6

Transmission Networks and Electricity Markets

6.1 Introduction

In most, if not all, regions of the world, the assumption that electrical energy can be traded as if all generators and loads were connected to the same busbar is not tenable. Transmission constraints and losses in the network connecting these generators and loads can introduce gross distortions in the market for electrical energy. In this chapter, we study the effects that a transmission network has on trading of electrical energy and the special techniques that can be used to hedge against the limitations and price fluctuations that are caused by this network. We consider first and briefly bilateral or decentralized trading. We then turn our attention to centralized or pool-based trading.

6.2 Decentralized Trading Over a Transmission Network

In a decentralized or bilateral trading system, all transactions for electrical energy involve only two parties: a buyer and a seller. These two parties agree on a quantity, a price and any other condition that they may want to attach to the trade. The system operator does not get involved in these transactions and does not set the prices at which transactions take place. Its role is limited to maintaining the balance and the security of the system. This involves the following:

- Buying or selling energy to balance the load and the generation. Under normal circumstances, the amounts involved in these balancing transactions should be small.
- Limiting the amount of power that generators can inject at some nodes of the system if security cannot be maintained through other means.

Let us consider the two-bus power system shown in Figure 6.1 in which trading in electrical energy operates on a bilateral basis. Let us suppose that Generator G_1 has

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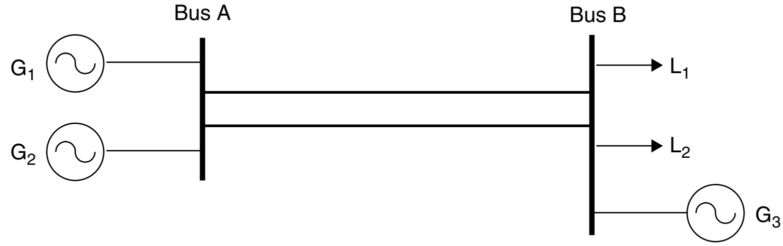

Figure 6.1 Bilateral trading in a two-bus power system

signed a contract for the delivery of 300 MW to load L_1 and that Generator G_2 has agreed to deliver 200 MW to load L_2. Since these transactions are bilateral, the agreed prices are a private matter between the buyer and the seller. On the other hand, the amount of power to be transmitted must be reported to the system operator because this power flows on the transmission system that is open to all parties. The system operator must check that the system will remain secure when all these transactions are implemented. In this case, security is not a problem as long as the capacity of the transmission lines connecting buses A and B is at least 500 MW even under contingency conditions. If the amount of power that can securely be transmitted between buses A and B is less than 500 MW, the system operator has to intervene. Some of the bilateral transactions that were concluded between generators at bus A and loads at bus B must be curtailed.

6.2.1 Physical transmission rights

With modern power system analysis software, determining that a set of transactions would make the operation of the system insecure can be computationally demanding, but is conceptually simple. Deciding which transactions should be curtailed to maintain the required level of security is a much more complex question. Administrative procedures can be established to determine the order in which transactions should be cut back. Such transmission load relief procedures take into account the nature of the transactions (firm or nonfirm), the order in which they were registered with the system operator and possibly some historical factors. They do not, however, factor in the relative economic benefits of the various transactions because a decentralized trading environment does not provide a framework for evaluating these benefits. Administrative curtailments are therefore economically inefficient and should be avoided.

Advocates of decentralized electricity trading believe that the parties considering transactions for electrical energy are best placed to decide whether they wish to use the transmission network. When they sign a contract, producers at bus A and consumers at bus B who do not wish to see their transaction interrupted by congestion should therefore also purchase the right to use the transmission system for this transaction. Since these transmission rights are purchased at a public auction, the parties have the opportunity to decide whether this additional cost is justifiable.

For example, let us suppose that Generator G_1 and load L_1 of Figure 6.1 have agreed on a price of 30.00 \$/MWh, while Generator G_2 and load L_2 agreed on 32.00 \$/MWh. At the same time, Generator G_3 offers energy at 35.00 \$/MWh. Load L_2 should therefore not agree to pay more than 3.00 \$/MWh for transmission rights because this would make the energy it purchases from G_1 more expensive than the energy it could purchase from G_3. The price of transmission rights would have to rise above 5.00 \$/MWh before L_1 reaches the same conclusion. The cost of transmission rights is also an argument that the consumers can use in their negotiations with the generators at bus B to convince them to lower their prices.

Transmission rights of this type are called *physical transmission rights* because they are intended to support the actual transmission of a certain amount of power over a given transmission link.

6.2.2 Problems with physical transmission rights

Our simple example makes physical transmission rights appear simpler than they turn out to be. The first difficulty is practical and arises because the path that power takes through a network is determined by physical laws and not by the wishes of market participants. The second problem is that physical transmission rights have the potential to exacerbate the exercise of market power by some participants. Let us consider these two issues in turn.

6.2.2.1 Parallel paths

Two fundamental laws govern current and power flows in electrical networks: Kirchoff's Current Law (KCL) and Kirchoff's Voltage Law (KVL). KCL specifies that the sum of all the currents entering a node must be equal to the sum of all the currents exiting this node. KCL implies that the active and reactive powers must both be in balance at each node. KVL specifies that the sum of the voltage drops across all the branches of any loop must be equal to zero or, equivalently, that the voltage drops along parallel paths must be equal. Since these voltage drops are proportional to the current flowing through the branch, KVL determines how the currents (and hence the active and reactive power flows) distribute themselves through the network. In the simple example shown in Figure 6.2, a current $\overline{I}$ can flow from node 1 to node 2 along two parallel paths of impedances z_A and z_B. The voltage difference between the two nodes is thus

$$\overline{V_{12}} = z_A \overline{I_A} = z_B \overline{I_B}$$

Since $\overline{I} = \overline{I_A} + \overline{I_B}$, we have

$$\overline{I_A} = \frac{z_B}{z_A + z_B} \overline{I} \tag{6.1}$$

$$\overline{I_B} = \frac{z_A}{z_A + z_B} \overline{I} \tag{6.2}$$

Currents in parallel paths therefore divide themselves in inverse proportion of the impedance of each path. To simplify the following discussion, we will assume that the resistance of any branch is much smaller than its reactance:

$$Z = R + jX \approx jX \tag{6.3}$$

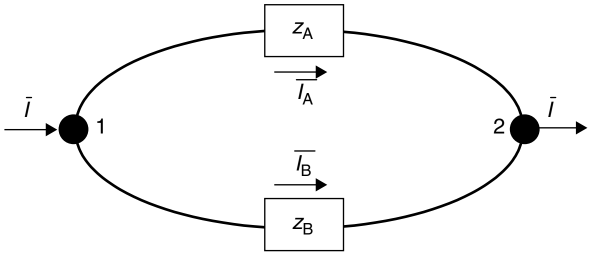

Figure 6.2 Illustration of Kirchoff's Voltage Law

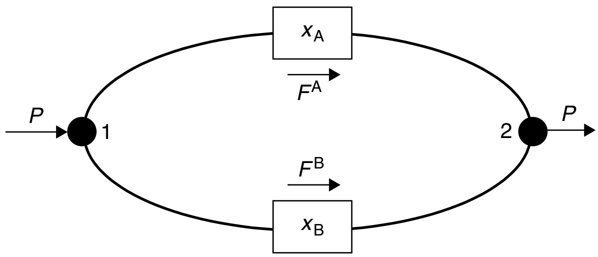

Figure 6.3 Active power flows on parallel paths

We also neglect the flow of reactive power through the network and the losses. Under these assumptions, the system of Figure 6.2 can be depicted in terms of active power flows as shown in Figure 6.3. The active power flows in the parallel paths are related by the following expressions:

$$F^A = \frac{x_B}{x_A + x_B} P \tag{6.4}$$

$$F^B = \frac{x_A}{x_A + x_B} P \tag{6.5}$$

The factors relating the active power injections and the branch flows are called power transfer distribution factors (PTDF).

6.2.2.2 Example

A two-bus system does not illustrate the effect of KVL because there is only one path that the power can follow[1]. We must therefore consider a network with three buses and one loop. Figure 6.4 illustrates such a system and Table 6.1 gives its parameters. To keep matters simple, we assume that network limitations take the form of constant capacity limits on the active power flowing in each line and that the resistance of the lines is negligible.

