PDE Estimation and Optimal Control for Electrochemical Model-based Battery Management Systems

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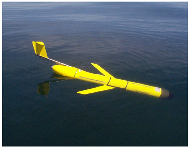
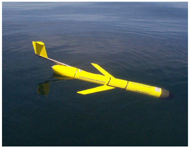
A Golden Era


ST. JUDE MEDICAL
Sylmar, CA USA
ACCENT SR RF
PM1210 SSR
ST. JUDE MEDICAL
Sylmar, CA USA
ANTHEM RF
PM3212 DDDRV

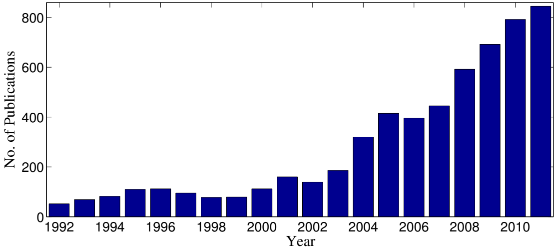



A123 SYSTEMS
2MWH-APU
HYBRID ANCILLARY POWER UNIT

A Golden Era

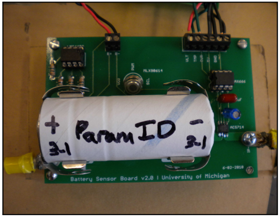


Keyword: “Battery Systems and Control”



Cell Level

- Modeling & Design
- Control under constraints
- SOC/SOH Estimation
- ...



Open Problems in Battery Systems and Control

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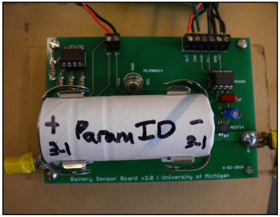
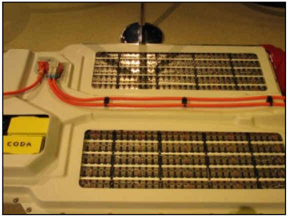
Pack Level

- Charge (Un)balancing
- Thermal Management
- Energy Management
- ...





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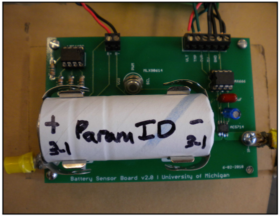
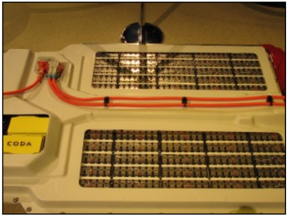
Smart-Grid Level

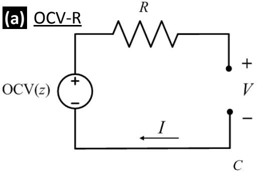
- Renewable Energy Integration
- Optimal Power Flow
- PEV Power Management
- ...





Equivalent Circuit Model

(a) OCV-R



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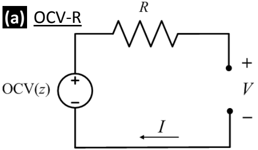
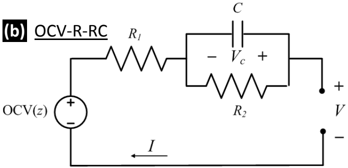
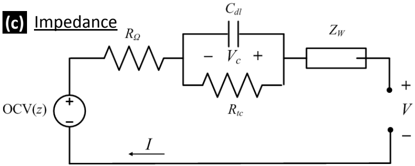


(b) OCV-R-RC

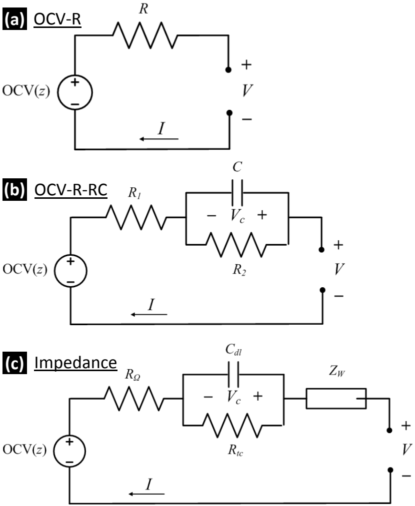


(c) Impedance

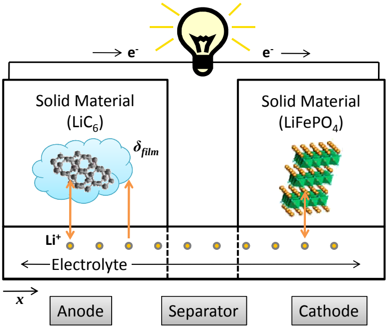


Battery Models

Equivalent Circuit Model



Electrochemical Model



Electrochemical Model Equations

Diffusion of lithium:

$$\frac{\partial c_s}{\partial t}(x,r,t) = \frac{D_s}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial c_s}{\partial r}(x,r,t)\right]$$

$$\varepsilon_e \frac{\partial c_e}{\partial t}(x,t) = D_e^{eff}\frac{\partial^2 c_e}{\partial x^2}(x,t) + \frac{1-t^+}{F}J(x,t)$$

