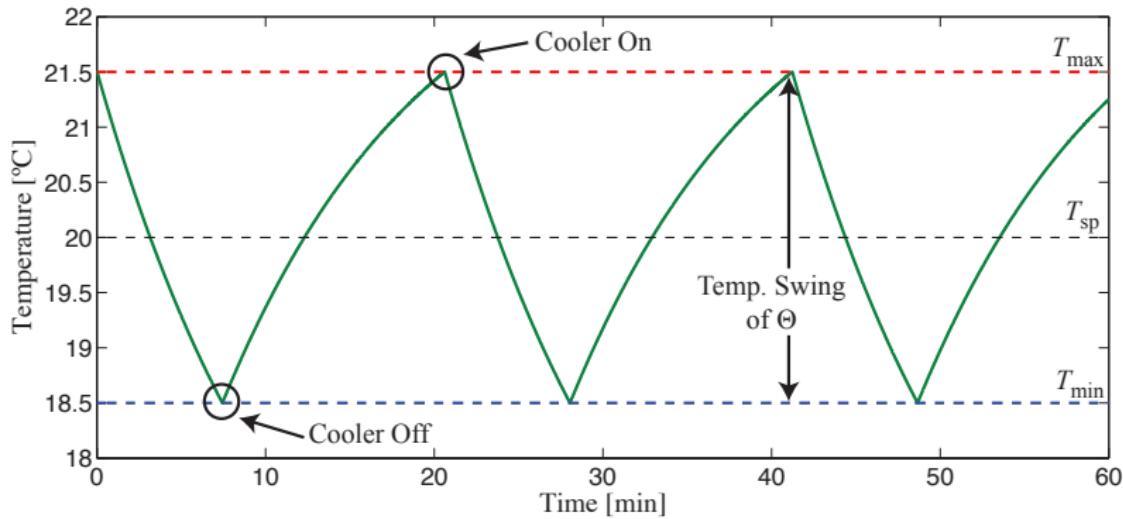
Linear Modified Reference Governor

Modified Reference Governor (MRG) : Simulations

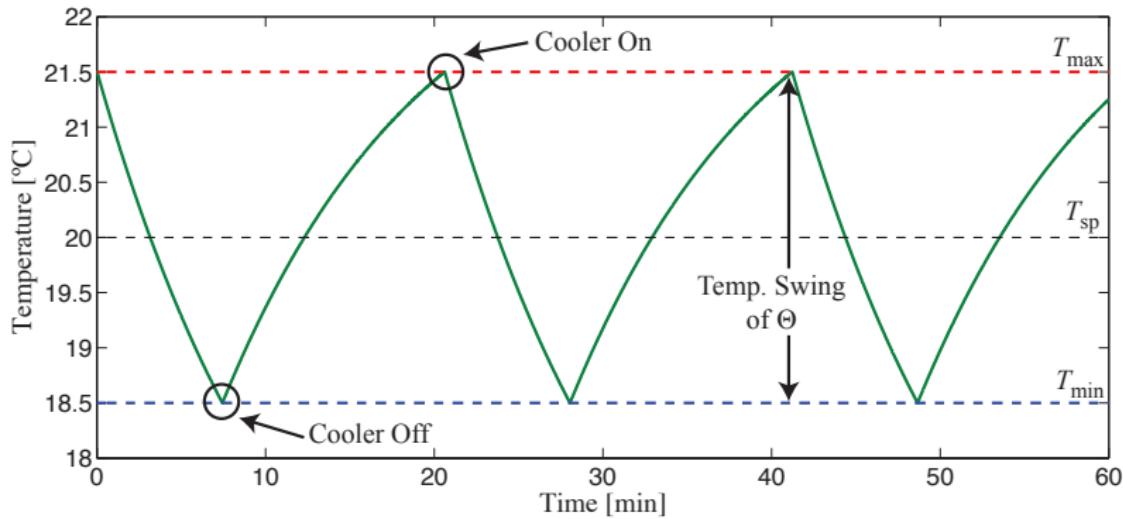
Linearized MRG (LMRG) : Explicit function evaluation



