Discrete Optimal Control & Analysis of a PEM Fuel Cell to Grid (V2G) System

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Introduction

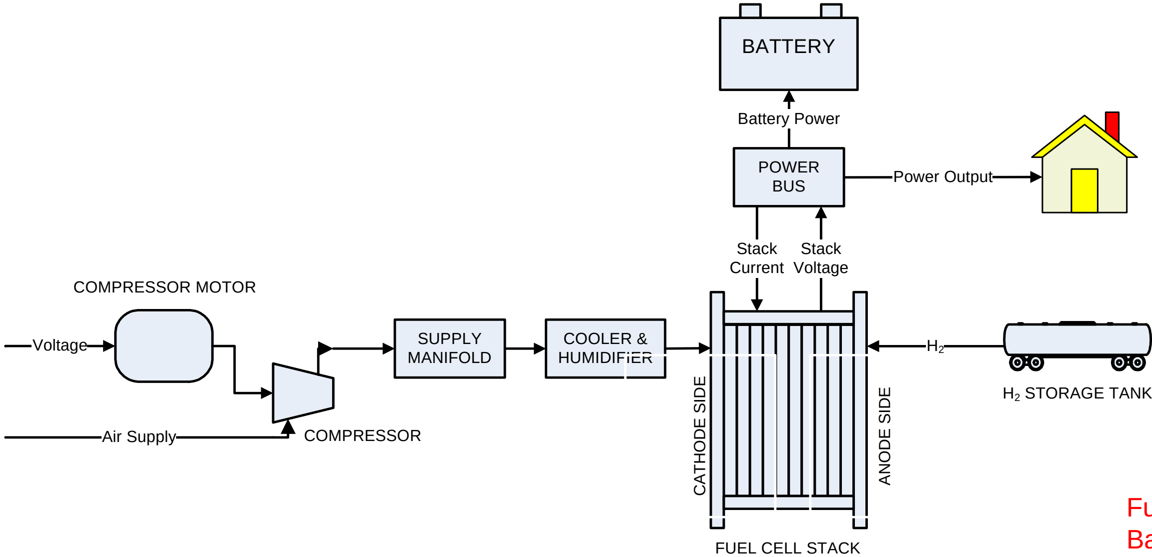
- Fuel Cell technology
  - Abundant energy source H_2
  - High efficiency (50-70%)
  - Clean energy source (zero emissions)

- Hybrid Technology
  - Hybrid concept is developing in many engineering fields, esp. the auto industry
  - Fuel Cell/Battery leverages advantages of each energy source

- V2G Concept
  - Enables the use of renewable energy sources
  - Adds energy storage capacity element to grid
  - Distributed generation (DG) decentralizes grid
  - 5% of California's vehicle fleet can provide 10% peak power for entire state [1]
  - Consumer may sell power back to the grid
  - More expensive FCV becomes a more profitable investment

[1] W. Kempton, J. Tomic, S. Letendre, A. N. Brooks and T. Lipman, "Vehicle-to-grid power: Battery, hybrid, and fuel cell vehicles as resources for distributed electric power in California," California Air Resources Board, Tech. Rep. UCD-ITS-RR-01-03, June 2001, 2001.

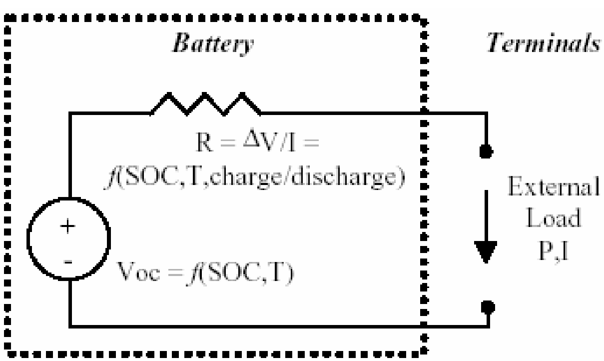
Fuel Cell System & Battery



- List of components
  - FC stack
  - Humidifier
  - Supply Manifold
  - Compressor

Fuel Cell stack model adapted from [Lin, Kim, Peng, Grizzle]
Battery model from ADVISOR