Electric Potential:

$$0 = \sigma_s^{eff}\frac{\partial^2 \phi}{\partial x^2}(x,t) - J(x,t)$$

$$0 = \frac{\partial}{\partial x}\left[\kappa^{eff}(c_e)\frac{\partial \phi_e}{\partial x}(x,t)\right] + \frac{\partial}{\partial x}\left[\kappa(c_e)\frac{\partial \ln c_e}{\partial x}(x,t)\right] + J(x,t)$$

Butler-Volmer Kinetics:

$$J(x,t) = 2ai_0(x,t)\sinh\left[\frac{\alpha F}{RT}\eta(x,t)\right]$$

Overpotential:

$$\eta(x,t) = \phi_s(x,t) - \phi_e(x,t) - U_{ref}(c_{ss}(x,t)) - \frac{R_f}{a}J(x,t)$$

Outline

1. SOC/SOH Estimation
   - Single Particle Model
   - State Estimation via PDE Backstepping
   - Parameter Identification via Adaptive & Nonlinear Control
2. PHEV Power Management
   - Models
   - Stochastic Optimal Control
   - Sample Results
3. Bonus
   - LQR for PDEs
   - Education on Battery Systems and Control
4. Future Research Directions

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A Short History

- Equivalent Circuit Model

- Electrochemical Model

A Short History

- Equivalent Circuit Model
  - G. Plett (2004) - Extended Kalman Filter (States & Params)
  - RLS, Bias-correcting RLS, EKF on Impedance-based ECMs, LPV, Neural nets, Sliding-mode, Particle filters, and many more...
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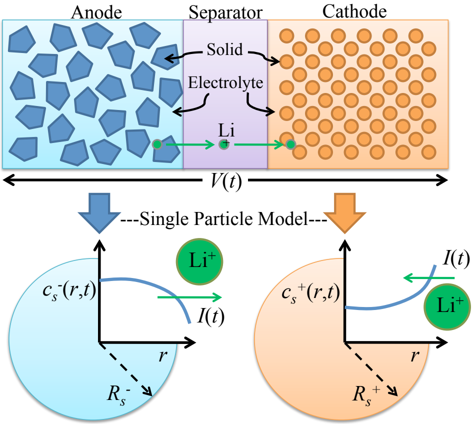
  - S. Moura (2012) - Adaptive PDE Backstepping on SPM (States & Params)

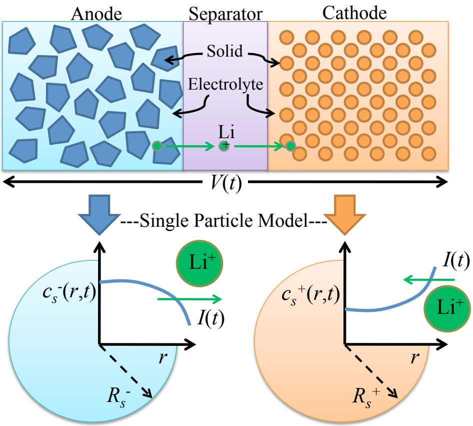
Single Particle Model (SPM)



Single Particle Model (SPM)

Mathematical Structure

- Two spherical diffusion PDEs
  - States: $c_s^-(r,t)$, $c_s^+(r,t)$



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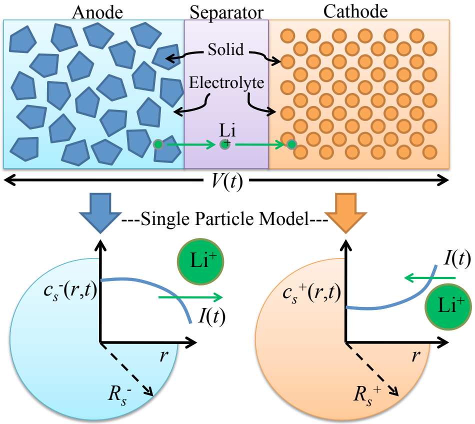

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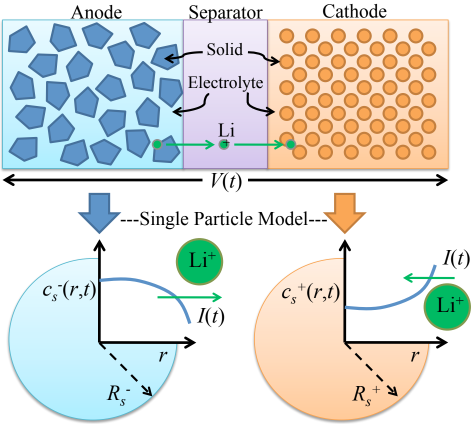
  - Output: $V(t) = h(c_{ss}^-(t), c_{ss}^+(t), I(t))$