Modeling TCLs



Modeling TCLs



$$\dot{T}_i(t) = \frac{1}{R_i C_i} [T_\infty - T_i(t) - s_i(t) R_i P_i], \quad i = 1, 2, \dots, N$$
$$s_i \in \{0, 1\}$$

Modeling Aggregated TCLs

Main Idea: Convert 1000+ ODEs into two coupled linear PDEs

Modeling Aggregated TCLs

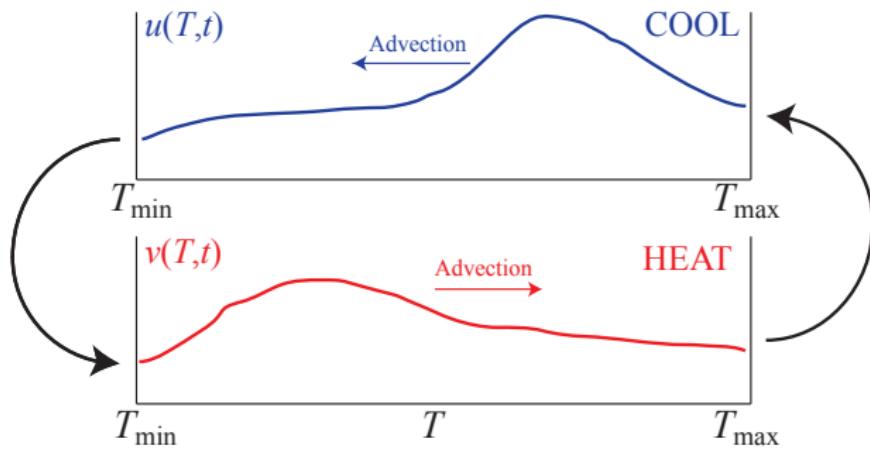
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$$\begin{array}{l|l} u(T, t) & \# \text{ TCLs} / {}^{\circ}\text{C}, \text{ in COOL state, @ temp } T, \text{ time } t \\ v(T, t) & \# \text{ TCLs} / {}^{\circ}\text{C}, \text{ in HEAT state, @ temp } T, \text{ time } t \end{array}$$

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Flux of TCLs in HEAT state:

#TCLs / sec

$$\psi(T, t) = v(T, t) \frac{dT}{dt}(t) = \frac{1}{RC} [T_\infty - T(t)] v(T, t)$$

Modeling Aggregated TCLs

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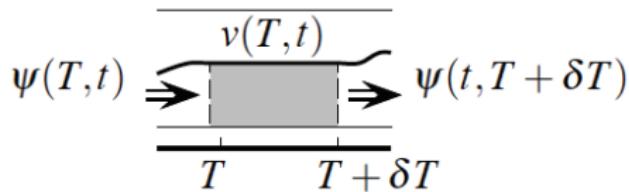
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Control volume:



Modeling Aggregated TCLs

Main Idea: Convert 1000+ ODEs into two coupled linear PDEs

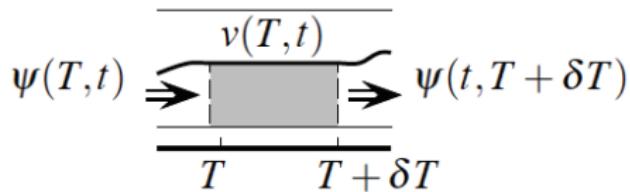
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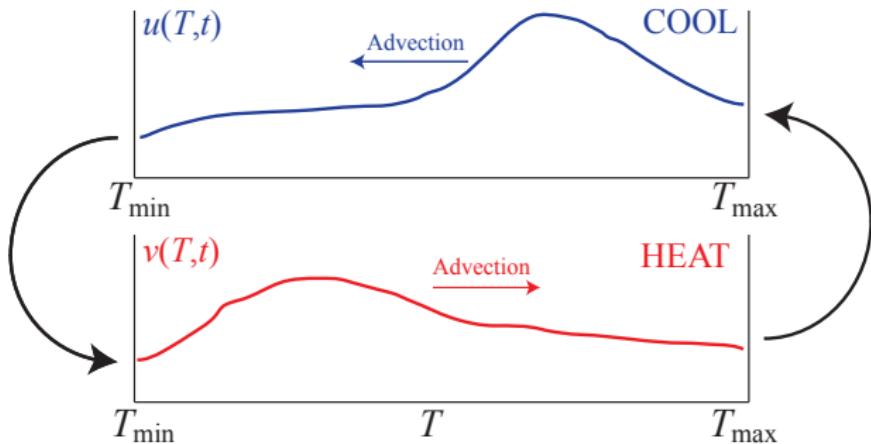
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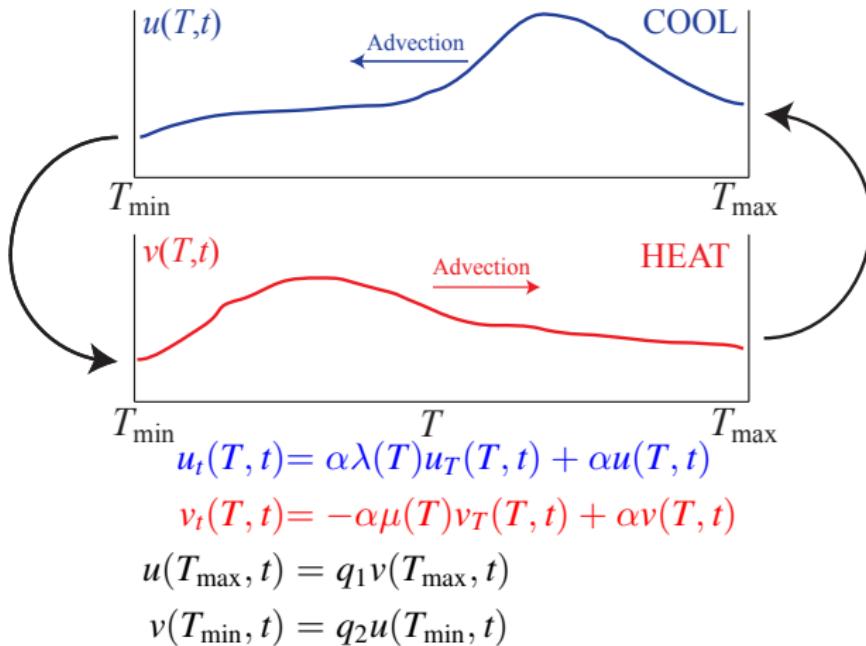


$$\begin{aligned} \frac{\partial v}{\partial t}(T, t) &= \lim_{\delta T \rightarrow 0} \left[\frac{\psi(T + \delta T, t) - \psi(T, t)}{\delta T} \right] \\ &= \frac{\partial \psi}{\partial T}(T, t) \\ &= -\frac{1}{RC} [T_\infty - T(t)] \frac{\partial v}{\partial T}(T, t) + \frac{1}{RC} v(T, t) \end{aligned}$$

PDE Model of Aggregated TCLs

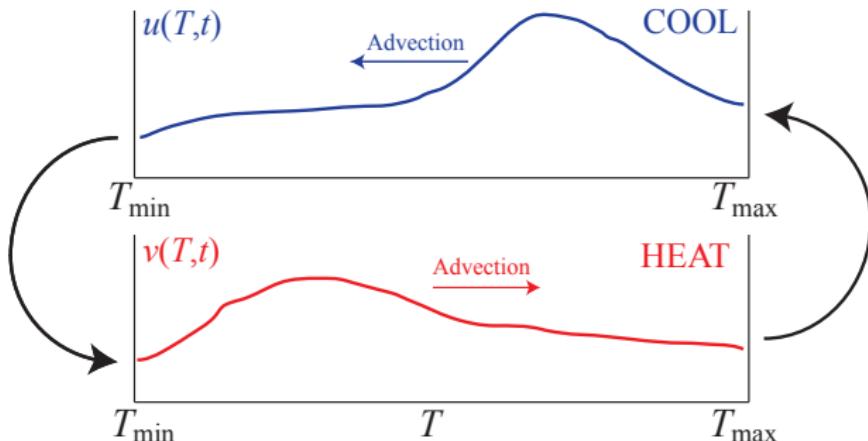


PDE Model of Aggregated TCLs



Video of 1,000 TCLs

PDE Model of Aggregated TCLs



$$u_t(T, t) = \alpha \lambda(T) u_T(T, t) + \alpha u(T, t)$$

$$v_t(T, t) = -\alpha \mu(T) v_T(T, t) + \alpha v(T, t)$$

$$u(T_{\max}, t) = q_1 v(T_{\max}, t)$$

$$v(T_{\min}, t) = q_2 u(T_{\min}, t)$$

Original Idea: Malhame and Chong, Trans. on Automatic Control (1985)
Remark: Assumes homogeneous populations