Resistive Equivalent Circuit Model



Isothermal Operation Assumption

- Main functions of battery model

$$V_{oc} = f(soc, T), \quad R_{\text{int}} = f(soc, T)$$

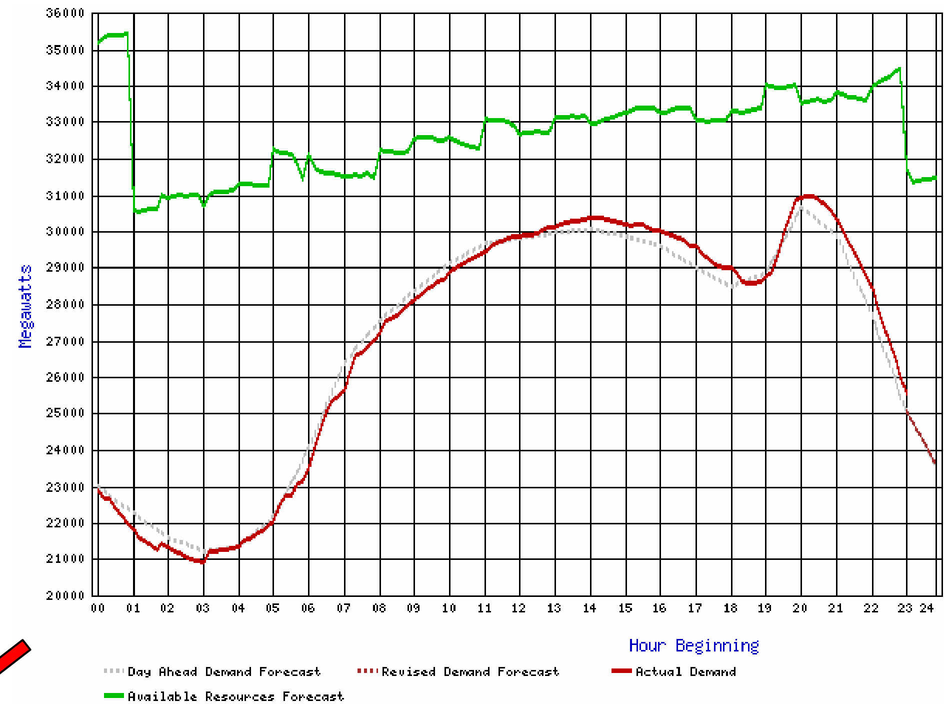
$$P_{batt} = f(soc, V_{oc}, R_{\text{int}})$$

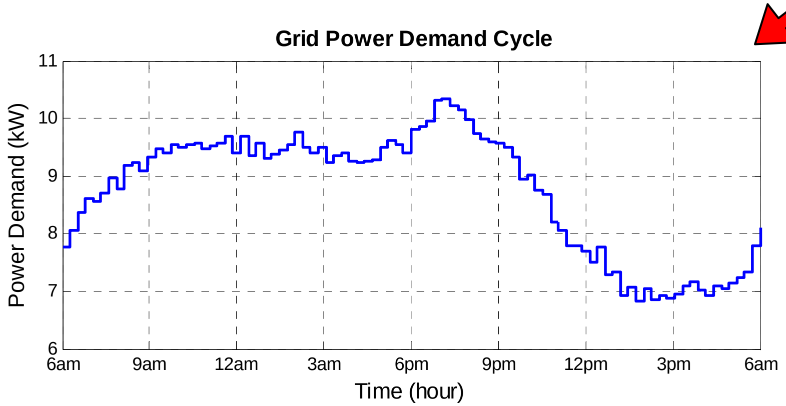
$$I_{batt} = -\frac{V_{oc} - \sqrt{V_{oc}^{\,2} - 4P_{batt}R_{\text{int}}}}{2R_{\text{int}}}$$

$$\dot{SOC} = -\frac{I_{batt}}{Q_{\max}} \Rightarrow SOC = \frac{Q_{\max} - \int I_{batt}\,dt}{Q_{\max}}$$