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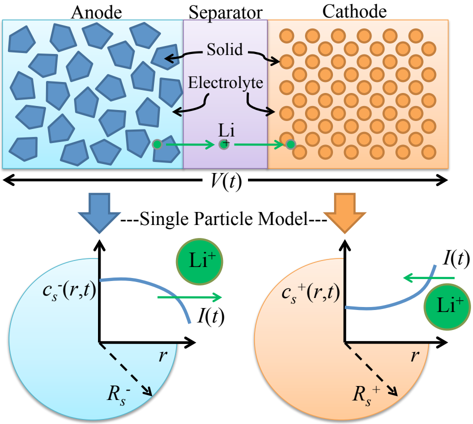
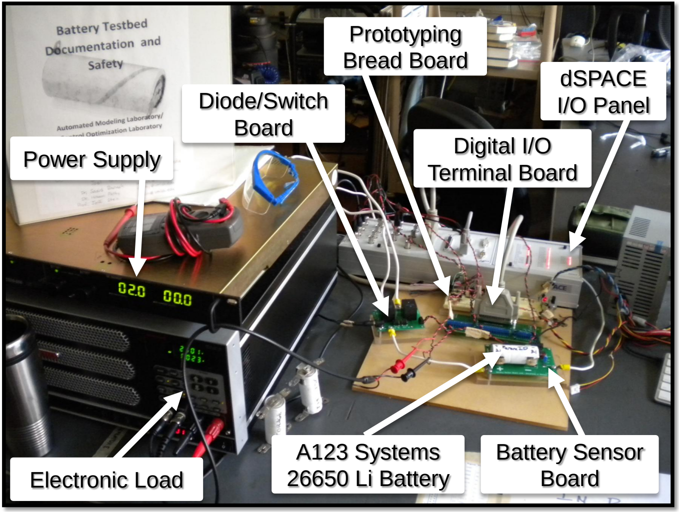


Definitions

- SOC: Bulk concentration SOC_{bulk}, Surface concentration $c_{ss}^-(t)$
- SOH: Physical parameters, e.g. ε, q, n_{Li}, R_f

Custom Battery-in-the-Loop Testbed

JPS 2012



The SOC Estimation Problem

ACC12, DSCC12

Problem Statement

Estimate states $c_s^-(r,t), c_s^+(r,t)$ from measurements $I(t), V(t)$ and SPM

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Simplify the Math

- Model reduction to achieve observability
- Normalize time and space
- State transformation

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Observer Model Equations

$$\frac{\partial c}{\partial t}(r,t) = \varepsilon \frac{\partial^2 c}{\partial r^2}(r,t) \qquad \text{Heat PDE}$$

$$c(0,t) = 0$$

$$\frac{\partial c}{\partial r}(1,t) - c(1,t) = -q\rho I(t)$$

$$\text{Measurement} = c(1,t)$$

Backstepping PDE Estimator

Estimator

$$\begin{aligned}
\frac{\partial \hat{c}}{\partial t}(r,t) &= \varepsilon \frac{\partial^2 \hat{c}}{\partial r^2}(r,t) + p_1(r)\tilde{c}(1,t) \\
\hat{c}(0,t) &= 0 \\
\frac{\partial \hat{c}}{\partial r}(1,t) - \hat{c}(1,t) &= -q\rho I(t) + p_{10}\tilde{c}(1,t) \\
\tilde{c}(1,t) &= c(1,t) - \hat{c}(1,t)
\end{aligned}$$

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Estimation Error Dynamics: $\tilde{c}(r,t) = c(r,t) - \hat{c}(r,t)$

$$\begin{aligned}
\frac{\partial \tilde{c}}{\partial t}(r,t) &= \varepsilon \frac{\partial^2 \tilde{c}}{\partial r^2}(r,t) - p_1(r)\tilde{c}(1,t) \\
\tilde{c}(0,t) &= 0 \\
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\end{aligned}
$$

The Backstepping Idea

$$\tilde{c}(r,t) = \tilde{w}(r,t) - \int_r^1 p(r,s)\tilde{w}(s)ds$$

Backstepping Transformation

$$
\begin{aligned}
\frac{\partial \tilde{w}}{\partial t}(r,t) &= \varepsilon \frac{\partial^2 \tilde{w}}{\partial r^2}(r,t) + \lambda \tilde{w}(r,t) \\
\tilde{w}(0,t) &= 0 \\
\frac{\partial \tilde{w}}{\partial r}(1,t) &= \frac{1}{2}\tilde{w}(1,t)
\end{aligned}
$$

Exp. Stable Target System

$$W = \int_0^1 \tilde{w}^2(x,t)dx$$

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Kernel PDE

$$p_{rr}(r,s) - p_{ss}(r,s) = \frac{\lambda}{\varepsilon} p(r,s)$$

$$p(0,s) = 0$$

$$p(r,r) = \frac{\lambda}{2\varepsilon} r$$

$$p_1(r) = -p_s(r,1) - \frac{1}{2} p(r,1)$$

$$p_{10} = \frac{3 - \lambda/\varepsilon}{2}$$

Backstepping PDE Estimator

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\end{aligned}$$

Explicit Solution to Estimator Gains

$$p_1(r) = \frac{-\lambda r}{2\varepsilon z}\left[I_1(z) - \frac{2\lambda}{\varepsilon z} I_2(z)\right] \qquad \text{where} \quad z = \sqrt{\frac{\lambda}{\varepsilon}(r^2 - 1)}$$

$$p_{10} = \frac{1}{2}\left(3 - \frac{\lambda}{\varepsilon}\right)$$

Problem Statement

Estimate physical parameters from measurements $I(t), V(t)$ and SPM

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Relate uncertain parameters to SOH-related concepts