Let us suppose that Generator B and load Y want to sign a contract for the delivery of 400 MW. If Generator B injects these 400 MW at bus 1 and load Y extracts them

[1]For the sake of simplicity, we treat a line with two identical circuits as a single branch.

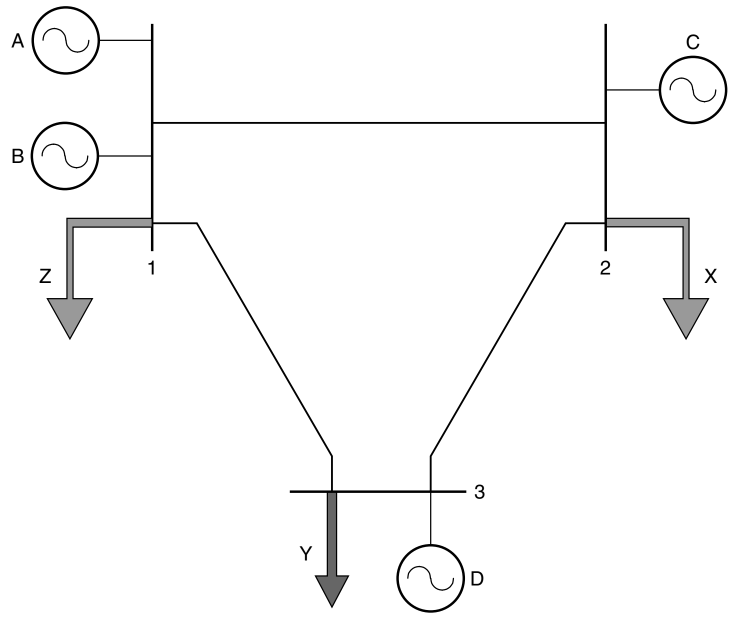

Figure 6.4 A simple three-bus system

Table 6.1 Branch data for the three-bus system of Figure 6.4

Branch	Reactance (p.u.)	Capacity (MW)
1-2	0.2	126
1-3	0.2	250
2-3	0.1	130

at bus 3, this power flows along the two paths shown in Figure 6.5. The amounts of power flowing along paths I and II are given by

$$F^{\mathrm{I}} = \frac{0.2}{0.2 + 0.3} \times 400 = 160\,\mathrm{MW}$$

$$F^{\mathrm{II}} = \frac{0.3}{0.2 + 0.3} \times 400 = 240\,\mathrm{MW}$$

To guarantee that this transaction can actually take place, the parties therefore need to secure 240 MW of transmission rights on Line 1-3 as well as 160 MW of transmission rights on Lines 1-2 and 2-3. This is clearly not possible if this transaction is the only one taking place in this network because the maximum capacity of Lines 1-2 and 2-3 are 126 MW and 130 MW respectively. In the absence of any other transaction, since the most constraining limitation is the capacity of Line 1-2, the maximum amount that

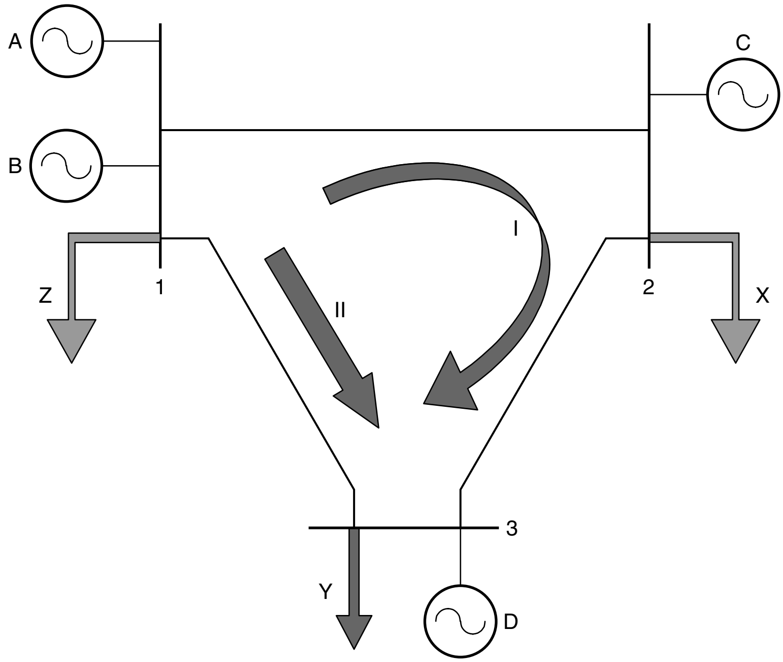

Figure 6.5 Paths for a transaction between Generator B and load Y

A and Y can trade is thus

$$P^{\text{MAX}} = \frac{0.5}{0.2} \times 126 = 315\,\text{MW}$$

However, suppose that load Z would like to purchase 200 MW from Generator D. This power would flow in the following proportions along the paths shown in Figure 6.6:

$$F^{\text{III}} = \frac{0.2}{0.2 + 0.3} \times 200 = 80\,\text{MW}$$

$$F^{\text{IV}} = \frac{0.3}{0.2 + 0.3} \times 200 = 120\,\text{MW}$$

Let us calculate what the flows in this network would be if both of these transactions were to take place at the same time. For this calculation, we can make use of the superposition theorem because our simplifying assumptions have linearized the relations between flows and injections. The flows in the various lines are thus given by

$$F_{12} = F_{23} = F^{\text{I}} - F^{\text{III}} = 160 - 80 = 80\,\text{MW}$$

$$F_{13} = F^{\text{II}} - F^{\text{IV}} = 240 - 120 = 120\,\text{MW}$$

The transaction between Generator D and load Z thus creates a counterflow that increases the power that Generator D and load Y can trade.

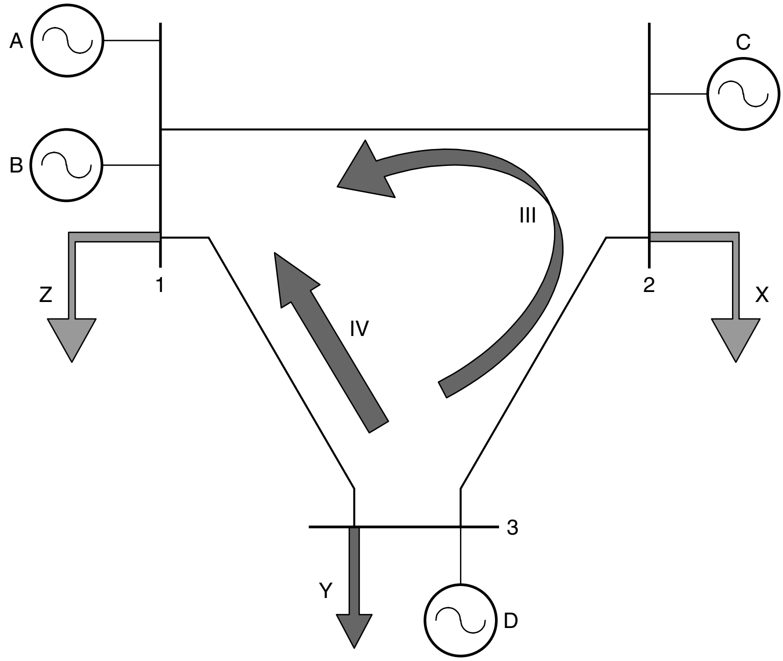

Figure 6.6 Paths for a transaction between Generator D and load Z

If we do not want the transmission network to limit trading opportunities unnecessarily, the amount of physical transmission rights that is made available must take into account possible counterflows. In keeping with the bilateral or decentralized trading philosophy, the system operator should only check that the system would be secure if all the proposed transactions were implemented. If this is not the case, the market participants have to adjust their position through further bilateral contracts until a secure system operating state is achieved. Bilateral energy trading is therefore closely coupled with bilateral trading in physical transmission rights.

In theory, if the market is sufficiently competitive, participants should be able to discover through iterative interactions a combination of bilateral trades in energy and transmission rights that achieves the economic optimum. In practice, in a power system with more than a few capacity constraints, the amount of information that needs to be exchanged is so large that it is unlikely that this optimum could be reached quickly enough through bilateral interactions.

