Demand-Side Management of Flexible Loads:
Modeling, Estimation, and Control with PDEs

Scott Moura

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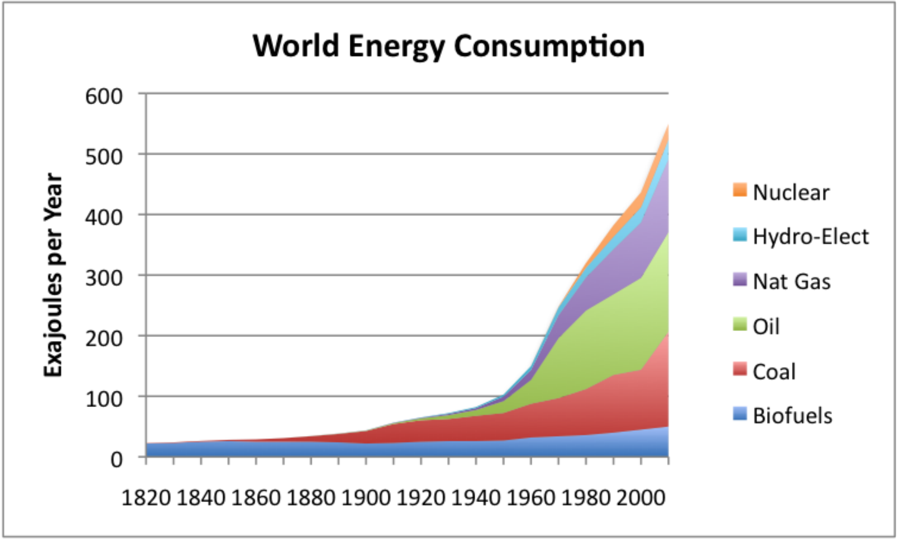
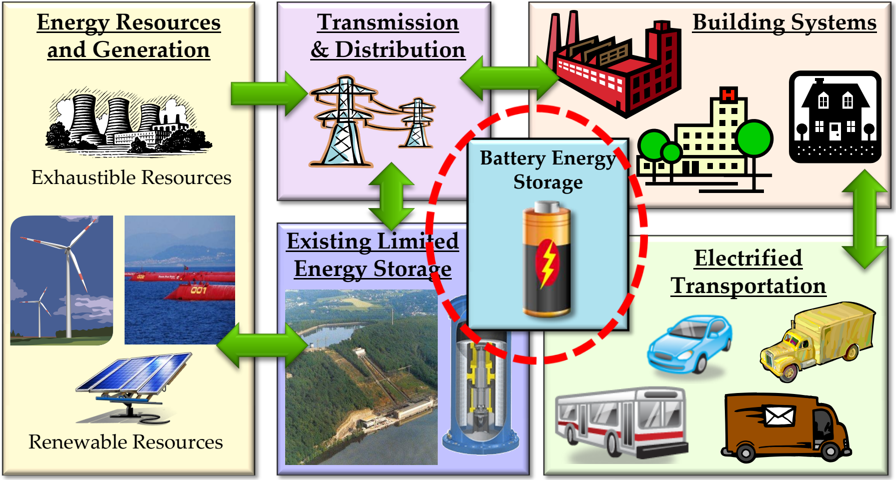


Source: Vaclav Smil Estimates from Energy Transitions

Vision for Future Energy Infrastructure



Flexible Loads of Interest

Buildings	Transportation
(e.g., HVAC, EWH)	(e.g., PEVs)

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Why Buildings?

U.S. buildings produce

- 48% of carbon emissions

U.S. buildings consume

- 39% of total energy
- 71% of electricity
- 54% of natural gas

The Building DSM Problem

Needs:

(1) Dispatch loads, (2) enhance efficiency, resilience, & economics

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Systems controlled by on-off actuation, e.g. HVAC, water heaters, freezers

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Some Interesting Facts

Thermostatically Controlled Loads (TCLs)	50% of U.S. electricity consumption is TCLs
	11% of thermostats are programmed
	Comfort is loosely coupled with control

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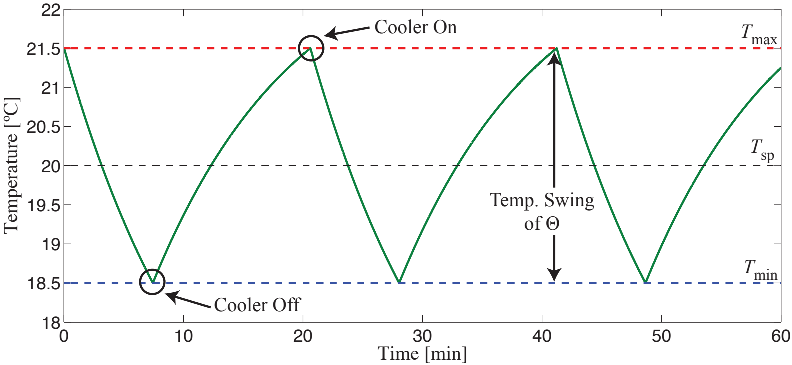
The Punchline

Exploit flexibility of TCLs for power system services

Modeling TCLs



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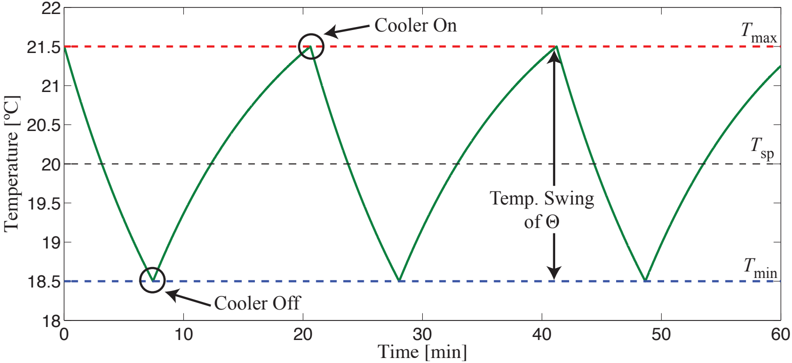

$$\dot{T}_i(t) = \frac{1}{R_i C_i}\left[T_\infty - T_i(t) - s_i(t) R_i P_i\right], \quad i = 1, 2, \ldots, N$$

$$s_i \in \{0, 1\}$$

Modeling Aggregated TCLs

Main Idea: Convert $> 10^3$ ODEs into two coupled linear PDEs

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$u(T,t)$	# TCLs / °C, in COOL state, @ temp T, time t
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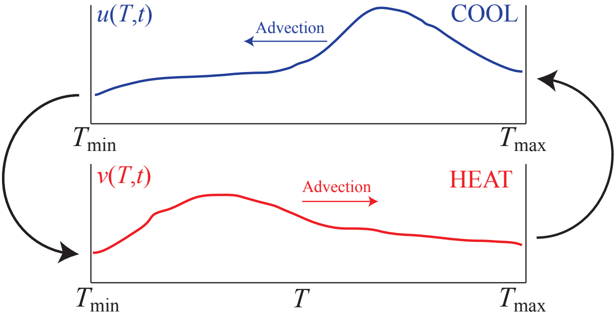

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Flux of TCLs in HEAT state:
#TCLs / sec

$$\psi(T,t) = v(T,t)\frac{dT}{dt}(t) = \frac{1}{RC}\left[T_\infty - T(t)\right] v(T,t)$$

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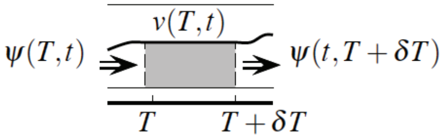
Control volume:



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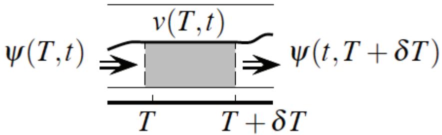
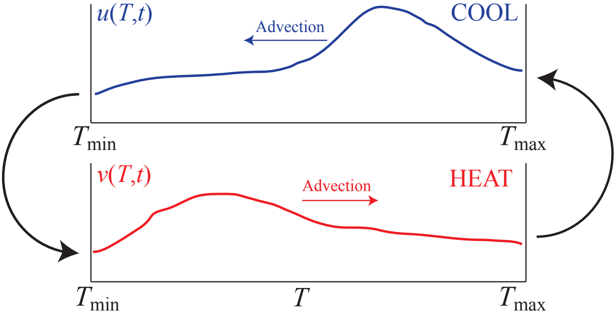


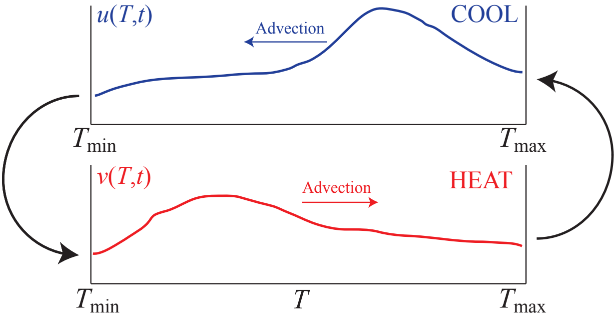
$$\begin{aligned}
\frac{\partial v}{\partial t}(T,t) &= \lim_{\delta T \to 0}\left[\frac{\psi(T+\delta T,t) - \psi(T,t)}{\delta T}\right] \\
&= \frac{\partial \psi}{\partial T}(T,t) \\
&= -\frac{1}{RC}\left[T_\infty - T(t)\right]\frac{\partial v}{\partial T}(T,t) + \frac{1}{RC}v(T,t)
\end{aligned}$$

PDE Model of Aggregated TCLs



Video of 1,000 TCLs

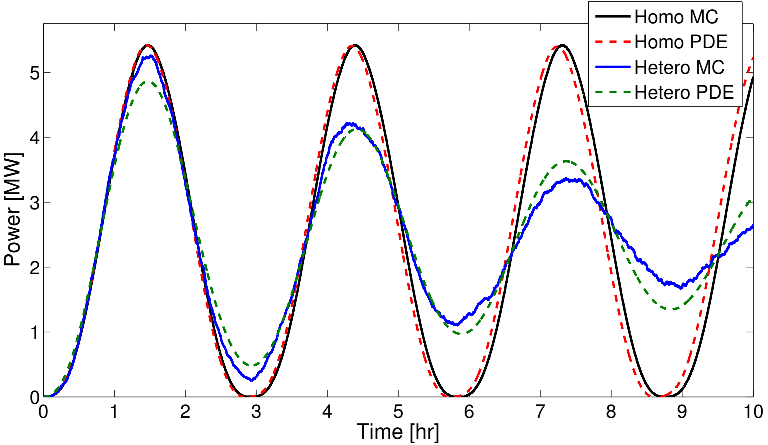
PDE Model of Aggregated TCLs



$$u_t(T,t) = \alpha\lambda(T)u_T(T,t) + \alpha u(T,t)$$
$$v_t(T,t) = -\alpha\mu(T)v_T(T,t) + \alpha v(T,t)$$
$$u(T_{\max},t) = q_1 v(T_{\max},t)$$
$$v(T_{\min},t) = q_2 u(T_{\min},t)$$

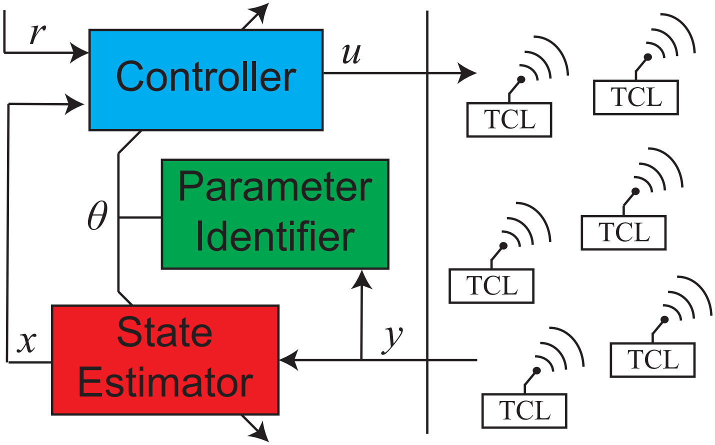
Video of 1,000 TCLs

Model Comparison



S. J. Moura, J. Bendsten, V. Ruiz, “Parameter Identification of Aggregated Thermostatically Controlled Loads for Smart Grids using PDE Techniques,” International Journal of Control, 2014 (Invited Paper).
DOI: 10.1080/00207179.2014.915083

Feedback Control System



PDE State Estimator

Estimation Error Dynamics: $(\tilde{u}, \tilde{v}) = (u - \hat{u}, v - \hat{v})$

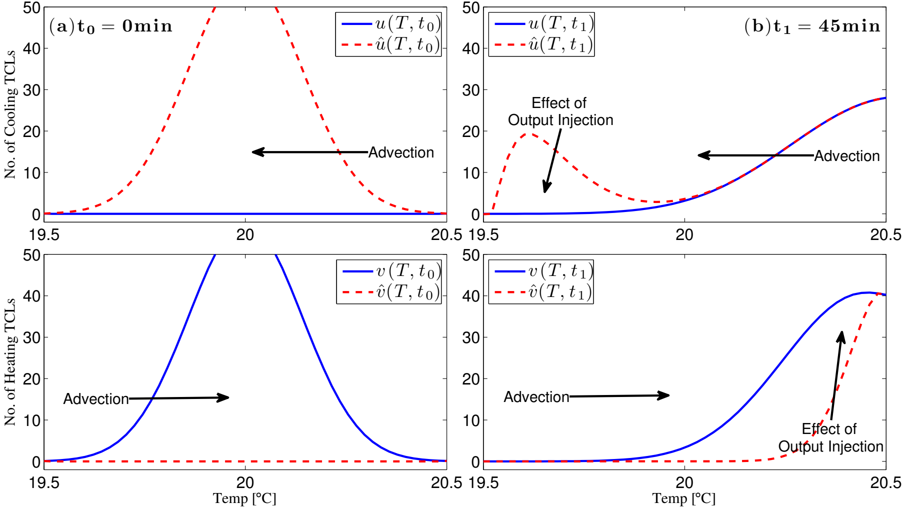
$$
\begin{aligned}
\tilde{u}_t(x,t) &= \alpha\lambda(x)\tilde{u}_x + \alpha\tilde{u} + \beta\tilde{u}_{xx} - p_1(x)\tilde{u}(0,t) \\
\tilde{u}_x(0,t) &= -p_{10}\tilde{u}(0,t) \\
\tilde{u}(1,t) &= 0 \\
\\
\tilde{v}_t(x,t) &= -\alpha\mu(x)\tilde{v}_x + \alpha\tilde{v} + \beta\tilde{v}_{xx} - p_2(x)\tilde{v}(1,t) \\
\tilde{v}(0,t) &= 0 \\
\tilde{v}_x(1,t) &= -p_{20}\tilde{v}(1,t)
\end{aligned}
$$

Goal: Design estimation gains:

- $p_1(x), p_2(x) : (0,1) \rightarrow \mathbb{R}$
- $p_{10}, p_{20} \in \mathbb{R}$

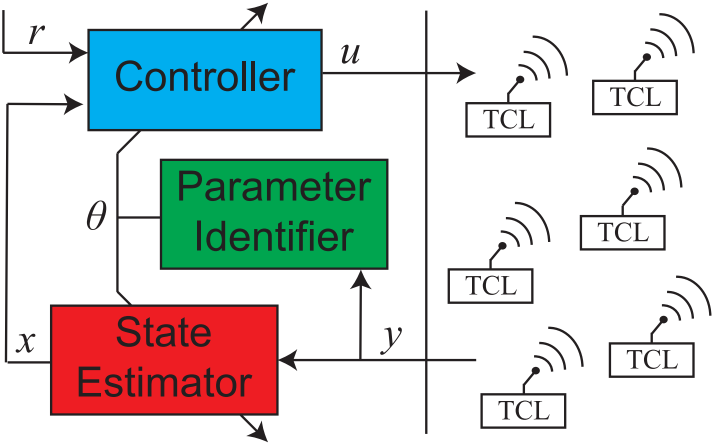
such that $(\tilde{u}, \tilde{v}) = (0,0)$ is exponentially stable

Simulations



S. J. Moura, J. Bendsten, V. Ruiz, "Observer Design for Boundary Coupled PDEs: Application to Thermostatically Controlled Loads in Smart Grids," IEEE Conf. on Decision and Control, Florence, Italy, 2013.