- Capacity fade
- Power fade

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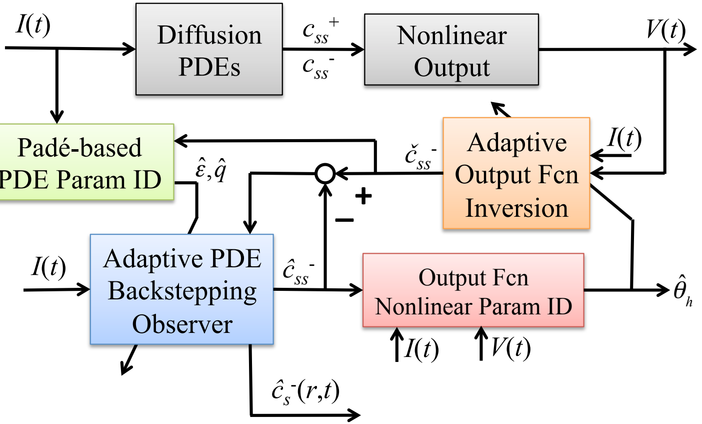
Technical Challenges

- PDE models
- Nonlinear in parameters

Adaptive Observer

Combined State & Parameter Estimation



Padé-based PDE Parameter Identification

PDE Model

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Challenge for adaptive observers:

- Cannot re-express model such that ε multiplies measured signals

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Main Idea:

- Approximate PDE transfer function via Padé representation

$$\frac{c_{ss}(s)}{I(s)} = \frac{-q\rho \sinh\left(\sqrt{\frac{s}{\varepsilon}}\right)}{\left(\sqrt{\frac{s}{\varepsilon}}\right)\cosh\left(\sqrt{\frac{s}{\varepsilon}}\right) - \sinh\left(\sqrt{\frac{s}{\varepsilon}}\right)} \approx \frac{-3\rho q\varepsilon^2 - \frac{2}{7}\rho q\varepsilon s}{\varepsilon s + \frac{1}{35}s^2}$$

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Enables the application of standard (e.g. least squares) parameter identification tools applied to vector $\theta_{pde} = [\varepsilon, q\varepsilon, q\varepsilon^2]^T$

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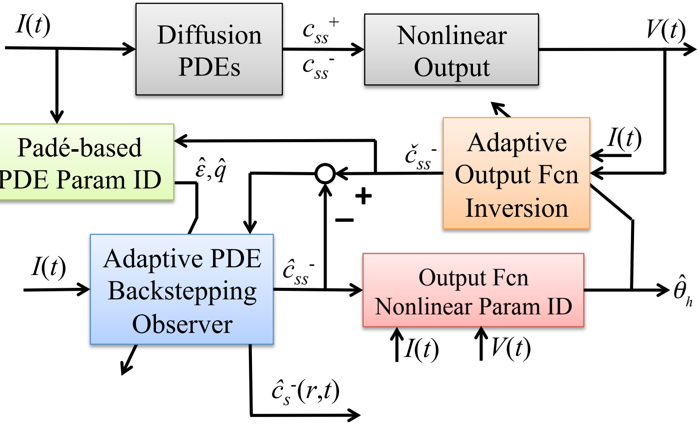

Output Function Nonlinear Parameter ID

Nonlinearly Parameterized Output

$$V(t) = h(c_{ss}^{-}(t), I(t); \theta)$$

- θ contains many parameters
- Linear dependence between parameters?

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Identifiability Analysis Result

- Linearly independent parameter subset : $\theta_h = [n_{Li}, R_f]^T$
  - n_{Li} : Total amount of cyclable Li (Capacity Fade)
  - R_f : Resistance of current collectors, electrolyte, etc. (Power Fade)

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Enables the application of nonlinear least squares parameter identification tools applied to vector θ_h

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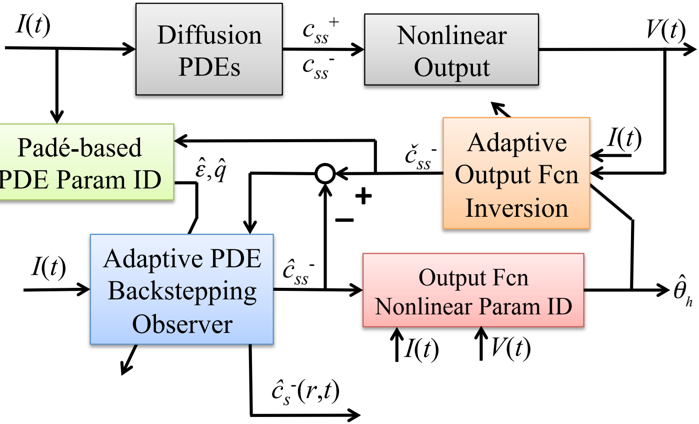

Adaptive Output Function Inversion

Recall state observer requires boundary value $c_{ss}^{-}(t)$ for output error injection

Adaptive Output Function Inversion

Recall state observer requires boundary value $c_{ss}^-(t)$ for output error injection

Nonlinear Function Inversion

Solve $g(c_{ss}^-, t) = 0$ for c_{ss}^-, where

$$g(c_{ss}^-, t) = h(c_{ss}^-(t), I(t); \hat{\theta}_h) - V(t)$$

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Newton's Method

Main Idea: Construct ODE with exp. stable equilibrium $g(c_{ss}^-, t) = 0$

$$\frac{d}{dt}\left[g(\check{c}_{ss}^-, t)\right] = -\gamma g(\check{c}_{ss}^-, t)$$

which expands to a Newton's method update law:

$$\frac{d}{dt}\check{c}_{ss}^- = -\frac{\gamma g(\check{c}_{ss}^-, t) + \frac{\partial g}{\partial t}(\check{c}_{ss}^-, t)}{\frac{\partial g}{\partial c_{ss}^-}(\check{c}_{ss}^-, t)}$$