6.2.2.3 Physical transmission rights and market power

We have defined physical transmission rights as giving their owner the right to transmit a certain amount of power for a certain time through a given branch of the transmission network. If physical transmission rights are treated like other types of property rights, their owners can use them or sell them. They can also decide to keep them but not use them. In a perfectly competitive market, buying physical transmission rights but not using them would be an irrational decision. On the other hand, in a less than perfectly competitive market, physical transmission rights can enhance the ability of some participants to exert market power. Consider, for example, the two-bus power system of Figure 6.1. If Generator G_3 is the only generator connected to bus B, it might want to purchase physical transmission rights for power flowing from bus A to bus B. If G_3 does not use or resell these rights, it effectively decreases the amount of power that can be sold at bus B by the other generators. This artificial reduction in transmission capacity enhances the market power that G_3 exerts at bus B and allows it to increase the profit margin on its production. It also has a detrimental effect on the economic efficiency of the overall system. See Joskow and Tirole (2000) for a comprehensive discussion of this issue.

To avoid this problem, it has been suggested that a "use them or lose them" provision be attached to physical transmission rights. Under this provision, transmission capacity that a participant has reserved but does not use is released to others who wish to use it. In theory, this approach should prevent market participants from hoarding transmission capacity for the purpose of enhancing market power. In practice, enforcing this condition is difficult because the unused transmission capacity may be released so late that other market participants are unable to readjust their trading positions.

6.3 Centralized Trading Over a Transmission Network

In a centralized or pool-based trading system, producers and consumers submit their bids and offers to the system operator, who also acts as market operator. The system operator, which must be independent from all the other parties, selects the bids and offers that optimally clear the market while respecting the security constraints imposed by the transmission network. As part of this process, the system operator also determines the market clearing prices. We shall show that, when losses or congestion in the transmission network is taken into account, the price of electrical energy depends on the bus in which the power is injected or extracted. The price that consumers and producers pay or are paid is the same for all participants connected to the same bus. This was not necessarily the case in a decentralized trading system in which prices are determined by bilateral contracts. In a centralized trading system, the system operator thus has a much more active role than it does in the bilateral model. Economic efficiency is indeed achieved only if it optimizes the use of the transmission network.

6.3.1 Centralized trading in a two-bus system

We will begin our analysis of the effects of a transmission network on centralized trading of electrical energy using a simple example involving the two fictitious countries of Borduria and Syldavia. After many years of hostility, these two countries have decided that the path to progress lies in economic cooperation. One of the projects that are under consideration is the reenergization of an existing electrical interconnection between the two national power systems. Before committing themselves to this project, the two governments have asked Bill, a highly regarded independent economist, to

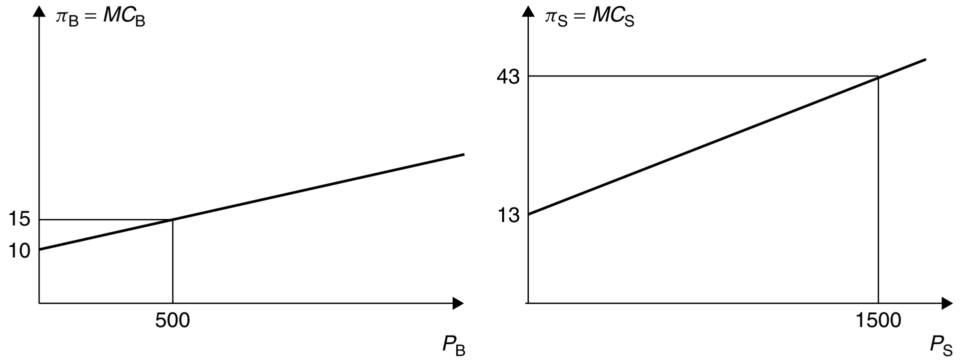

Figure 6.7 Supply functions for the electrical energy markets of Borduria and Syldavia

study the effect that this interconnection would have on their electricity markets and to evaluate the benefit that this interconnection would bring to both countries.

Bill begins his study by analyzing the two national power systems. He observes that both countries have developed centralized electricity markets that are quite competitive. The price of electrical energy in each market thus closely reflects its marginal cost of production. In both countries, the installed generation capacity exceeds the demand by a significant margin. Using regression analysis, Bill estimates the supply function for the electricity market in each country. In Borduria, this function is given by

$$\pi_B = MC_B = 10 + 0.01 P_B \text{ [\$/MWh]} \tag{6.6}$$

While in Syldavia, it is given by

$$\pi_S = MC_S = 13 + 0.02 P_S \text{ [\$/MWh]} \tag{6.7}$$

Like all supply curves, these functions increase monotonically with the demand for electrical energy. Figure 6.7 illustrates these supply functions. For the sake of simplicity, Bill assumes that the demands in Borduria and Syldavia are constant and equal to 500 MW and 1500 MW respectively. He also assumes that these demands have a price elasticity of zero. When the two national electricity markets operate independently, the prices are thus respectively

$$\pi_B = MC_B = 10 + 0.01 \times 500 = 15 \text{ \$/MWh} \tag{6.8}$$

$$\pi_S = MC_S = 13 + 0.02 \times 1500 = 43 \text{ \$/MWh} \tag{6.9}$$

Neither country is interconnected with other countries. Since the transmission infrastructure within each country is quite strong and very rarely affects the operation of the market for electrical energy, Bill decides that the simple model shown in Figure 6.8 is adequate for the study he needs to perform.

6.3.1.1 Unconstrained transmission

Under normal operating conditions, the interconnection can carry 1600 MW. If all the generators in Syldavia were to be shut down, the entire load of that country could still

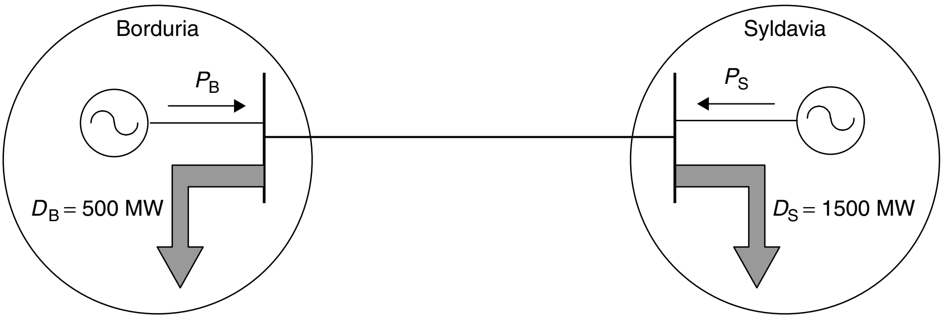

Figure 6.8 Model of the Borduria/Syldavia interconnection

be supplied from Borduria through the interconnection. The capacity of this link is thus larger than the power that could possibly need to be transmitted.

Equations (6.8) and (6.9) show that electricity prices in Borduria are significantly lower than in Syldavia. One might therefore envision that Bordurian generators might supply not only their domestic demand but also the entire demand of Syldavia. We would then have

$$P_B = 2000\,\text{MW} \tag{6.10}$$

$$P_S = 0\,\text{MW} \tag{6.11}$$

Replacing these values in Equations (6.6) and (6.7), we find that the marginal cost of producing electrical energy in the two systems would be

$$MC_B = 30\,\$/\text{MWh} \tag{6.12}$$

$$MC_S = 13\,\$/\text{MWh} \tag{6.13}$$

This situation is clearly not tenable because Bordurian generators would demand 30 $/MWh while Syldavian generators would be willing to sell energy at 13 $/MWh. The Bordurian generators are thus not able to capture the entire market because a process of price equalization would take place. In other words, the interconnection forces the markets for electrical energy in both countries to operate as a single market. A single market clearing price then applies to all the energy consumed in both countries:

$$\pi = \pi_B = \pi_S \tag{6.14}$$

Generators from both countries compete to supply the total demand, which is equal to the sum of the two national demands:

$$P_B + P_S = D_B + D_S = 500 + 1500 = 2000\,\text{MW} \tag{6.15}$$

Since the generators in both countries are willing to produce up to the point at which their marginal cost of production is equal to the market clearing price, Equations

(6.6) and (6.7) are still applicable. To determine the market equilibrium, Bill solves the system of Equations (6.6), (6.7), (6.14) and (6.15). He gets the following solution:

$$\pi = \pi_B = \pi_S = 24.30\,\$/\text{MWh} \tag{6.16}$$

$$P_B = 1433\,\text{MW} \tag{6.17}$$

$$P_S = 567\,\text{MW} \tag{6.18}$$

The flow of power in the interconnection is equal to the excess of generation over load in the Bordurian system and the deficit in the Syldavian system:

$$F_{BS} = P_B - D_B = D_S - P_S = 933\,\text{MW} \tag{6.19}$$

A flow of power from Borduria to Syldavia makes sense because the price of electricity in Borduria is lower than in Syldavia when the interconnection is not in service.