Power Grid Demand Cycle Modeling

- Representative grid power demand cycle
- Adapted from California Independent System Operator (CAISO) daily demand forecast
- 24 hour cycle (6am to 6am)





- Scaled for medium size office or apartment complex
- Augmented with white Gaussian noise to simulate stochastic nature of power demand

Control Problem Formulation

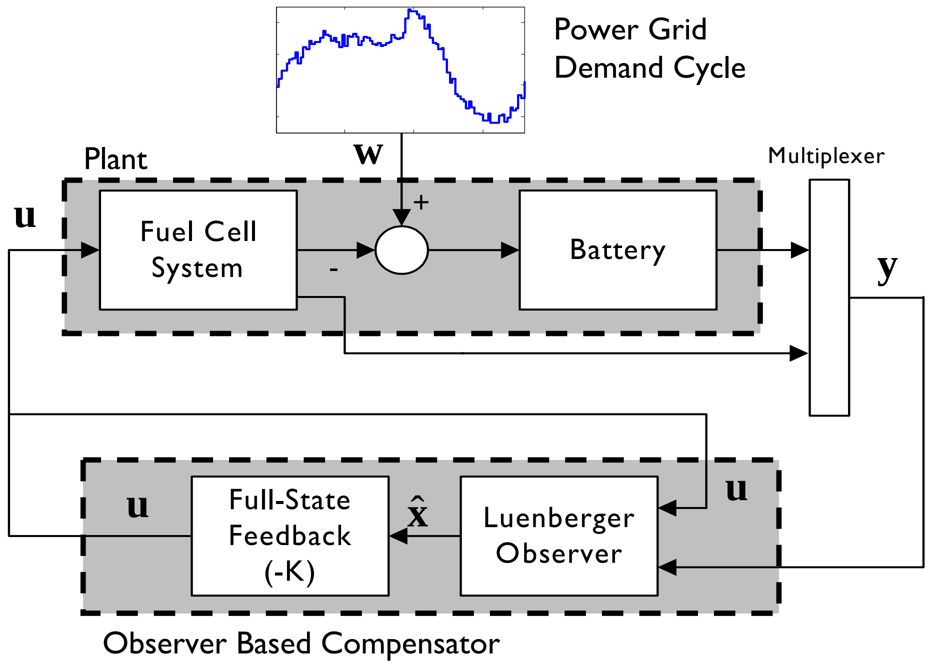
- Regulate battery state of charge (SOC)
- Maximize fuel cell system (FCS) efficiency
- Reject disturbances (grid power demand)
- Accurately estimate SOC

Control Inputs: $\mathbf{u} = \begin{bmatrix} I_{st} \\ e_{cm} \end{bmatrix}$

Process Disturbance: $\mathbf{w} = P_{grid}$

Dynamic States: $\mathbf{x} = \begin{bmatrix} \omega_{cp} \\ p_{sm} \\ Q_{used} \end{bmatrix}$

Measurements: $\mathbf{y} = \begin{bmatrix} V_{oc} \\ \eta_{fc} \end{bmatrix}$



Linearization

Operating Points

$$\mathbf{u}^0 = \begin{bmatrix} 30A \\ 42.6V \end{bmatrix} \quad \mathbf{w}^0 = 8500W \quad \mathbf{x}^0 = \begin{bmatrix} 2646\dfrac{rad}{\sec} \\ 111{,}570Pa \\ 1.8C \end{bmatrix} \quad \mathbf{y}^0 = \begin{bmatrix} 155.331V \\ 0.5892 \end{bmatrix}$$