Modeling Heterogeneous Aggregated TCLs

Reality: TCL populations are heterogeneous

e.g. variable heat capacity, power, deadband sizes

Video of 1,000 heterogeneous TCLs

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$$u_t(T, t) = \alpha\lambda(T)u_T(T, t) + \alpha u(T, t) + \beta u_{TT}(T, t)$$

$$v_t(T, t) = -\alpha\mu(T)v_T(T, t) + \alpha v(T, t) + \beta v_{TT}(T, t)$$

$$u(T_{\max}, t) = q_1 v(T_{\max}, t), \quad u_T(T_{\min}, t) = -v_T(T_{\min}, t)$$

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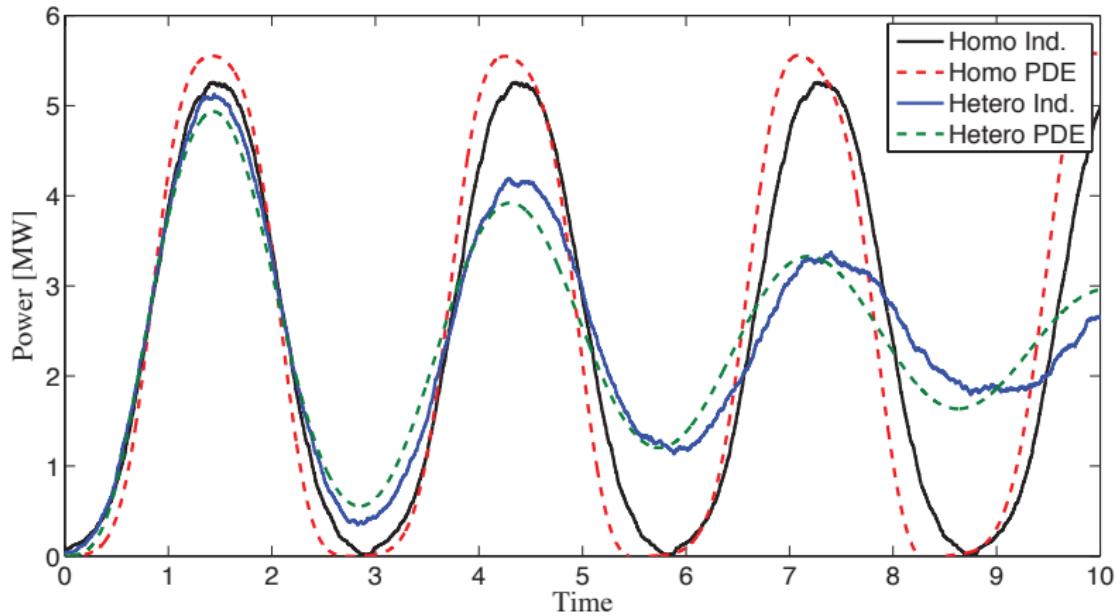
Proposition: The total number of TCLs is conserved over time.

$$Q(t) = \int_{T_{\min}}^{T_{\max}} u(T, t) dT + \int_{T_{\min}}^{T_{\max}} v(T, t) dT$$

$$\frac{dQ}{dt}(t) = 0, \quad \forall t$$

Video Evolution of Heterogeneous PDE

Model Comparison



Outline

- Estimation - looking inside w/ Models, Meas., and Math

The State Estimation Problem

Question: Possible to monitor TCLs with minimal sensing infrastructure?

