Project 2

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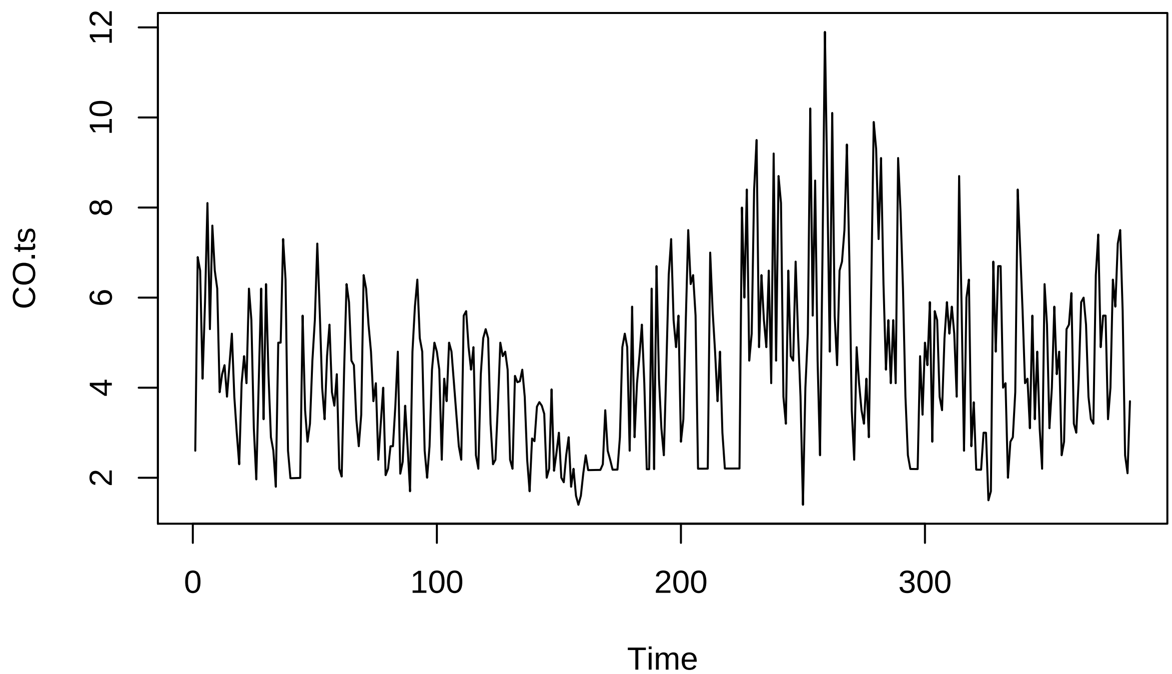
Load data and impute missing values

```
setwd(datadir)
airquality = read.csv('AirQualityUCI.csv')

# replace -200 with NA
airquality[airquality == -200] <- NA

# convert integer type to numeric
intcols = c(4,5,7,8,9,10,11,12)
for(i in 1:length(intcols)){
  airquality[,intcols[i]] <- as.numeric(airquality[,intcols[i]])
}

setwd(sourcedir)

# create new data frame with just CO and NO2
AQdata = airquality[,c(3,10)]

# impute missing air quality data
f <- ~ CO.GT. + NO2.GT.
t <- c(seq(1,dim(AQdata)[1],1))
i <- mnimput(f, AQdata, eps=1e-3, ts=TRUE, method='gam',
             ga.control=list(formula=paste(names(AQdata)[c(1:3)],'~ns(t,2)')))

# set airquality to imputed data
AQdata <- i$filled.dataset

# aggregate to daily maxima for model building
dailyAQ <- aggregate(AQdata, by=list(as.Date(airquality[,1],"%m/%d/%Y")), FUN=max)

# remove last 7 days
dailyAQ <- dailyAQ[1:(dim(dailyAQ)[1]-7),]
```

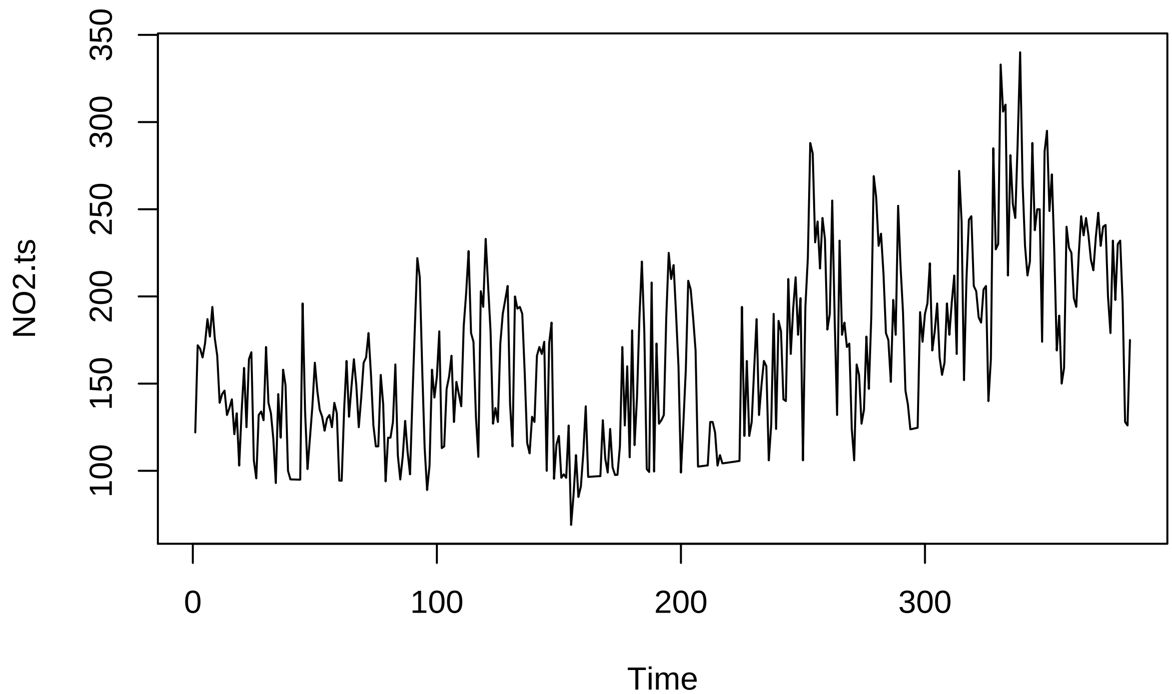
Part 1: Building Univariate Time Series Models

```
AQ.CO <- dailyAQ$CO.GT.
#AQ.CO <- AQdata$CO.GT.
AQ.NO2 <- dailyAQ$NO2.GT.
#AQ.NO2<-AQdata$NO2.GT.

CO.ts<-ts(AQ.CO)
NO2.ts<-ts(AQ.NO2)
```

```r
plot(CO.ts)
```

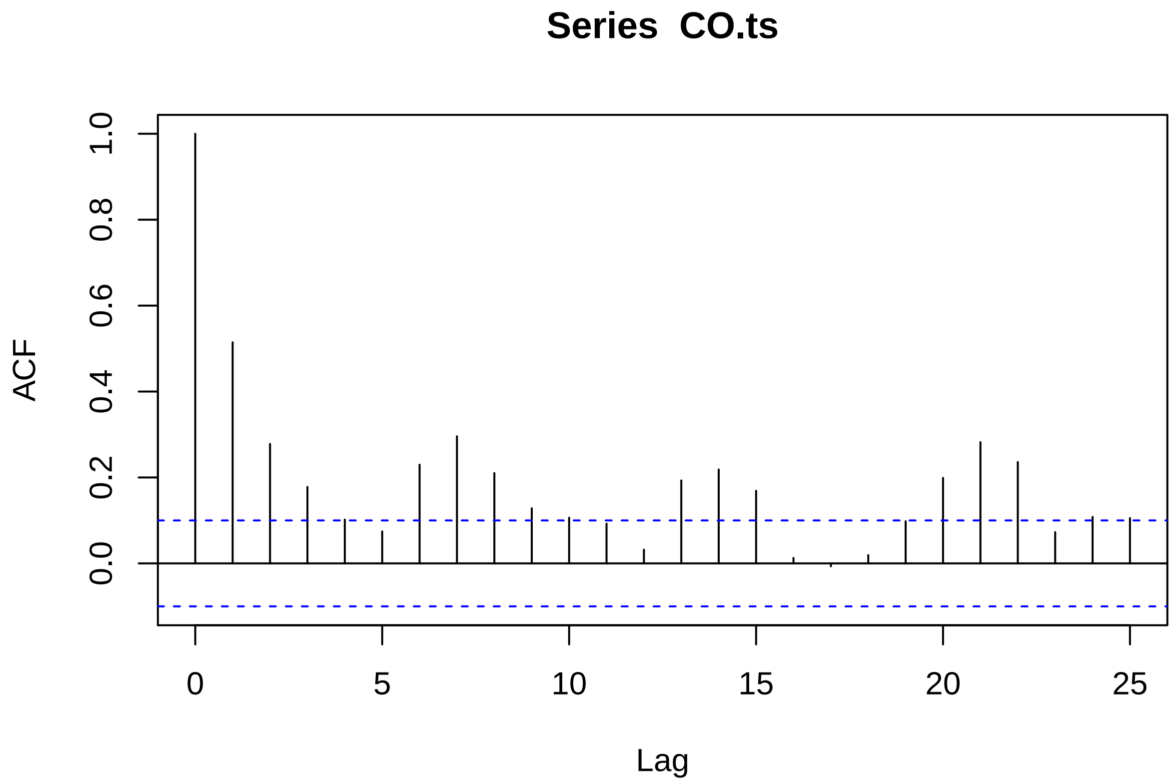



```r
plot(NO2.ts)
```

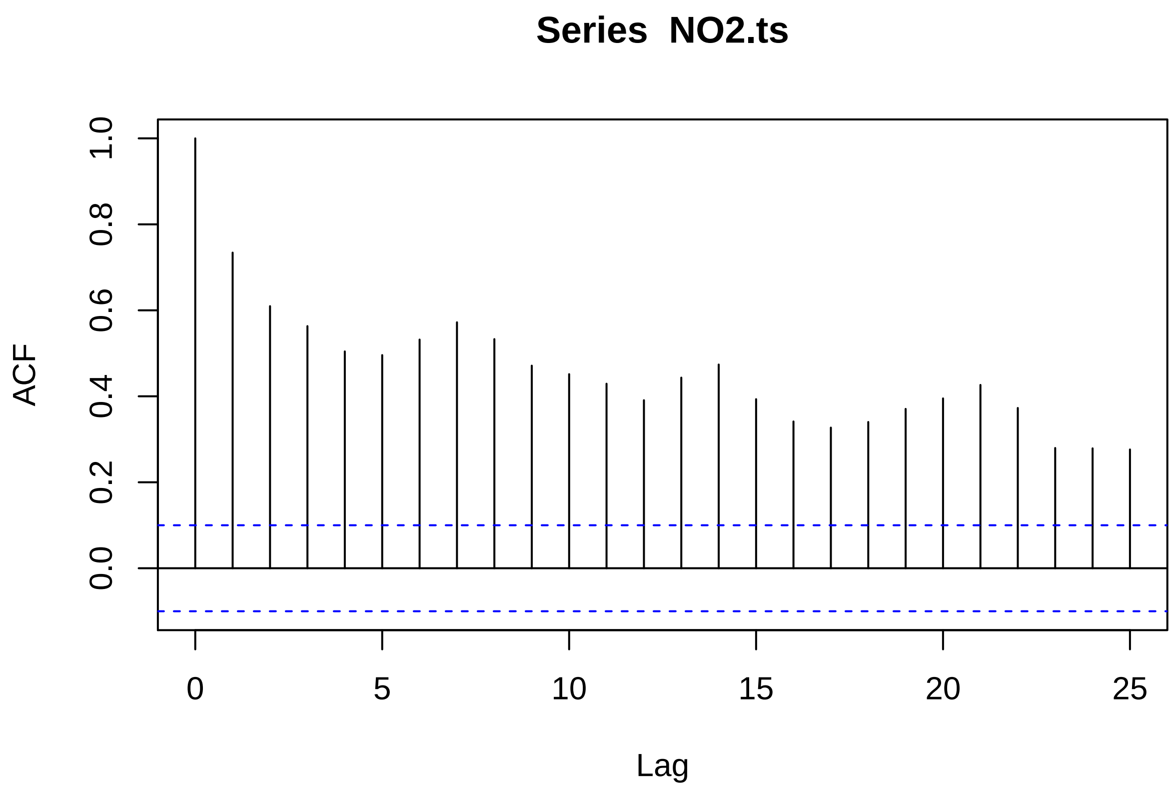


Part A: Seasonality

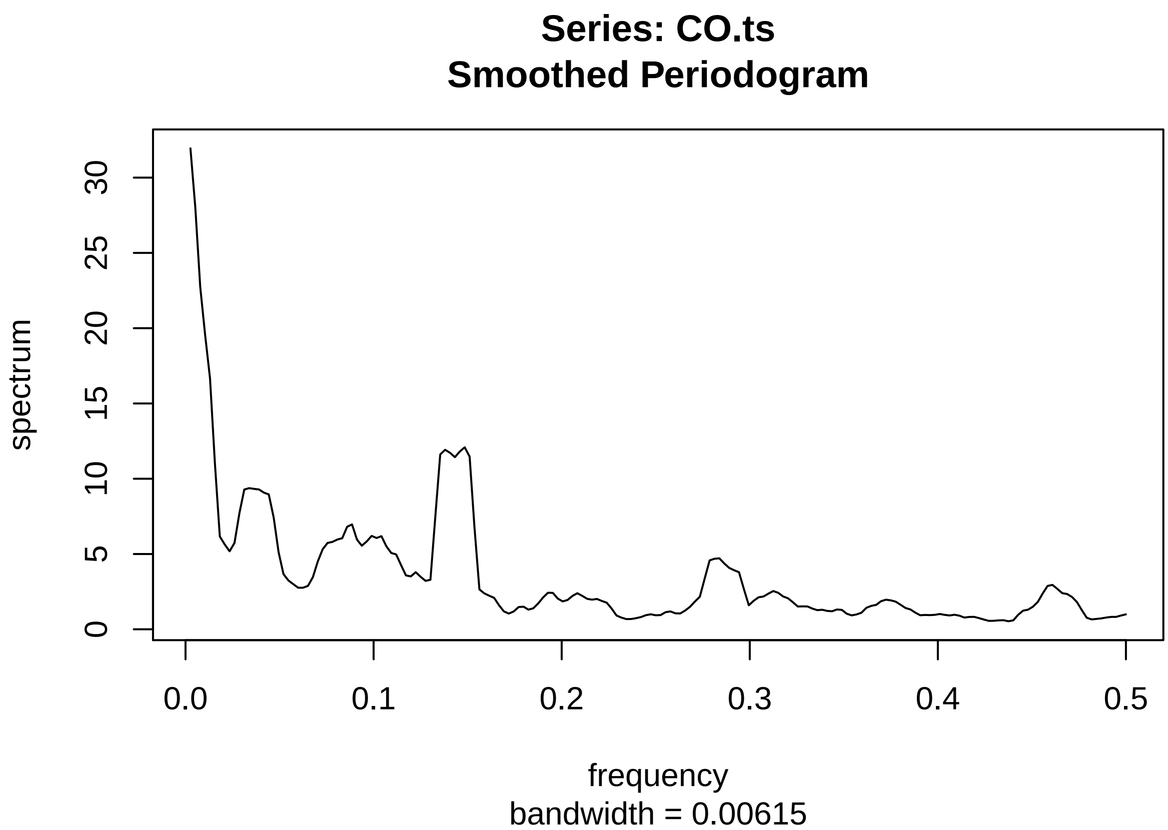
```
acf(CO.ts)
```




```r
acf(NO2.ts)
```




```
# both show sinusoidal exponential decay --> AR model

pg.CO <- spec.pgram(CO.ts,spans=9,demean=T,log='no')
```



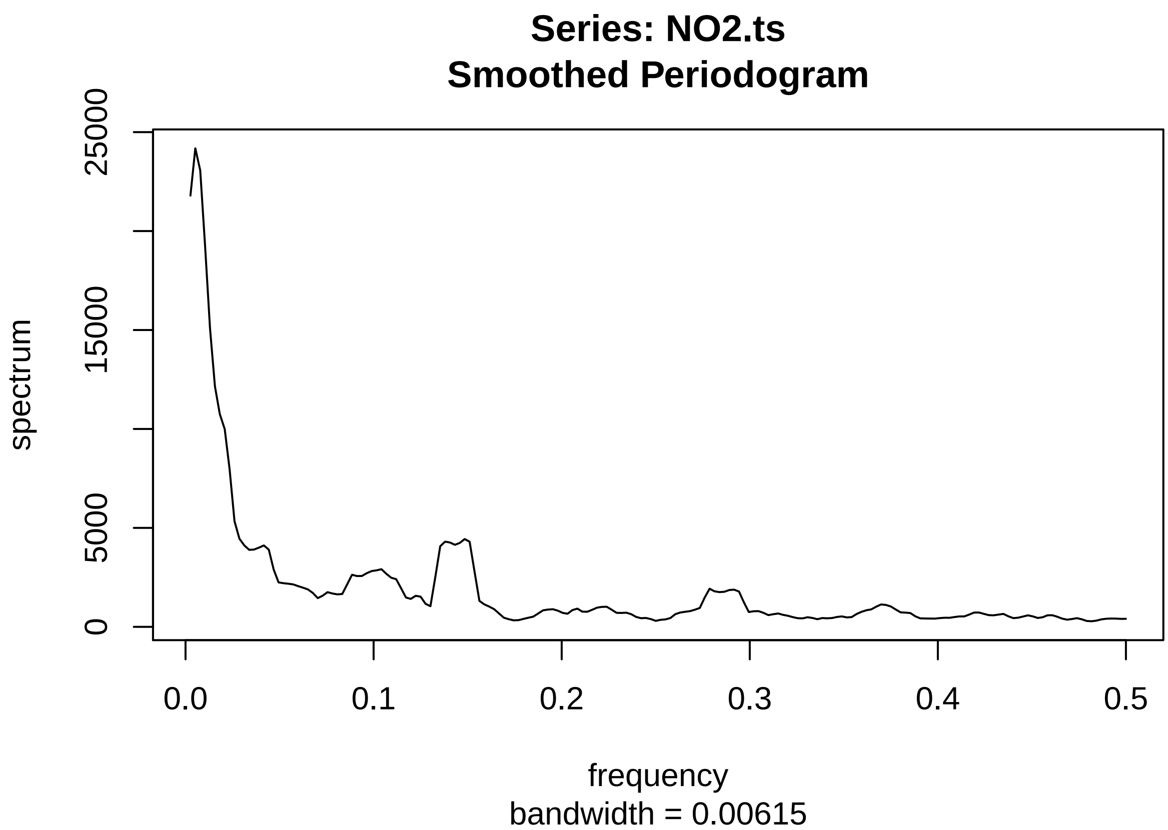

```r
# spikes in periodagram at repeated frequencies --> indicates seasonality present
max.pg.CO<-pg.CO$freq[which(pg.CO$spec==max(pg.CO$spec))]

# Where is the peak? -->0.002604167
max.pg.CO
```

```
## [1] 0.002604167
```

```r
# What is the period? -->384
1/max.pg.CO
```

```
## [1] 384
```

```r
pg.NO2 <- spec.pgram(NO2.ts,spans=9,demean=T,log='no')
```




```r
# spikes in periodagram at repeated frequencies --> indicates seasonality present
max.pg.NO2<-pg.CO$freq[which(pg.NO2$spec==max(pg.NO2$spec))]

# Where is the peak? -->0.00520833
max.pg.NO2
```

```
## [1] 0.005208333
```

```r
# What is the period? -->192
1/max.pg.NO2
```

```
## [1] 192
```

```r
# What are the periods of the next biggest peaks?
# sort spectrum from largest to smallest and find index
sorted.spec <- sort(pg.CO$spec, decreasing=T, index.return=T)
names(sorted.spec)
```

```
## [1] "x"  "ix"
```

```r
# corresponding periods
sorted.omegas <- pg.NO2$freq[sorted.spec$ix]
sorted.Ts <- 1/pg.NO2$freq[sorted.spec$ix]
```

```r
# look at first 20
sorted.omegas[1:20]
```

```
##  [1] 0.002604167 0.005208333 0.007812500 0.010416667 0.013020833
##  [6] 0.148437500 0.138020833 0.145833333 0.140625000 0.135416667
## [11] 0.151041667 0.143229167 0.015625000 0.033854167 0.036458333
## [16] 0.039062500 0.031250000 0.041666667 0.044270833 0.028645833
```

```r
sorted.Ts[1:192]
```

```
##   [1] 384.000000 192.000000 128.000000  96.000000  76.800000   6.736842
##   [7]   7.245283   6.857143   7.111111   7.384615   6.620690   6.981818
##  [13]  64.000000  29.538462  27.428571  25.600000  32.000000  24.000000
##  [19]  22.588235  34.909091   7.529412  21.333333  11.294118  11.636364
##  [25]   6.508475  10.105263   9.600000  54.857143   9.846154  12.000000
##  [31]  12.387097  10.971429  10.378378  12.800000  38.400000  13.241379
##  [37]  48.000000  10.666667   9.365854  13.714286  42.666667  20.210526
##  [43]   9.142857   8.930233   3.522936   3.555556   3.588785  14.222222
##  [49]   3.490909   8.727273   3.459459   3.428571   8.170213   3.398230
##  [55]  19.200000   8.533333   8.347826   8.000000  14.769231   3.622642
##  [61]   7.680000  18.285714   7.836735  17.454545   2.169492  15.360000
##  [67]   2.181818  16.000000  16.695652   2.157303   3.368421   6.400000
##  [73]   3.200000   5.189189   3.173554   5.120000   2.145251   4.800000
##  [79]   6.295082   2.194286   3.226891   2.133333   6.193548   4.740741
##  [85]   4.860759   3.147541   3.254237   3.657143   2.121547   3.282051
##  [91]   5.260274   6.095238   3.121951   5.052632   4.682927   4.571429
##  [97]   4.626506   2.685315   4.923077   2.666667   3.310345   4.517647
## [103]   2.704225   4.987013   3.692308   2.648276   2.206897   2.109890
## [109]   3.096774   4.465116   5.333333   2.630137   2.723404   6.000000
## [115]   3.339130   2.742857   3.047619   3.023622   3.072000   5.565217
## [121]   2.219653   3.728155   5.647059   2.762590   2.612245   5.408451
## [127]   4.413793   3.000000   2.594595   2.887218   5.485714   2.232558
## [133]   2.953846   2.865672   2.976744   2.098361   3.764706   2.245614
## [139]   2.931298   2.909091   5.907692   3.878788   5.731343   3.918367
## [145]   2.577181   2.782609   3.840000   3.801980   5.818182   2.844444
## [151]   2.493506   4.042105   2.000000   2.802920   2.445860   2.258824
## [157]   2.509804   2.477419   2.543046   3.958763   2.526316   4.085106
## [163]   2.560000   4.000000   2.823529   2.461538   4.363636   2.010471
## [169]   2.430380   2.385093   2.021053   2.031746   2.400000   4.129032
## [175]   2.042553   2.415094   4.314607   2.086957   2.370370   4.173913
## [181]   2.053476   2.064516   4.219780   4.266667   2.075676   2.355828
## [187]   2.299401   2.272189   2.313253   2.327273   2.341463   2.285714
```

```r
# evens around 7
period<-7
```

Part B: Trends

```r
# Build a new model, CO.trend which predicts CO.ts based on the time variable
time<-c(1:(length(CO.ts)))
CO.trend<-lm(CO.ts ~ time)
NO2.trend<-lm(NO2.ts ~ time)

summary(CO.trend)
```

```
##
## Call:
## lm(formula = CO.ts ~ time)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.2485 -1.6980 -0.0525  1.0863  7.3442
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 3.810929   0.192140  19.834  < 2e-16 ***
## time        0.002876   0.000865   3.325  0.00097 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.879 on 382 degrees of freedom
## Multiple R-squared:  0.02813,    Adjusted R-squared:  0.02558
## F-statistic: 11.06 on 1 and 382 DF,  p-value: 0.0009695
```