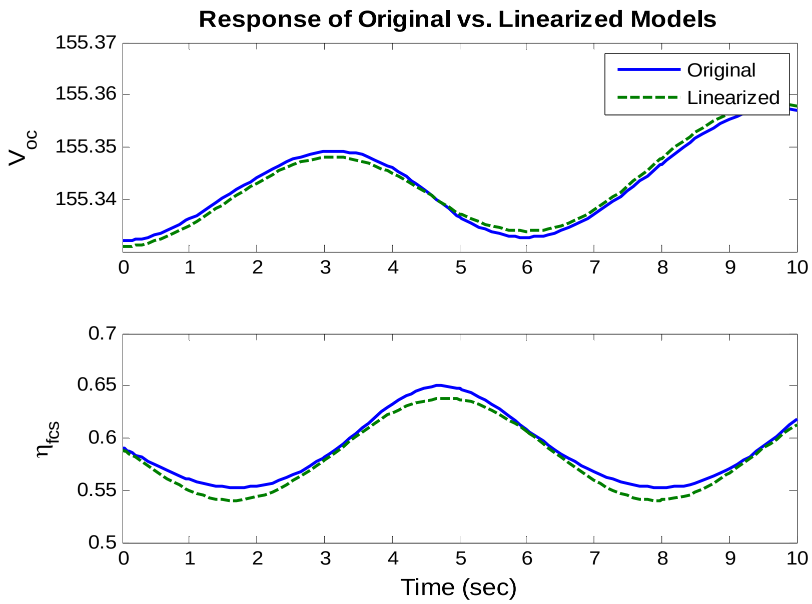
Linear System Model

$$\delta\dot{\mathbf{x}} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta\mathbf{y} + \mathbf{G}\delta\mathbf{w}$$
$$\delta\mathbf{y} = \mathbf{C}\delta\mathbf{x} + \mathbf{D}\delta\mathbf{y} + \mathbf{H}\delta\mathbf{w}$$

In terms of perturbation variables

$\delta(\bullet) = (\bullet) - (\bullet)^0$ represents the perturbation about the nominal operating point

Verification



Response of original vs. linearized model for sinusoidal control inputs and power demand cycle disturbance.

Discrete System Analysis & LQR Design

Continuous to Discrete Transformation

- Sampling Time T = 0.05s
- 4 times faster than fastest plant dynamics

		Output	
		Open Circuit Voltage	*FCS Efficiency*
Input	*Stack Current*	$\frac{2.79\times10^{-5}(z-0.768)(z-0.212)}{(z-1)(z-0.7678)(z-0.2087)}$	$\frac{-0.0033(z-1)(z-0.766)(z-0.203)}{(z-1)(z-0.7678)(z-0.2087)}$
	CM Voltage	$\frac{1.81\times10^{-8}(z+2.48)(z+0.16)}{(z-1)(z-0.7678)(z-0.2087)}$	$\frac{2.80\times10^{-5}(z-1)(z+0.55)}{(z-1)(z-0.7678)(z-0.2087)}$
	Power Demand Cycle	$-\frac{-1.16\times10^{-7}(z-0.77)(z-0.21)}{(z-1)(z-0.7678)(z-0.2087)}$	0

Table 1: Discrete-time transfer functions for MIMO system using a sampling time of T = 0.05s.

Linear Quadratic Regulator

Performance Index

Continuous

$$J=\int_0^\infty\left[\mathbf{x}^\mathrm{T}(t)\mathbf{Q}\mathbf{x}(t)+\mathbf{u}^\mathrm{T}(t)\mathbf{R}\mathbf{u}(t)\right]dt$$

Discrete

$$J=\frac{1}{2}\sum_{i=0}^{\infty}\left[\mathbf{x}^T(i)\mathbf{Q}\mathbf{x}(i)+\mathbf{u}^T(i)\mathbf{R}\mathbf{u}(i)\right]$$