Figure 6.9 offers a graphical representation of the operation of this single market. The productions of the Bordurian and Syldavian generators are plotted respectively from left to right and right to left. Since the two vertical axes are separated by the total load in the system, any point on the horizontal axis represents a feasible dispatch of this load between generators in the two countries. This diagram also shows the supply curves of the two national markets. The prices in Borduria and Syldavia are measured along the left and right axes respectively.

When the two systems operate as a single market, the prices in both systems must be identical. Given the way this diagram has been constructed, the intersection of the two supply curves gives this operating point. The diagram then shows the production in each country and the flow on the interconnection.

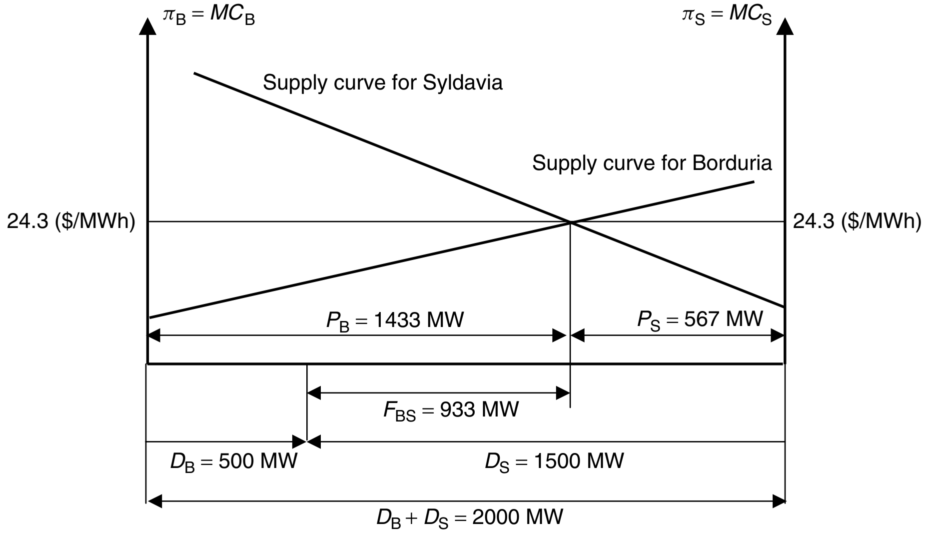

Figure 6.9 Graphical representation of the combination of the Syldavian and Bordurian electricity markets into a single market

6.3.1.2 Constrained transmission

Over the course of a year, various components of the transmission system must be taken out of service for maintenance. These components include not only lines and transformers but also some generating plants that provide essential reactive support services. The Borduria–Syldavia interconnection is therefore not always able to carry its nominal 1600 MW capacity. After consulting transmission engineers, Bill estimates that, during a significant part of each year, the interconnection is able to carry only a maximum of 400 MW. He therefore needs to study how the system behaves under these conditions.

When the capacity of the interconnection is limited to 400 MW, the production in Borduria must be reduced to 900 MW (500 MW of local load and 400 MW sold to consumers in Syldavia). The production in Syldavia is then 1100 MW. Using Equations (6.6) and (6.7), we find that

$$\pi_B = MC_B = 10 + 0.01 \times 900 = 19\ \$/\text{MWh} \tag{6.20}$$

$$\pi_S = MC_S = 13 + 0.02 \times 1100 = 35\ \$/\text{MWh} \tag{6.21}$$

Figure 6.10 illustrates this situation. The constraint on the capacity of the transmission corridor creates a difference of 16 \$/MWh between the prices of electrical energy in Borduria and Syldavia. If electricity were a normal commodity, traders would spot a business opportunity in this price disparity. If they could find a way of shipping more power from Borduria to Syldavia, they could make money by buying energy in one market and selling it in the other. However, this opportunity for *arbitrage* cannot be realized because the interconnection is the only way to transmit power between the two countries and it is already fully loaded. The price difference can thus persist

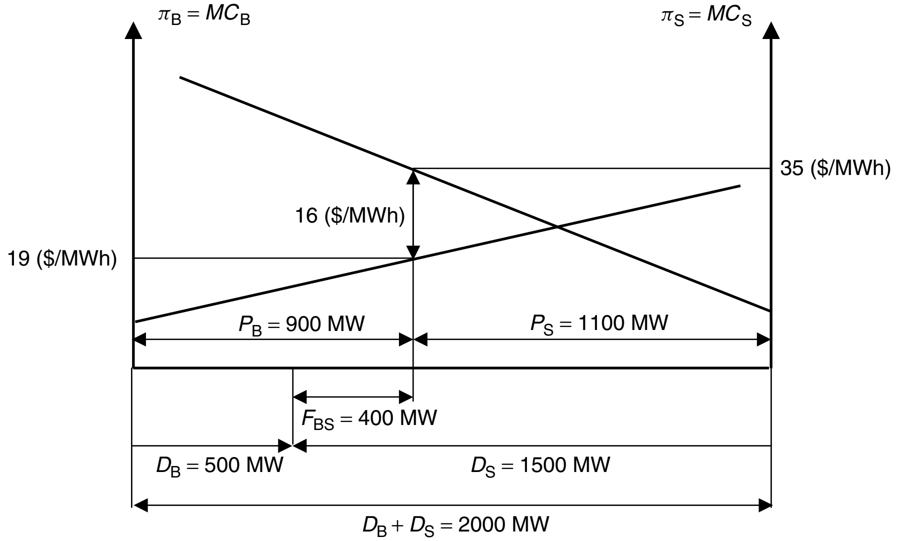

Figure 6.10 Graphical representation of the effect of congestion on the Syldavian and Bordurian electricity markets

as long as the capacity of the interconnection remains below the capacity needed to ensure free interchanges. Constraints imposed by the need to maintain the security of the system can thus create congestion in the transmission network. This congestion divides what should be a single market into separate markets. Because of the congestion, an additional megawatt of load in each country would have to be provided solely by the local generators. The marginal cost of producing electrical energy is therefore different in each country. If these separate markets are still sufficiently competitive, the prices are still equal to the marginal costs. We thus have what is called *locational marginal pricing* because the marginal cost depends on the location where the energy is produced or consumed. If a different price is defined at each bus or node in the system, locational marginal pricing is called *nodal pricing*. Our example shows that locational marginal prices are higher in areas that normally import power and lower in areas that export power.

Bill summarizes his findings so far in Table 6.2. He uses the following notations in this table: R represents the revenue accruing to a group of generators from the sale of electrical energy; E represents the payment made by a group of consumers for the purchase of electrical energy; the subscripts B and S denote respectively Borduria and Syldavia. F_{BS} represents the power flowing on the interconnection. This quantity is positive if power flows from Borduria to Syldavia.

Table 6.2 shows that the biggest beneficiaries of the reenergization of the interconnection are likely to be the Bordurian generators and the Syldavian consumers. Bordurian consumers would see an increase in the price of electrical energy. Syldavian generators would loose a substantial share of their market. Overall, the interconnection would have a positive effect because it would reduce the total amount of money spent by consumers on electrical energy. This saving arises because the energy produced by less efficient generators is replaced by energy produced by more efficient ones. Congestion on the interconnection reduces its overall benefit. Note that this congestion partially shields Syldavian generators from the competition of their Bordurian counterparts.

We have assumed so far that the markets are perfectly competitive. If competition were less than perfect, congestion in the interconnection would allow Syldavian

Table 6.2 Operation of the Borduria/Syldavia interconnection as separate markets, as a single market and as a single market with congestion

	Separate markets	**Single market**	**Single market with congestion**
P_{B} (MW)	500	1433	900
π_{B} (\$/MWh)	15	24.33	19
R_{B} (\$/h)	7500	34 865	17 100
E_{B} (\$/h)	7500	12 165	9500
P_{S} (MW)	1500	567	1100
π_{S} (\$/MWh)	43	24.33	35
R_{S} (\$/h)	64 500	13 795	38 500
E_{S} (\$/h)	64 500	36 495	52 500
F_{BS} (MW)	0	933	400
$R_{\text{TOTAL}} = R_{\text{B}} + R_{\text{S}}$	72 000	48 660	55 600
$E_{\text{TOTAL}} = E_{\text{B}} + E_{\text{S}}$	72 000	48 660	62 000

generators to raise their prices above their marginal cost of production. On the other hand, this congestion would intensify competition in the Bordurian market.