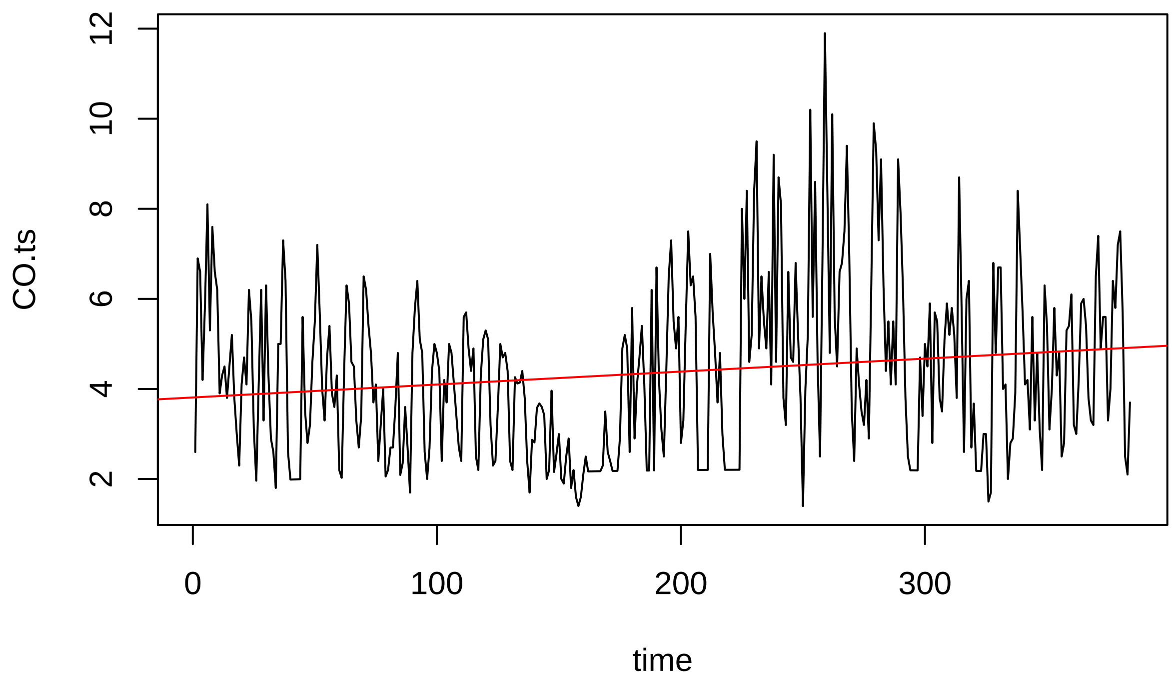
```r
summary(NO2.trend)
```

```
##
## Call:
## lm(formula = NO2.ts ~ time)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -87.389 -34.365   2.159  27.847 137.895
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 115.16473    4.40111   26.17   <2e-16 ***
## time          0.25646    0.01981   12.94   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 43.04 on 382 degrees of freedom
## Multiple R-squared:  0.3049, Adjusted R-squared:  0.3031
## F-statistic: 167.6 on 1 and 382 DF,  p-value: < 2.2e-16
```

```r
# Here we built two new models, CO.trend and No2.trend, that both model the trend components.
# For CO.trend, the p-value is 0.00097, and for NO2.trend, the p-value is <2.2e-16. Therefore, the tren
# component is significant in both models and must be considered.
```

Plot CO.trend model

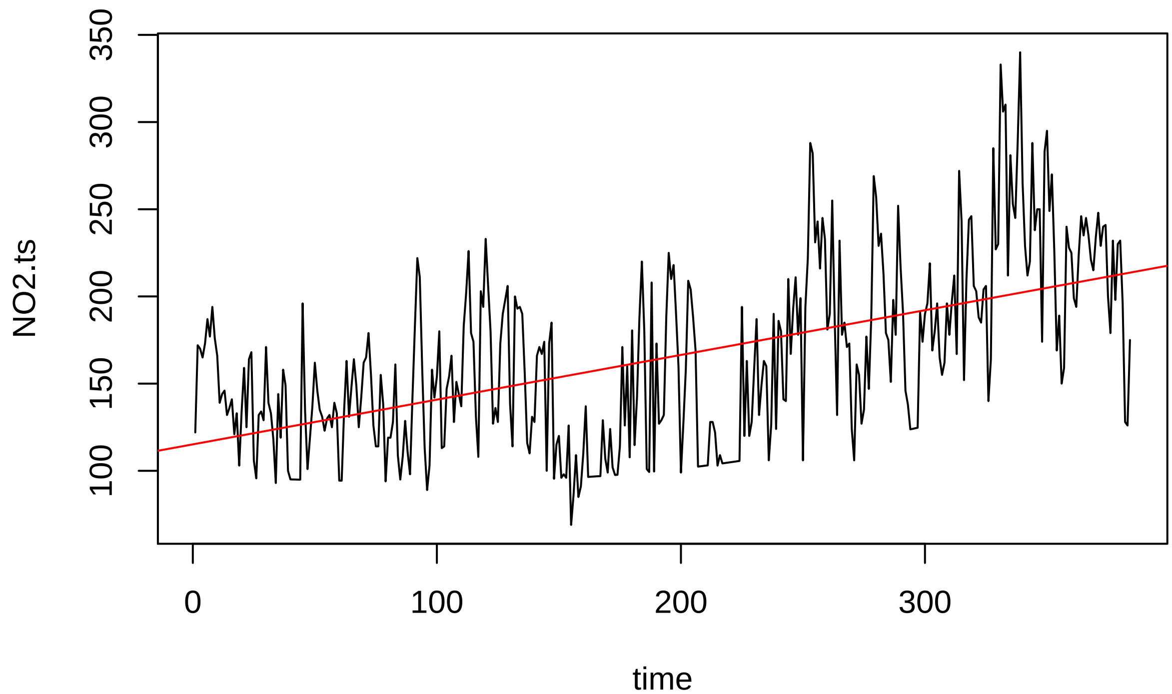
```
{plot(time, CO.ts, type = "l")
abline(CO.trend, col = "red")}
```




```
# As seen in the plot of the CO.trend model, we can see that there is a clear upward trend line, which
# supports the results of our statistical test.
# The adjusted R^2 for the model CO.trend is 0.02558.
```

Plot NO2.trend model

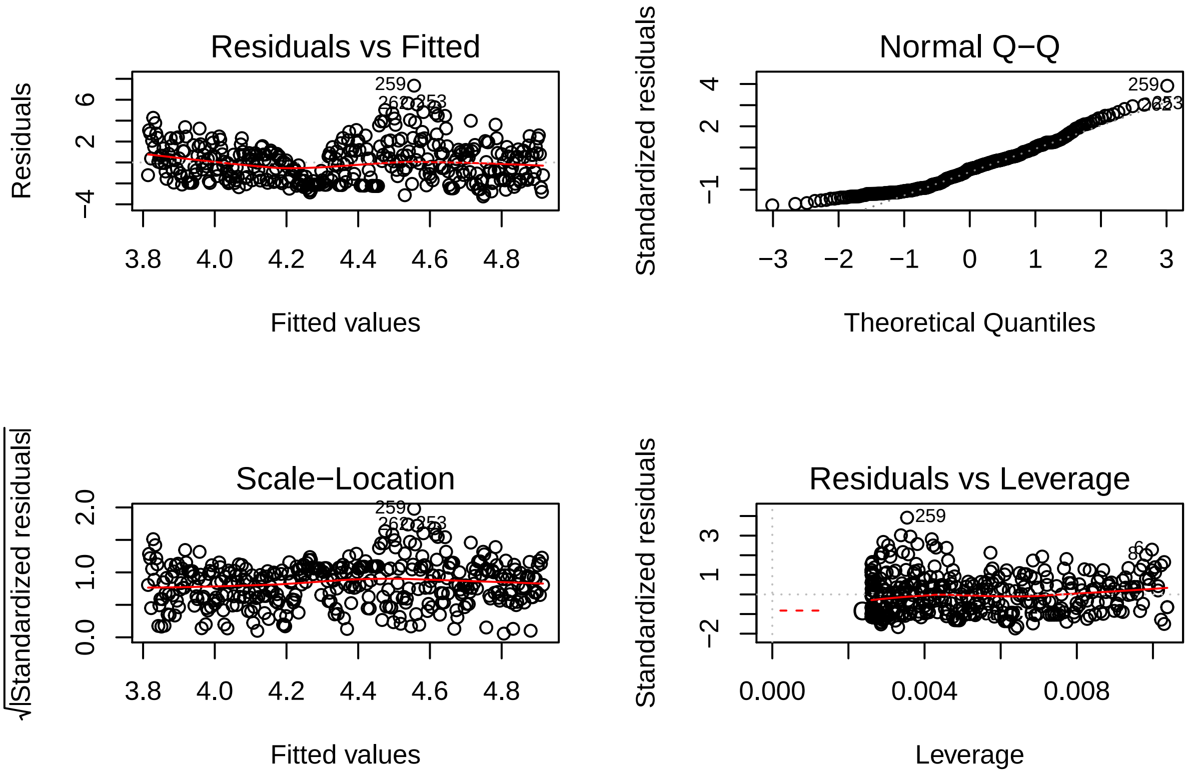
```
{plot(time, NO2.ts, type = "l")
abline(NO2.trend, col = "red")}
```




```
# As seen in the plot of the NO2.trend model, we can see that there is a clear upward trend line.
# From the naked eye, the slope seems more drastic than with the trend line from CO.trend model.
# This supports the results of our statistical test.
# The adjusted R^2 for the model NO2.trend is 0.3031.
```

Model diagnostics for CO.trend

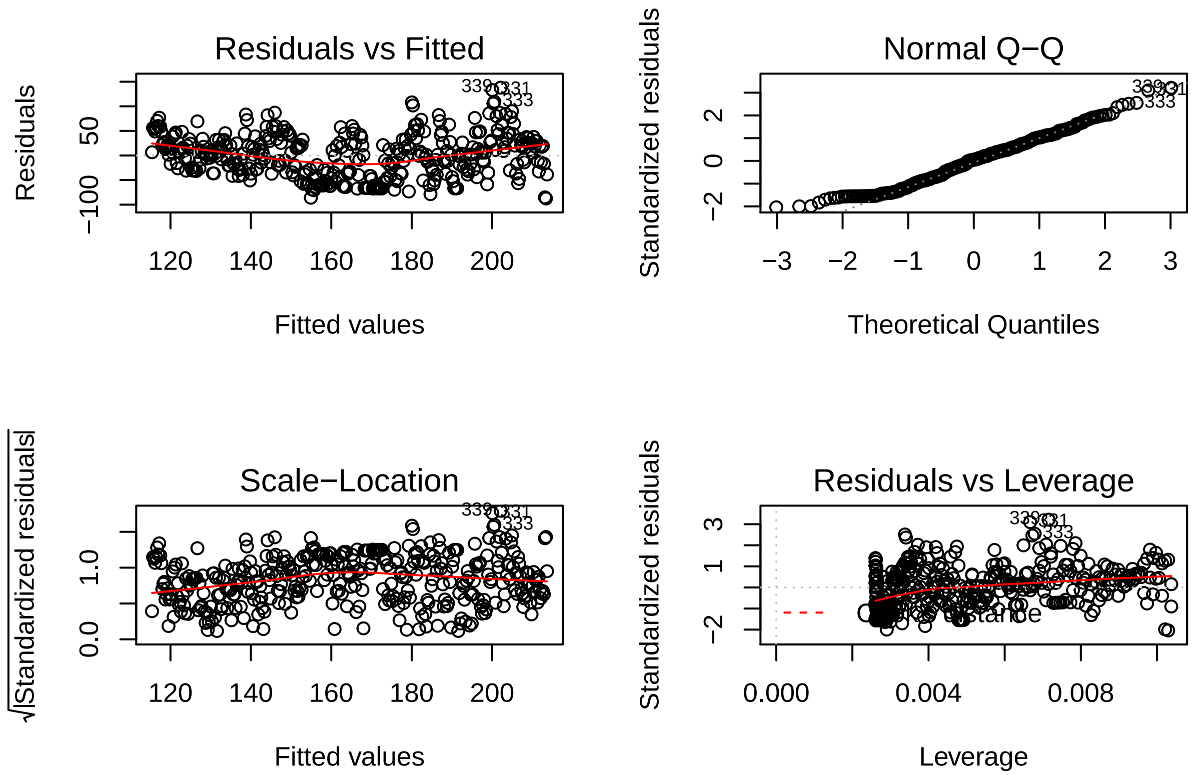
```
par(mfrow=c(2,2))
plot(CO.trend, labels.id = NULL)
```




```
par(mfrow=c(1,1))
# Residuals versus fitted plot: Does not violate assumptions. The mean is about zero and there seems to
# Q-Q plot: The fit to the line is fairly solid, thus no drastic violation of assumptions. The Q-Q plot
# Scale-location: Does not violate assumptions. The mean is about zero and there seems to be constant v
# Residuals versus leverage: No clear influential points with regards to Cook's distance.
```

Model diagnostics for NO2.trend

```
par(mfrow=c(2,2))
plot(NO2.trend, labels.id = NULL)
```




```
par(mfrow=c(1,1))
# Residuals versus fitted plot: Does not violate assumptions. The mean is about zero and there seems to
# Q-Q plot: The fit to the line is fairly solid, thus no drastic violation of assumptions. As seen by t
# Scale-location: Does not violate assumptions. The mean is about zero and there seems to be constant v
# Residuals versus leverage: No clear influential points with regards to Cook's distance.
```

Add seasonality component to CO.trend

```
# Because the seasonality component was significant, we added a seasonality component to CO.trend. We d
CO.trend.seasonal <- lm(CO.ts[time] ~ time + sin(2*pi*time/7) + cos(2*pi*time/7))
summary(CO.trend.seasonal)
```

```
##
## Call:
## lm(formula = CO.ts[time] ~ time + sin(2 * pi * time/7) + cos(2 *
##     pi * time/7))
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.4604 -1.1866 -0.1247  1.0272  6.9821
##
## Coefficients:
```

```
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)          3.7979449  0.1805339  21.037  < 2e-16 ***
## time                 0.0029483  0.0008127   3.628 0.000325 ***
## sin(2 * pi * time/7) 0.8531164  0.1272445   6.705 7.33e-11 ***
## cos(2 * pi * time/7) 0.3563039  0.1275659   2.793 0.005485 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.765 on 380 degrees of freedom
## Multiple R-squared:  0.1466, Adjusted R-squared:  0.1399
## F-statistic: 21.76 on 3 and 380 DF,  p-value: 5.027e-13
```

```
# The p-value for this model is 2.449e-10.
# The adjusted R^2 for the model CO.trend.seasonal is 0.1109.
```

add seasonality component to NO2.trend

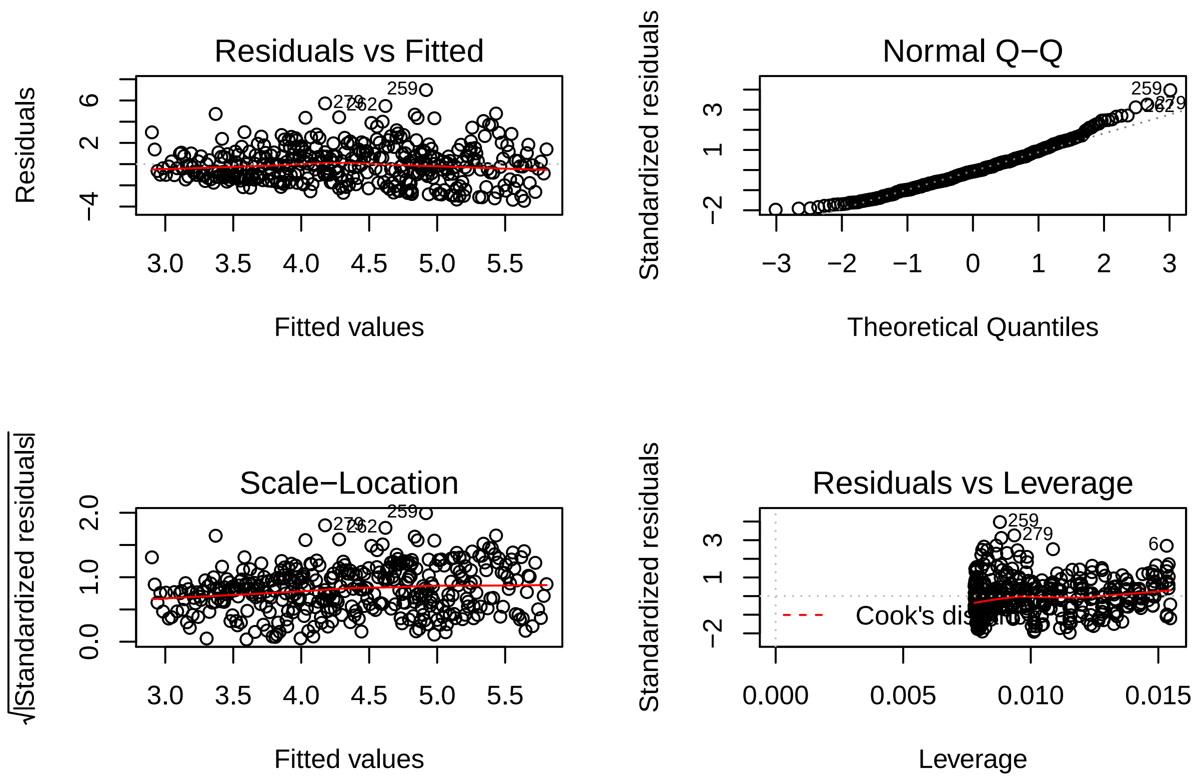
```
# Because the seasonality component was significant, we added a seasonality component to NO2.trend. We
NO2.trend.seasonal <- lm(NO2.ts[time] ~ time + sin(2*pi*time/7) + cos(2*pi*time/7))
summary(NO2.trend.seasonal)
```

```
##
## Call:
## lm(formula = NO2.ts[time] ~ time + sin(2 * pi * time/7) + cos(2 *
##     pi * time/7))
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -101.641  -28.675    1.226   26.385  135.816
##
## Coefficients:
##                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)          114.9221     4.2424  27.089  < 2e-16 ***
## time                   0.2578     0.0191  13.498  < 2e-16 ***
## sin(2 * pi * time/7)  15.7600     2.9902   5.271 2.28e-07 ***
## cos(2 * pi * time/7)   5.5152     2.9977   1.840   0.0666 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 41.48 on 380 degrees of freedom
## Multiple R-squared:  0.3576, Adjusted R-squared:  0.3525
## F-statistic:  70.5 on 3 and 380 DF,  p-value: < 2.2e-16
```

```
# The p-value for this model is 2.2e-16.
# The adjusted R^2 for the model NO2.trend.seasonal is 0.388
```

Model diagnostics for CO.trend.seasonal

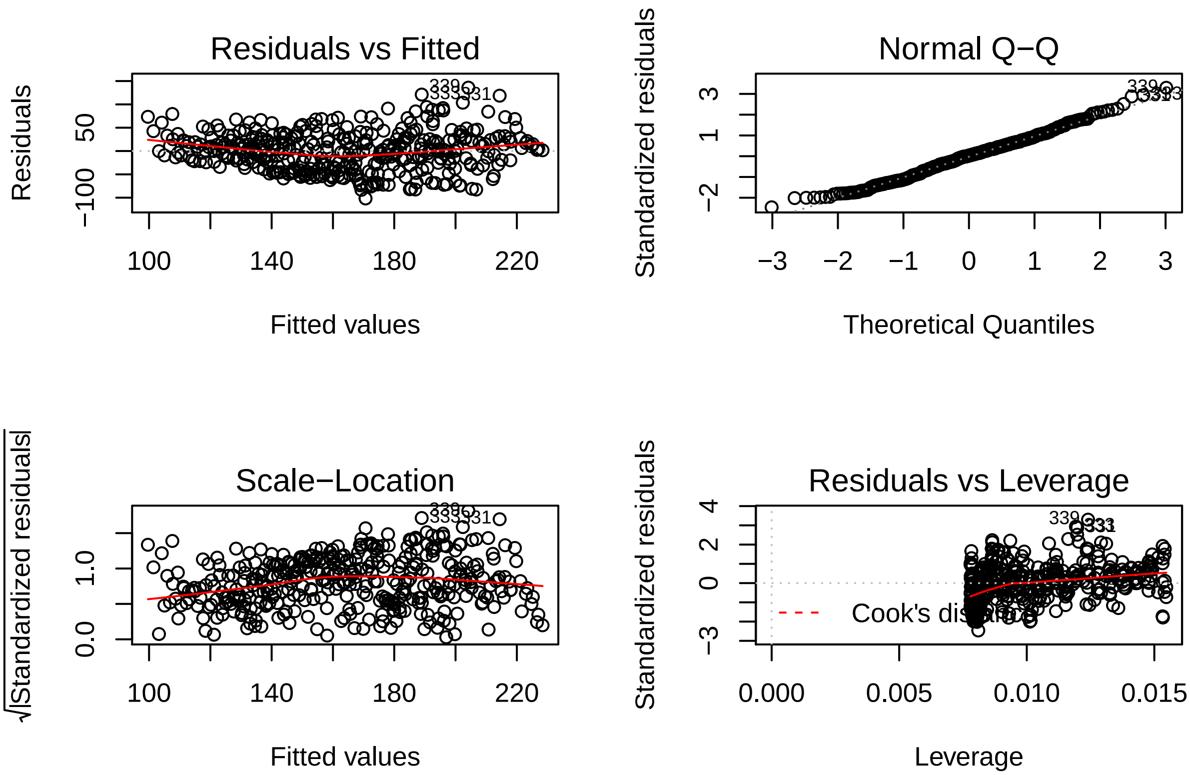
```
par(mfrow=c(2,2))
plot(CO.trend.seasonal, labels.id = NULL)
```




```
par(mfrow=c(1,1))
# Residuals versus fitted plot: Does not violate assumptions. The mean is about zero and there seems to
# Q-Q plot: The fit to the line is fairly solid, thus no drastic violation of assumptions.
# Scale-location: Does not violate assumptions. The mean is about zero and there seems to be constant v
# Residuals versus leverage: No clear influential points with regards to Cook's distance.
# The spread of points above and below the mean line in the residuals versus fitted plot and scale-loca
```

Model diagnostics for NO2.trend.seasonal

```
par(mfrow=c(2,2))
plot(NO2.trend.seasonal, labels.id = NULL)
```




```
par(mfrow=c(1,1))
# Residuals versus fitted plot: Does not violate assumptions. The mean is about zero and there seems to
# Q-Q plot: The fit to the line is fairly solid, thus no drastic violation of assumptions. The Q-Q plot
# Scale-location: Does not violate assumptions. The mean is about zero and there seems to be constant v
# Residuals versus leverage: No clear influential points with regards to Cook's distance.
```

Part C: Auto-Regressive and Moving Average

#Get the residuals from the CO.trend.seasonal model above and store in e.ts:

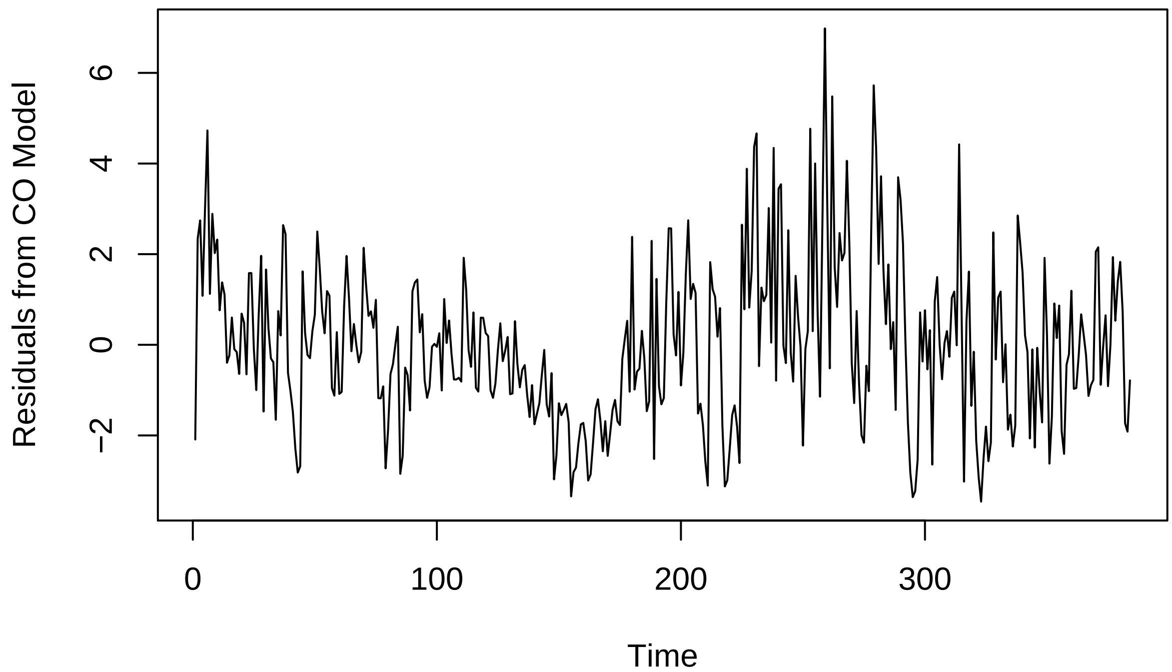
```
e.ts.CO<-ts(CO.trend.seasonal$residuals)
```

#Get the residuals from the NO2.trend.seasonal model above and store in e.ts:

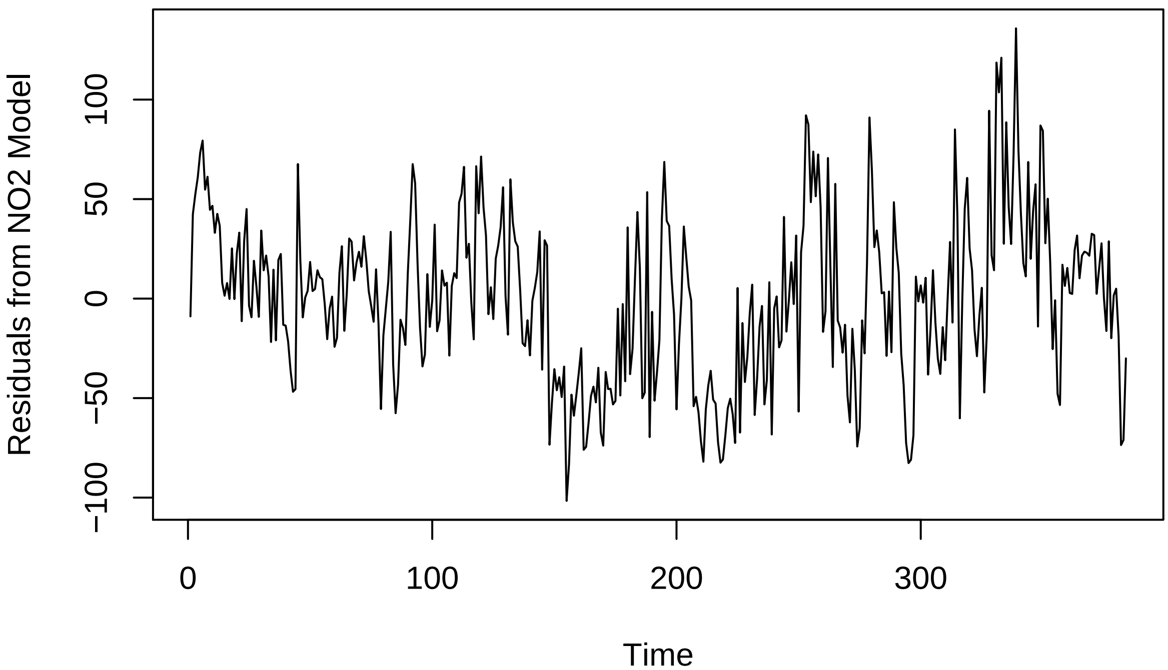
```
e.ts.NO2<-ts(NO2.trend.seasonal$residuals)
```

#Plot the residuals for the CO.trend.seasonal model NO2.trend.seasonal

```
plot(e.ts.CO, ylab = "Residuals from CO Model")
```

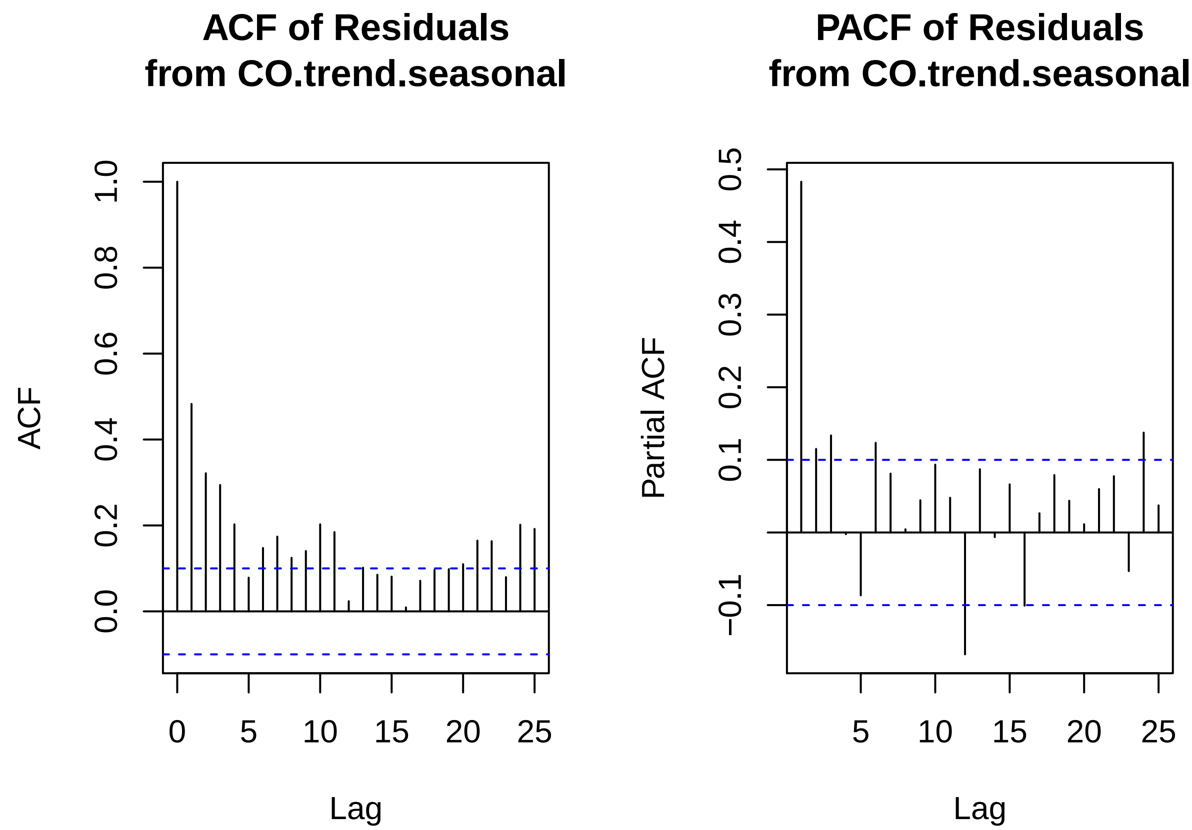



```r
plot(e.ts.NO2, ylab = "Residuals from NO2 Model")
```



Plot the autocorrelation (ACF) and partial autocorrelation (PACF) of the residuals of CO.trend.seasonal

```
par(mfrow=c(1,2))
acf(e.ts.CO, main="ACF of Residuals\nfrom CO.trend.seasonal")
pacf(e.ts.CO,main="PACF of Residuals\nfrom CO.trend.seasonal")
```

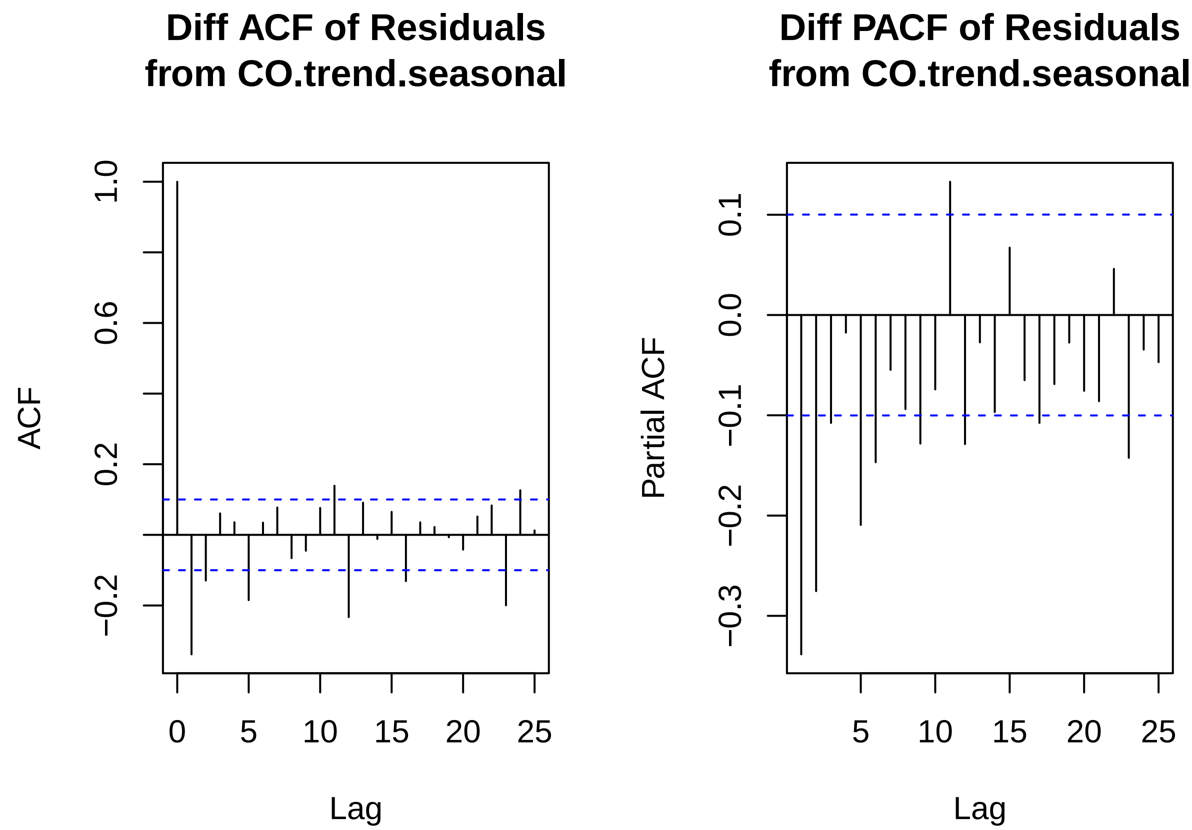



```r
par(mfrow=c(1,1))
```

The ACF plot for the residuals of the CO.trend.seasonal shows sinusoidal decay. It also seems to show a cutoff at lag 3, so we will set q=3 for future use. The PACF plot for the residuals of the CO.trend.seasonal shows a cutoff at lag 1, so we will set p=1. The above statements point to the model being AR(1), but we will calculate AIC values to assess several model choices.

Do we need to consider a first order difference of our residuals?

```r
par(mfrow=c(1,2))
acf(diff(e.ts.CO), main="Diff ACF of Residuals\nfrom CO.trend.seasonal")
pacf(diff(e.ts.CO),main="Diff PACF of Residuals\nfrom CO.trend.seasonal")
```

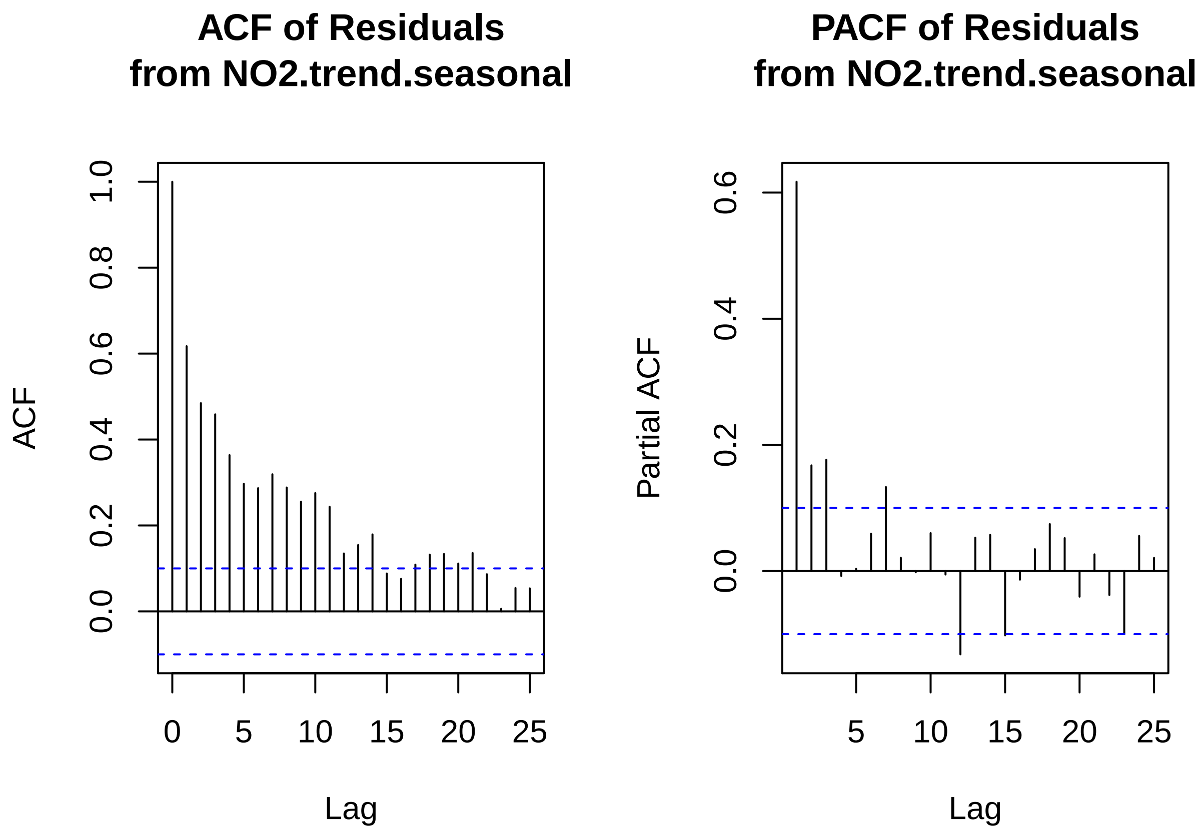



```r
par(mfrow=c(1,1))
```

No, we do not need to consider a first order difference the residuals of the CO.trend.seasonal model because the ACF shows decay with a lag of 3 and the differentiated ACF does not improve this. Thus, the value of d is 0.

Plot the autocorrelation (ACF) and partial autocorrelation (PACF) of the residuals of NO2.trend.seasonal

```r
par(mfrow=c(1,2))
acf(e.ts.NO2, main="ACF of Residuals\nfrom NO2.trend.seasonal")
pacf(e.ts.NO2,main="PACF of Residuals\nfrom NO2.trend.seasonal")
```

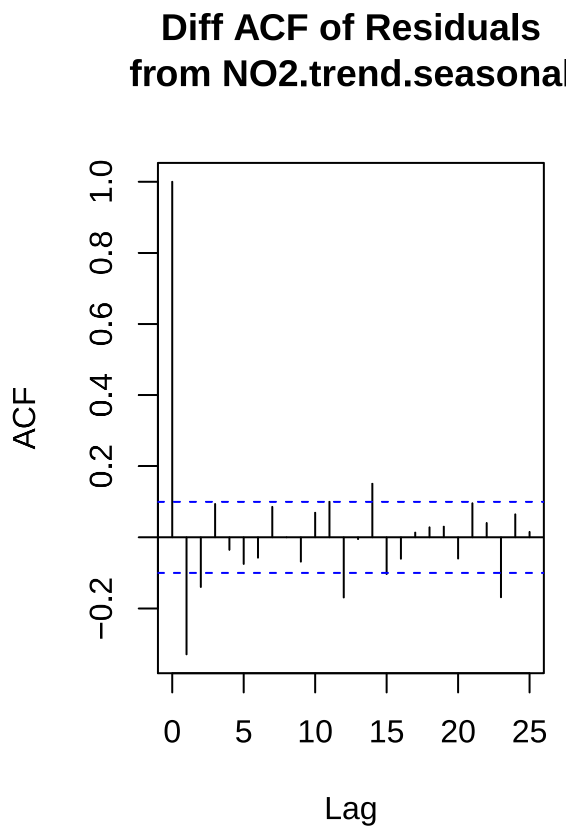


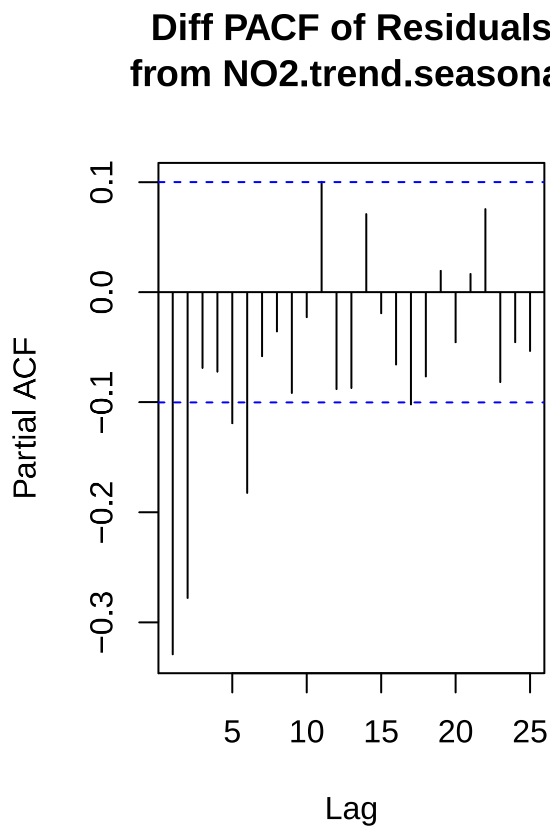

```r
par(mfrow=c(1,1))
```

The ACF plot for the residuals of the CO.trend.seasonal shows a cutoff at lag 3, so we will set q=10 (note that this seems high). The PACF plot for the residuals of the CO.trend.seasonal shows a cutoff at lag 1, so we will set p=1.

Do we need to consider a first order difference of our residuals?

```r
par(mfrow=c(1,2))
acf(diff(e.ts.NO2), main="Diff ACF of Residuals\nfrom NO2.trend.seasonal")
pacf(diff(e.ts.NO2),main="Diff PACF of Residuals\nfrom NO2.trend.seasonal")
```






```r
par(mfrow=c(1,1))
```

Both plots show sinusoidal decay, which points to using an ARMA model. Yes, we should consider a first order difference of our residuals for NO2 because the ACF of the original has positive autocorrelations out to a high number of lags (10). By taking the first order difference of the residuals, we reduce this number of lags to 3. Thus, the value of d is 1, to represent first order differentation.

Modeling e.ts.CO

Now we will try out some models for e.ts.CO using p=1 and q=3.

ar(1) p=1

```r
CO.ar1 <- arima(e.ts.CO, order=c(1,0,0), include.mean=FALSE)
summary(CO.ar1)
```

```
##
## Call:
## arima(x = e.ts.CO, order = c(1, 0, 0), include.mean = FALSE)
##
## Coefficients:
##          ar1
##       0.4837
## s.e.  0.0446
```

```
##
## sigma^2 estimated as 2.361:  log likelihood = -709.99,  aic = 1423.98
##
## Training set error measures:
##                         ME     RMSE      MAE      MPE     MAPE      MASE
## Training set -0.0003100464 1.536712 1.181932 146.0159 267.2095 0.9043589
##                     ACF1
## Training set -0.05561519
```

```
# AIC = 1452.36
```

ma(3) p=0, q=3

```
CO.ma3 <- arima(e.ts.CO, order=c(0,0,3), include.mean=FALSE)
summary(CO.ma3)
```

```
##
## Call:
## arima(x = e.ts.CO, order = c(0, 0, 3), include.mean = FALSE)
##
## Coefficients:
##          ma1     ma2     ma3
##       0.3772  0.2265  0.2156
## s.e.  0.0503  0.0475  0.0517
##
## sigma^2 estimated as 2.351:  log likelihood = -709.12,  aic = 1426.24
##
## Training set error measures:
##                         ME     RMSE      MAE    MPE    MAPE      MASE
## Training set -0.0007200315 1.533204 1.193358 96.042 195.776 0.9131019
##                    ACF1
## Training set 0.03672645
```

```
# AIC = 1455.71
```

arma(1,3) p=1, q=3

```
CO.arma13 <- arima(e.ts.CO, order=c(1,0,3), include.mean=FALSE)
summary(CO.arma13)
```

```
##
## Call:
## arima(x = e.ts.CO, order = c(1, 0, 3), include.mean = FALSE)
##
## Coefficients:
##          ar1      ma1     ma2     ma3
##       0.5889  -0.1777  0.0119  0.1140
## s.e.  0.1797   0.1781  0.0981  0.0773
##
## sigma^2 estimated as 2.294:  log likelihood = -704.45,  aic = 1418.89
##
## Training set error measures:
```

```
##                        ME     RMSE      MAE      MPE     MAPE      MASE
## Training set -0.001465841 1.514552 1.162444 161.5466 268.2974 0.8894476
##                     ACF1
## Training set 0.002203679
```

```
# AIC = 1457.24.
```

Based on the above AIC values, we would choose model AR(1) because it has the lowest value. As a final step, we will use the auto.arima function on e.ts.CO.

```
CO.auto <- auto.arima(e.ts.CO,approximation=FALSE)
summary(CO.auto)
```

```
## Series: e.ts.CO
## ARIMA(3,0,0) with zero mean
##
## Coefficients:
##          ar1     ar2     ar3
##       0.4120  0.0565  0.1344
## s.e.  0.0508  0.0551  0.0510
##
## sigma^2 estimated as 2.308:  log likelihood=-704.08
## AIC=1416.17   AICc=1416.27   BIC=1431.97
##
## Training set error measures:
##                        ME     RMSE     MAE      MPE     MAPE      MASE
## Training set -0.001624096 1.513104 1.15694 177.2785 280.2686 0.8852366
##                     ACF1
## Training set 0.001369499
```

The auto arima function supports the use of AR(1) as our model, with an AIC of 1452.36

Modeling e.ts.NO2

Now we will try out some models for e.ts.CO using p=2 and q=3.

ar(2) p=2

```
NO2.ar2 <- arima(e.ts.NO2, order=c(2,1,0), include.mean=FALSE)
summary(NO2.ar2)
```

```
##
## Call:
## arima(x = e.ts.NO2, order = c(2, 1, 0), include.mean = FALSE)
##
## Coefficients:
##           ar1      ar2
##       -0.4250  -0.2812
## s.e.   0.0493   0.0492
##
## sigma^2 estimated as 1070:  log likelihood = -1879.45,  aic = 3764.89
```

```
##
## Training set error measures:
##                     ME     RMSE      MAE      MPE     MAPE      MASE
## Training set -0.199042 32.67334 25.33623 200.1778 517.0787 0.9273205
##                     ACF1
## Training set -0.01800656
```

```r
# AIC = 3820.68
```

ma(3) p=0, q=3

```r
NO2.ma3 <- arima(e.ts.NO2, order=c(0,1,3), include.mean=FALSE)
summary(NO2.ma3)
```

```
##
## Call:
## arima(x = e.ts.NO2, order = c(0, 1, 3), include.mean = FALSE)
##
## Coefficients:
##           ma1      ma2      ma3
##       -0.4995  -0.1971  -0.0198
## s.e.   0.0515   0.0626   0.0497
##
## sigma^2 estimated as 1025:  log likelihood = -1871.4,  aic = 3750.79
##
## Training set error measures:
##                      ME     RMSE     MAE      MPE     MAPE      MASE
## Training set -0.3873864 31.98022 24.5708 225.9336 462.4165 0.8993053
##                     ACF1
## Training set 0.004824268
```

```r
# AIC = 3786.07
```

arma(2,3) p=2, q=3

```r
NO2.arma23 <- arima(e.ts.NO2, order=c(2,1,3), include.mean=FALSE)
summary(NO2.arma23)
```

```
##
## Call:
## arima(x = e.ts.NO2, order = c(2, 1, 3), include.mean = FALSE)
##
## Coefficients:
##          ar1     ar2      ma1      ma2     ma3
##       0.3585  0.4734  -0.8804  -0.4739  0.3542
## s.e.  0.3354  0.2736   0.3310   0.4187  0.1150
##
## sigma^2 estimated as 990.8:  log likelihood = -1866.35,  aic = 3744.7
##
## Training set error measures:
##                     ME     RMSE      MAE      MPE     MAPE      MASE
## Training set -1.319191 31.43666 24.07713 205.7028 425.4323 0.8812367
##                     ACF1
## Training set 0.000659074
```

```r
# AIC = 3786.98
```

As a final step, we will use the auto.arima function on e.ts.NO2.

```r
NO2.auto <- auto.arima(e.ts.NO2,approximation=FALSE)
summary(NO2.auto)
```

```
## Series: e.ts.NO2
## ARIMA(1,1,2)
##
## Coefficients:
##          ar1      ma1     ma2
##       0.8271  -1.3734  0.3832
## s.e.  0.0810   0.1150  0.1037
##
## sigma^2 estimated as 1018:  log likelihood=-1869.01
## AIC=3746.02   AICc=3746.13   BIC=3761.82
##
## Training set error measures:
##                      ME    RMSE      MAE      MPE    MAPE      MASE
## Training set -0.6548743 31.7411 24.27322 171.3796 441.267 0.8884137
##                    ACF1
## Training set 0.02459958
```

```r
# AIC = 3781.53
```

Is the auto.arima function always the right model to go with? how does it incorporate differencing? Auto.arima said ARIMA(2,0,1)? What does this mean

Part D: Assessment of Models

We used AIC and diagnostics to assess the models for CO.

```r
AIC(CO.ar1) # AIC = 1452.36
```

```
## [1] 1423.979
```

```r
AIC(CO.ma3) # AIC = 1455.71
```

```
## [1] 1426.24
```

```r
AIC(CO.arma13) # AIC = 1457.24
```

```
## [1] 1418.894
```

```r
AIC(CO.auto) # AIC = 1452.36
```

```
## [1] 1416.168
```

The lowest AIC is the CO.ar1, which is what the auto.arima function produced as well. Therefore the model we would choose is AR(1).