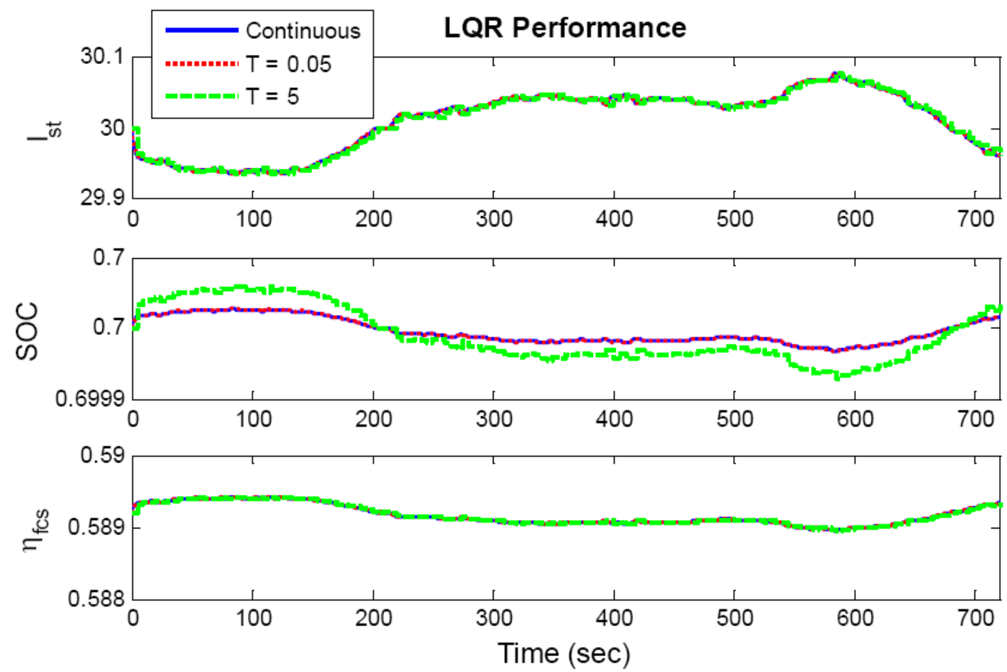


Figure 7: Control and performance variable trajectories for both continuous & discrete time LQR.

Luenberger Estimator Design

Observer Pole Placement

Closed loop full state feedback dynamics $\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{BK})\mathbf{x}$ $\quad$ $\mathbf{x}(k+1) = (\mathbf{A} - \mathbf{BK})\mathbf{x}$

Rule of thumb: Select observer dynamics to be 3x faster than closed loop system

$$\mathbf{p}_{observer} = 3 \cdot eig(\mathbf{A} - \mathbf{BK}) \qquad \mathbf{p}_{observer} = \left[eig(\mathbf{A} - \mathbf{BK})\right]^3$$

Why cube the eigenvalues for discrete time?

	Continuous Time	Discrete Time
Closed-loop poles	$(1.738\times10^{-6}, -5.285, -31.337)$	$(0.209, 0.768, 0.999)$
Observer poles	$(-5.213\times10^{-6}, -15.854, -94.013)$	$(0.009, 0.453, 0.950)$

Proof:

Z-transform definition $z = e^{sT}$

3x faster in continuous time $p_{observer}(s) = 3 \cdot s$

Substitute into Z-transform definition $p_{observer}(s) = e^{3sT} = \left(e^{sT}\right)^3$

Re-apply Z-transform definition $p_{observer}(z) = z^3$

State Estimator Dynamics: $\dot{\hat{\mathbf{x}}}(t) = (\mathbf{A} - \mathbf{LC})\hat{\mathbf{x}}(t) + \mathbf{Ly}(t) + \mathbf{Bu}(t)$ $\quad$ $\hat{\mathbf{x}}(k+1) = (\mathbf{A} - \mathbf{LC})\hat{\mathbf{x}}(k) + \mathbf{Ly}(k) + \mathbf{Bu}(k)$

Complete Closed Loop Dynamics:

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK} \\ \mathbf{0} & \mathbf{A} - \mathbf{LC} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \tilde{\mathbf{x}}(t) \end{bmatrix} \qquad \begin{bmatrix} \mathbf{x}(k+1) \\ \tilde{\mathbf{x}}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK} \\ \mathbf{0} & \mathbf{A} - \mathbf{LC} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \tilde{\mathbf{x}}(k) \end{bmatrix}$$

Separation Principle

Poles of FB Control & Poles of Observer are independent!

Open Loop System Analysis

Zero-Pole Cancellation

Analyze physical significance

- Open Circuit Voltage (Integrator)
- FCS Efficiency (Gain)

		Output	
		Open Circuit Voltage	*FCS Efficiency*
Input	*Stack Current*	$\approx \dfrac{2.79\times10^{-5}}{(z-1)}$	≈ -0.0033
	CM Voltage	$\approx \dfrac{1.81\times10^{-8}(z+2.48)(z+0.16)}{(z-1)(z-0.7678)(z-0.2087)}$	$\approx \dfrac{2.80\times10^{-5}(z+0.55)}{(z-0.7678)(z-0.2087)}$
	Power Demand Cycle	$\approx \dfrac{-1.16\times10^{-7}}{(z-1)}$	0

Table 2: Approximate discrete-time transfer functions after near or exact zero-pole cancellations.