6.3.1.3 Congestion surplus

Bill decides that it would be interesting to quantify the effect that congestion on the interconnection would have on producers and consumers in both countries. He calculates the prices in Borduria and Syldavia as a function of the amount of power flowing on the interconnection:

$$\pi_B = MC_B = 10 + 0.01(D_B + F_{BS}) \tag{6.22}$$

$$\pi_S = MC_S = 13 + 0.02(D_S - F_{BS}) \tag{6.23}$$

Bill assumes that consumers pay the going price in their local market independently of where the energy they consume is produced. The total payment made by consumers is thus given by

$$E_{TOTAL} = \pi_B \cdot D_B + \pi_S \cdot D_S \tag{6.24}$$

Combining Equations (6.22), (6.23) and (6.24), Figure 6.11 shows how this payment varies as a function of F_{BS}. As Bill expected, this payment decreases monotonically as the flow between the two countries increases. The curve does not extend beyond $F_{BS} = 933$ MW because we saw earlier that a greater interchange does not make economic sense.

Similarly, Bill assumes that generators are paid the going price in their local market for the electrical energy they produce, independently of where this energy is consumed.

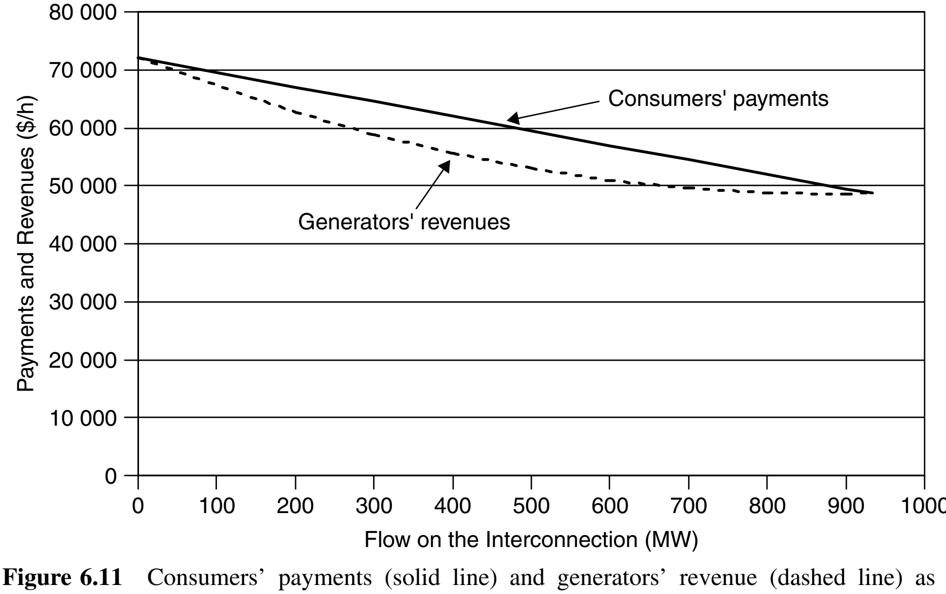

Figure 6.11 Consumers' payments (solid line) and generators' revenue (dashed line) as a function of the flow on the interconnection between Borduria and Syldavia

The total revenue collected by the generators from the sale of electrical energy in both markets is thus given by

$$R_{\text{TOTAL}} = \pi_{\text{B}} \cdot P_{\text{B}} + \pi_{\text{S}} \cdot P_{\text{S}} = \pi_{\text{B}} \cdot (D_{\text{B}} + F_{\text{BS}}) + \pi_{\text{S}} \cdot (D_{\text{S}} - F_{\text{BS}}) \tag{6.25}$$

This quantity has also been plotted on Figure 6.11 as a function of the power flowing on the interconnection. We observe that this revenue is less than the payment made by the consumers except when the interconnection is not congested ($F_{\text{BS}} = 933\,\text{MW}$) or when it is not in service ($F_{\text{BS}} = 0\,\text{MW}$). Combining Equations (6.24) and (6.25) while recalling that the flow on the interconnection is equal to the surplus of production over consumption in each country, we can write

$$\begin{aligned} E_{\text{TOTAL}} - R_{\text{TOTAL}} &= \pi_{\text{S}} \cdot D_{\text{S}} + \pi_{\text{B}} \cdot D_{\text{B}} - \pi_{\text{S}} \cdot P_{\text{S}} - \pi_{\text{B}} \cdot P_{\text{B}} \\ &= \pi_{\text{S}} \cdot (D_{\text{S}} - P_{\text{S}}) + \pi_{\text{B}} \cdot (D_{\text{B}} - P_{\text{B}}) \\ &= \pi_{\text{S}} \cdot F_{\text{BS}} + \pi_{\text{B}} \cdot (-F_{\text{BS}}) \\ &= (\pi_{\text{S}} - \pi_{\text{B}}) \cdot F_{\text{BS}} \end{aligned} \tag{6.26}$$

This difference between payments and revenues is called the *merchandizing surplus*. It is thus equal to the product of the differences between the prices in the two markets and the flow on the interconnection between these two markets. In this case, since this surplus is due to the congestion in the network, it is also called the *congestion surplus*.

In particular, for the case where the flow on the interconnection is limited to 400 MW, we have

$$E_{\text{TOTAL}} - R_{\text{TOTAL}} = (\pi_{\text{S}} - \pi_{\text{B}}) \cdot F_{\text{BS}} = (35 - 19) \cdot 400 = \$6400 \tag{6.27}$$

Note that its value is identical to the one we obtain if we take the difference between the total payment and the total revenue given in the last column of Table 6.2.

In a pool system in which all market participants buy or sell at the centrally determined nodal price applicable to their location, this congestion surplus is collected by the market operator. It should not, however, be kept by the market operator because this would give a perverse incentive to increase congestion or at least not work very hard to reduce congestion. On the other hand, simply returning the congestion surplus to the market participants would blunt the effect of nodal marginal pricing, which is designed to encourage efficient economic behavior. We will return to this issue when we discuss the management of congestion risks and financial transmission rights (FTRs) later in this chapter.

6.3.2 Centralized trading in a three-bus system

In our discussion of decentralized or bilateral trading, we already mentioned that Kirchoff's current and voltage laws dictate power flows in a transmission network with more than two buses. Now we need to explore the effect that these physical laws have on centralized trading. We will carry out this investigation using the same three-bus system that we used when we discussed bilateral trading. Figure 6.12 shows the diagram

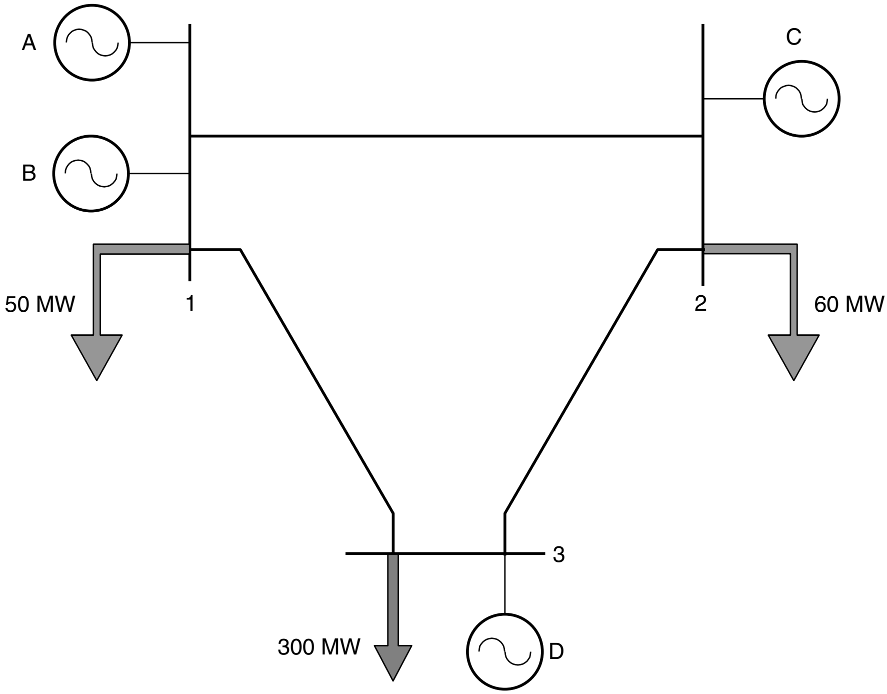

Figure 6.12 Simple three-bus system used to illustrate centralized trading

Table 6.3 Branch data for the three-bus system of Figure 6.12

Branch	Reactance (p.u.)	Capacity (MW)
1-2	0.2	126
1-3	0.2	250
2-3	0.1	130

of the network and Table 6.3 gives its parameters. We will again assume that network limitations take the form of constant capacity limits on the active power flowing in each line and that the resistance of the lines is negligible.