We also used AIC and diagnostics to assess the models for N02.

```r
AIC(NO2.ar2) # AIC = 3820.68
```

```
## [1] 3764.891
```

```r
AIC(NO2.ma3) # AIC = 3786.07
```

```
## [1] 3750.791
```

```r
AIC(NO2.arma23) # AIC = 3786.98
```

```
## [1] 3744.7
```

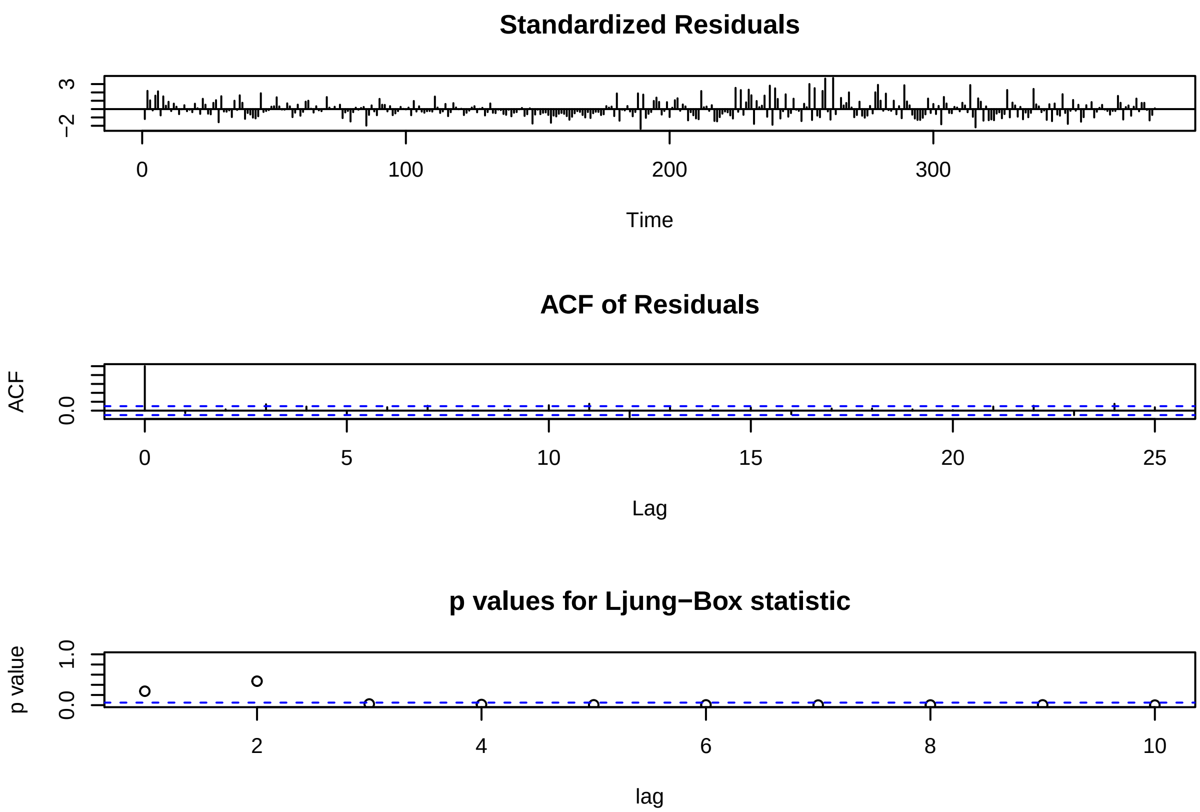
```r
AIC(NO2.auto) # AIC = 3781.53
```

```
## [1] 3746.025
```

NOT SURE BC auto arima is weird so double check this ???

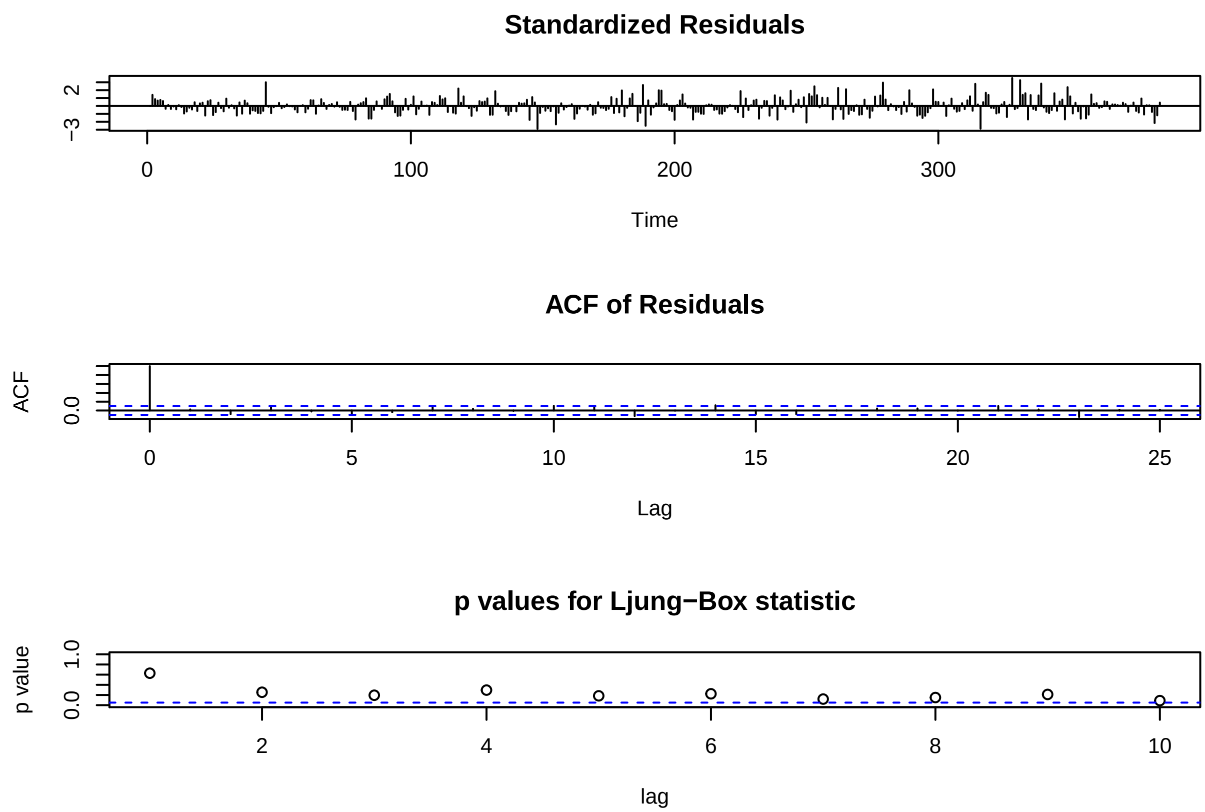
Part E: Diagnostics

```r
tsdiag(CO.ar1, lag = 30)
```



The above graph shows that ...

```
tsdiag(NO2.auto, lag = 30)
```



The above graph shows that . . .

Part 2: Building Multivariate Time Series Models

Part A: Seasonality

Same as part 1a

Part B: Trends

Same as part 1b

Part C: Auto-Regressive and Moving Average

```
allResiduals <- data.frame(e.ts.CO, e.ts.NO2)
colnames(allResiduals) <- c("CO","NO2")
cor(allResiduals)
```

```
##            CO       NO2
## CO  1.0000000 0.5891419
## NO2 0.5891419 1.0000000
```

Correlation between residuals of NO2 and CO is 0.663.

Build VARMA model to CO and NO2 residuals

Part D: Assessment of Models

We will analyze the AICmatrix to find which model has the lowest AIC.

```
AICmatrix
```

```
##          [,1]      [,2]     [,3]     [,4]     [,5]
## [1,] 7.245625  7.493295 7.669472 7.541659 7.751891
## [2,] 7.406789 10.508200 7.163330 8.063202 9.748072
## [3,] 7.484692  9.111086 7.466969 7.188300 8.756757
## [4,] 7.747901 10.328910 7.225039 7.207042 7.231542
```