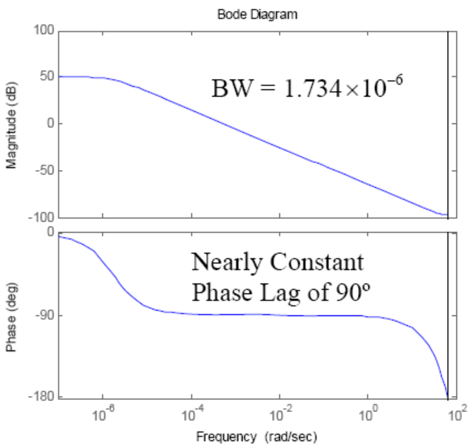
A Closer Look:

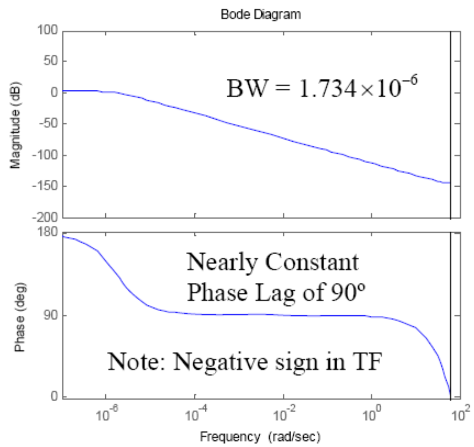
Open Circuit Voltage (Integrator)

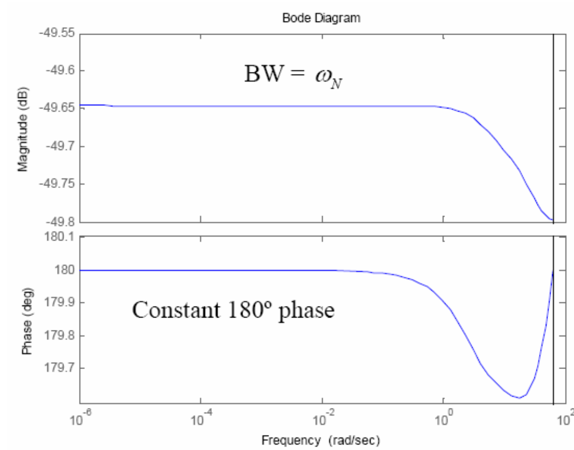
- Battery modeled as capacitive element
- Zero bandwidth
- Constant 90° phase lag

FCS Efficiency (Gain)

- Proportional to I_{st} ➡ $\eta_{fcs} = \dfrac{P_{fcs,net}}{LHV \cdot \dot{m}_{H_2}} \approx K \cdot I_{st}$
- Infinite Bandwidth
- Constant 180° phase lag







LQR Weight Selection by Parameter Sweep

Select Q and R to meet desired specifications:

- V_{oc} acheives steady state in less than 1 min
- FCS operates at "optimal" efficiency for as long as possible
- Stack current cannot exceed 310A (saturated control action)

Recall Performance Index

$$J = \int_0^\infty \left[\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t) \right] dt$$

$$J = \frac{1}{2}\sum_{i=0}^{\infty} \left[\mathbf{x}^T(i)\mathbf{Q}\mathbf{x}(i) + \mathbf{u}^T(i)\mathbf{R}\mathbf{u}(i) \right]$$