Feedback Control System



Parameter Identification

Uncertain parameters

$$u_t(x,t) = \alpha\lambda(x)u_x + \alpha u + \beta u_{xx} \qquad v_t(x,t) = -\alpha\mu(x)v_x + \alpha v + \beta v_{xx}$$
$$u_x(0,t) = -v_x(0,t) \qquad v(0,t) = q_2 u(0,t)$$
$$u(1,t) = q_1 v(1,t) \qquad v_x(1,t) = -u_x(1,t)$$

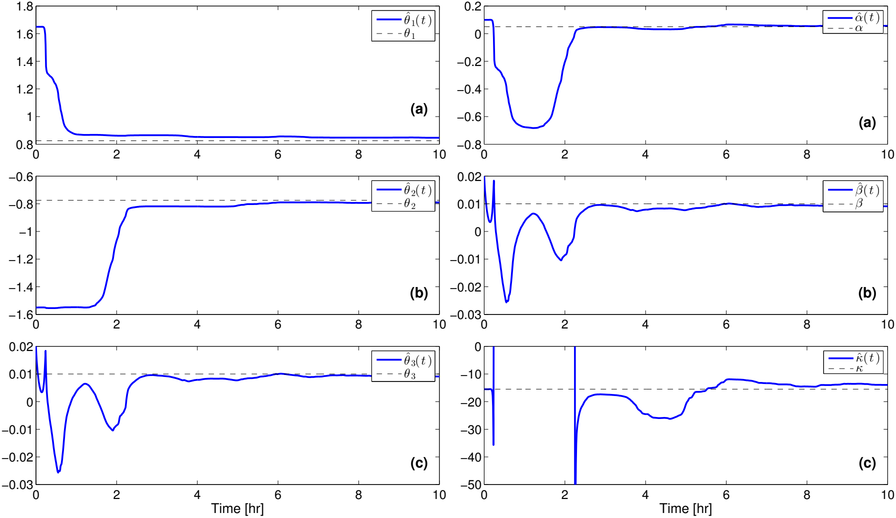
$$P(t) = \frac{\overline{P}}{\eta}\int_0^1 u(x,t)dx$$

Assumptions:

1. Aggregate Power $P(t)$ is measured
2. No. of TCLs switching $u(0,t), u(1,t), u_x(0,t), u_x(1,t)$ is measured

Simulations

Identified from Population of 1,000 Heterogeneous TCLs



S. J. Moura, V. Ruiz, J. Bendsten, "Modeling Heterogeneous Populations of Thermostatically Controlled Loads using Diffusion-Advection PDEs," ASME Dynamic Systems and Control Conference, Stanford, CA, 2013.

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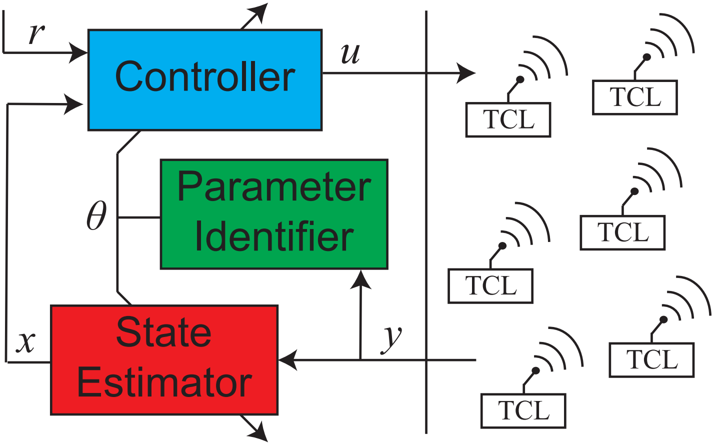
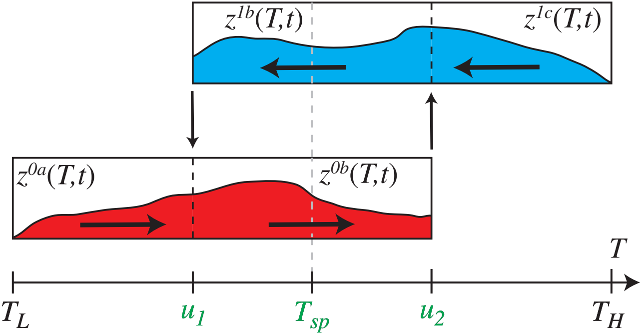


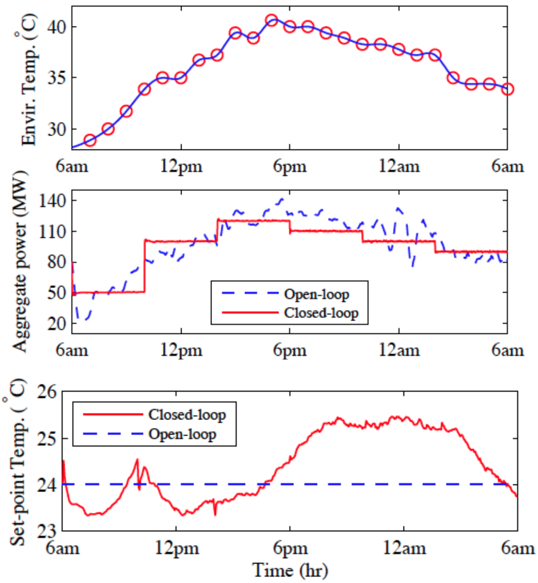


$$
\begin{aligned}
z_t^{1j}(T,t) &= \alpha\lambda(T) z_T^{1j}(T,t) + \alpha z^{1j}(T,t), \qquad j \in \{b,c\} \\
z_t^{0j}(T,t) &= -\alpha\mu(T) z_T^{0j}(T,t) + \alpha z^{0j}(T,t), \qquad j \in \{a,b\}
\end{aligned}
$$

with boundary conditions

$$
\begin{aligned}
z^{0a}(T_L,t) &= 0, & z^{0b}(u_1,t) &= z^{0a}(u_1,t) + z^{1b}(u_1,t), \\
z^{1b}(u_2,t) &= z^{1c}(u_2,t) + z^{0b}(u_2,t), & z^{1c}(T_H,t) &= 0
\end{aligned}
$$

Aggregate Power Control



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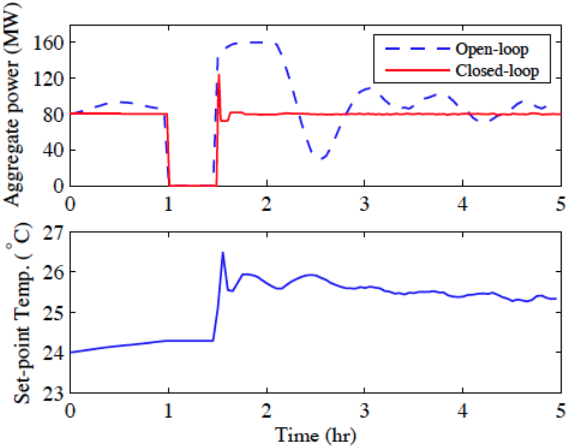


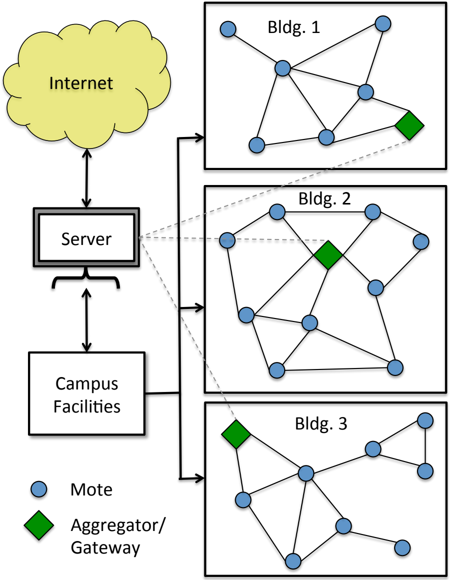
A. Ghaffari, S. J. Moura, M. Krstic, "Analytic Modeling and Integral Control of Heterogeneous Thermostatically Controlled Load Populations," ASME Dynamic Systems and Control Conference, San Antonio, TX, 2014.