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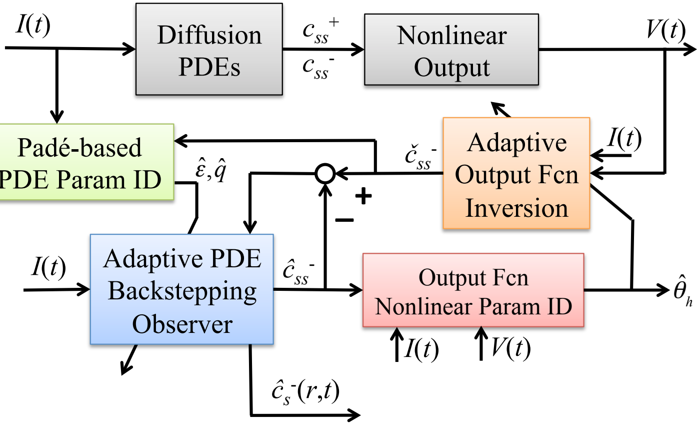
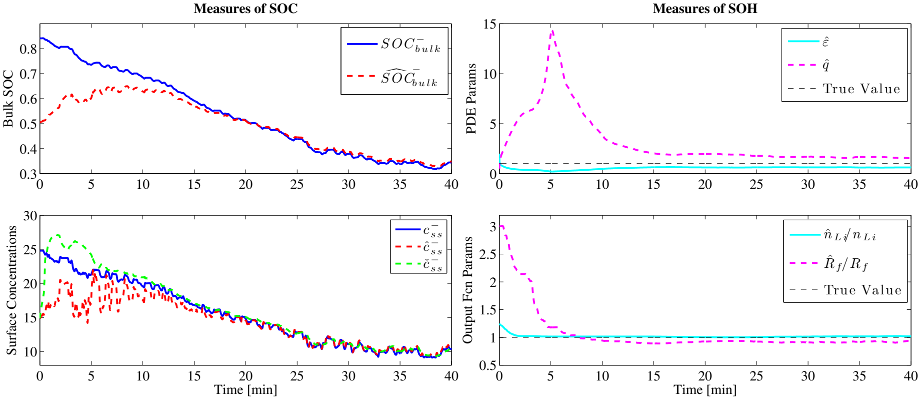


Results

UDDS Drive Cycle Input



Outlook:

- Validate on “Doyle-Fuller-Newman” model
- Robustness

Outline

PHEV Power Management

JPS 2010, TCST 2011, TCST 2012

Problem Statement

Design a supervisory control algorithm for plug-in hybrid electric vehicles (PHEVs) that splits engine and battery power in some optimal sense.

J. Voelcker, "Plugging Away in a Prius," *IEEE Spectrum*, vol. 45, pp. 30-48, 2008.

A Short History

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- Rizzoni (2004) - Equivalent Consumption Minimization Strategy
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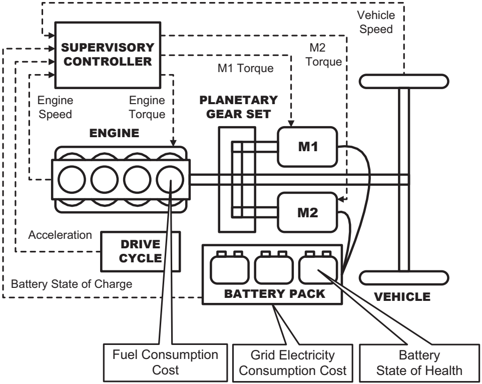
- Moura (2011) - SDP with Electrochemical Battery Model for Health

Power-Split PHEV Model

Ex: Toyota Prius, Ford Escape Hybrid

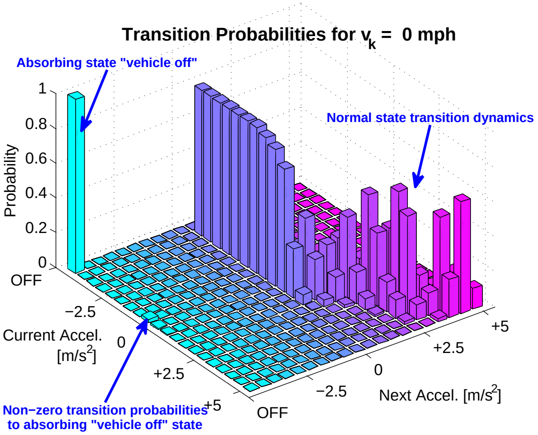
- Control Inputs
  - Engine Torque
  - M1 Torque
- State Variables
  - Engine speed
  - Vehicle speed
  - Battery SOC
  - Vehicle acceleration (Markov Chain)