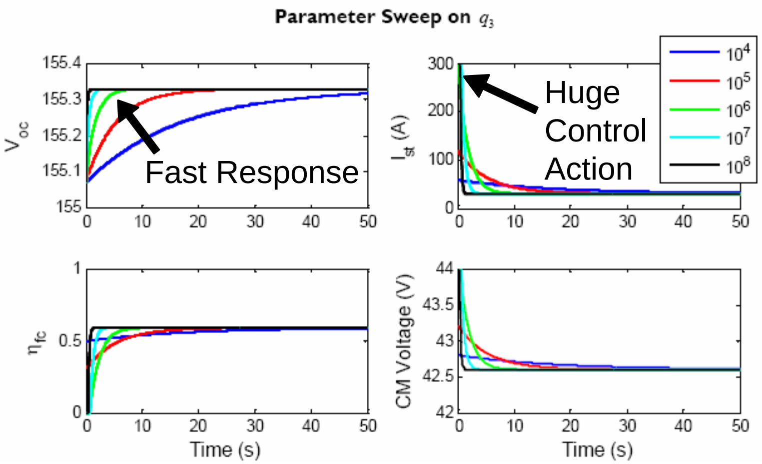
Select diagonal matrices

- Set $q_1 = q_2 = r_2 = 0$
- Initially, no desire to regulate I_{st}, V_{oc}, or e_{cm}

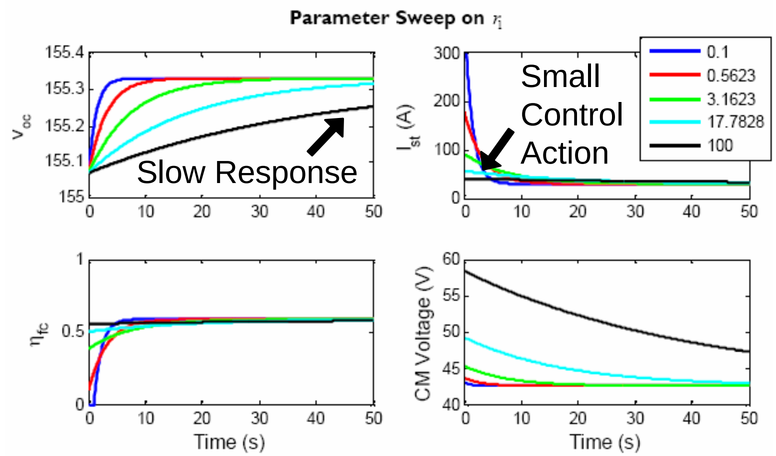
$$\mathbf{Q} = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$$

Parameter Sweep Methodology

1. Fix one parameter and vary the other
2. Observe performance



Tradeoff between response speed and control actuation

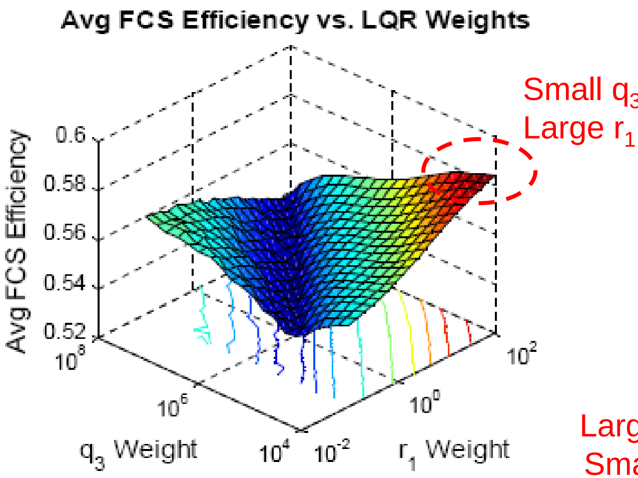


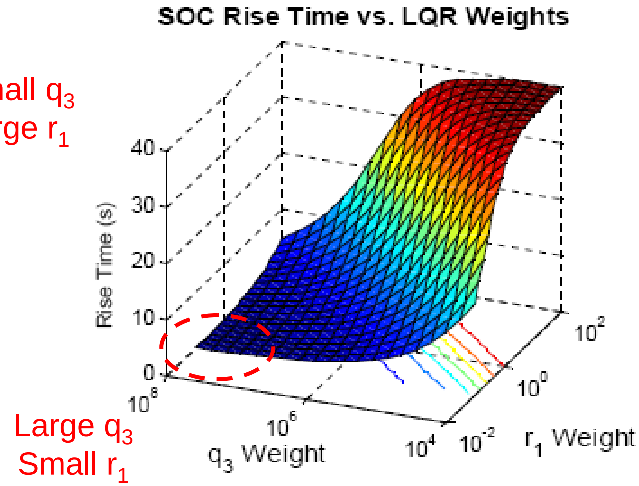
LQR Weight Selection by Multiobjective Optimization