UC San Diego Campus: A Living Laboratory

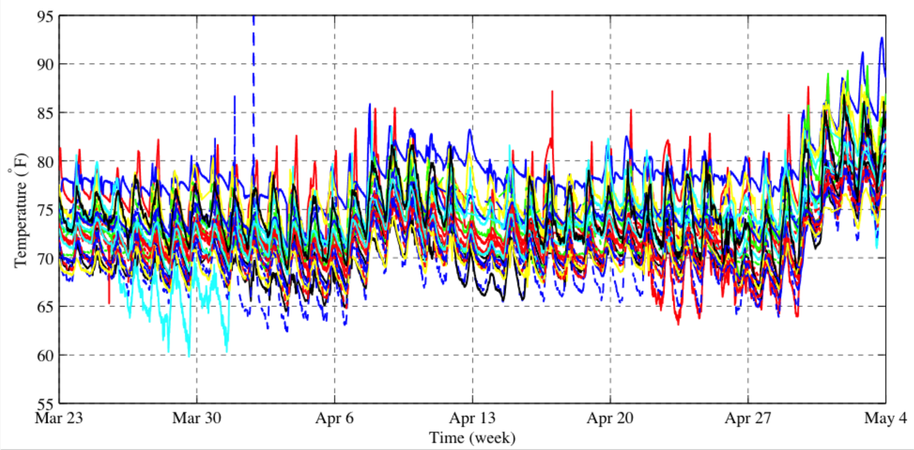
Goal: DR for Bldg Energy Mgmt

1. Deploy wireless sensor network
2. Model/estimator verification
3. Control design
4. Campus implementation

Sensor Nodes (Temp & Humidity)

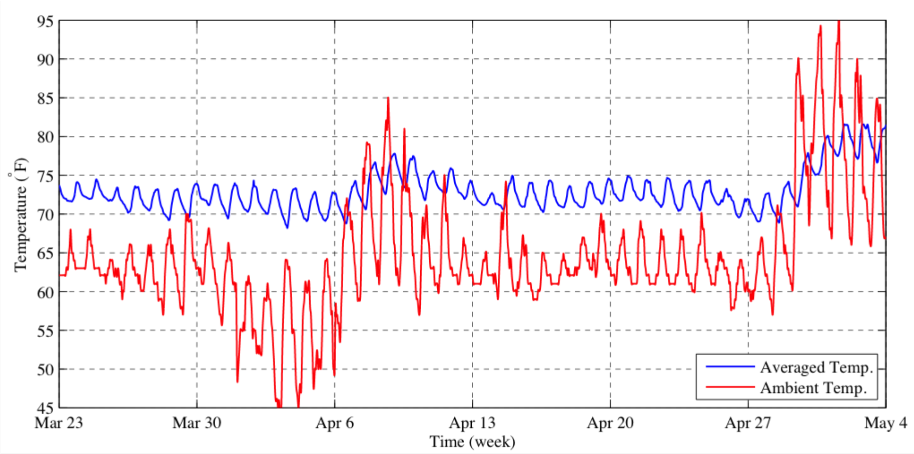


UCSD Office Temperature Data



Temperature variation of monitored spaces over six week. The peak point of room temperature happens at midnight.

UCSD Office Temperature Data



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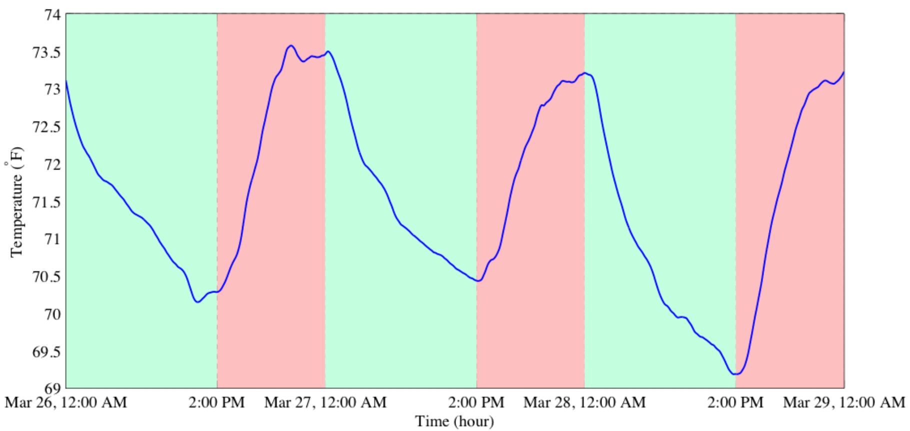
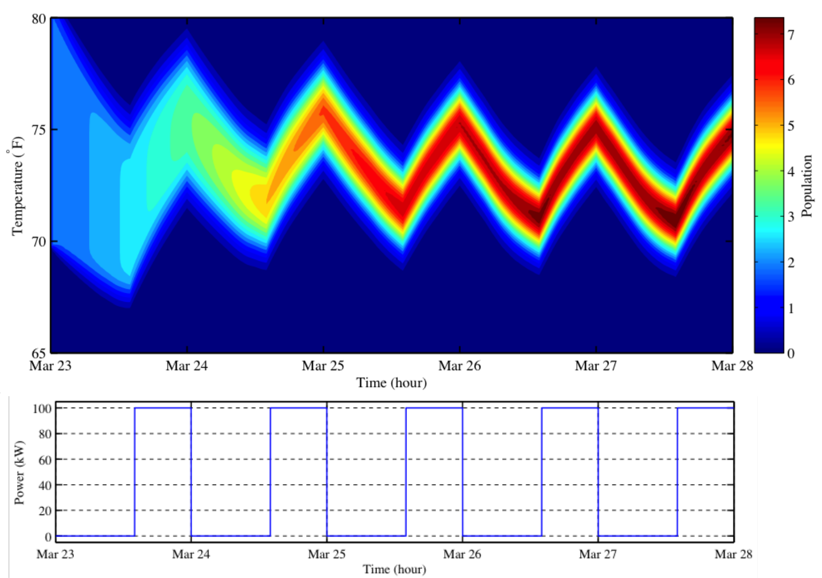


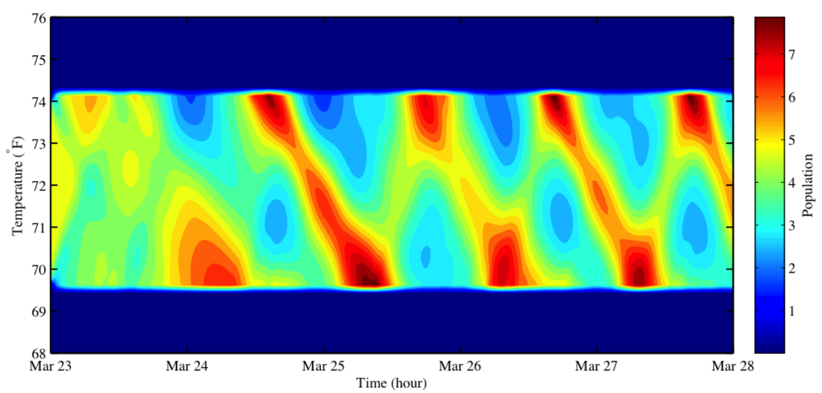
The HVAC system works in a time scheduled manner. The green parts indicate off regions and the red bands show on regions.