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Estimate states $u(T, t), v(T, t)$ from measurements of HVAC on/off signals

The State Estimation Problem

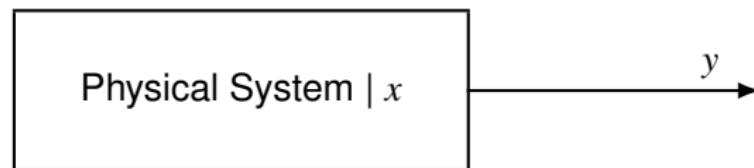
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Intro to Estimation



The State Estimation Problem

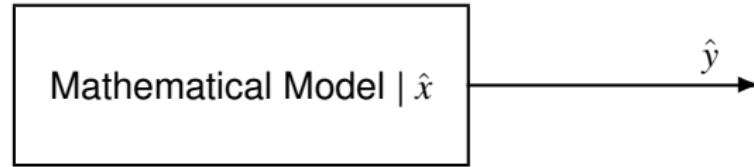
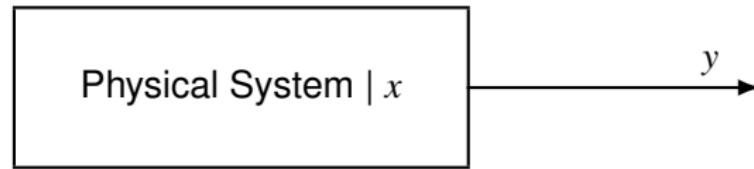
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Intro to Estimation



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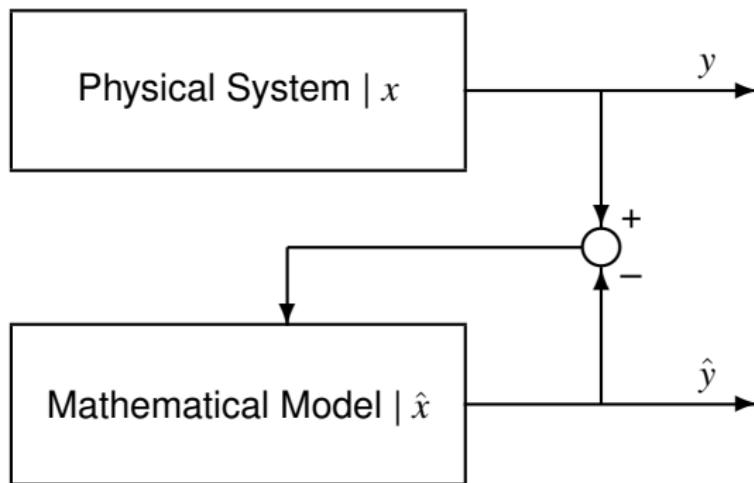
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The State Estimation Problem

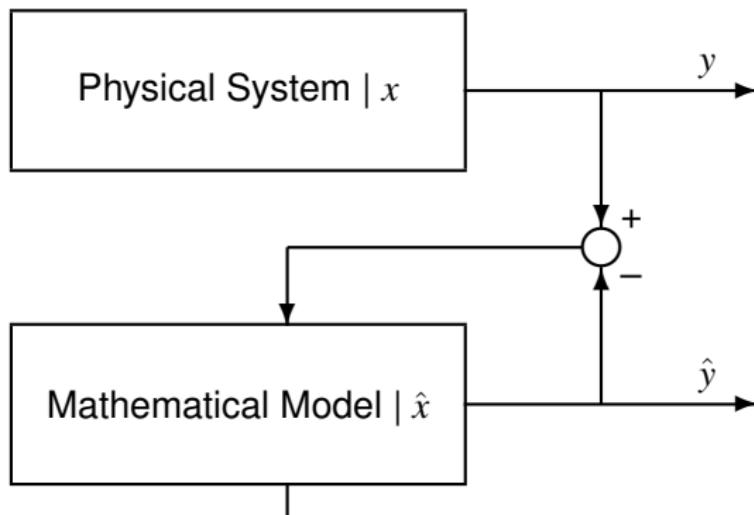
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Intro to Estimation