Markov Chain of Drive Cycle Dynamics

State transition dynamics

$$p_{ijm} = \Pr(a_{k+1} = j | a_k = i, v_k = m)$$



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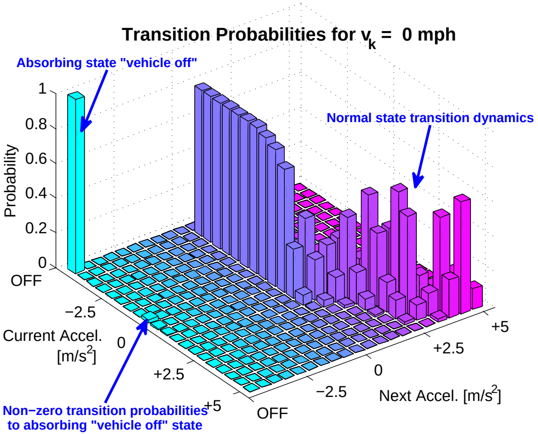

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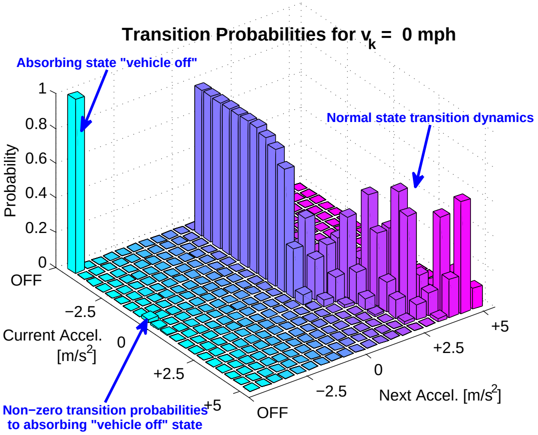

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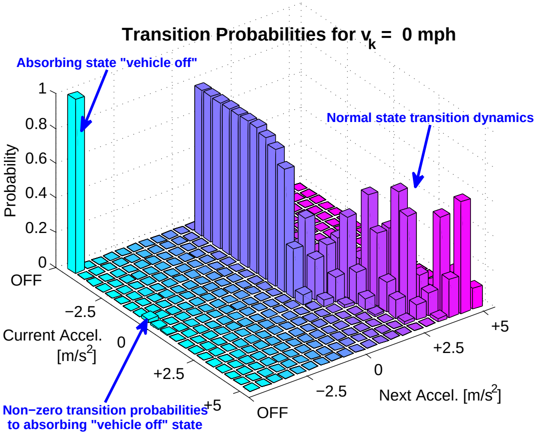
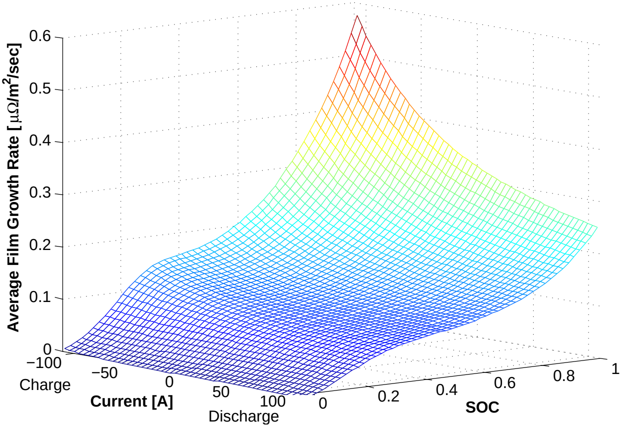


Identification data:

- federal certification cycles, "naturalistic" driving data, 2009 NHTS

Resistive film layer at solid/electrolyte interface



Ramadass, Haran, White, Popov (2003)

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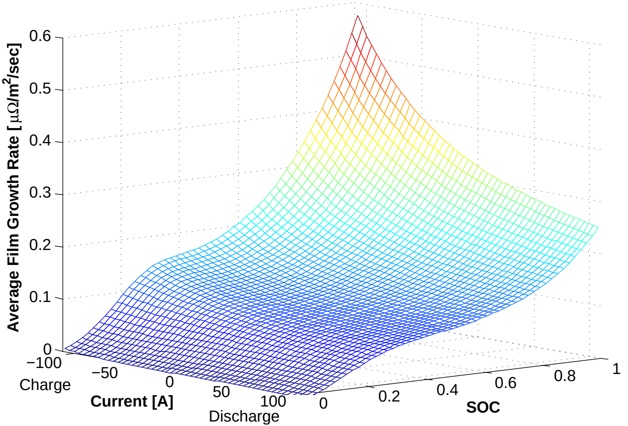

Ramadass, Haran, White, Popov (2003)

Degradation depends on multitude of physical phenomena (e.g. temperature, stress, manufacturing, operating conditions, etc.)

Optimal Control Problem

Multiobjective Shortest-Path Stochastic Dynamic Program

Cost Functional:

$$J^g = \lim_{N \to \infty} \mathrm{E}\left[\sum_{k=0}^{N} c(x_k, u_k)\right]$$

Constraints:

$$\begin{aligned} x_{k+1} &= f(x_k, u_k, w_k) \\ x &\in X \\ u &\in U(x) \end{aligned}$$