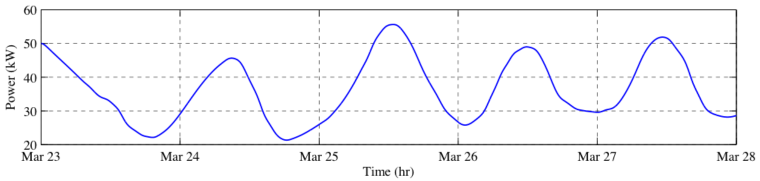
UCSD Office TCL Simulation



Time Scheduled Control

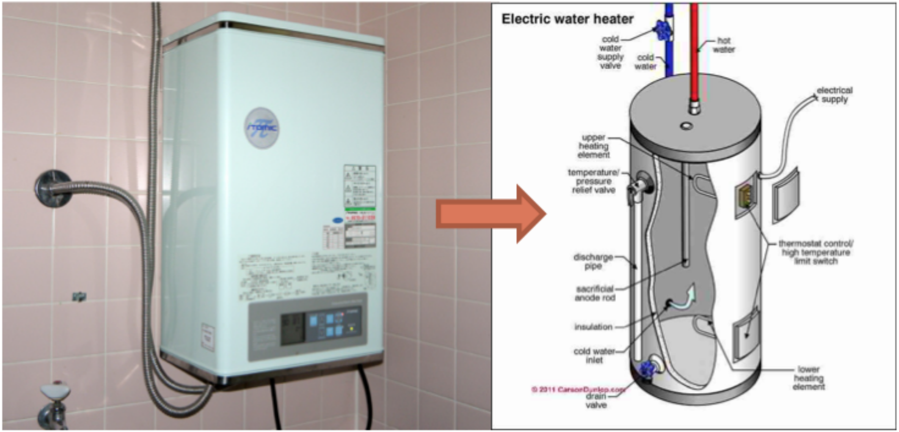
UCSD Office TCL Simulation





PDE Temp. Set-point Control | **13% reduction in energy & guaranteed comfort!**

Aggregating Electric Water Heaters (EWH) | EDF



Flexible Loads of Interest

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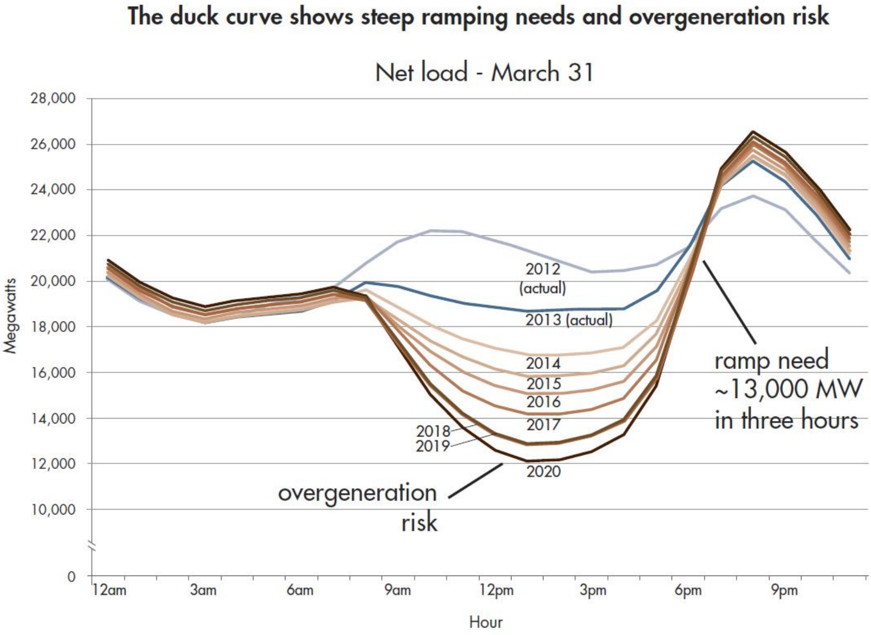


The duck curve shows steep ramping needs and overgeneration risk
Net load - March 31
28,000
26,000
24,000
22,000
20,000
18,000
16,000
14,000
12,000
10,000
0
Megawatts
2012
(actual)
2013 (actual)
2014
2015
2016
2017
2018
2019
2020
ramp need
~13,000 MW
in three hours
overgeneration
risk
12am
3am
6am
9am
12pm
3pm
6pm
9pm
Hour

The Vehicle-Grid Integration (VGI) Problem

Needs: Resilient and sustainable energy/transportation infrastructure

Obstacle: Unprecedented constraints and demands on grid

Some Interesting Facts

Plug-in Electric Vehicles (PEVs)	Potentially dispatchable loads "carbitrage" opportunity Firm variable renewables

The Punchline

Exploit flexibility of PEV charging to enhance efficiency across infrastructures

Government Initiatives

THE WHITE HOUSE
WASHINGTON

Vehicle-Grid Integration Roadmap

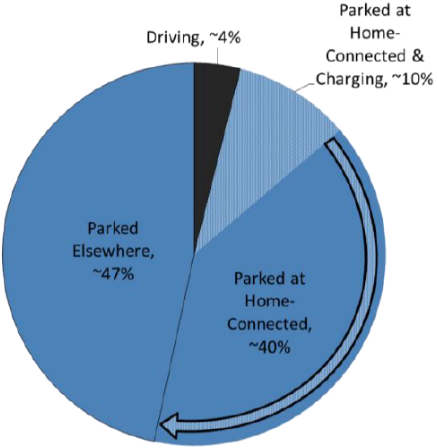
– 1.5M ZEVs in California by 2025

“Vehicle electrification and smart grid technology implementation present an opportunity for EVs, through charging strategies and aggregation, to support and provide valuable services to contribute to reliable management of the electricity grid”

DOE Congressional Budget Request

“The lack of understanding of the impact that the large-scale market penetration of PEVs may have on the electric grid (such as charging during on-peak hours, coordination of charging events, and time-of-day pricing) represents a challenge that must be overcome in order to achieve market success. ”

PEV Energy Storage: How much, when, and where?



Estimated percent of time PEVs spend by location and activity.

A. Langton and N. Crisostomo, “Vehicle-grid integration: A vision for zero-emission transportation interconnected throughout Californias electricity system,” California Public Utilities Commission, Tech. Rep. R. 13-11-XXX, 2013.

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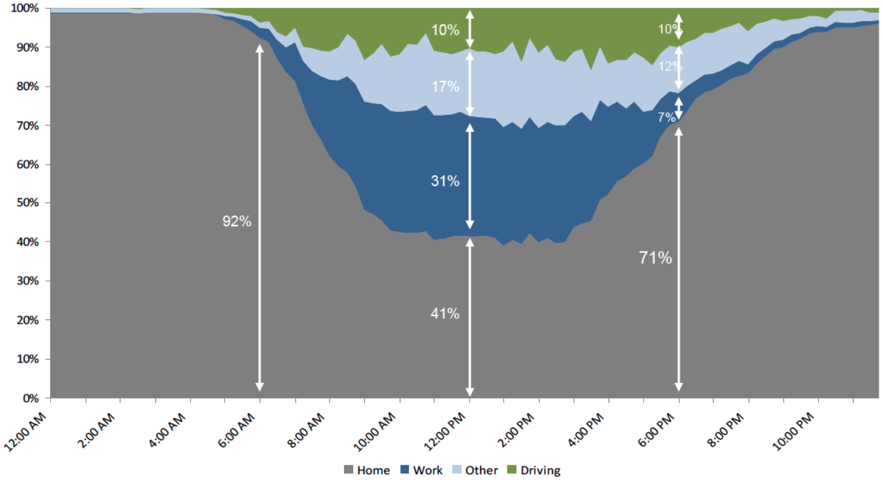