Multiobjective Optimization:

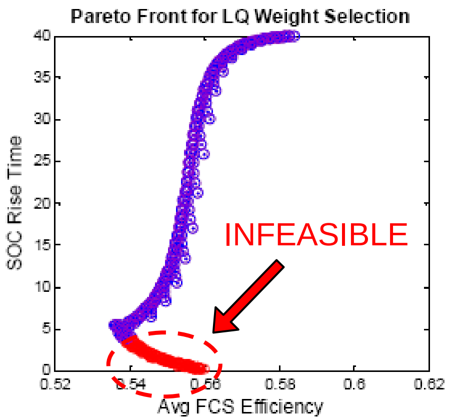
- Minimize SOC rise time
- Maximize average FCS efficiency

Tradeoff observed between each objective





Pareto Front Analysis: Plot each objective on an axis



- Lower region exhibits typical Pareto Front behavior
- Red points indicate the design exceeds the $I_{st} < 310$A limit
- Pareto Front visualization method is very intuitive
- Effective method for LQR weight selection

Observer Pole Placement & Estimation Analysis

Observer Pole Placement

Fast poles

Advantage: Estimation error decays rapidly

Disadvantage: Assumes perfect sensors and/or noise-free environment

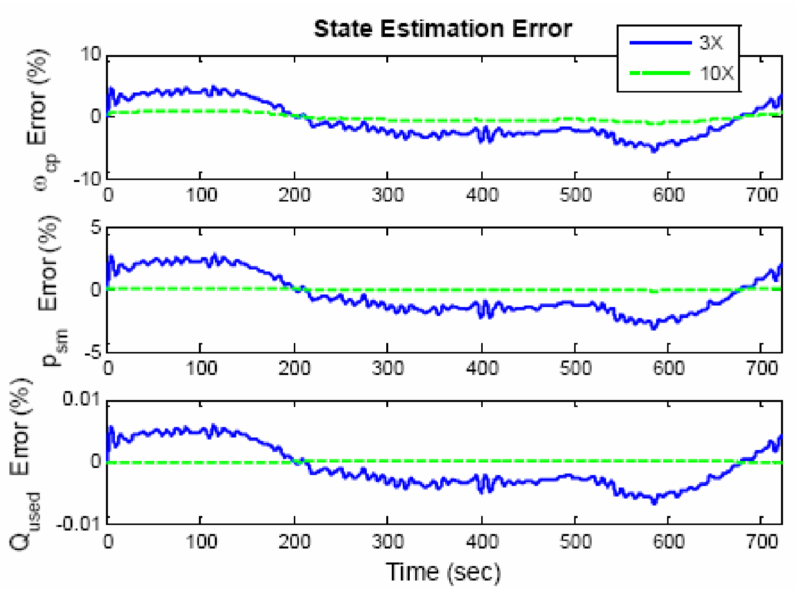
Slow poles

Advantage: Less sensitive to process disturbances and measurement noise

Disadvantage: Estimation error decays slowly

Estimation Analysis

- No state experiences greater than 5% error
- Estimation error imitates power grid cycle
- Estimator picks up on process disturbance
- Error reduces dramatically for 10x faster poles
- NOTE: Perfect sensors assumed



Conclusions & Recommendations

Conclusions

- Pole-zero cancellation analysis elucidates physical significance of transfer function models
- LQR weight selection is a multi-objective optimization problem
- Fast observer poles track process disturbance
- Linearization assumes inputs, as well as states, do not deviate far from the linearization points
- A tradeoff exists between fast SOC response and low stack current control input

Recommendations

- Gain scheduling to accommodate nonlinearity of power disturbance cycle
- Kalman estimator design to reject process disturbance and sensor noise

Acknowledgements & References

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Key References

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