According to the AICmatrix, the model with p=2 and q=3 has the lowest AIC, this we should use these values to build our model. The AIC for p=2 and q=3 is 7.227414. The next best model is p=2 and q=2, with an AIC of 7.249367. We will build these 2 models and compare them using diagnostics.

```
varma.model <- VARMACpp(allResiduals, p=2, q=3, include.mean=F)
```

```
## Number of parameters:  20
## initial estimates:  0.7339 -0.0105 -0.1195 0.0073 5.0425 0.6528 -9.9506 0.2203 -0.3209 0.01 0.0334 -
## Par. lower-bounds:  0.1837 -0.0371 -0.5907 -0.0179 -6.3507 0.1016 -19.7087 -0.301 -0.888 -0.0174 -0.
## Par. upper-bounds:  1.2841 0.0162 0.3518 0.0325 16.4358 1.2041 -0.1926 0.7416 0.2461 0.0374 0.35 0.0
## Final   Estimates:  0.7346295 -0.01014744 -0.1132653 0.01643692 5.04286 0.6523543 -9.94569 0.2175392
##
## Coefficient(s):
##       Estimate  Std. Error  t value Pr(>|t|)
## CO    0.734629          NA       NA       NA
## NO2  -0.010147          NA       NA       NA
## CO   -0.113265          NA       NA       NA
## NO2   0.016437          NA       NA       NA
## CO    5.042860          NA       NA       NA
## NO2   0.652354          NA       NA       NA
## CO   -9.945690          NA       NA       NA
## NO2   0.217539          NA       NA       NA
##       0.246124          NA       NA       NA
##      -0.017382          NA       NA       NA
##      -0.076440          NA       NA       NA
##       0.014295          NA       NA       NA
##      -0.089836          NA       NA       NA
##      -0.008902          NA       NA       NA
##      -6.114664          NA       NA       NA
##      -0.133709          NA       NA       NA
##       4.734502          NA       NA       NA
##       0.130516          NA       NA       NA
##       4.300280          NA       NA       NA
##      -0.005112          NA       NA       NA
## ---
## Estimates in matrix form:
## AR coefficient matrix
## AR( 1 )-matrix
```

```
##       [,1]    [,2]
## [1,] 0.735 -0.0101
## [2,] 5.043  0.6524
## AR( 2 )-matrix
##        [,1]   [,2]
## [1,] -0.113 0.0164
## [2,] -9.946 0.2175
## MA coefficient matrix
## MA( 1 )-matrix
##        [,1]   [,2]
## [1,] -0.246 0.0174
## [2,]  6.115 0.1337
## MA( 2 )-matrix
##         [,1]    [,2]
## [1,]  0.0764 -0.0143
## [2,] -4.7345 -0.1305
## MA( 3 )-matrix
##         [,1]    [,2]
## [1,]  0.0898 0.00890
## [2,] -4.3003 0.00511
##
## Residuals cov-matrix:
##           [,1]       [,2]
## [1,]  3.554225   32.26891
## [2,] 32.268907 1098.01537
## ----
## aic=  8.063202
## bic=  8.268965
```

```
varma.model
```

```
## $data
##              CO          NO2
## 1   -2.090038263   -8.94021914
## 2    2.343716565   42.42475108
## 3    2.744075476   52.43560271
## 4    1.081433762   60.85379355
## 5    2.998325429   73.38102627
## 6    4.729206463   79.41407636
## 7    1.125112849   54.75809366
## 8    2.889323315   61.25520611
## 9    2.023078142   44.62017633
## 10   2.323437053   46.63102796
## 11   0.760795339   33.04921880
## 12   1.377687006   42.57645152
## 13   1.108568040   36.60950162
## 14  -0.395525573    7.95351891
## 15  -0.231315108    1.45063137
## 16   0.602439719    7.81560159
## 17  -0.097201369   -0.17354679
## 18  -0.159843084   25.24464405
## 19  -0.642951417   -0.22812322
## 20   0.687929617   22.80492687
## 21   0.483836004   33.14894416
```

```
## 22  -0.651953531  -11.35394338
## 23   1.581801296   29.01102684
## 24   1.582160208   45.02187846
## 25  -0.080481506   -3.55993070
## 26  -0.999987702   -9.36940461
## 27   0.667291195   19.00035212
## 28   1.963197581    6.34436941
## 29  -1.472591954   -9.15851813
## 30   1.661162873   34.20645209
## 31   0.361521785   14.21730371
## 32  -0.301119929   21.63549455
## 33  -0.384228262   11.16272728
## 34  -1.653347228  -21.80422263
## 35   0.742559158   14.53979466
## 36   0.206769623  -20.96309288
## 37   2.640524451   19.40187734
## 38   2.440883362   22.41272896
## 39  -0.621758352  -13.16908020
## 40  -1.015520814  -13.56612959
## 41  -1.482819267  -21.57237470
## 42  -2.285095184  -36.26723994
## 43  -2.819069910  -46.80858683
## 44  -2.683503231  -45.48164225
## 45   1.620244939   67.60815422
## 46   0.257603225   19.02634505
## 47  -0.225505108   -9.44642222
## 48  -0.294624074    0.58662787
## 49   0.301282312    3.93064517
## 50   0.665492778   18.42775762
## 51   2.499247605    3.79272784
## 52   1.699606516    4.80357947
## 53   0.736964802   14.22177031
## 54   0.253856469   10.74900303
## 55   1.184737503    9.78205312
## 56   1.080643890   -2.87392958
## 57  -0.955145645  -20.37681713
## 58  -1.121390818   -5.01184691
## 59   0.278968094    0.99900472
## 60  -1.083673621  -24.20626142
## 61  -1.039890068  -19.70440860
## 62   0.864099080   12.97747838
## 63   1.960005467   26.32149567
## 64   1.024215932  -16.18139187
## 65  -0.142029241    2.18357835
## 66   0.458329671   30.19442997
## 67  -0.004312043   28.61262081
## 68  -0.387420377    9.13985354
## 69  -0.156539342   18.17290363
## 70   2.139367044   23.51692092
## 71   1.303577509   16.01403338
## 72   0.637332337   31.37900360
## 73   0.737691248   19.38985522
## 74   0.375049534    3.80804606
## 75   0.991941201   -3.66472121
```

```
## 76  -1.177177765  -11.63167112
## 77  -1.181271379   14.71234617
## 78  -0.917060914  -12.59582947
## 79  -2.725687913  -55.48049012
## 80  -1.882947175  -18.41471953
## 81  -0.645588889   -4.99652869
## 82  -0.428697222    8.53070404
## 83   0.002183812   33.56375413
## 84   0.398090198  -33.09222858
## 85  -2.847354773  -57.59511612
## 86  -2.450425326  -43.23014590
## 87  -0.503585598  -10.58744799
## 88  -0.666227312  -14.80110344
## 89  -1.449335645  -23.27387071
## 90   1.181545389   12.75917938
## 91   1.377451775   39.10319667
## 92   1.441662241   67.60030913
## 93   0.275417068   57.96527935
## 94   0.675775979   16.97613098
## 95  -0.786865735  -14.60567819
## 96  -1.169974068  -34.07844546
## 97  -0.939093034  -28.04539537
## 98  -0.043186647   12.29862193
## 99   0.021023818  -14.20426562
## 100 -0.045221355   -0.83929540
## 101  0.255137557   37.17155623
## 102 -1.007504158  -16.41025293
## 103  1.009387509  -10.88302021
## 104  0.040268543   14.15002988
## 105  0.536174930    6.49404718
## 106 -0.199614605    7.99115963
## 107 -0.765859778  -28.64387015
## 108 -0.765500866    6.36698148
## 109 -0.728142580   12.78517232
## 110 -0.811250914   10.31240504
## 111  1.919630121   48.34545514
## 112  1.215536507   52.68947243
## 113 -0.120253028   66.18658489
## 114 -0.486498200   20.55155511
## 115  0.713860711   27.56240673
## 116 -0.948781003   -2.01940243
## 117 -1.031889336  -20.49216970
## 118  0.598991698   66.54088039
## 119  0.594898084   42.88489768
## 120  0.259108549   71.38201014
## 121  0.192863377   45.74698036
## 122 -1.006777712   31.75783198
## 123 -1.169419426   -7.82397718
## 124 -0.852527759    5.70325555
## 125 -0.121646725  -10.26369436
## 126  0.474259661   20.08032293
## 127 -0.361529873   26.57743539
## 128 -0.127775046   35.94240561
## 129  0.172583865   55.95325723
```

```
## 130 -1.090057849    1.37144807
## 131 -1.073166182  -18.10131920
## 132  0.520627305   59.93173089
## 133 -0.424380775   38.27574818
## 134 -0.939638569   28.77286064
## 135 -0.548413469   26.13783086
## 136 -0.448054557    4.14868248
## 137 -1.110696272  -22.43312668
## 138 -1.593804605  -23.90589395
## 139 -0.889685393  -10.87284386
## 140 -1.754167757  -28.52882657
## 141 -1.523485425   -1.03171411
## 142 -1.288853805    5.33325611
## 143 -0.669151197   13.34410774
## 144 -0.113315596   33.76229857
## 145 -1.314443028  -35.71046870
## 146 -1.583561994   29.32258139
## 147 -0.625415495   26.66659869
## 148 -2.968063964  -73.41031921
## 149 -2.439655771  -52.47131864
## 150 -1.289331403  -35.46046701
## 151 -1.551973117  -46.04227617
## 152 -1.435081451  -39.51504345
## 153 -1.304200416  -49.48199336
## 154 -1.708294030  -34.13797606
## 155 -3.344083565 -101.64086361
## 156 -2.810328737  -83.27589339
## 157 -2.709969826  -48.26504176
## 158 -2.172611540  -58.84685092
## 159 -1.755719873  -48.31961820
## 160 -1.724838839  -37.28656810
## 161 -2.128932453  -24.94255081
## 162 -2.995073149  -75.95847064
## 163 -2.860036185  -74.47350640
## 164 -2.158728920  -62.37067093
## 165 -1.420433716  -48.85878029
## 166 -1.202616634  -44.23612129
## 167 -1.671315316  -52.15875429
## 168 -2.349570876  -34.74712556
## 169 -1.685360410  -67.25001310
## 170 -2.451605583  -73.88504288
## 171 -1.951246672  -36.87419126
## 172 -1.434839758  -45.45600042
## 173 -1.217105170  -45.32348773
## 174 -1.685393280  -53.18082108
## 175 -1.770209299  -51.41902655
## 176 -0.305998833   -5.05458785
## 177  0.127755994  -48.68961763
## 178  0.528114906   -2.74644222
## 179 -1.034526809  -41.52389126
## 180  2.382364858   35.79181021
## 181 -0.986754108  -37.94871397
## 182 -0.590847721  -25.20315153
## 183 -0.526637256   10.14083740
```

```
## 184  0.307117571   43.50580762
## 185 -0.292523517   15.51665924
## 186 -1.465439134  -50.06514992
## 187 -1.247879833  -47.22845854
## 188  2.292607470   53.49513290
## 189 -2.519796351  -69.57728549
## 190  1.452724321   -6.66373735
## 191 -0.913520852  -51.29876713
## 192 -1.313161940  -37.28791550
## 193 -1.175803654  -20.86972467
## 194  0.941088013   39.65750806
## 195  2.571969047   68.69055815
## 196  2.567875433   39.03457545
## 197  0.232085898   36.53168790
## 198 -0.234159274    9.89665812
## 199  1.166199637   -8.09249025
## 200 -0.896442077  -55.67429941
## 201 -0.179550410  -23.14706669
## 202  1.551330624   -0.11401660
## 203  2.747237010   36.23000070
## 204  1.011447475   20.72711315
## 205  1.345202303    6.09208337
## 206  1.145561214   -0.89706500
## 207 -1.516133271  -54.06712604
## 208 -1.298852898  -49.36392390
## 209 -1.767595954  -57.15298429
## 210 -2.571326381  -71.62916790
## 211 -3.106765385  -81.95035726
## 212  1.824563880  -55.71249138
## 213  1.224922791  -43.70163975
## 214  1.062281077  -36.28344891
## 215  0.179172744  -50.75621618
## 216  0.810053778  -52.72316609
## 217 -1.794039835  -72.12256358
## 218 -3.125298536  -82.43075380
## 219 -2.991291893  -80.86927130
## 220 -2.290693190  -68.66010294
## 221 -1.553107067  -55.04180143
## 222 -1.335999447  -50.31267460
## 223 -1.804914274  -58.07595751
## 224 -2.608815492  -72.52651082
## 225  2.649532207    5.31338891
## 226  0.783287034  -67.32164087
## 227  3.883645946  -12.31078925
## 228  0.821004232  -41.89259841
## 229  1.637895898  -29.36536568
## 230  4.368776933   -7.33231559
## 231  4.664683319    7.01170170
## 232 -0.471106216  -58.49118584
## 233  1.262648611  -40.12621562
## 234  0.963007523  -14.11536400
## 235  1.100365809   -3.69717316
## 236  3.017257476  -53.16994043
## 237  0.048138510  -40.81284872
```

```
## 238  4.344044896    8.20712695
## 239 -0.791744639  -68.29576059
## 240  3.442010189   -4.93079037
## 241  3.542369100    1.08006126
## 242 -0.020272614  -24.50174791
## 243 -0.403380947  -20.97451518
## 244  2.527500087   41.04931897
## 245 -0.176593527  -16.57163871
## 246 -0.812383061   -1.10033534
## 247  1.521371766   18.26463488
## 248  0.621730677   -2.72451349
## 249 -0.040911037   31.69367734
## 250 -2.224019370  -56.77908993
## 251 -0.093138336   23.25396016
## 252  0.302768050   36.59797746
## 253  4.766978516   92.09508991
## 254  0.300733343   87.46006013
## 255  4.001092254   48.47091176
## 256  0.738450540   73.88910260
## 257 -1.144657793   51.41633532
## 258  2.786223241   72.44938542
## 259  6.982129628   45.79340271
## 260  2.946340093  -16.70948484
## 261 -0.519905080   -6.34451462
## 262  5.480453832   70.66633701
## 263  1.717812117   16.08452785
## 264  0.834703784  -34.38823943
## 265  2.465584818   57.64481067
## 266  1.861491205  -11.01117204
## 267  2.025701670  -14.51405958
## 268  4.059456497  -27.14908936
## 269  2.259815409  -13.13823774
## 270 -0.402826305  -48.72004690
## 271 -1.285934639  -62.19281417
## 272  0.744946396  -15.15976408
## 273 -0.859147218  -35.81574679
## 274 -1.994936753  -74.31863433
## 275 -2.161181925  -64.95366411
## 276 -0.460823014  -10.94281249
## 277 -1.023464728  -27.52462165
## 278  2.593426939   18.00261108
## 279  5.724307973   91.03566117
## 280  4.320214359   64.37967846
## 281  1.784424824   25.87679092
## 282  3.718179652   34.24176114
## 283  1.618538563   23.25261276
## 284  0.455896849    2.67080360
## 285  1.772788516    3.19803633
## 286 -0.096330450  -28.76891358
## 287  0.499575936    3.57510371
## 288 -1.436213598  -26.92778383
## 289  3.697541229   48.43718639
## 290  3.197900140   25.44803802
## 291  2.235258426   12.82676459
```

```
## 292  0.052150093  -27.60653842
## 293 -1.716968873  -43.57348833
## 294 -2.826319406  -72.43969899
## 295 -3.362545944  -82.63890066
## 296 -3.229253746  -80.96923918
## 297 -2.529364160  -68.65270141
## 298  0.714620003   11.06165410
## 299 -0.368488330   -1.41111317
## 300  0.762392704    6.62193692
## 301 -0.541700909   -2.03404578
## 302  0.322509556   10.46306667
## 303 -2.643735617  -38.17196311
## 304  0.956623295  -15.16111148
## 305  1.493981580   14.25707936
## 306  0.010873247  -12.21568792
## 307 -0.758245719  -30.18263782
## 308  0.037660668  -37.83862053
## 309  0.301871133  -14.34150808
## 310 -0.264374040  -30.97653786
## 311  1.035984872    0.03431377
## 312  1.173343158   28.45250461
## 313 -0.009765176  -12.02026267
## 314  4.421115859   85.01278743
## 315  0.717022245   41.35680472
## 316 -3.018767290  -60.14608282
## 317  0.514987538   -0.78111260
## 318  1.615346449   45.22973902
## 319 -1.347295265   60.64792986
## 320 -0.154053058   25.17516259
## 321 -2.119315562   14.20821268
## 322 -2.924023761  -15.44777003
## 323 -3.460432747  -28.95065757
## 324 -2.505650885   -8.58568735
## 325 -1.805291974    5.42516427
## 326 -2.567933688  -47.15664489
## 327 -2.151042021  -18.62941216
## 328  2.479839013   94.40363793
## 329 -0.324254601   21.74765522
## 330  1.039955865   14.24476768
## 331  1.173710692  118.60973790
## 332 -0.825930397  103.62058952
## 333  0.011427889  121.03878036
## 334 -1.871680444   27.56601309
## 335 -1.540799410   88.59906318
## 336 -2.244893024   45.94308047
## 337 -1.780682558   27.44019293
## 338  2.853072269   72.80516315
## 339  2.253431181  135.81601478
## 340  1.590789466   74.23420561
## 341  0.207681133   42.76143834
## 342 -0.161437833   17.79448843
## 343 -2.065531446   11.13850572
## 344 -0.101320981   68.63561818
## 345 -2.267566154   20.00058840
```

```
## 346 -0.067207242   44.01144003
## 347 -1.029848956   57.42963086
## 348 -1.712957290  -14.04313641
## 349  1.917923744   86.98991368
## 350  0.213830131   84.33393098
## 351 -2.621959404   27.83104343
## 352 -1.588204577   50.19601365
## 353  0.912154335   17.92236887
## 354  0.149512621  -25.37494388
## 355  0.866404287   -0.84771116
## 356 -1.902714678  -47.81466106
## 357 -2.406808292  -53.47064377
## 358 -0.442597827   17.02646868
## 359 -0.208842999    6.39143890
## 360  1.191515912   15.40229053
## 361 -0.971125802    2.82048137
## 362 -0.954234135    2.34771409
## 363 -0.223353101   24.38076419
## 364  0.672553285   31.72478148
## 365  0.236763750   10.22189394
## 366 -0.229481422   21.58686416
## 367 -1.129122511   23.59771578
## 368 -0.891764225   23.01590662
## 369 -0.774872558   21.54313935
## 370  2.056008476   32.57618944
## 371  2.151914862   31.92020673
## 372 -0.883874672    2.41731919
## 373 -0.050119845   14.78228941
## 374  0.650239066   27.79314103
## 375 -0.912402648    0.21133187
## 376  0.004489019  -16.26143540
## 377  1.935370053   28.77161469
## 378  0.531276439  -19.88436802
## 379  1.395486905    1.61274444
## 380  1.829241732    4.97771466
## 381  0.729600644  -19.01143372
## 382 -1.733041071  -73.59324288
## 383 -1.916149404  -71.06601015
## 384 -0.785268370  -30.03296006
##
## $ARorder
## [1] 2
##
## $MAorder
## [1] 3
##
## $cnst
## [1] FALSE
##
## $coef
##              [,1]         [,2]
##  [1,]  0.734629473  5.042860274
##  [2,] -0.010147444  0.652354321
##  [3,] -0.113265329 -9.945690491
```

```
##  [4,]  0.016436921  0.217539206
##  [5,]  0.246124244 -6.114663704
##  [6,] -0.017382253 -0.133708542
##  [7,] -0.076440220  4.734501646
##  [8,]  0.014294859  0.130515528
##  [9,] -0.089836026  4.300279981
## [10,] -0.008902294 -0.005111609
##
## $secoef
##       [,1] [,2]
##  [1,]  NaN  NaN
##  [2,]  NaN  NaN
##  [3,]  NaN  NaN
##  [4,]  NaN  NaN
##  [5,]  NaN  NaN
##  [6,]  NaN  NaN
##  [7,]  NaN  NaN
##  [8,]  NaN  NaN
##  [9,]  NaN  NaN
## [10,]  NaN  NaN
##
## $residuals
##               [,1]         [,2]
##   [1,] -1.058699963  21.88975377
##   [2,]  3.244291832  13.03820995
##   [3,]  1.850636919  33.17952036
##   [4,] -2.127180677  -3.68437118
##   [5,]  2.382968035   9.14265931
##   [6,] -0.326431392   7.92446781
##   [7,]  0.656070685  18.44843441
##   [8,] -0.660722118  -1.88433820
##   [9,]  0.608362520  21.78240784
##  [10,]  0.499962392   9.54378568
##  [11,] -1.680366090 -15.63759046
##  [12,] -0.044108920 -17.16505214
##  [13,]  0.549014934   7.70241351
##  [14,] -0.559914982   3.05590592
##  [15,] -0.184407867  23.62176210
##  [16,]  0.209879060 -14.85812271
##  [17,]  0.040111041  18.62477486
##  [18,]  0.876827514  13.05279925
##  [19,] -1.333451430 -30.03018795
##  [20,]  1.311513344  19.20529942
##  [21,]  1.360644947  31.22153308
##  [22,] -1.437216089 -22.12845817
##  [23,] -1.622220751 -28.50499156
##  [24,]  1.866661822  20.39965447
##  [25,]  1.559056360  14.28668376
##  [26,] -3.770964600 -13.92060695
##  [27,]  3.718517083  23.66396604
##  [28,] -0.853865859   9.81265600
##  [29,] -0.932531230  11.15997851
##  [30,]  0.626471738 -18.24772124
##  [31,] -2.339078398 -26.77875182
```

```
## [32,]  1.943438638  16.45127103
## [33,] -0.114602349 -20.02793582
## [34,]  1.265958545  31.45160823
## [35,]  1.899642139   9.88635913
## [36,] -3.044415488 -14.82857995
## [37,]  0.104835332 -18.60483433
## [38,] -0.839686029  -4.53624700
## [39,] -1.310437369 -12.65866321
## [40,] -1.374814194 -27.36473620
## [41,] -0.922538245 -16.18829352
## [42,]  3.581527576 100.80153818
## [43,]  0.861434426  -2.41339350
## [44,] -2.797562816 -43.03416637
## [45,]  0.751604342 -35.20068322
## [46,]  0.313771699  19.87999006
## [47,]  0.635583924  28.70923145
## [48,]  2.003424716  -9.21048517
## [49,] -1.135281371  -4.64906717
## [50,]  0.553955330   8.11590390
## [51,]  0.052718184  15.40556830
## [52,]  0.994792909   9.28408273
## [53,] -0.016003444 -12.32006425
## [54,] -1.929781895 -22.11115528
## [55,] -0.019314682   7.16754478
## [56,]  1.465454909  17.72520335
## [57,] -1.849829247 -17.65517296
## [58,] -0.405132325 -18.69497501
## [59,]  1.878391921  25.43778921
## [60,]  1.555766438  34.55171344
## [61,] -0.469197043 -33.89922265
## [62,] -1.722842788  -5.88224292
## [63,]  2.184665907  32.00388091
## [64,] -0.459536523  33.12288342
## [65,] -0.347078262 -17.03680349
## [66,] -0.488132561  -7.88175651
## [67,]  2.698394889   8.58334742
## [68,] -0.968298614   6.76657685
## [69,]  0.023448396  13.66698244
## [70,]  0.852205611  -0.70099238
## [71,] -0.855967790  -5.62754165
## [72,]  0.831698469 -14.88210224
## [73,] -2.341239753  -7.12701921
## [74,]  0.339745853  25.28738671
## [75,]  0.378962545 -11.59325313
## [76,] -3.459498779 -51.69826439
## [77,]  0.062767061  -4.53624952
## [78,]  1.465133517  22.68695396
## [79,] -0.583354245  27.22420990
## [80,]  0.782426741  14.70900039
## [81,]  0.511228304 -61.34296633
## [82,] -5.179649681 -53.34698097
## [83,]  1.110654411 -16.64920370
## [84,]  0.785801324  47.72018426
## [85,]  0.048054817   9.77960209
```

```
##  [86,] -1.505474282 -26.17231360
##  [87,]  2.452811502  14.50044114
##  [88,]  0.856100717  42.79351262
##  [89,]  0.895762323  47.91196760
##  [90,] -0.168918327   3.56049595
##  [91,]  0.058706235 -37.10783347
##  [92,] -2.248899419 -46.83322526
##  [93,] -0.651720850 -32.25769582
##  [94,] -0.502267009   3.45678994
##  [95,]  0.765789985  45.22528589
##  [96,]  0.696277490  -9.95442826
##  [97,] -1.358547487   0.79177812
##  [98,]  1.530756897  27.98574989
##  [99,] -0.840599489 -25.82744711
## [100,]  0.360833816 -14.29286740
## [101,] -0.301726957  10.91533435
## [102,]  1.133843573  12.72106548
## [103,] -1.087494714   5.38634428
## [104,] -0.247962675 -40.53502881
## [105,] -1.236078443  17.89019844
## [106,]  1.408840819  17.10848219
## [107,] -1.120677992  11.92387562
## [108,]  2.722059372  26.93158548
## [109,] -0.144567106  19.18297994
## [110,] -0.858192312  24.41815791
## [111,]  0.379781023 -36.76162478
## [112,] -0.811064295   0.01746592
## [113,] -0.691683800 -31.05096469
## [114,] -1.452452011 -19.65086333
## [115,]  1.409030403  75.42228667
## [116,]  1.845562225  21.60847768
## [117,] -2.123129438  35.69290651
## [118,]  1.862520083 -30.98003628
## [119,] -3.139467070  -7.19547424
## [120,]  0.522222550 -47.11901025
## [121,] -1.902026582   4.31102514
## [122,]  1.469121300 -13.52644738
## [123,] -0.907107350  30.80019308
## [124,]  0.580558108  13.69194984
## [125,] -0.271785796  20.04489892
## [126,]  0.610048260  24.31401112
## [127,] -1.115100403 -41.86465467
## [128,] -1.758764845 -41.16108100
## [129,]  1.481184492  57.99880398
## [130,]  0.601960666  24.91010579
## [131,] -2.121053536  -2.13405132
## [132,]  0.580902414 -25.87008896
## [133,] -0.807754980 -17.78909094
## [134,] -1.136923145 -31.60173767
## [135,] -1.379488549 -16.69070966
## [136,]  0.464824822   8.81617116
## [137,] -1.209975165 -10.14215431
## [138,] -0.687868453  13.68277194
## [139,]  0.669984477   5.17069731
```

```
## [140,] -0.345985128  12.84715402
## [141,]  0.625806296  13.23617544
## [142,] -1.211551862 -64.01904406
## [143,] -2.420774557  31.67035693
## [144,]  3.416452057  11.11941579
## [145,] -4.863469487 -75.19325973
## [146,] -1.456650858 -48.48248845
## [147,]  1.465726098   0.33812164
## [148,] -1.257963948  13.50472594
## [149,] -0.236243419 -13.61579030
## [150,] -0.402965474 -27.25985768
## [151,] -0.956213355   1.30723262
## [152,] -1.295414651 -72.22316469
## [153,] -2.324652960 -21.29788297
## [154,]  0.862518537  11.69996946
## [155,] -0.262556582   2.29581223
## [156,] -0.665645458 -12.44873303
## [157,] -0.127978346 -14.15836124
## [158,] -0.735498700   4.37060541
## [159,] -0.987704939 -56.65491523
## [160,] -2.259636775 -35.79062602
## [161,]  0.737145101 -15.97552291
## [162,] -0.280613771  12.16966449
## [163,]  0.169458695  -5.06132806
## [164,] -0.995614850 -23.88691021
## [165,] -1.061890790  -2.78521947
## [166,]  0.804601072 -37.79858004
## [167,] -2.789398600 -27.90817970
## [168,]  0.697590900   6.44389247
## [169,]  0.423010132  -1.54833599
## [170,] -0.908054704  -9.73098127
## [171,] -0.437700064 -33.47928885
## [172,] -0.844904318 -13.88300252
## [173,]  1.399463038  35.49669357
## [174,]  1.015005649 -29.44272053
## [175,] -1.372927479  21.00179180
## [176,]  1.007572965 -42.91924738
## [177,]  1.255847977  73.61305966
## [178,] -0.083729600 -50.20321741
## [179,] -2.669390902  -6.89321065
## [180,]  2.180239699  12.68355867
## [181,]  0.268145931  67.82111249
## [182,]  0.494059705 -11.68306687
## [183,] -2.738301410 -83.09524723
## [184,] -0.905330993 -42.93813600
## [185,]  3.782381202  97.26244257
## [186,] -2.707666467 -60.79944557
## [187,]  0.022971957  10.98243159
## [188,]  0.862919223 -57.60735900
## [189,] -3.042007244  24.04645266
## [190,]  2.306206919   0.15607703
## [191,]  0.480767233  75.63389886
## [192,]  3.804098337  46.23775647
## [193,] -0.138210419 -10.63434432
```

```
## [194,] -1.900498848 -19.23809521
## [195,]  0.643466077 -26.33082245
## [196,]  0.271047896  -6.54391257
## [197,] -2.121351279 -51.51107783
## [198,] -0.256441500   7.86233733
## [199,]  3.001991006  34.13784047
## [200,]  1.039952331  63.69862993
## [201,]  0.178174770  -3.95723988
## [202,]  0.158031866 -18.50006018
## [203,]  0.363235872 -12.08522093
## [204,] -2.356725311 -45.72004651
## [205,] -0.753846340 -16.29912996
## [206,] -0.296683496 -13.00509889
## [207,] -1.784878567 -15.56659297
## [208,] -1.086309742 -34.13773509
## [209,]  3.779053551   3.90831021
## [210,] -0.441822153  11.19487581
## [211,]  0.976202734   1.20475241
## [212,] -0.151172752 -20.22403445
## [213,]  0.683126048  -8.50786704
## [214,] -2.009617124 -26.78689986
## [215,] -1.571631327 -24.25521079
## [216,] -0.369411038 -16.31185484
## [217,] -0.297926271   0.70035779
## [218,]  0.356733615  -2.07334320
## [219,] -0.219494834  -9.69167855
## [220,] -0.683253180 -24.76158406
## [221,] -1.323425741 -34.61017428
## [222,]  4.500104928  55.10594701
## [223,] -0.250546379 -45.83888656
## [224,]  1.231980430  22.20751792
## [225,]  0.652066100 -32.40951047
## [226,] -0.125806308  26.89246849
## [227,]  4.969046779  19.62824522
## [228,]  0.543239343  39.56648978
## [229,] -2.330012444 -58.34025789
## [230,]  1.084748394  -5.77714117
## [231,]  1.225796589  36.10477812
## [232,]  0.814947734  39.60427856
## [233,]  2.623908267 -48.59010547
## [234,] -4.085283001 -14.16129358
## [235,]  7.190509036  39.90725081
## [236,] -4.606205528 -27.23734239
## [237,]  3.834105960  31.47027371
## [238,]  2.638472125  16.21955059
## [239,] -3.321254947   7.94667390
## [240,]  1.296356650 -20.07354919
## [241,]  2.357393721  70.56163575
## [242,] -0.513030464 -20.89067487
## [243,] -2.366826118  -5.16937013
## [244,]  3.951421908   5.14301103
## [245,] -1.606735664   8.67715889
## [246,] -0.137026645  23.60143996
## [247,] -1.159273528 -78.64555978
```

```
## [248,] -1.057712490  51.12597285
## [249,]  3.670949977  28.96147610
## [250,]  2.507104398  89.94567604
## [251,] -1.660839649   9.39832632
## [252,]  3.759461409 -30.87010213
## [253,] -3.809763382  21.23233604
## [254,]  0.688974910   1.70334893
## [255,]  2.346221034  39.33294110
## [256,]  4.676999733  -5.47646524
## [257,] -4.145685493 -61.17237564
## [258,] -1.862036471 -15.66734533
## [259,]  7.521303372 104.35608500
## [260,] -2.418420646  26.87634670
## [261,] -0.966001666 -66.90629016
## [262,]  1.544031273  50.73388019
## [263,]  2.701347250  -5.19921864
## [264,] -2.166627793   0.96677427
## [265,]  4.237103494 -38.59725697
## [266,] -2.226671883  26.64197286
## [267,]  0.410498461 -21.01917946
## [268,] -1.992884051 -14.69859653
## [269,]  2.417701567  34.84908611
## [270,] -0.765395932  -0.01894196
## [271,] -1.828847631 -48.09747288
## [272,] -0.875119211 -31.37112721
## [273,]  1.612251110  47.33654447
## [274,]  0.241958106   6.61740510
## [275,]  2.337813630  31.07273135
## [276,]  4.793141559  71.77115461
## [277,]  0.919591493  20.77678093
## [278,] -1.622140885 -34.33322040
## [279,]  2.746275506  -4.49524500
## [280,] -1.108398014  18.78695938
## [281,] -0.216685996  -1.01015297
## [282,]  1.155284330  -0.30120667
## [283,] -1.582513427 -22.79779123
## [284,]  0.484016740  22.53578408
## [285,] -0.726354479 -15.00789716
## [286,]  3.764834335  72.44494163
## [287,]  1.988782357   2.91603930
## [288,] -1.618598898  -5.34567937
## [289,] -0.116914924 -57.20481482
## [290,] -2.801330451 -14.92649488
## [291,] -0.501650035 -33.00397792
## [292,] -2.470319960 -20.78624887
## [293,] -0.431446079 -21.68544276
## [294,] -0.502545983  -4.15351559
## [295,]  2.762771288  65.86112159
## [296,]  0.314707319   7.91292763
## [297,]  0.165574297  -2.45226776
## [298,] -0.391048124 -26.93866729
## [299,]  0.451745369  12.89390594
## [300,] -2.341233722 -42.43998749
## [301,]  1.789790668   1.73580254
```

```
## [302,]  1.138096189  27.25648969
## [303,] -0.866897805  -4.48367144
## [304,] -0.946280500 -33.04504400
## [305,]  0.491653730 -21.82946771
## [306,]  0.082259398  21.76294525
## [307,]  0.323036427  -6.70255512
## [308,]  0.534803879  23.32072888
## [309,]  1.487214489  32.80622478
## [310,] -0.585167542 -19.72112835
## [311,]  3.673143798  78.66514753
## [312,] -0.345562942  -1.78408172
## [313,] -5.040260049 -92.85727961
## [314,]  2.179599862  -4.94660835
## [315,]  2.148531010  76.32814760
## [316,] -2.248010802  63.15741629
## [317,]  1.766668980 -36.20161131
## [318,] -4.166386481 -28.51794552
## [319,] -0.111608455 -43.97655367
## [320,] -2.754026737 -19.19360514
## [321,]  0.001578244   6.66253139
## [322,] -0.190022997  12.24288391
## [323,] -1.583161622 -53.30165236
## [324,] -1.703662099 -11.53024151
## [325,]  5.307001348 105.49546018
## [326,] -1.180060085  -7.02079787
## [327,] -0.961645483 -33.96194728
## [328,]  1.232503055  69.91668472
## [329,]  0.554483997  58.47941365
## [330,] -0.560907317  43.68851185
## [331,] -1.611564540 -94.99640721
## [332,] -3.225883586  26.14992499
## [333,]  1.942803755 -22.26159474
## [334,] -3.772115126   1.39922415
## [335,]  4.792689767  16.39775537
## [336,] -0.983011085  90.96045009
## [337,]  2.068574921 -15.64334960
## [338,] -3.764751367 -38.00785644
## [339,]  0.449035393 -42.63773590
## [340,] -2.995258227   3.96163954
## [341,]  1.991660000  67.95778243
## [342,] -1.847452354 -14.44946298
## [343,] -0.187545655   7.83341348
## [344,] -0.087899360   4.47001043
## [345,] -1.426242441 -48.76039699
## [346,]  1.458935294  67.49208296
## [347,]  1.158382000  35.39479681
## [348,] -4.221223950 -26.12791274
## [349,]  0.157673651 -16.29442015
## [350,]  1.981765539 -21.56519789
## [351,] -2.573122600 -39.74850114
## [352,]  0.579058446  -8.25294542
## [353,] -2.042194104 -33.44862877
## [354,] -1.883700242 -11.64416384
## [355,]  1.915148635  53.95014409
```