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Project-based Course on V2G System

- Fleet of eScooters
- Collect shared mobility data, design VGI system
- Learn hardware, software, algorithms, big data, cloud-based computing
- Berkeley Energy and Climate Lectures Curriculum Innovation Award

Google App Engine

CE 186

DESIGN OF CYBER-PHYSICAL SYSTEMS

Spring 2014: Mon & Wed 2-4

Topics Include:

- Energy Management and Power Systems
- Vehicle-to-Grid and Battery Models
- Internet-based Systems
- Data Collection and Analysis

Cloud Enabled Smart Charging of PEVs

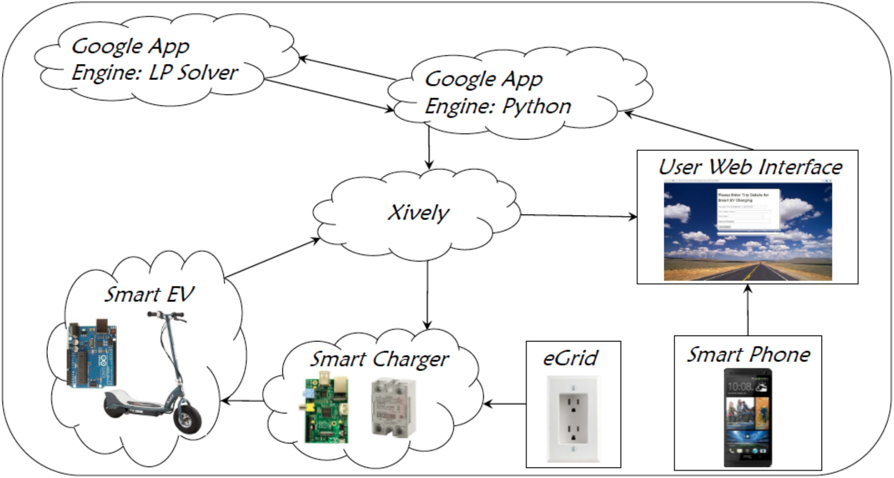


Figure 2. On the Cloud Optimization System for Smart EV Charging

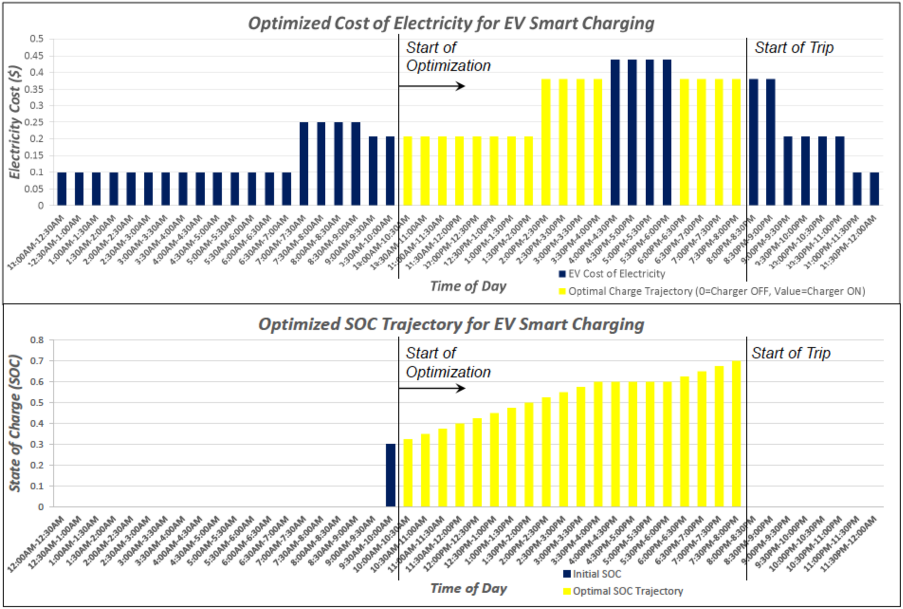


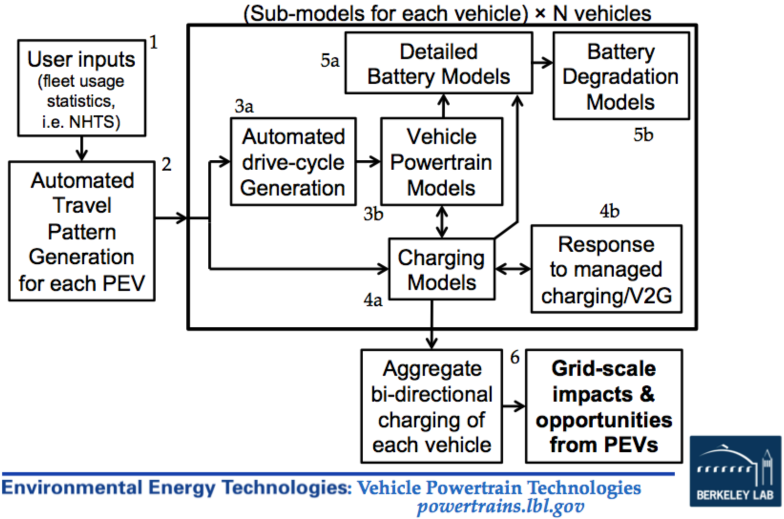
Figure 3. Optimal Charge Cost and SOC Trajectory

V2G-Sim (Lawrence Berkeley National Lab)

A simulation platform for model-based design and analysis of vehicle-grid integration

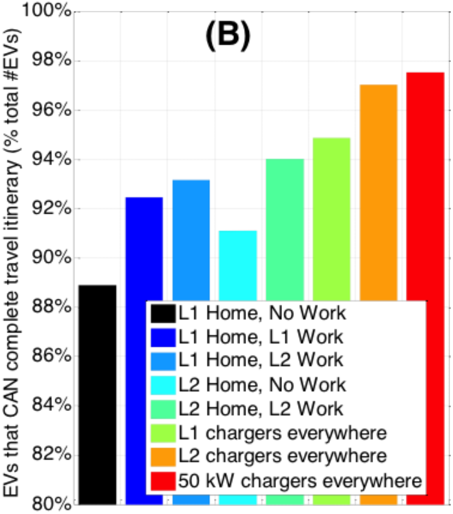
V2G-Sim (Lawrence Berkeley National Lab)

A simulation platform for model-based design and analysis of vehicle-grid integration

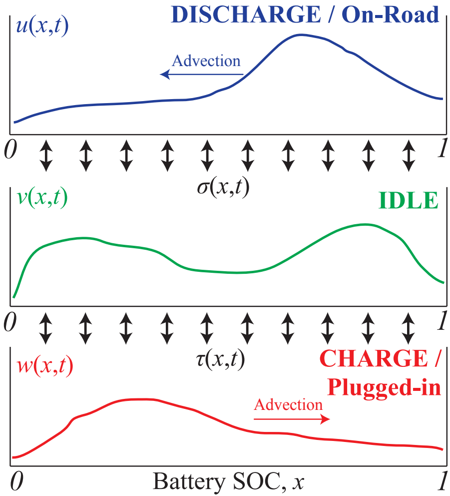


V2G-Sim (Lawrence Berkeley National Lab)

Do we need fast charging, or are standard home outlets enough?