When we analyzed this system in the context of bilateral trading, we did not need to consider price or cost information because this data remains private to the parties involved in each bilateral transaction. On the other hand, in a centralized trading system, producers and consumers submit their bids and offers to the system operator, who uses this information to optimize the operation of the system. Since we are taking the perspective of the system operator, we assume that we have access to the data given in Table 6.4. We also assume that, since the market is perfectly competitive, the generators' bids are equal to their marginal cost. For the sake of simplicity, the marginal cost of each generator is assumed constant and the demand side is represented by the constant loads shown in Figure 6.12.

Table 6.4 Generator data for the three-bus system of Figure 6.12

Generator	Capacity (MW)	Marginal cost ($/MWh)
A	140	7.5
B	285	6
C	90	14
D	85	10

6.3.2.1 Economic dispatch

If we ignore the constraints that the network might impose, the total load of 410 MW should be dispatched solely on the basis of the bids or marginal costs of the generators in a way that minimizes the total cost of supplying the demand. Since we have assumed that these generators have a constant marginal cost over their entire range of operation and that the demand is not price sensitive, this dispatch is easy to compute: the generators are ranked in order of increasing marginal cost and loaded up to their capacity until the demand is satisfied. We get

$$\begin{aligned} P_{\mathrm{A}} &= 125\,\mathrm{MW} \\ P_{\mathrm{B}} &= 285\,\mathrm{MW} \\ P_{\mathrm{C}} &= 0\,\mathrm{MW} \\ P_{\mathrm{D}} &= 0\,\mathrm{MW} \end{aligned} \tag{6.28}$$

The total cost of the economic dispatch is

$$C_{\mathrm{ED}} = MC_{\mathrm{A}} \cdot P_{\mathrm{A}} + MC_{\mathrm{B}} \cdot P_{\mathrm{B}} = 2647.50\,\$/\mathrm{h} \tag{6.29}$$

We need to check whether this dispatch would cause one or more flows to exceed the capacity of a line. In a large network, we would calculate the branch flows using a power flow program. For such a simple system, we can do this computation by hand. This exercise will give us a more intuitive understanding of the way power flows through the network. Given the assumed flow directions shown in Figure 6.13, we can write the power balance equation at each bus or node as follows:

$$\text{Node 1: } F_{12} + F_{13} = 360 \tag{6.30}$$

$$\text{Node 2: } F_{12} - F_{23} = 60 \tag{6.31}$$

$$\text{Node 3: } F_{13} + F_{23} = 300 \tag{6.32}$$

In this case, we get three equations in three unknowns. However, these equations are linearly dependent because the power balance also holds for the system as a whole. For example, subtracting Equation (6.31) from Equation (6.30) gives Equation (6.32). Since one of these equations can be eliminated with no loss of information, we are left

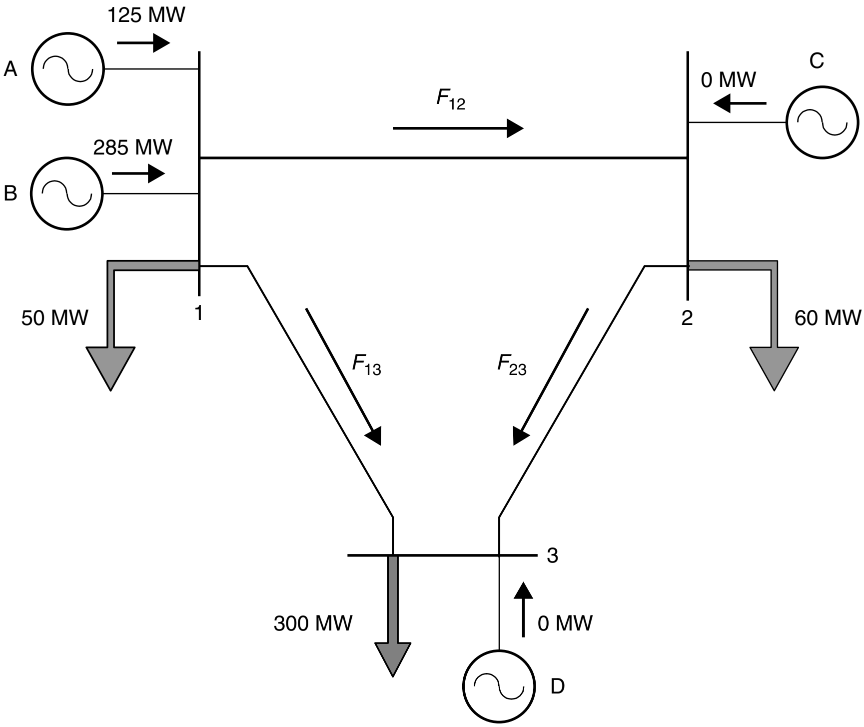

Figure 6.13 Basic dispatch in the three-bus system

with two equations and three unknowns. This is hardly surprising because we have not taken into account the impedances of the branches.

Rather than simply add an equation on the basis of KVL, let us again make use of the superposition theorem. Figure 6.14 shows how our original problem can be decomposed in two simpler problems. If we succeed in determining the flows in these two simpler problems, we can easily find the flows in the original problem because we know from the superposition theorem that

$$F_{12} = F_1^A + F_2^A \tag{6.33}$$

$$F_{13} = F_1^B + F_2^B \tag{6.34}$$

$$F_{23} = F_1^A - F_2^B \tag{6.35}$$

Let us consider the first problem. 300 MW is injected at bus 1 and taken out at bus 3. Since this power can flow along two paths (A and B), we have

$$F_1^A + F_1^B = 300\,\text{MW} \tag{6.36}$$

The reactances of paths A and B are respectively

$$x_1^A = x_{12} + x_{23} = 0.3 \text{ p.u.} \tag{6.37}$$

$$x_1^B = x_{13} = 0.2 \text{ p.u.} \tag{6.38}$$

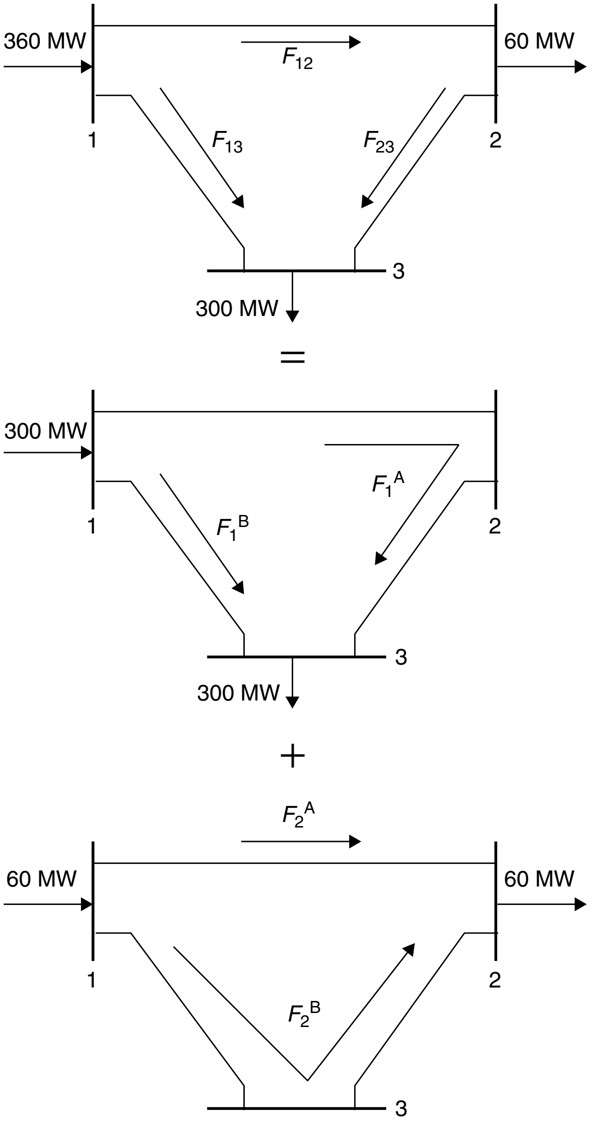

Figure 6.14 Application of the superposition theorem to the calculation of the line flows in the three-bus system