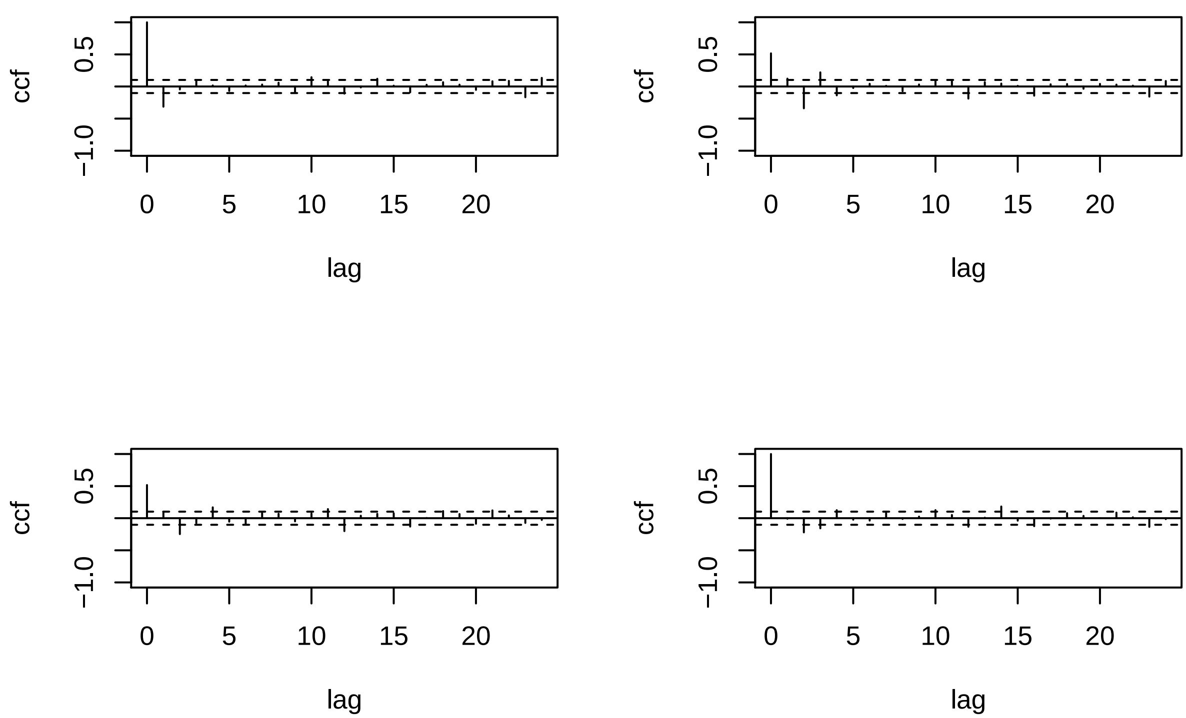
```
## [356,]  0.903001987  23.18393971
## [357,]  0.362840385   4.73386472
## [358,] -1.435939536 -29.11282079
## [359,] -0.235453461  -4.86842655
## [360,]  0.699588912  24.36055697
## [361,]  0.852047594  22.25624010
## [362,] -0.543523465 -18.70884107
## [363,] -0.909703192  -2.13548329
## [364,] -0.195336679   6.42055825
## [365,] -0.298345133  12.82894249
## [366,] -0.312989353  -1.45871479
## [367,]  2.249638345   7.05602426
## [368,]  0.183497513   5.62771559
## [369,] -2.360750476 -24.25479850
## [370,]  0.700725025   3.20143036
## [371,]  0.813244956  27.38102972
## [372,] -1.501129778  -9.98662871
## [373,]  0.251808669 -32.31231073
## [374,]  1.381429508  32.50933254
## [375,]  0.151722230 -22.60825714
## [376,] -0.504763087  10.76832696
## [377,]  2.267155665   1.30933062
## [378,] -1.347545442  -3.70071385
## [379,] -1.864134360 -65.17378043
## [380,] -1.503497444 -25.91814230
## [381,]  1.469289597  35.21224335
##
## $Sigma
##           [,1]       [,2]
## [1,]  3.554225   32.26891
## [2,] 32.268907 1098.01537
##
## $aic
## [1] 8.063202
##
## $bic
## [1] 8.268965
##
## $Phi
##           [,1]        [,2]       [,3]       [,4]
## [1,] 0.7346295 -0.01014744 -0.1132653 0.01643692
## [2,] 5.0428603  0.65235432 -9.9456905 0.21753921
##
## $Theta
##            [,1]       [,2]        [,3]        [,4]        [,5]        [,6]
## [1,] -0.2461242 0.01738225  0.07644022 -0.01429486  0.08983603 0.008902294
## [2,]  6.1146637 0.13370854 -4.73450165 -0.13051553 -4.30027998 0.005111609
##
## $Ph0
## [1] 0 0
```

```
MTSdiag(varma.model)
```

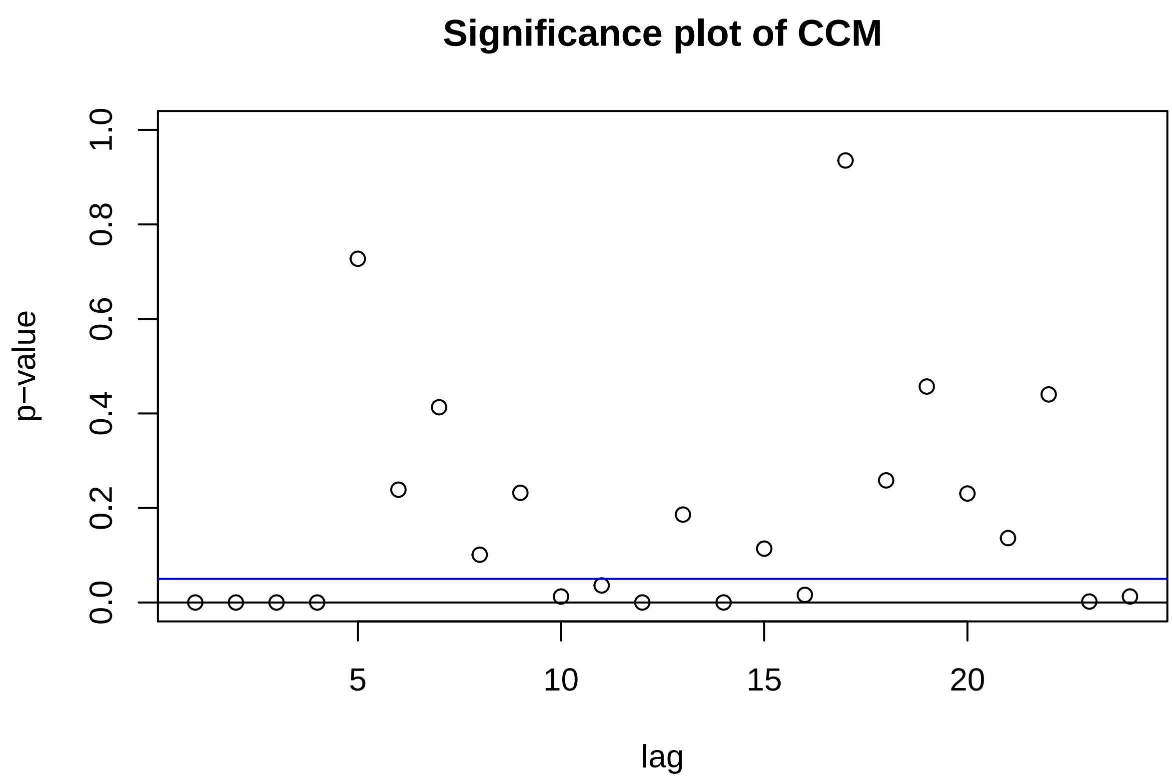
```
## [1] "Covariance matrix:"
```

```
##        CO    NO2
## CO   3.56   32.4
## NO2 32.35 1100.8
## CCM at lag:  0
##       [,1]  [,2]
## [1,] 1.000 0.517
## [2,] 0.517 1.000
## Simplified matrix:
## CCM at lag:  1
## - +
## . .
## CCM at lag:  2
## . -
## - -
## CCM at lag:  3
## . +
## . -
## CCM at lag:  4
## . -
## + +
## CCM at lag:  5
## . .
## . .
## CCM at lag:  6
## . .
## . .
## CCM at lag:  7
## . .
## . .
## CCM at lag:  8
## . .
## . .
## CCM at lag:  9
## . .
## . .
## CCM at lag:  10
## + .
## . +
## CCM at lag:  11
## . .
## + .
## CCM at lag:  12
## - -
## - -
## CCM at lag:  13
## . .
## . .
## CCM at lag:  14
## + .
## . +
## CCM at lag:  15
## . .
## . .
## CCM at lag:  16
```

```
## . -
## - -
## CCM at lag:  17
## . .
## . .
## CCM at lag:  18
## . .
## + .
## CCM at lag:  19
## . .
## . .
## CCM at lag:  20
## . .
## . .
## CCM at lag:  21
## . .
## + .
## CCM at lag:  22
## . .
## . .
## CCM at lag:  23
## - -
## . -
## CCM at lag:  24
## + .
## . .
```

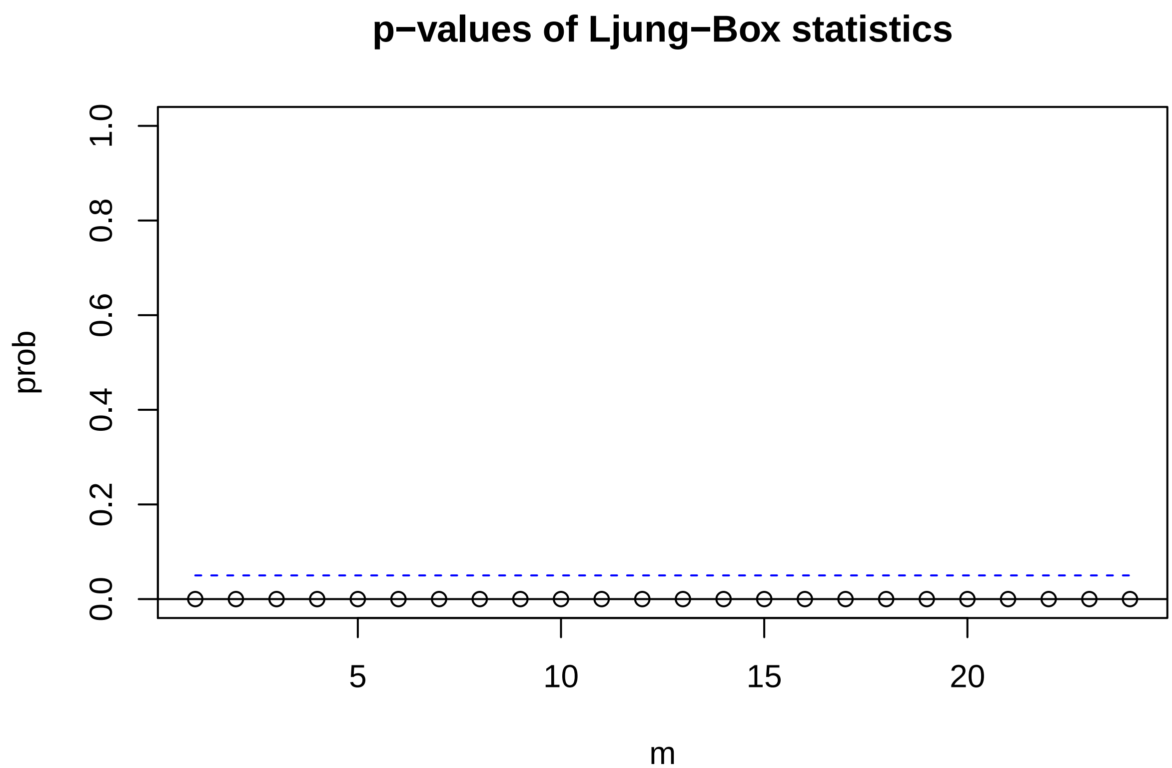


```
## Hit Enter for p-value plot of individual ccm:
```

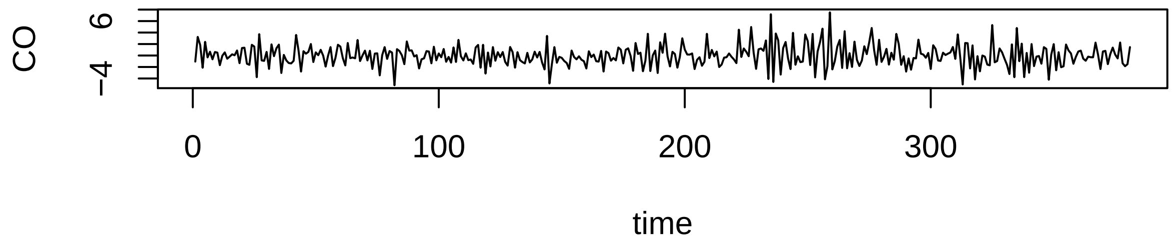


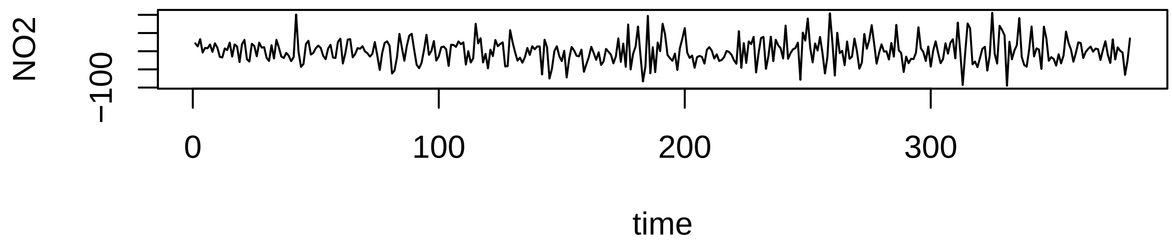

```
## Hit Enter to compute MQ-statistics:
##
## Ljung-Box Statistics:
##          m       Q(m)     df    p-value
##  [1,]     1        147       4          0
##  [2,]     2        230       8          0
##  [3,]     3        288      12          0
##  [4,]     4        322      16          0
##  [5,]     5        325      20          0
##  [6,]     6        330      24          0
##  [7,]     7        334      28          0
##  [8,]     8        342      32          0
##  [9,]     9        347      36          0
## [10,]    10        360      40          0
## [11,]    11        371      44          0
## [12,]    12        397      48          0
## [13,]    13        403      52          0
## [14,]    14        427      56          0
## [15,]    15        435      60          0
## [16,]    16        447      64          0
## [17,]    17        448      68          0
## [18,]    18        453      72          0
## [19,]    19        457      76          0
```

```
## [20,]    20      462     80        0
## [21,]    21      469     84        0
## [22,]    22      473     88        0
## [23,]    23      490     92        0
## [24,]    24      503     96        0
```




```
## Hit Enter to obtain residual plots:
```






```r
varma.model2 <- VARMACpp(allResiduals, p=2, q=2, include.mean=F)
```

```
## Number of parameters:  16
## initial estimates:  0.7605 -0.0125 -0.0601 0.0092 5.8783 0.5354 -6.8113 0.3256 -0.3465 0.012 -0.0364
## Par. lower-bounds:  0.2433 -0.0368 -0.4857 -0.0125 -4.9125 0.0292 -15.6904 -0.1279 -0.8821 -0.0131 -0
## Par. upper-bounds:  1.2777 0.0117 0.3654 0.031 16.669 1.0415 2.0678 0.779 0.1892 0.0371 0.2411 0.0114
## Final   Estimates:  0.7604869 -0.01254823 -0.06011082 0.009244218 5.878253 0.5353698 -6.811301 0.3255
##
## Coefficient(s):
##       Estimate  Std. Error  t value Pr(>|t|)
## CO    0.760487          NA       NA       NA
## NO2 -0.012548          NA       NA       NA
## CO  -0.060111          NA       NA       NA
## NO2  0.009244          NA       NA       NA
## CO   5.878253          NA       NA       NA
## NO2  0.535370          NA       NA       NA
## CO  -6.811301          NA       NA       NA
## NO2  0.325555          NA       NA       NA
##     -0.346455          NA       NA       NA
##      0.012014          NA       NA       NA
##     -0.036432          NA       NA       NA
##     -0.001438          NA       NA       NA
##     -6.906632          NA       NA       NA
##     -0.061093          NA       NA       NA
##      2.676609          NA       NA       NA
```

```
##     -0.223547          NA       NA       NA
## ---
## Estimates in matrix form:
## AR coefficient matrix
## AR( 1 )-matrix
##      [,1]    [,2]
## [1,] 0.76 -0.0125
## [2,] 5.88  0.5354
## AR( 2 )-matrix
##         [,1]    [,2]
## [1,] -0.0601 0.00924
## [2,] -6.8113 0.32556
## MA coefficient matrix
## MA( 1 )-matrix
##       [,1]    [,2]
## [1,] 0.346 -0.0120
## [2,] 6.907  0.0611
## MA( 2 )-matrix
##         [,1]    [,2]
## [1,]  0.0364 0.00144
## [2,] -2.6766 0.22355
##
## Residuals cov-matrix:
##          [,1]      [,2]
## [1,]  2.27996  32.42389
## [2,] 32.42389 982.15475
## ----
## aic=  7.16333
## bic=  7.32794
```

```
varma.model2
```

```
## $data
##               CO          NO2
## 1   -2.090038263   -8.94021914
## 2    2.343716565   42.42475108
## 3    2.744075476   52.43560271
## 4    1.081433762   60.85379355
## 5    2.998325429   73.38102627
## 6    4.729206463   79.41407636
## 7    1.125112849   54.75809366
## 8    2.889323315   61.25520611
## 9    2.023078142   44.62017633
## 10   2.323437053   46.63102796
## 11   0.760795339   33.04921880
## 12   1.377687006   42.57645152
## 13   1.108568040   36.60950162
## 14  -0.395525573    7.95351891
## 15  -0.231315108    1.45063137
## 16   0.602439719    7.81560159
## 17  -0.097201369   -0.17354679
## 18  -0.159843084   25.24464405
## 19  -0.642951417   -0.22812322
## 20   0.687929617   22.80492687
```

```
## 21   0.483836004   33.14894416
## 22  -0.651953531  -11.35394338
## 23   1.581801296   29.01102684
## 24   1.582160208   45.02187846
## 25  -0.080481506   -3.55993070
## 26  -0.999987702   -9.36940461
## 27   0.667291195   19.00035212
## 28   1.963197581    6.34436941
## 29  -1.472591954   -9.15851813
## 30   1.661162873   34.20645209
## 31   0.361521785   14.21730371
## 32  -0.301119929   21.63549455
## 33  -0.384228262   11.16272728
## 34  -1.653347228  -21.80422263
## 35   0.742559158   14.53979466
## 36   0.206769623  -20.96309288
## 37   2.640524451   19.40187734
## 38   2.440883362   22.41272896
## 39  -0.621758352  -13.16908020
## 40  -1.015520814  -13.56612959
## 41  -1.482819267  -21.57237470
## 42  -2.285095184  -36.26723994
## 43  -2.819069910  -46.80858683
## 44  -2.683503231  -45.48164225
## 45   1.620244939   67.60815422
## 46   0.257603225   19.02634505
## 47  -0.225505108   -9.44642222
## 48  -0.294624074    0.58662787
## 49   0.301282312    3.93064517
## 50   0.665492778   18.42775762
## 51   2.499247605    3.79272784
## 52   1.699606516    4.80357947
## 53   0.736964802   14.22177031
## 54   0.253856469   10.74900303
## 55   1.184737503    9.78205312
## 56   1.080643890   -2.87392958
## 57  -0.955145645  -20.37681713
## 58  -1.121390818   -5.01184691
## 59   0.278968094    0.99900472
## 60  -1.083673621  -24.20626142
## 61  -1.039890068  -19.70440860
## 62   0.864099080   12.97747838
## 63   1.960005467   26.32149567
## 64   1.024215932  -16.18139187
## 65  -0.142029241    2.18357835
## 66   0.458329671   30.19442997
## 67  -0.004312043   28.61262081
## 68  -0.387420377    9.13985354
## 69  -0.156539342   18.17290363
## 70   2.139367044   23.51692092
## 71   1.303577509   16.01403338
## 72   0.637332337   31.37900360
## 73   0.737691248   19.38985522
## 74   0.375049534    3.80804606
```

```
## 75   0.991941201   -3.66472121
## 76  -1.177177765  -11.63167112
## 77  -1.181271379   14.71234617
## 78  -0.917060914  -12.59582947
## 79  -2.725687913  -55.48049012
## 80  -1.882947175  -18.41471953
## 81  -0.645588889   -4.99652869
## 82  -0.428697222    8.53070404
## 83   0.002183812   33.56375413
## 84   0.398090198  -33.09222858
## 85  -2.847354773  -57.59511612
## 86  -2.450425326  -43.23014590
## 87  -0.503585598  -10.58744799
## 88  -0.666227312  -14.80110344
## 89  -1.449335645  -23.27387071
## 90   1.181545389   12.75917938
## 91   1.377451775   39.10319667
## 92   1.441662241   67.60030913
## 93   0.275417068   57.96527935
## 94   0.675775979   16.97613098
## 95  -0.786865735  -14.60567819
## 96  -1.169974068  -34.07844546
## 97  -0.939093034  -28.04539537
## 98  -0.043186647   12.29862193
## 99   0.021023818  -14.20426562
## 100 -0.045221355   -0.83929540
## 101  0.255137557   37.17155623
## 102 -1.007504158  -16.41025293
## 103  1.009387509  -10.88302021
## 104  0.040268543   14.15002988
## 105  0.536174930    6.49404718
## 106 -0.199614605    7.99115963
## 107 -0.765859778  -28.64387015
## 108 -0.765500866    6.36698148
## 109 -0.728142580   12.78517232
## 110 -0.811250914   10.31240504
## 111  1.919630121   48.34545514
## 112  1.215536507   52.68947243
## 113 -0.120253028   66.18658489
## 114 -0.486498200   20.55155511
## 115  0.713860711   27.56240673
## 116 -0.948781003   -2.01940243
## 117 -1.031889336  -20.49216970
## 118  0.598991698   66.54088039
## 119  0.594898084   42.88489768
## 120  0.259108549   71.38201014
## 121  0.192863377   45.74698036
## 122 -1.006777712   31.75783198
## 123 -1.169419426   -7.82397718
## 124 -0.852527759    5.70325555
## 125 -0.121646725  -10.26369436
## 126  0.474259661   20.08032293
## 127 -0.361529873   26.57743539
## 128 -0.127775046   35.94240561
```