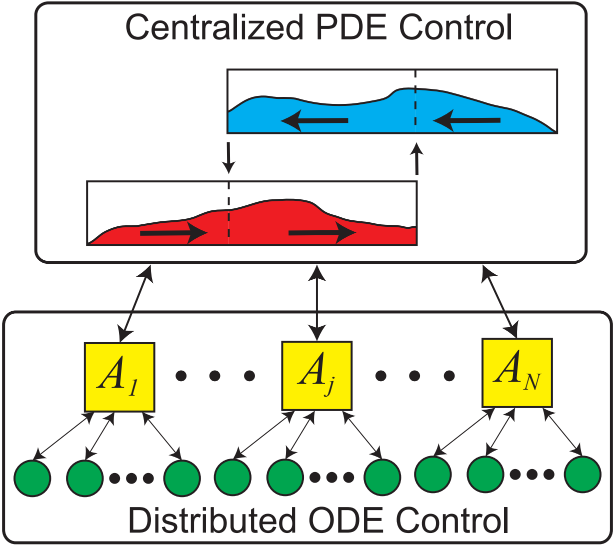


Modeling Aggregated PEVs w/ PDEs

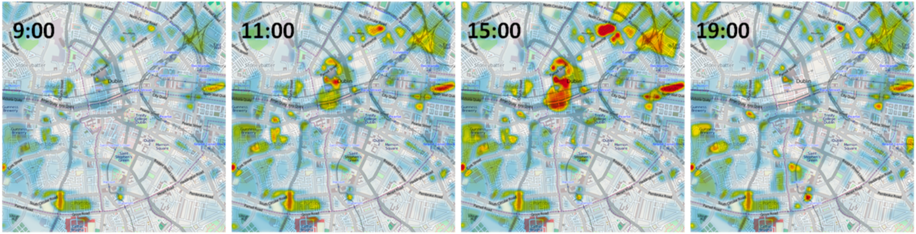


$u(x,t)$	# PEVs / SOC, in DISCHARGE state, @ SOC x, time t
$v(x,t)$	# PEVs / SOC, in IDLE state, @ SOC x, time t
$w(x,t)$	# PEVs / SOC, in CHARGE state, @ SOC x, time t

Hierarchical Control

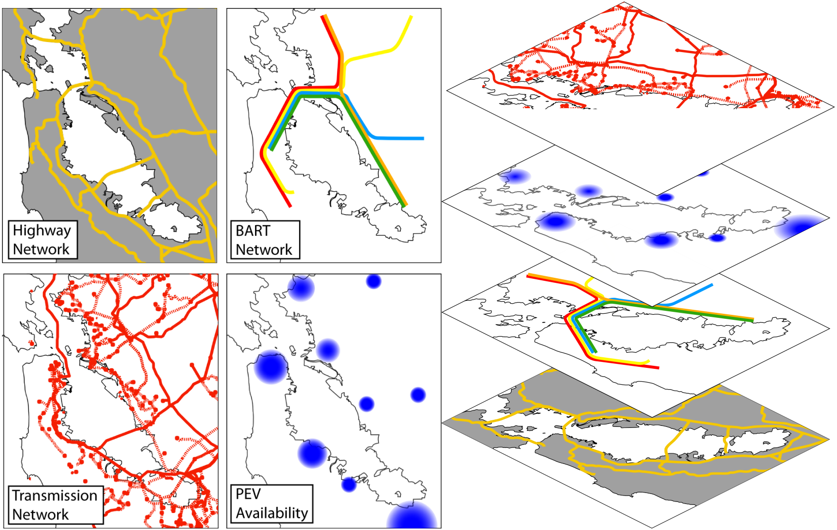


Spatio-Temporal Evolution



Population densities estimated from cell phone usage at different time of the day: morning (left), shopping time (both images in the centre), evening (right) (Source: Kaiser and Pozdnoukhov 2013)

Coupled Transportation-Energy Networks



Summary

Today we discussed

- Modeling, Estimation, Control of TCL Populations
- Vehicle-to-Grid Integration

Not discussed today

- Modeling and Control of PEV Populations
- Smart Home Energy Management
- Advanced Battery Controls
- Traffic-Enabled PHEV Energy Management

QUESTIONS?

Energy, Controls, and Applications Lab (eCAL)

Pubs available at ecal.berkeley.edu

smoura@berkeley.edu

Question:

Given non-pervasive sensing, can we estimate the states of the TCL population in real-time?

Normalization

$$x = \frac{T - T_{\min}}{T_{\max} - T_{\min}}$$

renders heterogenous PDE into

$$\begin{aligned}
u_t(x,t) &= \alpha \breve{\lambda}(x) u_x(x,t) + \alpha u(x,t) + \breve{\beta} u_{xx}(x,t) \\
v_t(x,t) &= -\alpha \breve{\mu}(x) v_x(x,t) + \alpha v(x,t) + \breve{\beta} v_{xx}(x,t) \\
u(1,t) &= q_1 v(1,t), \qquad u_x(0,t) = -v_x(0,t) \\
v(0,t) &= q_2 u(0,t), \qquad v_x(1,t) = -u_x(1,t)
\end{aligned}$$

where parameters $\breve{\lambda}(x), \breve{\mu}(x), \breve{\beta}$ are

$$\breve{\lambda}(x) = x - \frac{T_\infty - T_{\min} - RP}{T_{\max} - T_{\min}}, \quad \breve{\mu}(x) = \frac{T_\infty - T_{\min}}{T_{\max} - T_{\min}} - x, \quad \breve{\beta} = \frac{\beta}{(T_{\max} - T_{\min})^2}$$

Henceforth the breves are dropped from $\breve{\lambda}(x), \breve{\mu}(x), \breve{\beta}$.

PDE State Estimator

Heterogeneous PDE Model: (u, v)

$$
\begin{aligned}
u_t(x,t) &= \alpha\lambda(x)u_x + \alpha u + \beta u_{xx} \\
u_x(0,t) &= -v_x(0,t) \\
u(1,t) &= q_1 v(1,t) \\
\\
v_t(x,t) &= -\alpha\mu(x)v_x + \alpha v + \beta v_{xx} \\
v(0,t) &= q_2 u(0,t) \\
v_x(1,t) &= -u_x(1,t)
\end{aligned}
$$

Measurements

TCLs only announce switching between HEAT & COOL states:

- $u(0,t), v(1,t)$
- $u_x(1,t), v_x(0,t)$

PDE State Estimator

Estimator: $(\hat{u}, \hat{v})$

$$
\begin{aligned}
\hat{u}_t(x,t) &= \alpha\lambda(x)\hat{u}_x + \alpha\hat{u} + \beta\hat{u}_{xx} + p_1(x)\left[u(0,t) - \hat{u}(0,t)\right] \\
\hat{u}_x(0,t) &= -v_x(0,t) + p_{10}\left[u(0,t) - \hat{u}(0,t)\right] \\
\hat{u}(1,t) &= q_1 v(1,t) \\
\\
\hat{v}_t(x,t) &= -\alpha\mu(x)\hat{v}_x + \alpha\hat{v} + \beta\hat{v}_{xx} + p_2(x)\left[v(1,t) - \hat{v}(1,t)\right] \\
\hat{v}(0,t) &= q_2 u(0,t) \\
\hat{v}_x(1,t) &= -u_x(1,t) + p_{20}\left[v(1,t) - \hat{v}(1,t)\right]
\end{aligned}
$$

PDE State Estimator

Estimation Error Dynamics: $(\tilde{u}, \tilde{v}) = (u - \hat{u}, v - \hat{v})$

$$
\begin{aligned}
\tilde{u}_t(x,t) &= \alpha\lambda(x)\tilde{u}_x + \alpha\tilde{u} + \beta\tilde{u}_{xx} - p_1(x)\tilde{u}(0,t) \\
\tilde{u}_x(0,t) &= -p_{10}\tilde{u}(0,t) \\
\tilde{u}(1,t) &= 0 \\
\\
\tilde{v}_t(x,t) &= -\alpha\mu(x)\tilde{v}_x + \alpha\tilde{v} + \beta\tilde{v}_{xx} - p_2(x)\tilde{v}(1,t) \\
\tilde{v}(0,t) &= 0 \\
\tilde{v}_x(1,t) &= -p_{20}\tilde{v}(1,t)
\end{aligned}
$$

Decoupled systems!