Objective:

$$g^* = \arg\min_{g \in G} J^g$$

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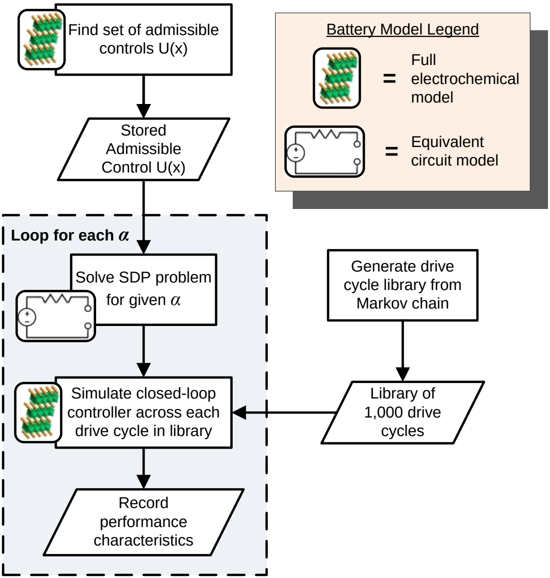
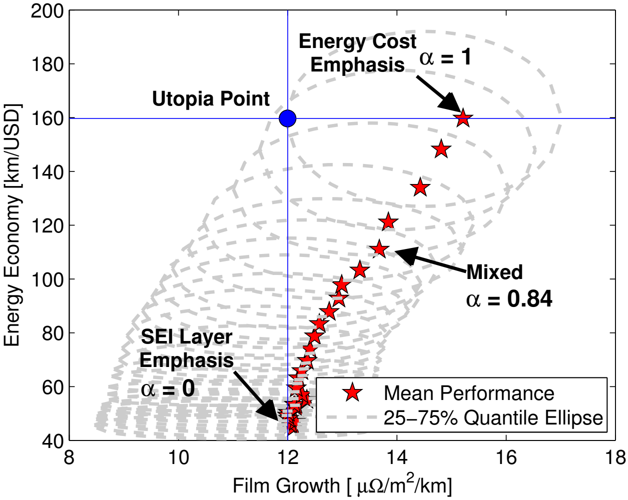
Energy: $c_E(x_k, u_k) = \beta W_{fuel} + \dfrac{-V_{oc} Q_{batt} \dot{SOC}}{\eta_{EVSE}}$

Health: $c_H(x_k, u_k) = \dot{\delta}_{film}(I, SOC)$

Optimization Procedure

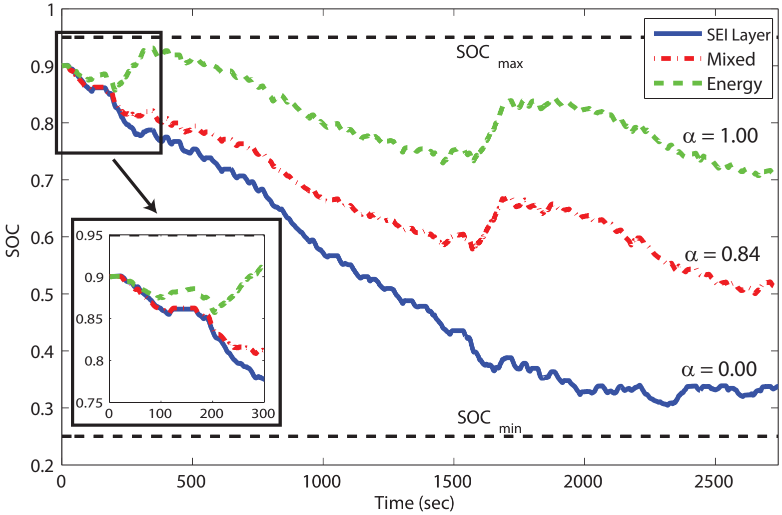




Energy Cost
Emphasis
α = 1
Utopia Point
Mixed
α = 0.84
SEI Layer
Emphasis
α = 0
Mean Performance
25–75% Quantile Ellipse
Energy Economy [km/USD]
Film Growth [μΩ/m2/km]
200
180
160
140
120
100
80
60
40
8
10
12
14
16
18

SOC Trajectories

Anode-side SEI Layer Growth | UDDSx2



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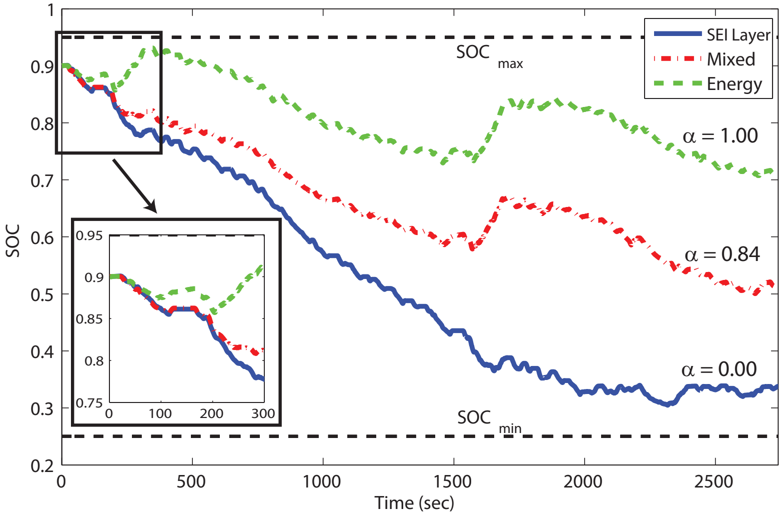