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## 129  0.172583865   55.95325723
## 130 -1.090057849    1.37144807
## 131 -1.073166182  -18.10131920
## 132  0.520627305   59.93173089
## 133 -0.424380775   38.27574818
## 134 -0.939638569   28.77286064
## 135 -0.548413469   26.13783086
## 136 -0.448054557    4.14868248
## 137 -1.110696272  -22.43312668
## 138 -1.593804605  -23.90589395
## 139 -0.889685393  -10.87284386
## 140 -1.754167757  -28.52882657
## 141 -1.523485425   -1.03171411
## 142 -1.288853805    5.33325611
## 143 -0.669151197   13.34410774
## 144 -0.113315596   33.76229857
## 145 -1.314443028  -35.71046870
## 146 -1.583561994   29.32258139
## 147 -0.625415495   26.66659869
## 148 -2.968063964  -73.41031921
## 149 -2.439655771  -52.47131864
## 150 -1.289331403  -35.46046701
## 151 -1.551973117  -46.04227617
## 152 -1.435081451  -39.51504345
## 153 -1.304200416  -49.48199336
## 154 -1.708294030  -34.13797606
## 155 -3.344083565 -101.64086361
## 156 -2.810328737  -83.27589339
## 157 -2.709969826  -48.26504176
## 158 -2.172611540  -58.84685092
## 159 -1.755719873  -48.31961820
## 160 -1.724838839  -37.28656810
## 161 -2.128932453  -24.94255081
## 162 -2.995073149  -75.95847064
## 163 -2.860036185  -74.47350640
## 164 -2.158728920  -62.37067093
## 165 -1.420433716  -48.85878029
## 166 -1.202616634  -44.23612129
## 167 -1.671315316  -52.15875429
## 168 -2.349570876  -34.74712556
## 169 -1.685360410  -67.25001310
## 170 -2.451605583  -73.88504288
## 171 -1.951246672  -36.87419126
## 172 -1.434839758  -45.45600042
## 173 -1.217105170  -45.32348773
## 174 -1.685393280  -53.18082108
## 175 -1.770209299  -51.41902655
## 176 -0.305998833   -5.05458785
## 177  0.127755994  -48.68961763
## 178  0.528114906   -2.74644222
## 179 -1.034526809  -41.52389126
## 180  2.382364858   35.79181021
## 181 -0.986754108  -37.94871397
## 182 -0.590847721  -25.20315153
```

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## 183 -0.526637256   10.14083740
## 184  0.307117571   43.50580762
## 185 -0.292523517   15.51665924
## 186 -1.465439134  -50.06514992
## 187 -1.247879833  -47.22845854
## 188  2.292607470   53.49513290
## 189 -2.519796351  -69.57728549
## 190  1.452724321   -6.66373735
## 191 -0.913520852  -51.29876713
## 192 -1.313161940  -37.28791550
## 193 -1.175803654  -20.86972467
## 194  0.941088013   39.65750806
## 195  2.571969047   68.69055815
## 196  2.567875433   39.03457545
## 197  0.232085898   36.53168790
## 198 -0.234159274    9.89665812
## 199  1.166199637   -8.09249025
## 200 -0.896442077  -55.67429941
## 201 -0.179550410  -23.14706669
## 202  1.551330624   -0.11401660
## 203  2.747237010   36.23000070
## 204  1.011447475   20.72711315
## 205  1.345202303    6.09208337
## 206  1.145561214   -0.89706500
## 207 -1.516133271  -54.06712604
## 208 -1.298852898  -49.36392390
## 209 -1.767595954  -57.15298429
## 210 -2.571326381  -71.62916790
## 211 -3.106765385  -81.95035726
## 212  1.824563880  -55.71249138
## 213  1.224922791  -43.70163975
## 214  1.062281077  -36.28344891
## 215  0.179172744  -50.75621618
## 216  0.810053778  -52.72316609
## 217 -1.794039835  -72.12256358
## 218 -3.125298536  -82.43075380
## 219 -2.991291893  -80.86927130
## 220 -2.290693190  -68.66010294
## 221 -1.553107067  -55.04180143
## 222 -1.335999447  -50.31267460
## 223 -1.804914274  -58.07595751
## 224 -2.608815492  -72.52651082
## 225  2.649532207    5.31338891
## 226  0.783287034  -67.32164087
## 227  3.883645946  -12.31078925
## 228  0.821004232  -41.89259841
## 229  1.637895898  -29.36536568
## 230  4.368776933   -7.33231559
## 231  4.664683319    7.01170170
## 232 -0.471106216  -58.49118584
## 233  1.262648611  -40.12621562
## 234  0.963007523  -14.11536400
## 235  1.100365809   -3.69717316
## 236  3.017257476  -53.16994043
```

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## 237  0.048138510  -40.81284872
## 238  4.344044896    8.20712695
## 239 -0.791744639  -68.29576059
## 240  3.442010189   -4.93079037
## 241  3.542369100    1.08006126
## 242 -0.020272614  -24.50174791
## 243 -0.403380947  -20.97451518
## 244  2.527500087   41.04931897
## 245 -0.176593527  -16.57163871
## 246 -0.812383061   -1.10033534
## 247  1.521371766   18.26463488
## 248  0.621730677   -2.72451349
## 249 -0.040911037   31.69367734
## 250 -2.224019370  -56.77908993
## 251 -0.093138336   23.25396016
## 252  0.302768050   36.59797746
## 253  4.766978516   92.09508991
## 254  0.300733343   87.46006013
## 255  4.001092254   48.47091176
## 256  0.738450540   73.88910260
## 257 -1.144657793   51.41633532
## 258  2.786223241   72.44938542
## 259  6.982129628   45.79340271
## 260  2.946340093  -16.70948484
## 261 -0.519905080   -6.34451462
## 262  5.480453832   70.66633701
## 263  1.717812117   16.08452785
## 264  0.834703784  -34.38823943
## 265  2.465584818   57.64481067
## 266  1.861491205  -11.01117204
## 267  2.025701670  -14.51405958
## 268  4.059456497  -27.14908936
## 269  2.259815409  -13.13823774
## 270 -0.402826305  -48.72004690
## 271 -1.285934639  -62.19281417
## 272  0.744946396  -15.15976408
## 273 -0.859147218  -35.81574679
## 274 -1.994936753  -74.31863433
## 275 -2.161181925  -64.95366411
## 276 -0.460823014  -10.94281249
## 277 -1.023464728  -27.52462165
## 278  2.593426939   18.00261108
## 279  5.724307973   91.03566117
## 280  4.320214359   64.37967846
## 281  1.784424824   25.87679092
## 282  3.718179652   34.24176114
## 283  1.618538563   23.25261276
## 284  0.455896849    2.67080360
## 285  1.772788516    3.19803633
## 286 -0.096330450  -28.76891358
## 287  0.499575936    3.57510371
## 288 -1.436213598  -26.92778383
## 289  3.697541229   48.43718639
## 290  3.197900140   25.44803802
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## 291  2.235258426   12.82676459
## 292  0.052150093  -27.60653842
## 293 -1.716968873  -43.57348833
## 294 -2.826319406  -72.43969899
## 295 -3.362545944  -82.63890066
## 296 -3.229253746  -80.96923918
## 297 -2.529364160  -68.65270141
## 298  0.714620003   11.06165410
## 299 -0.368488330   -1.41111317
## 300  0.762392704    6.62193692
## 301 -0.541700909   -2.03404578
## 302  0.322509556   10.46306667
## 303 -2.643735617  -38.17196311
## 304  0.956623295  -15.16111148
## 305  1.493981580   14.25707936
## 306  0.010873247  -12.21568792
## 307 -0.758245719  -30.18263782
## 308  0.037660668  -37.83862053
## 309  0.301871133  -14.34150808
## 310 -0.264374040  -30.97653786
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## 312  1.173343158   28.45250461
## 313 -0.009765176  -12.02026267
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## 315  0.717022245   41.35680472
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## 317  0.514987538   -0.78111260
## 318  1.615346449   45.22973902
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## 320 -0.154053058   25.17516259
## 321 -2.119315562   14.20821268
## 322 -2.924023761  -15.44777003
## 323 -3.460432747  -28.95065757
## 324 -2.505650885   -8.58568735
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## 326 -2.567933688  -47.15664489
## 327 -2.151042021  -18.62941216
## 328  2.479839013   94.40363793
## 329 -0.324254601   21.74765522
## 330  1.039955865   14.24476768
## 331  1.173710692  118.60973790
## 332 -0.825930397  103.62058952
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## 334 -1.871680444   27.56601309
## 335 -1.540799410   88.59906318
## 336 -2.244893024   45.94308047
## 337 -1.780682558   27.44019293
## 338  2.853072269   72.80516315
## 339  2.253431181  135.81601478
## 340  1.590789466   74.23420561
## 341  0.207681133   42.76143834
## 342 -0.161437833   17.79448843
## 343 -2.065531446   11.13850572
## 344 -0.101320981   68.63561818
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## 345 -2.267566154   20.00058840
## 346 -0.067207242   44.01144003
## 347 -1.029848956   57.42963086
## 348 -1.712957290  -14.04313641
## 349  1.917923744   86.98991368
## 350  0.213830131   84.33393098
## 351 -2.621959404   27.83104343
## 352 -1.588204577   50.19601365
## 353  0.912154335   17.92236887
## 354  0.149512621  -25.37494388
## 355  0.866404287   -0.84771116
## 356 -1.902714678  -47.81466106
## 357 -2.406808292  -53.47064377
## 358 -0.442597827   17.02646868
## 359 -0.208842999    6.39143890
## 360  1.191515912   15.40229053
## 361 -0.971125802    2.82048137
## 362 -0.954234135    2.34771409
## 363 -0.223353101   24.38076419
## 364  0.672553285   31.72478148
## 365  0.236763750   10.22189394
## 366 -0.229481422   21.58686416
## 367 -1.129122511   23.59771578
## 368 -0.891764225   23.01590662
## 369 -0.774872558   21.54313935
## 370  2.056008476   32.57618944
## 371  2.151914862   31.92020673
## 372 -0.883874672    2.41731919
## 373 -0.050119845   14.78228941
## 374  0.650239066   27.79314103
## 375 -0.912402648    0.21133187
## 376  0.004489019  -16.26143540
## 377  1.935370053   28.77161469
## 378  0.531276439  -19.88436802
## 379  1.395486905    1.61274444
## 380  1.829241732    4.97771466
## 381  0.729600644  -19.01143372
## 382 -1.733041071  -73.59324288
## 383 -1.916149404  -71.06601015
## 384 -0.785268370  -30.03296006
##
## $ARorder
## [1] 2
##
## $MAorder
## [1] 2
##
## $cnst
## [1] FALSE
##
## $coef
##              [,1]        [,2]
## [1,]  0.760486894  5.87825294
## [2,] -0.012548229  0.53536978
```

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## [3,] -0.060110816 -6.81130054
## [4,]  0.009244218  0.32555532
## [5,] -0.346454644 -6.90663197
## [6,]  0.012013901 -0.06109274
## [7,] -0.036431563  2.67660866
## [8,] -0.001438387 -0.22354678
##
## $secoef
##      [,1] [,2]
## [1,]  NaN  NaN
## [2,]  NaN  NaN
## [3,]  NaN  NaN
## [4,]  NaN  NaN
## [5,]  NaN  NaN
## [6,]  NaN  NaN
## [7,]  NaN  NaN
## [8,]  NaN  NaN
##
## $residuals
##                [,1]         [,2]
##   [1,]  1.937407165  33.06640708
##   [2,] -0.143991888  35.57891151
##   [3,]  2.260555917  39.45015866
##   [4,]  3.227441515  36.41977847
##   [5,] -1.153280412   8.25970462
##   [6,]  1.942138121  23.72630210
##   [7,]  0.513547859   4.47495866
##   [8,]  1.181282008  14.51452845
##   [9,] -0.451850418   2.35129073
##  [10,]  0.801537215  18.16112863
##  [11,]  0.381762781   8.52010602
##  [12,] -1.004736969 -17.56802327
##  [13,] -0.213381660 -11.97988530
##  [14,]  0.707379599  -0.32826783
##  [15,] -0.260578225  -7.18826406
##  [16,] -0.102757074  23.26228708
##  [17,] -0.543760082 -13.60730913
##  [18,]  0.935853144  18.28761206
##  [19,]  0.275438977  18.58537895
##  [20,] -0.840862169 -30.06255508
##  [21,]  1.764394851  27.19899997
##  [22,]  1.019668114  28.82566737
##  [23,] -0.781487273 -25.47302612
##  [24,] -1.190649415 -14.11000731
##  [25,]  1.030169615  17.81732579
##  [26,]  1.799833470  -3.27514114
##  [27,] -2.295432576 -12.27947363
##  [28,]  2.138608181  36.91890763
##  [29,] -0.280293985  -0.48419986
##  [30,] -0.574284078  12.64086770
##  [31,] -0.355177380  -3.36839643
##  [32,] -1.524502879 -32.91229225
##  [33,]  1.449465924  17.33852456
##  [34,]  0.117680971 -29.48192802
```

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##  [35,]  2.603165885  28.74169140
##  [36,]  1.400934765  17.56633739
##  [37,] -1.806910019 -17.64091379
##  [38,] -1.106165767  -6.91219235
##  [39,] -1.187793339 -15.45705266
##  [40,] -1.639819961 -26.23453816
##  [41,] -1.744538829 -30.24125061
##  [42,] -1.267663850 -22.97990935
##  [43,]  3.083387532  91.51913405
##  [44,]  0.022440538  -5.02165075
##  [45,] -0.398175519 -20.06704016
##  [46,] -0.305337051  -2.62862257
##  [47,]  0.488894939   1.19820451
##  [48,]  0.602641084  16.03410910
##  [49,]  2.241849665  -5.11132748
##  [50,]  0.599331215   3.75817949
##  [51,] -0.143303546  14.67352843
##  [52,] -0.269072386   7.95838643
##  [53,]  0.866447143   5.21644823
##  [54,]  0.457466637  -8.04324255
##  [55,] -1.538044321 -18.79034619
##  [56,] -0.861202728   5.01492631
##  [57,]  0.758156803   4.67659879
##  [58,] -1.122045381 -23.43925028
##  [59,] -0.584775612  -8.96546886
##  [60,]  1.396813319  23.31556303
##  [61,]  1.754978502  24.25892514
##  [62,]  0.196921694 -25.05735044
##  [63,] -0.781392658  10.16194703
##  [64,]  0.383222113  31.19992999
##  [65,] -0.258616439  16.99099457
##  [66,] -0.511564770  -7.66050172
##  [67,] -0.082166403   6.70200218
##  [68,]  2.240031975   8.59223955
##  [69,]  0.473798420   1.57966024
##  [70,] -0.002731345  21.35256452
##  [71,]  0.339140670   2.88026601
##  [72,] -0.080903055  -9.48468074
##  [73,]  0.722021499 -10.59798916
##  [74,] -1.629298849 -11.75017344
##  [75,] -0.750748480  19.53628600
##  [76,] -0.368407544 -20.01725842
##  [77,] -2.279743121 -53.57265536
##  [78,] -0.633391048   2.65744516
##  [79,]  0.492845690   5.34082184
##  [80,]  0.143942846  14.18991260
##  [81,]  0.347660547  30.48183963
##  [82,]  0.492863531 -49.72107377
##  [83,] -3.050875972 -46.88059970
##  [84,] -1.225258354 -18.54265724
##  [85,]  0.798426543  14.40785658
##  [86,] -0.131564544  -3.26044008
##  [87,] -1.017402954 -11.44092994
##  [88,]  1.863960796  25.91707185
```

```
##  [89,]  1.047925748  37.65460937
##  [90,]  0.853744872  52.80531651
##  [91,] -0.497637243  24.68684843
##  [92,]  0.293497663 -20.27323699
##  [93,] -1.244428419 -37.02234608
##  [94,] -0.875981361 -38.73156831
##  [95,] -0.326007797 -16.88972171
##  [96,]  0.566101915  26.36176630
##  [97,]  0.254250768 -15.18350883
##  [98,] -0.026693266   7.54988585
##  [99,]  0.299038622  38.85631428
## [100,] -1.083380814 -31.64693454
## [101,]  1.313021858  -8.06879762
## [102,] -0.305931534  16.92389636
## [103,]  0.571305127   2.70287820
## [104,] -0.475608455   5.74339685
## [105,] -0.750659097 -34.06974664
## [106,] -0.488200093  17.53375712
## [107,] -0.303482320  10.07735910
## [108,] -0.420726223   4.20690361
## [109,]  2.311154993  38.88763905
## [110,]  0.542479296  27.04438183
## [111,] -0.711848031  36.07512944
## [112,] -0.599891437 -21.16868229
## [113,]  0.795089666   1.58634583
## [114,] -1.160929609 -28.51429970
## [115,] -0.575969217 -29.47828788
## [116,]  1.159518330  68.72662749
## [117,]  0.614410279  10.54165513
## [118,] -0.006969208  44.49050917
## [119,]  0.031482815  -0.51934268
## [120,] -1.142812452  -5.19126938
## [121,] -0.749740825 -40.89808751
## [122,] -0.232978314  -6.20866709
## [123,]  0.508019409 -22.84807399
## [124,]  0.767092899  19.97523284
## [125,] -0.371232102  15.60303447
## [126,]  0.064154791  21.33334318
## [127,]  0.228197875  32.57501650
## [128,] -1.138401376 -34.00682347
## [129,] -0.664532911 -32.73712097
## [130,]  1.104095711  56.91503299
## [131,] -0.338001406   7.27614956
## [132,] -0.741763670   2.68886339
## [133,] -0.143260641  -1.52187582
## [134,] -0.079980476 -20.88425400
## [135,] -0.776703912 -36.05017652
## [136,] -0.964857772 -21.79072559
## [137,]  0.010378872  -2.94276762
## [138,] -1.116367057 -22.94802856
## [139,] -0.615348787  12.23507607
## [140,] -0.418787640   6.53619136
## [141,]  0.067452160   9.91307667
## [142,]  0.334653077  23.69022545
```

```
## [143,] -1.120143700 -56.22760072
## [144,] -1.017264472  37.63281310
## [145,]  0.271673694   8.65132677
## [146,] -2.516822185 -90.80230224
## [147,] -1.146490994 -30.38728136
## [148,]  0.153343013 -12.68209867
## [149,] -0.558002192 -22.45368100
## [150,] -0.518495128 -11.45134388
## [151,] -0.471034842 -23.27925248
## [152,] -0.977234961  -2.73839081
## [153,] -2.450607656 -76.95673732
## [154,] -1.293732735 -29.34840789
## [155,] -1.174745814   1.77792005
## [156,] -0.634169433 -20.21124642
## [157,] -0.575762493  -8.86206239
## [158,] -0.727749892  -4.07626700
## [159,] -1.180839663   3.21523734
## [160,] -1.928140321 -56.62266636
## [161,] -1.459013217 -35.47924558
## [162,] -0.627007361 -21.10077859
## [163,] -0.112773879  -7.65885030
## [164,] -0.288933772  -8.41313153
## [165,] -0.959701169 -19.09535235
## [166,] -1.522130303   0.30917470
## [167,] -0.546348112 -41.43321677
## [168,] -1.580323413 -34.82739709
## [169,] -0.702192605   6.66457342
## [170,] -0.309020958 -14.88797895
## [171,] -0.416933398 -13.51380885
## [172,] -1.009342213 -22.94245818
## [173,] -0.918686406 -16.85299437
## [174,]  0.599724136  28.91119909
## [175,]  0.468688536 -34.90262581
## [176,]  0.493451938  28.09336442
## [177,] -1.212517926 -30.36904747
## [178,]  2.708346388  63.32458527
## [179,] -1.938030376 -45.61210271
## [180,] -0.437990489  -3.77638491
## [181,] -0.344656689  24.47548372
## [182,]  0.597492398  44.79598329
## [183,] -0.414083945  -3.21154222
## [184,] -1.450664933 -63.36583296
## [185,] -0.683708655 -32.35485531
## [186,]  3.031534049  75.45146869
## [187,] -3.158070842 -84.67327458
## [188,]  2.281314526  25.36599965
## [189,] -1.361415176 -43.95215930
## [190,] -0.937258237  -4.91349218
## [191,] -0.604259971   4.33565349
## [192,]  1.536505500  58.43889176
## [193,]  2.290633804  57.48153012
## [194,]  1.406896784   8.92383344
## [195,] -1.164974326  12.67579253
## [196,] -0.650541711 -15.28537348
```

```
## [197,]  1.078748853 -21.80175757
## [198,] -1.400455222 -58.57103829
## [199,]  0.174891737   1.49487821
## [200,]  1.765565301  17.30713665
## [201,]  2.181507158  46.60211545
## [202,] -0.243725443  12.84255476
## [203,]  0.574097545  -0.35276640
## [204,]  0.280920461  -4.45777254
## [205,] -2.202740181 -53.08911366
## [206,] -0.868666639 -23.61630768
## [207,] -1.164434441 -29.22941694
## [208,] -1.683886467 -36.19929479
## [209,] -1.861002487 -39.17943269
## [210,]  3.379007162  -6.60318286
## [211,]  0.834937819   0.07505019
## [212,]  0.609030607   5.72862639
## [213,] -0.433648167 -12.66658923
## [214,]  0.468524961 -11.67366456
## [215,] -2.323134780 -30.06163463
## [216,] -2.573313252 -32.33611859
## [217,] -1.720956891 -27.35739032
## [218,] -0.864303647 -16.13138226
## [219,] -0.312552296  -7.32941580
## [220,] -0.423466921  -8.86451301
## [221,] -1.066923305 -20.21443171
## [222,] -1.735123464 -32.99697021
## [223,]  3.879129760  50.42742226
## [224,] -0.023904251 -52.75857312
## [225,]  3.392757919  32.94549358
## [226,] -0.914641558 -17.16347144
## [227,]  0.895418035   9.61523532
## [228,]  3.328006486  23.37455486
## [229,]  2.538828851  30.13878110
## [230,] -2.927789204 -41.82754771
## [231,]  0.726548729   0.61279660
## [232,]  0.089179513  19.31967642
## [233,]  0.463865416  19.85107062
## [234,]  2.275681031 -38.00749187
## [235,] -1.522822109  -4.79354914
## [236,]  4.026429108  42.23734397
## [237,] -2.787006418 -51.21640171
## [238,]  3.229566596  39.49056204
## [239,]  1.915899880  21.05656672
## [240,] -1.862899934  -6.15058994
## [241,] -0.963898099   2.37556152
## [242,]  2.357151862  59.58735696
## [243,] -1.344912286 -26.29330982
## [244,] -1.092050954   8.77766341
## [245,]  1.697330521  18.53714444
## [246,] -0.006539888  -8.88048665
## [247,] -0.432384749  32.92710816
## [248,] -2.291045370 -71.32708176
## [249,]  0.685062447  44.46551456
## [250,]  0.573661673  25.66755774
```

```
## [251,]  4.754700642  76.15369800
## [252,] -1.698776722  41.97504219
## [253,]  3.494983808  -2.50408531
## [254,] -1.247101629  35.91103776
## [255,] -1.726399045   2.65636988
## [256,]  3.039434123  32.23052145
## [257,]  5.834981976  -5.73138221
## [258,] -0.043603460 -47.85751096
## [259,] -2.209672866  -2.19309115
## [260,]  5.318165433  76.65033919
## [261,] -0.697888500  -8.60176487
## [262,] -0.428101978 -41.21920298
## [263,]  1.662949920  72.08290656
## [264,]  0.713107869 -31.66453356
## [265,]  0.878959508  -6.88052650
## [266,]  2.918111967 -18.35928122
## [267,]  0.341601708  11.19880077
## [268,] -1.727546744 -27.35260860
## [269,] -1.575000504 -26.08560696
## [270,]  1.034080471  24.85052660
## [271,] -1.153462759 -13.54582983
## [272,] -1.769537683 -46.09095369
## [273,] -1.418020369 -22.60899979
## [274,]  0.584353331  30.39998838
## [275,] -0.586744097  -7.89733655
## [276,]  3.056443938  39.87517194
## [277,]  4.717991347  91.49332595
## [278,]  1.802847626  32.70484075
## [279,] -0.655472482  -2.35816256
## [280,]  2.264369718  16.18031089
## [281,] -0.348521416   4.64908646
## [282,] -0.647069344  -9.68092899
## [283,]  1.228053802  -0.54572794
## [284,] -1.007145751 -30.65000223
## [285,]  0.352086214  18.33973021
## [286,] -1.690245987 -23.67250284
## [287,]  4.186859945  63.57206833
## [288,]  1.747543732   8.79825824
## [289,]  0.640841723   5.43256578
## [290,] -1.296722953 -32.06862030
## [291,] -2.120077153 -29.46713588
## [292,] -2.282885593 -49.81716506
## [293,] -2.134603274 -44.47544890
## [294,] -1.569342500 -35.11487683
## [295,] -0.791352121 -19.53460553
## [296,]  2.183919505  56.74072446
## [297,] -0.272521851   9.88930343
## [298,]  0.913569462  16.37044812
## [299,] -0.923368803  -1.86129733
## [300,]  0.452841578  12.49658013
## [301,] -2.801026518 -42.75038848
## [302,]  1.988465435  -0.76995392
## [303,]  1.304815705  22.79751486
## [304,] -0.499215265 -12.26868973
```