PDE State Estimator

Estimation Error Dynamics: $(\tilde{u}, \tilde{v}) = (u - \hat{u}, v - \hat{v})$

$$
\begin{aligned}
\tilde{u}_t(x,t) &= \alpha\lambda(x)\tilde{u}_x + \alpha\tilde{u} + \beta\tilde{u}_{xx} - p_1(x)\tilde{u}(0,t) \\
\tilde{u}_x(0,t) &= -p_{10}\tilde{u}(0,t) \\
\tilde{u}(1,t) &= 0 \\
\\
\tilde{v}_t(x,t) &= -\alpha\mu(x)\tilde{v}_x + \alpha\tilde{v} + \beta\tilde{v}_{xx} - p_2(x)\tilde{v}(1,t) \\
\tilde{v}(0,t) &= 0 \\
\tilde{v}_x(1,t) &= -p_{20}\tilde{v}(1,t)
\end{aligned}
$$

Goal: Design estimation gains:

- $p_1(x), p_2(x) : (0,1) \to \mathbb{R}$
- $p_{10}, p_{20} \in \mathbb{R}$

such that $(\tilde{u}, \tilde{v}) = (0,0)$ is exponentially stable

Backstepping Observer Design

Error Dynamics: $\tilde{u}$

$$\begin{aligned}
\tilde{u}_t(x,t) &= \alpha\lambda(x)\tilde{u}_x + \alpha\tilde{u} + \beta\tilde{u}_{xx} - p_1(x)\tilde{u}(0,t) \\
\tilde{u}_x(0,t) &= -p_{10}\tilde{u}(0,t) \\
\tilde{u}(1,t) &= 0
\end{aligned}$$

Backstepping Observer Design

Error Dynamics: $\tilde{u}$

$$\begin{aligned} \tilde{u}_t(x,t) &= \alpha\lambda(x)\tilde{u}_x + \alpha\tilde{u} + \beta\tilde{u}_{xx} - p_1(x)\tilde{u}(0,t) \\ \tilde{u}_x(0,t) &= -p_{10}\tilde{u}(0,t) \\ \tilde{u}(1,t) &= 0 \end{aligned}$$

Eliminate advection terms, “Gauge” Transformation

$$\xi(x,t) = \tilde{u}(x,t)e^{\frac{\alpha}{2\beta}\int_0^x \lambda(s)ds}$$

Backstepping Observer Design

Transformed error state: ξ

$$\begin{aligned}
\xi_t(x,t) &= \beta\xi_{xx} + g(x)\xi - p_1^{\xi}(x)\xi(0,t)\\
\xi_x(0,t) &= p_{10}^{\xi}\xi(0,t)\\
\xi(1,t) &= 0
\end{aligned}$$

$$\begin{aligned}
g(x) &= \alpha\left[1 - \frac{\lambda'(x)}{2}\right] - \frac{\alpha^2\lambda^2(x)}{4\beta}\\
p_1^{\xi}(x) &= p_1(x)e^{\frac{\alpha}{2\beta}\int_0^x \lambda(s)ds}\\
p_{10}^{\xi} &= \frac{\alpha}{2\beta}\lambda(0) - p_{10}
\end{aligned}$$

Backstepping Observer Design

Transformed error state: ξ

$$\begin{aligned} \xi_t(x,t) &= \beta\xi_{xx} + g(x)\xi - p_1^{\xi}(x)\xi(0,t) \\ \xi_x(0,t) &= p_{10}^{\xi}\xi(0,t) \\ \xi(1,t) &= 0 \end{aligned}$$

Backstepping transformation

$$\xi(x,t) = w_1(x,t) - \int_0^x p(x,y) w_1(y,t) dy$$

Backstepping Observer Design

Transformed error state: ξ

$$\begin{aligned}
\xi_t(x,t) &= \beta \xi_{xx} + g(x)\xi - p_1^{\xi}(x)\xi(0,t) \\
\xi_x(0,t) &= p_{10}^{\xi}\xi(0,t) \\
\xi(1,t) &= 0
\end{aligned}$$

Backstepping transformation

$$\xi(x,t) = w_1(x,t) - \int_0^x p(x,y) w_1(y,t) dy$$

Target system: w_1, exp. stable in $\mathcal{L}^2$-norm

$$\begin{aligned}
w_{1t}(x,t) &= \beta w_{1xx}(x,t) - c_1 w_1(x,t), \qquad c_1 \geq 0 \\
w_{1x}(0,t) &= w_1(0,t) \\
w_1(1,t) &= 0
\end{aligned}$$

Backstepping Observer Design

Kernel PDE: $p(x, y)$

$$
\begin{aligned}
\beta p_{xx}(x, y) - \beta p_{yy}(x, y) &= -\left[c_1 + g(x)\right] p(x, y) \\
p(x, x) &= -\frac{1}{2\beta} \int_x^1 \left[c_1 + g(s)\right] ds \\
p(1, y) &= 0
\end{aligned}
$$

Backstepping Observer Design

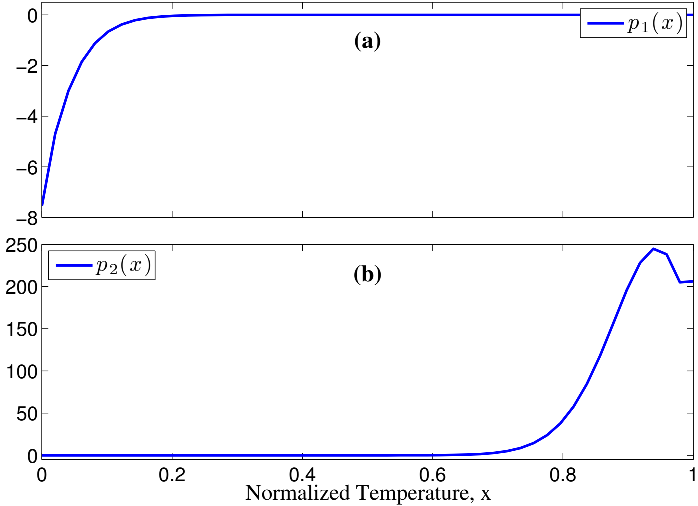
Kernel PDE: $p(x, y)$

$$\begin{aligned}
\beta p_{xx}(x,y) - \beta p_{yy}(x,y) &= -\left[c_1 + g(x)\right] p(x,y) \\
p(x,x) &= -\frac{1}{2\beta} \int_x^1 \left[c_1 + g(s)\right] ds \\
p(1,y) &= 0
\end{aligned}$$

Estimation gains

$$\begin{aligned}
p_1^{\xi}(x) &= -\beta \left[p(x,0) + p_y(x,0)\right] \\
p_{10}^{\xi} &= 1 - p(0,0) \\
p_1(x) &= p_1^{\xi}(x) e^{-\frac{\alpha}{2\beta}\int_0^x \lambda(s) ds} \\
p_{10} &= \frac{\alpha}{2\beta}\lambda(0) - p_{10}^{\xi}
\end{aligned}$$

Observer Gains



Control & Optimization w/ Application to Energy Systems

Sample Course Projects:

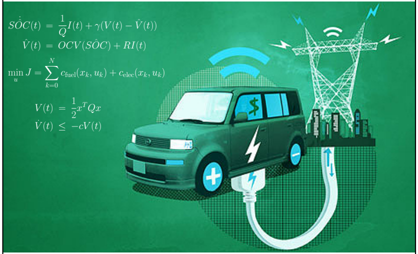
- Aggregate Modeling & Control of PEV Fleets
- Optimal Charging & Vending of Shared eBike Fleets
- Smart Home Thermostat
- Optimal Energy Storage Placement in CA
- Smart Home Energy Management w/ Solar
- Battery Estimation
- Building Lighting Controls
- Rooftop & Centralized Solar Generation in Nicaragua

CE 290:002

Energy Systems & Control

Spring 2014: MWF 10-11

Prof. Scott Moura



Topics Include:

- Energy Storage & Renewables
- Electrified Transportation
- State estimation
- Optimal control