Outlook: Need better battery health models

Outline

LQR for PDEs

ACC11

Problem Statement

Diffusion-Reaction PDE System

$$
\begin{aligned}
u_t(x,t) &= u_{xx}(x,t) + cu(x,t) \\
u(0,t) &= 0 \\
u(1,t) &= U(t) \leftarrow \text{Control} \\
u(x,0) &= u_0(x)
\end{aligned}
$$

Cost Function

$$
J = \frac{1}{2}\int_0^T \left[\langle u(x,t), Q[u](x,t)\rangle + RU^2(t)\right] dt + \frac{1}{2}\langle u(x,T), P_f[u](x,T)\rangle
$$

Q and P_f positive semidefinite, R positive

LQR for PDEs

Theorem: Riccati PDE

The optimal control law in state-feedback form is

$$U^*(t) = \frac{1}{R} \int_0^1 P_x(1, y, t) u^*(y, t) dy$$

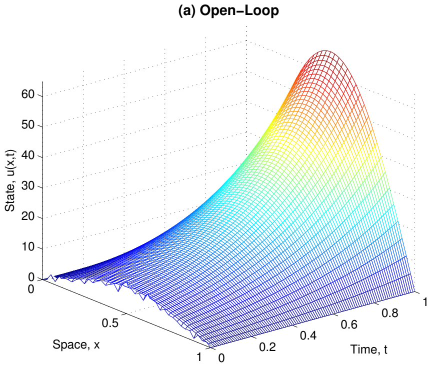
where $P(x, y, t)$ satisfies the Riccati PDE

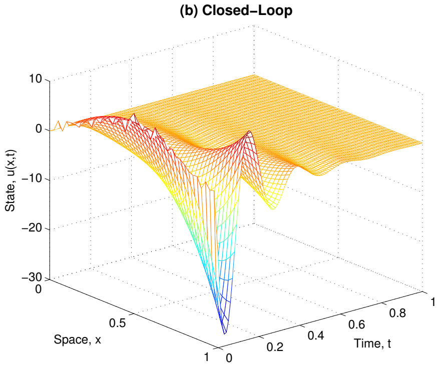
$$-P_t = P_{xx} + P_{yy} + 2cP + Q - \frac{1}{R} P_y(x, 1) P_x(1, y)$$

with boundary conditions and final condition

$$P(0, y, t) = P(1, y, t) = P(x, 0, t) = P(x, 1, t) = 0 \qquad P(x, y, T) = P_f(x, y)$$

LQR for PDEs

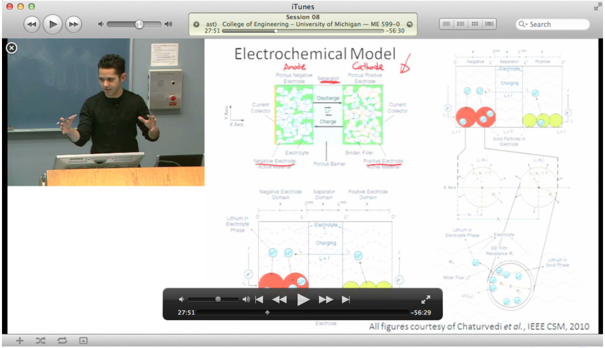

(a) Open−Loop
State, u(x,t)
60
50
40
30
20
10
0
0
0.5
1
Space, x
0
0.2
0.4
0.6
0.8
1
Time, t


(b) Closed−Loop
State, u(x,t)
10
0
−10
−20
−30
0
0.5
1
Space, x
0
0.2
0.4
0.6
0.8
1
Time, t

Enrollment

- Winter 2010: 59 + 5 distance
- Winter 2011: 50 + 26 distance
- ME, EE, ChemE, CS, Energy Systems, MatSci, Physics, Math

- Undergraduates
- Graduate students
- Professionals
  - Tesla Motors, General Motors, Roush, US Army



Not Covered Today

- Offline Parameter Identification of Electrochemical Models
  [ACC11, JPS]
- Charge Un-balancing in Battery Packs
  [DSCC09, IEEE TIE]
- Sensor Placement, Estimation, & Control of Battery Pack Thermal Dynamics
  [CDC12]
- Optimal PEV Charging on the Grid
  [DSCC10, JPS]
- Extremum Seeking with Application to Photovoltaic Systems
  [ACC09, IEEE TEC]

Outline

Future Research Directions

Task 1: Control Techniques for Battery Management Systems

- Thermal management - Sensor placement, estimation, control
- Stress mechanics - Modeling and estimation
- Optimal-fast charging via model predictive control

Funding: ARPA-E AMPED, Bosch, Ford Motor Company

Task 2: Control & Estimation of PDEs

- Moving boundary conditions (Stefan problem)
- Coupled parabolic and hyperbolic systems

Funding: NSF EPAS, NSF Control Systems

Task 3: Smart Grid Power Management

- Energy storage for renewable integration & ancillary services
- Stochastic distributions of PEV loads

Funding: NSF GOALI, PNNL

Summary

Simultaneous SOC/SOH estimation of physically meaningful variables via electrochemical models, PDE estimation theory, and adaptive control.

PHEV power management via electrochemical models and constrained multi-objective stochastic optimal control.

Control Theory + Batteries:
A critically important and technically rich research area

Thanks for your attention!
Questions?

Scott J. Moura, Ph.D.
UC Presidential Postdoctoral Fellow
UC San Diego
http://flyingv.ucsd.edu/smoura/