Since these 300 MW divide themselves between the two paths in accordance with Equations (6.4) and (6.5), we have

$$F_1^A = \frac{0.2}{0.3 + 0.2} \cdot 300 = 120\,\text{MW} \tag{6.39}$$

$$F_1^B = \frac{0.3}{0.3 + 0.2} \cdot 300 = 180\,\text{MW} \tag{6.40}$$

Similarly, for the second circuit, 60 MW is injected at bus 1 and taken out at bus 2. In this case, the impedances of the two paths are

$$x_2^{\mathrm{A}} = x_{12} = 0.2\,\mathrm{p.u.} \tag{6.41}$$

$$x_2^{\mathrm{B}} = x_{13} + x_{23} = 0.3\,\mathrm{p.u.} \tag{6.42}$$

Hence,

$$F_2^{\mathrm{A}} = \frac{0.3}{0.3+0.2} \cdot 60 = 36\,\mathrm{MW} \tag{6.43}$$

$$F_2^{\mathrm{B}} = \frac{0.2}{0.3+0.2} \cdot 60 = 24\,\mathrm{MW} \tag{6.44}$$

Equations (6.33) to (6.35) give the flows in the original system:

$$F_{12} = F_1^{\mathrm{A}} + F_2^{\mathrm{A}} = 120 + 36 = 156\,\mathrm{MW} \tag{6.45}$$

$$F_{13} = F_1^{\mathrm{B}} + F_2^{\mathrm{B}} = 180 + 24 = 204\,\mathrm{MW} \tag{6.46}$$

$$F_{23} = F_1^{\mathrm{A}} - F_2^{\mathrm{B}} = 120 - 24 = 96\,\mathrm{MW} \tag{6.47}$$

Figure 6.15 gives a graphical representation of this solution. From these results, we conclude that the economic dispatch would overload branch 1-2 by 30 MW because it would have to carry 156 MW when its capacity is only 126 MW. This is clearly not acceptable.

6.3.2.2 Correcting the economic dispatch

While the economic dispatch minimizes the total production cost, this solution is not viable because it does not satisfy the security criteria. We must therefore determine the least cost modifications that will remove the line overload. We begin by noting that the economic dispatch concentrates all the generation at bus 1. To reduce the flow on branch 1-2, we can increase the generation either at bus 2 or at bus 3. Let us first

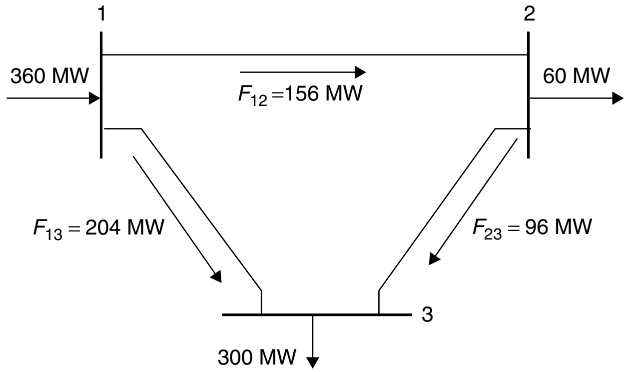

Figure 6.15 Flows for the economic dispatch in the three-bus system

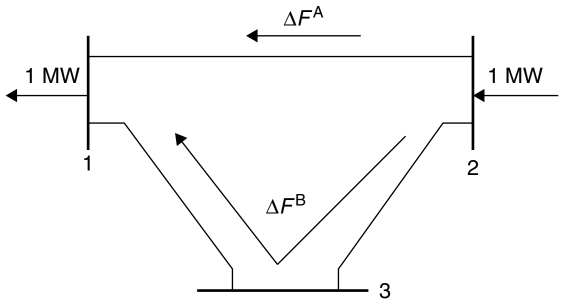

Figure 6.16 Effect of an incremental change in the generation at bus 2

consider what happens when we increase the generation at bus 2 by 1 MW. Since we neglect losses, this implies that we must reduce the generation at bus 1 by 1 MW. Figure 6.16 illustrates this incremental redispatch. Since the incremental flow ΔF^{A} is in the opposite direction as the flow F_{12}, increasing the generation at bus 2 and reducing it at bus 1 will reduce the overload on this branch. To quantify this effect, we can again make use of the superposition theorem. Since the reactances of paths A and B are respectively

$$x^{\mathrm{A}} = x_{12} = 0.2\,\text{p.u.} \tag{6.48}$$

$$x^{\mathrm{B}} = x_{23} + x_{13} = 0.3\,\text{p.u.} \tag{6.49}$$

and since the sum of the two flows must be equal to 1 MW, we get

$$\Delta F^{\mathrm{A}} = 0.6\,\text{MW} \tag{6.50}$$

$$\Delta F^{\mathrm{B}} = 0.4\,\text{MW} \tag{6.51}$$

Every megawatt injected at bus 2 and taken out at bus 1 thus reduces the flow on branch 1-2 by 0.6 MW. Given that this line is overloaded by 30 MW, a total of 50 MW of generation must be shifted from bus 1 to bus 2 to satisfy the line capacity constraint. Figure 6.17 illustrates this redispatch and its superposition with the economic dispatch to yield what we will call a constrained dispatch. We observe that the flow on branch 1-3 has also been reduced by this redispatch but that the flow on branch 2-3 has increased. This increase is tolerable, however, because this flow remains smaller than the capacity specified in Table 6.3. To implement this constrained dispatch, the generators connected at bus 1 must produce a total of 360 MW to meet the local load of 50 MW and inject 310 MW net into the network. At the same time, the generator at bus 2 must produce 50 MW. An additional 10 MW is taken from the network to supply the local load of 60 MW. Under these conditions, the least cost generation dispatch is

$$\begin{aligned} P_{\mathrm{A}} &= 75\,\text{MW} \\ P_{\mathrm{B}} &= 285\,\text{MW} \\ P_{\mathrm{C}} &= 50\,\text{MW} \\ P_{\mathrm{D}} &= 0\,\text{MW} \end{aligned} \tag{6.52}$$

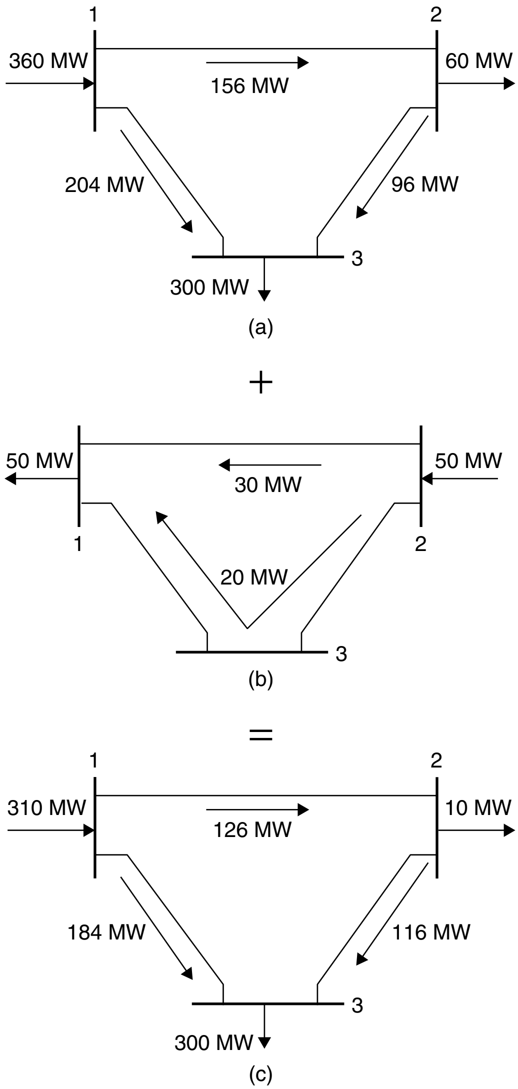

Figure 6.17 Superposition of the redispatch of generation from bus 1 to bus 2 (b) on the economic dispatch (a) to produce a constrained dispatch that meets the constraints on line flows (c)

Comparing with Equation (6.28), we see that the output of Generator A has been reduced rather than the output of Generator B because Generator A has a higher marginal cost. The total cost of this constrained dispatch is

$$C_2 = MC_A \cdot P_A + MC_B \cdot P_B + MC_C \cdot P_C = 2972.50\ \$/\text{h} \tag{6.53}$$

This cost is necessarily higher than the cost of the economic dispatch that we calculated in Equation (6.29). The difference represents the cost of achieving security using this redispatch.