PDE State Estimator

Heterogeneous PDE Model: (u, v)

$$u_t(x, t) = \alpha\lambda(x)u_x + \alpha u + \beta u_{xx}$$

$$v_t(x, t) = -\alpha\mu(x)v_x + \alpha v + \beta v_{xx}$$

$$u(1, t) = q_1 v(1, t), \quad u_x(0, t) = -v_x(0, t)$$

$$v(0, t) = q_2 u(0, t), \quad v_x(1, t) = -u_x(1, t)$$

PDE State Estimator

Estimator: (\hat{u}, \hat{v})

$$\hat{u}_t(x, t) = \alpha\lambda(x)\hat{u}_x + \alpha\hat{u} + \beta\hat{u}_{xx}$$

$$\hat{v}_t(x, t) = -\alpha\mu(x)\hat{v}_x + \alpha\hat{v} + \beta\hat{v}_{xx}$$

$$\hat{u}(1, t) = q_1 \textcolor{red}{v(1, t)}, \quad \hat{u}_x(0, t) = -\hat{v}_x(0, t) \textcolor{red}{+ p_{10}} [u(0, t) - \hat{u}(0, t)]$$

$$\hat{v}(0, t) = q_2 \textcolor{red}{u(0, t)}, \quad \hat{v}_x(1, t) = -\hat{u}_x(1, t) \textcolor{red}{+ p_{20}} [v(1, t) - \hat{v}(1, t)]$$

PDE State Estimator

Estimator: (\hat{u}, \hat{v})

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Estimation Error Dynamics: $(\tilde{u}, \tilde{v}) = (u - \hat{u}, v - \hat{v})$

$$\tilde{u}_t(x, t) = \alpha\lambda(x)\tilde{u}_x + \alpha\tilde{u} + \beta\tilde{u}_{xx}$$

$$\tilde{v}_t(x, t) = -\alpha\mu(x)\tilde{v}_x + \alpha\tilde{v} + \beta\tilde{v}_{xx}$$

$$\tilde{u}(1, t) = 0, \quad \tilde{u}_x(0, t) = -\tilde{v}_x(0, t) \textcolor{red}{- p_{10}\tilde{u}(0, t)}$$

$$\tilde{v}(0, t) = 0, \quad \tilde{v}_x(1, t) = -\tilde{u}_x(1, t) \textcolor{red}{- p_{20}\tilde{v}(1, t)}$$

Goal: Pick $p_{10}, p_{20} \in \mathbb{R}$ such that $(\tilde{u}, \tilde{v}) = (0, 0)$ is exponentially stable in \mathcal{L}_2 -norm

Lyapunov Stability Analysis

Consider the \mathcal{L}_2 -norm as a candidate Lyapunov functional

$$V(t) = \frac{1}{2} \int_0^1 \tilde{u}(x, t)^2 dx + \frac{1}{2} \int_0^1 \tilde{v}(x, t)^2 dx$$

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The time derivative along the solution trajectories

$$\begin{aligned} \frac{dV}{dt}(t) &\leq \left[-\frac{\alpha}{2}\lambda' - \frac{\beta}{4} + \alpha \right] \int_0^1 \tilde{u}^2 dx + \left[\frac{\alpha}{2}\mu' - \frac{\beta}{4} + \alpha \right] \int_0^1 \tilde{v}^2 dx \\ &+ \left[\beta p_{10} - \frac{\alpha}{2}\lambda(0) \right] \tilde{u}^2(0) + \left[-\beta p_{20} + \frac{\alpha}{2}\mu(1) \right] \tilde{v}^2(1) \end{aligned}$$

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Then the evolution of the \mathcal{L}_2 -norm is bounded as

$$\begin{aligned} \|\tilde{u}(x, t)\|_{\mathcal{L}_2} &\leq \|\tilde{u}(x, 0)\|_{\mathcal{L}_2} e^{[-\frac{\alpha}{2}\lambda' - \frac{\beta}{4} + \alpha]t}, \\ \|\tilde{v}(x, t)\|_{\mathcal{L}_2} &\leq \|\tilde{v}(x, 0)\|_{\mathcal{L}_2} e^{[\frac{\alpha}{2}\mu' - \frac{\beta}{4} + \alpha]t}, \end{aligned}$$

Video Evolution of PDE estimator

Key point: Converges to true distribution, using only HVAC on/off signals.