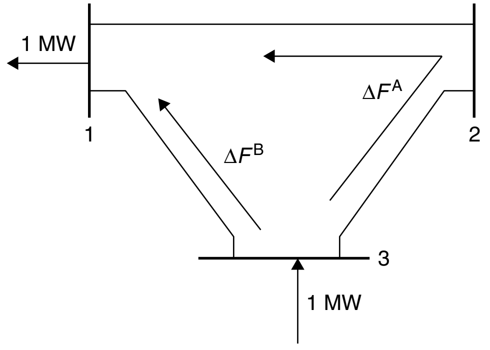

Figure 6.18 Effect of an incremental change in the generation at bus 3

We mentioned above that we could also relieve the overload on branch 1-2 by increasing the output of Generator D connected at bus 3. Let us calculate the extent and the cost of this other redispatch using the same procedure. Figure 6.18 shows the two paths along which an extra MW injected at bus 3 and taken out at bus 1 would divide itself. Given that the reactances of paths A and B are respectively

$$x^{A} = x_{23} + x_{12} = 0.3\,\text{p.u.} \tag{6.54}$$

$$x^{B} = x_{13} = 0.2\,\text{p.u.} \tag{6.55}$$

and that the sum of the two flows must be equal to 1 MW, we get

$$\Delta F^{A} = 0.4\,\text{MW} \tag{6.56}$$

$$\Delta F^{B} = 0.6\,\text{MW} \tag{6.57}$$

Every MW injected at bus 3 and taken out at bus 1 thus reduces the flow on branch 1-2 by 0.4 MW. This means that we need to shift 75 MW of generation from bus 1 to bus 3 to reduce the flow on branch 1-2 by 30 MW and remove the overload. Figure 6.19 shows how superposing this redispatch on the economic dispatch reduces the flows through all the branches of the network. As expected, the flow on branch 1-2 is equal to the maximum capacity of that branch. Since the total power to be produced at bus 1 is now reduced by 75 MW, the generation dispatch for this case is

$$\begin{aligned} P_{A} &= 50\,\text{MW} \\ P_{B} &= 285\,\text{MW} \\ P_{C} &= 0\,\text{MW} \\ P_{D} &= 75\,\text{MW} \end{aligned} \tag{6.58}$$

The total cost of this constrained dispatch is

$$C_3 = MC_{A} \cdot P_{A} + MC_{B} \cdot P_{B} + MC_{D} \cdot P_{D} = 2835\,\$/\text{h} \tag{6.59}$$

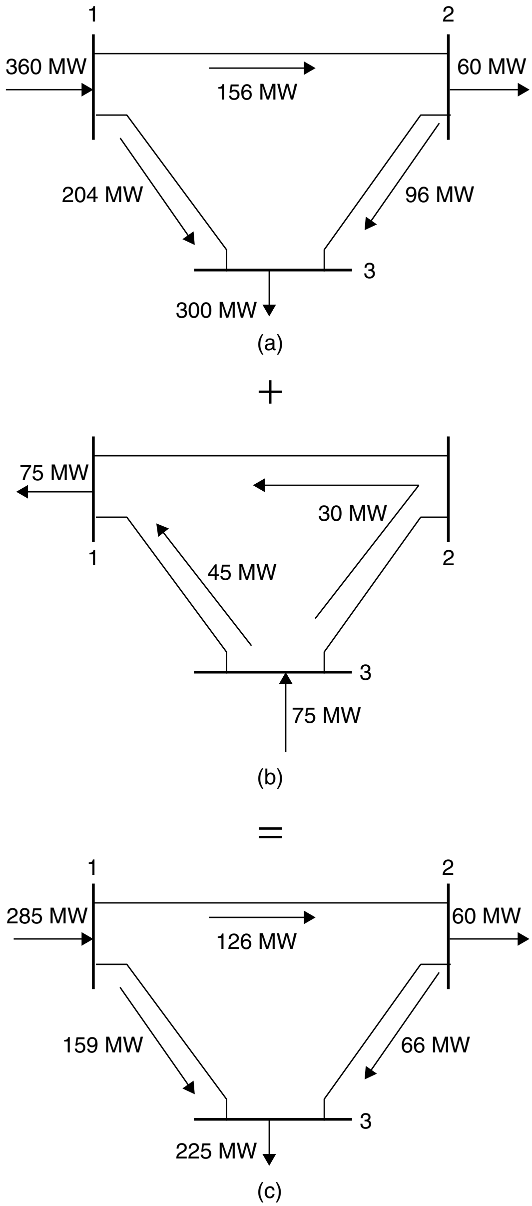

Figure 6.19 Superposition of the redispatch of generation from bus 1 to bus 3 (b) on the economic dispatch (a) to produce a constrained dispatch that meets the constraints on line flows (c)

Let us now compare these two ways of removing the overload on branch 1-2. If we make use of the generation at bus 3, we need to redispatch 75 MW. On the other hand, if we call upon the generation at bus 2, we need to shift only 50 MW. This is because the flow on branch 1-2 is less sensitive to an increase in generation at bus 3 than it is to an increase at bus 2. However, since the marginal cost of Generator D is less than the marginal cost of Generator C, increasing the generation at bus 3 is the preferred solution because it is cheaper. The cost of making the system secure is thus equal to the difference between the cost of this constrained dispatch and the cost of the economic dispatch:

$$C_{\mathrm{S}} = C_3 - C_{\mathrm{ED}} = 2835.00 - 2647.50 = 187.50\,\$/\mathrm{h} \tag{6.60}$$

6.3.2.3 Nodal prices

We have already alluded to the concept of nodal marginal price when we discussed the Borduria–Syldavia interconnection. We are now in a position to clarify this concept. The nodal marginal price is equal to the cost of supplying an additional megawatt of load at the node under consideration by the cheapest possible means.

In our three-bus example, this means that we do not start from the economic dispatch but from the constrained dispatch given by Equation (6.58). The output of Generator D has thus been increased to remove the overload on branch 1-2. At node 1 it is clear that an additional megawatt of load should be produced by Generator A. The marginal cost of Generator A is indeed lower than the marginal cost of Generators C and D. While it is higher than the marginal cost of Generator B, this generator is already loaded up to its maximum capacity and is therefore unable to produce an additional megawatt. The network has no impact on the marginal price at this node because the additional megawatt is produced and consumed locally. The nodal marginal price at bus 1 is therefore

$$\pi_1 = MC_{\mathrm{A}} = 7.50\,\$/\mathrm{MWh} \tag{6.61}$$

What is the cheapest way of supplying an additional megawatt at bus 3? Generator A has the lowest marginal cost and is not fully loaded. Unfortunately, increasing the generation at bus 1 would inevitably overload branch 1-2. The next cheapest option is to increase the output of Generator D. Since this generator is located at bus 3, this additional megawatt would not flow through the network. Therefore, we have

$$\pi_3 = MC_{\mathrm{D}} = 10\,\$/\mathrm{MWh} \tag{6.62}$$

Supplying an additional megawatt at bus 2 is a more complex matter. We could obviously generate it locally using Generator C, but this looks rather expensive because at 14 \$/MWh the marginal cost of this generator is much higher than the marginal cost of the other generators. If we choose to adjust the output of the generators at the other buses, we must consider what might happen in the network. Figure 6.20 shows how an additional megawatt of load at bus 2 would flow through the network if it were produced at bus 1 or bus 3. We can see that in both cases we increase the flow on branch 1-2. Since the flow on this branch is already at its maximum value, neither solution is acceptable. Any combination of generation increases at buses 1 and 3 would also be unacceptable. We could, however, increase generation at bus 3 and reduce it at bus 1. For example, as shown in Figure 6.21, we could increase the generation at bus 3 by 2 MW and reduce it at bus 1 by 1 MW. The net increase is then equal to the additional load at bus 2. We can then, once again, use superposition to determine the resulting incremental flows. The first diagram in Figure 6.22 shows that if 1 MW