```
## [305,] -0.907023525 -20.76574779
## [306,]  0.248536843 -22.11118139
## [307,]  0.320693952   8.50759658
## [308,] -0.335704710 -15.37137218
## [309,]  1.091343204  22.68316826
## [310,]  0.727630290  37.01381071
## [311,] -0.603294900 -17.66844806
## [312,]  4.168220160  91.31519058
## [313,] -1.168233052   5.73395450
## [314,] -3.755644080 -82.52665078
## [315,]  1.372776950  14.01236553
## [316,]  1.640179065  43.58098337
## [317,] -1.855177215  44.14853705
## [318,]  0.259867354  -7.85993603
## [319,] -2.147509797 -11.13586501
## [320,] -1.988076898 -37.80676151
## [321,] -2.018157347 -35.33506909
## [322,] -0.671776969  -6.86004435
## [323,] -0.222570217   3.04948594
## [324,] -1.346293345 -54.80714198
## [325,] -0.760267690  -3.72001809
## [326,]  3.816890071 100.75614483
## [327,] -0.903771398 -18.23614511
## [328,]  1.025772185  -4.38361857
## [329,]  0.689944689 100.74097026
## [330,] -1.239541361  42.86134538
## [331,]  0.139553896  54.53000977
## [332,] -1.959363802 -59.46704443
## [333,] -0.770608293  40.16715144
## [334,] -1.235172704 -25.07341483
## [335,] -0.505614634 -22.31987182
## [336,]  4.003845311  31.17947671
## [337,]  1.598620283  84.92690795
## [338,]  0.804069191  -3.50995422
## [339,] -0.689503092 -15.15397071
## [340,] -0.405988212 -28.27625027
## [341,] -1.950148306 -16.01936455
## [342,]  0.896411006  48.23940648
## [343,] -1.919451675 -23.06744741
## [344,]  0.981817644  17.31646125
## [345,] -0.718659654  20.12585510
## [346,] -1.050117802 -56.01231631
## [347,]  2.763466867  74.61416587
## [348,] -0.184121929  33.32648382
## [349,] -2.671368214 -23.78483532
## [350,] -0.610286959  12.74936871
## [351,]  1.838809513 -28.13763908
## [352,]  0.092445770 -52.02667567
## [353,]  1.007032861  -1.51518664
## [354,] -2.033054250 -48.18964090
## [355,] -1.590798481 -30.52990683
## [356,]  0.716682936  44.22405537
## [357,]  0.306143546   5.97655860
## [358,]  1.370522459  15.09782560
```

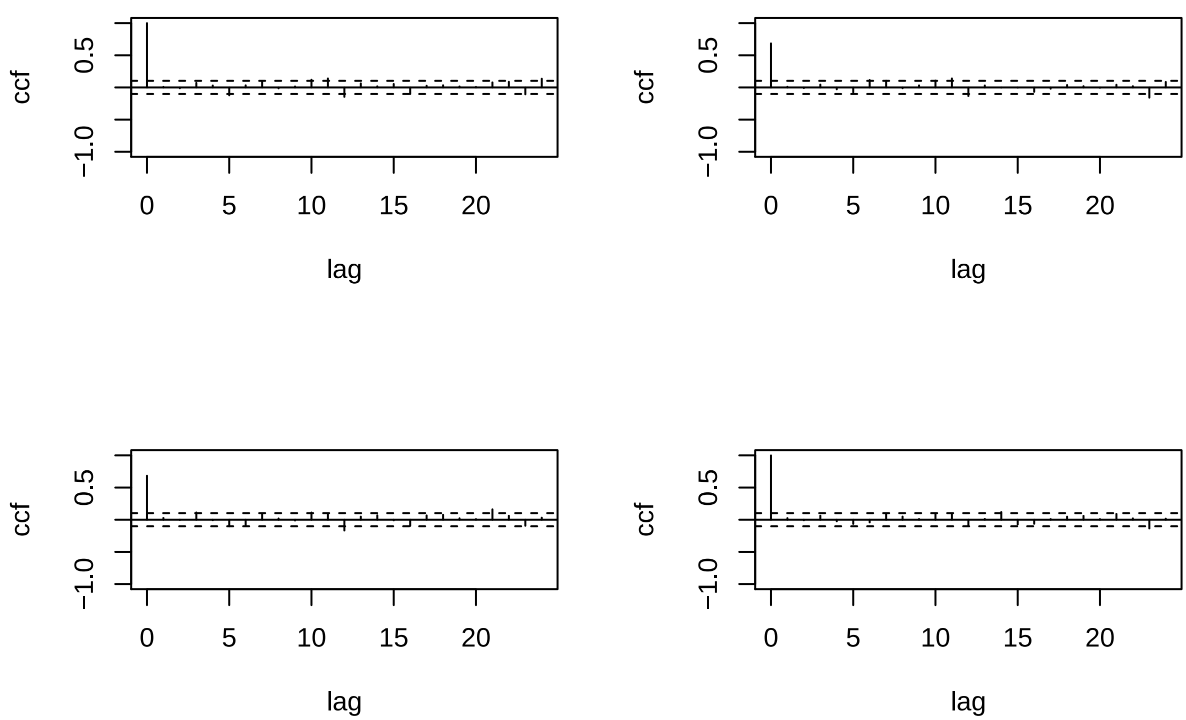
```
## [359,] -1.442434169  -5.02805508
## [360,] -0.618757450  -0.91510951
## [361,]  0.184181023  19.60763594
## [362,]  0.873670818  16.64265336
## [363,]  0.022236071  -9.23353478
## [364,] -0.359711215   9.94681613
## [365,] -1.020584320   7.67422906
## [366,] -0.394900880   5.03530931
## [367,] -0.317355508   1.11741932
## [368,]  2.518730432  12.08954638
## [369,]  1.468821919   9.33654761
## [370,] -1.791526154 -17.24662368
## [371,]  0.140122074   7.67795279
## [372,]  0.664597816  15.74308596
## [373,] -1.140543832 -16.75112947
## [374,]  0.336128361 -22.79071474
## [375,]  1.995717363  31.40495543
## [376,] -0.135326888 -31.63201778
## [377,]  1.043329019  11.76247440
## [378,]  1.173703936   7.21889303
## [379,] -0.155239714 -15.06481125
## [380,] -2.282163423 -60.38519920
## [381,] -1.394585473 -32.72368972
## [382,]  0.096301527  12.41012618
##
## $Sigma
##          [,1]      [,2]
## [1,]  2.27996  32.42389
## [2,] 32.42389 982.15475
##
## $aic
## [1] 7.16333
##
## $bic
## [1] 7.32794
##
## $Phi
##           [,1]        [,2]        [,3]        [,4]
## [1,] 0.7604869 -0.01254823 -0.06011082 0.009244218
## [2,] 5.8782529  0.53536978 -6.81130054 0.325555319
##
## $Theta
##           [,1]        [,2]        [,3]        [,4]
## [1,] 0.3464546 -0.01201390  0.03643156 0.001438387
## [2,] 6.9066320  0.06109274 -2.67660866 0.223546781
##
## $Ph0
## [1] 0 0
```

```
MTSdiag(varma.model2)
```

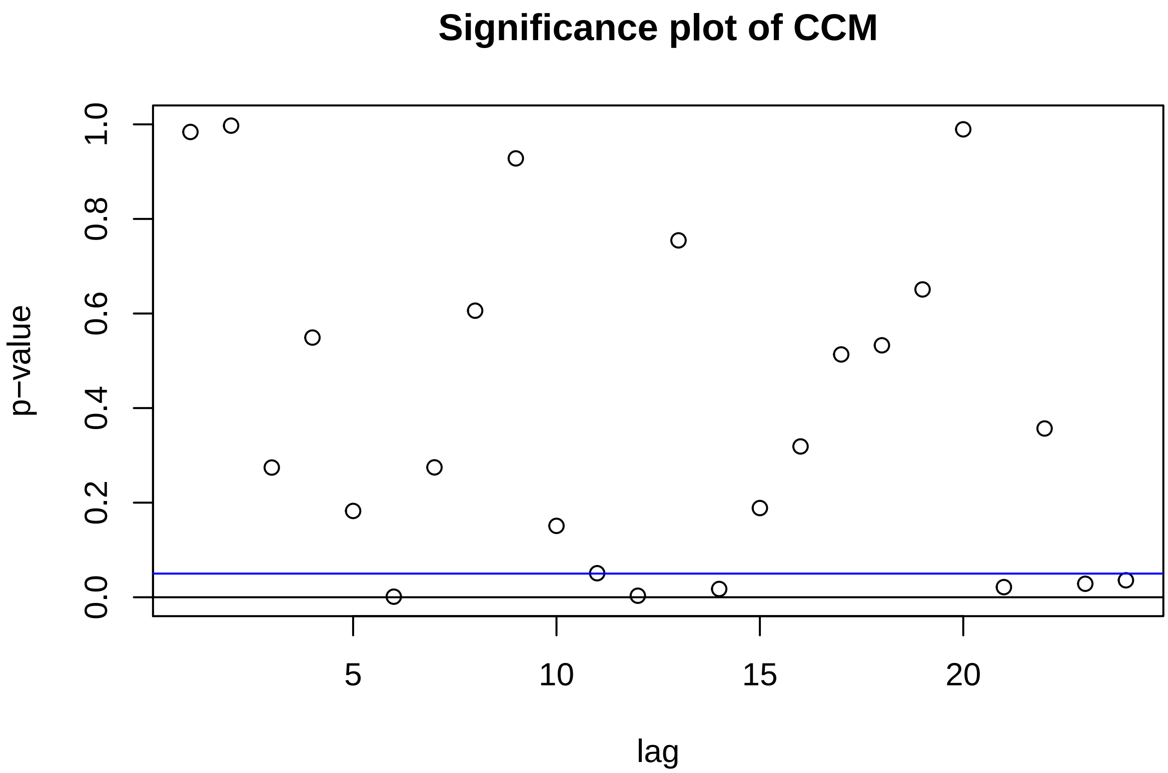
```
## [1] "Covariance matrix:"
##        CO   NO2
## CO   2.29  32.5
```

```
## NO2 32.51 984.7
## CCM at lag:  0
##       [,1]  [,2]
## [1,] 1.000 0.685
## [2,] 0.685 1.000
## Simplified matrix:
## CCM at lag:  1
## . .
## . .
## CCM at lag:  2
## . .
## . .
## CCM at lag:  3
## . .
## + .
## CCM at lag:  4
## . .
## . .
## CCM at lag:  5
## - .
## - .
## CCM at lag:  6
## . +
## . .
## CCM at lag:  7
## . +
## . .
## CCM at lag:  8
## . .
## . .
## CCM at lag:  9
## . .
## . .
## CCM at lag:  10
## + +
## + .
## CCM at lag:  11
## + +
## . .
## CCM at lag:  12
## - -
## - .
## CCM at lag:  13
## . .
## . .
## CCM at lag:  14
## . .
## . +
## CCM at lag:  15
## . .
## . .
## CCM at lag:  16
## - .
## . .
```

```
## CCM at lag:  17
## . .
## . .
## CCM at lag:  18
## . .
## . .
## CCM at lag:  19
## . .
## . .
## CCM at lag:  20
## . .
## . .
## CCM at lag:  21
## . .
## + .
## CCM at lag:  22
## . .
## . .
## CCM at lag:  23
## - -
## . -
## CCM at lag:  24
## + .
## . .
```

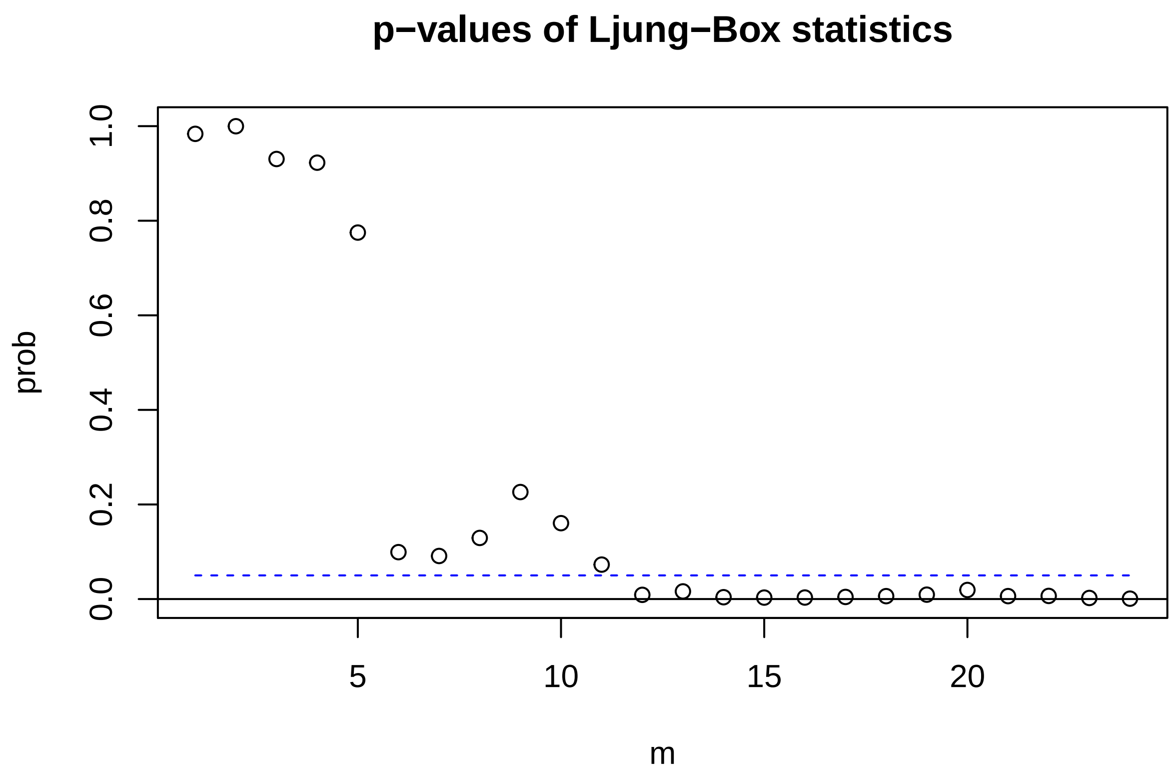



```
## Hit Enter for p-value plot of individual ccm:
```

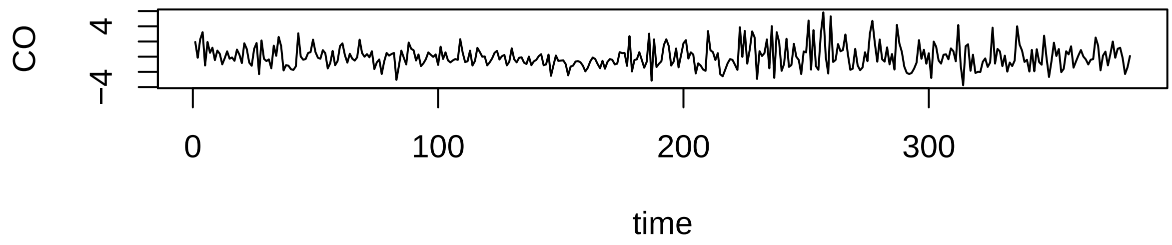


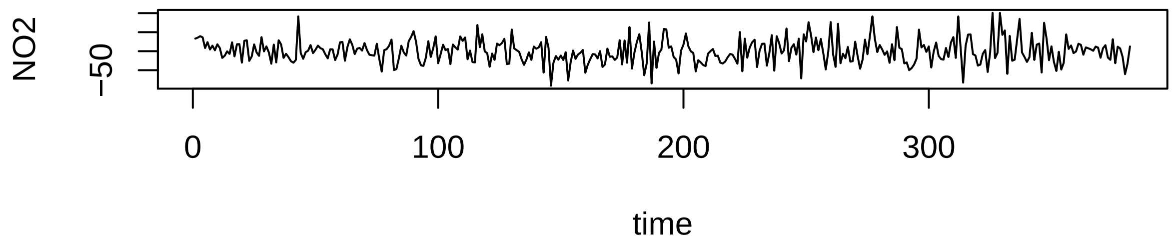

```
## Hit Enter to compute MQ-statistics:
##
## Ljung-Box Statistics:
##            m       Q(m)     df    p-value
##  [1,]   1.000     0.384   4.000     0.98
##  [2,]   2.000     0.538   8.000     1.00
##  [3,]   3.000     5.698  12.000     0.93
##  [4,]   4.000     8.765  16.000     0.92
##  [5,]   5.000    15.026  20.000     0.77
##  [6,]   6.000    33.237  24.000     0.10
##  [7,]   7.000    38.400  28.000     0.09
##  [8,]   8.000    41.136  32.000     0.13
##  [9,]   9.000    42.022  36.000     0.23
## [10,]  10.000    48.794  40.000     0.16
## [11,]  11.000    58.306  44.000     0.07
## [12,]  12.000    74.248  48.000     0.01
## [13,]  13.000    76.154  52.000     0.02
## [14,]  14.000    88.205  56.000     0.00
## [15,]  15.000    94.364  60.000     0.00
## [16,]  16.000    99.101  64.000     0.00
## [17,]  17.000   102.378  68.000     0.00
## [18,]  18.000   105.546  72.000     0.01
## [19,]  19.000   108.030  76.000     0.01
## [20,]  20.000   108.336  80.000     0.02
## [21,]  21.000   119.895  84.000     0.01
## [22,]  22.000   124.289  88.000     0.01
```

```
## [23,]  23.000   135.167  92.000     0.00
## [24,]  24.000   145.510  96.000     0.00
```




```
## Hit Enter to obtain residual plots:
```





According to the above diagnostics ______

** Also in the assignment they said " Please mask the output of tested models except for those whose diagnostics you discuss using ‘include=FALSE’ as an argument in the relevant chunk. " so need to figure out what that means ??

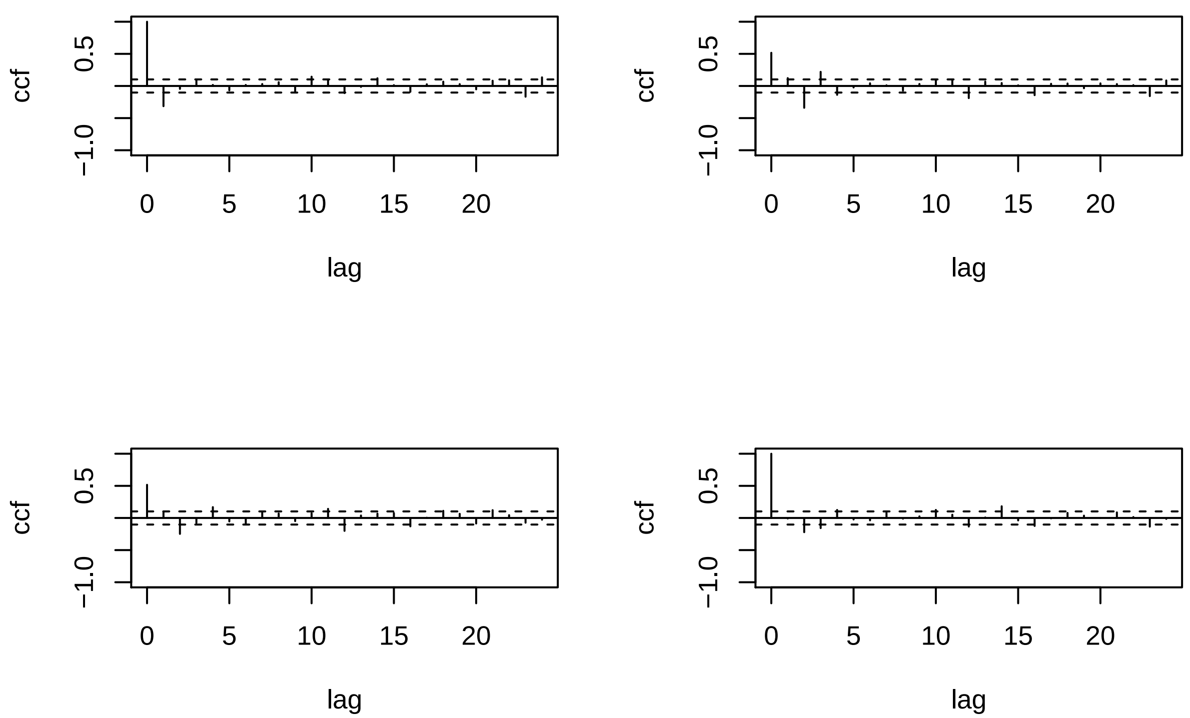
Part E: Diagnostics

```
MTSdiag(varma.model)
```

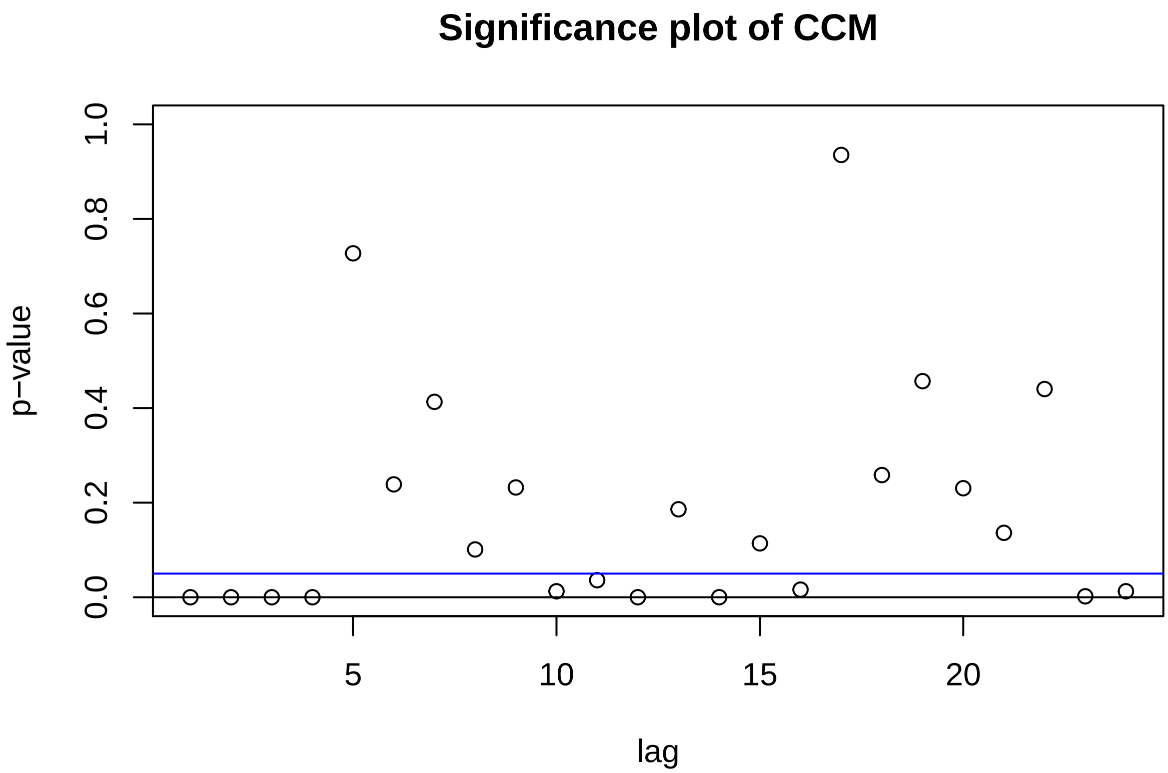
```
## [1] "Covariance matrix:"
##         CO    NO2
## CO   3.56   32.4
## NO2 32.35 1100.8
## CCM at lag:  0
##        [,1]  [,2]
## [1,] 1.000 0.517
## [2,] 0.517 1.000
## Simplified matrix:
## CCM at lag:  1
## - +
## . .
## CCM at lag:  2
## . -
## - -
```

```
## CCM at lag:  3
## . +
## . -
## CCM at lag:  4
## . -
## + +
## CCM at lag:  5
## . .
## . .
## CCM at lag:  6
## . .
## . .
## CCM at lag:  7
## . .
## . .
## CCM at lag:  8
## . .
## . .
## CCM at lag:  9
## . .
## . .
## CCM at lag:  10
## + .
## . +
## CCM at lag:  11
## . .
## + .
## CCM at lag:  12
## - -
## - -
## CCM at lag:  13
## . .
## . .
## CCM at lag:  14
## + .
## . +
## CCM at lag:  15
## . .
## . .
## CCM at lag:  16
## . -
## - -
## CCM at lag:  17
## . .
## . .
## CCM at lag:  18
## . .
## + .
## CCM at lag:  19
## . .
## . .
## CCM at lag:  20
## . .
## . .
```

```
## CCM at lag:  21
## . .
## + .
## CCM at lag:  22
## . .
## . .
## CCM at lag:  23
## - -
## . -
## CCM at lag:  24
## + .
## . .
```

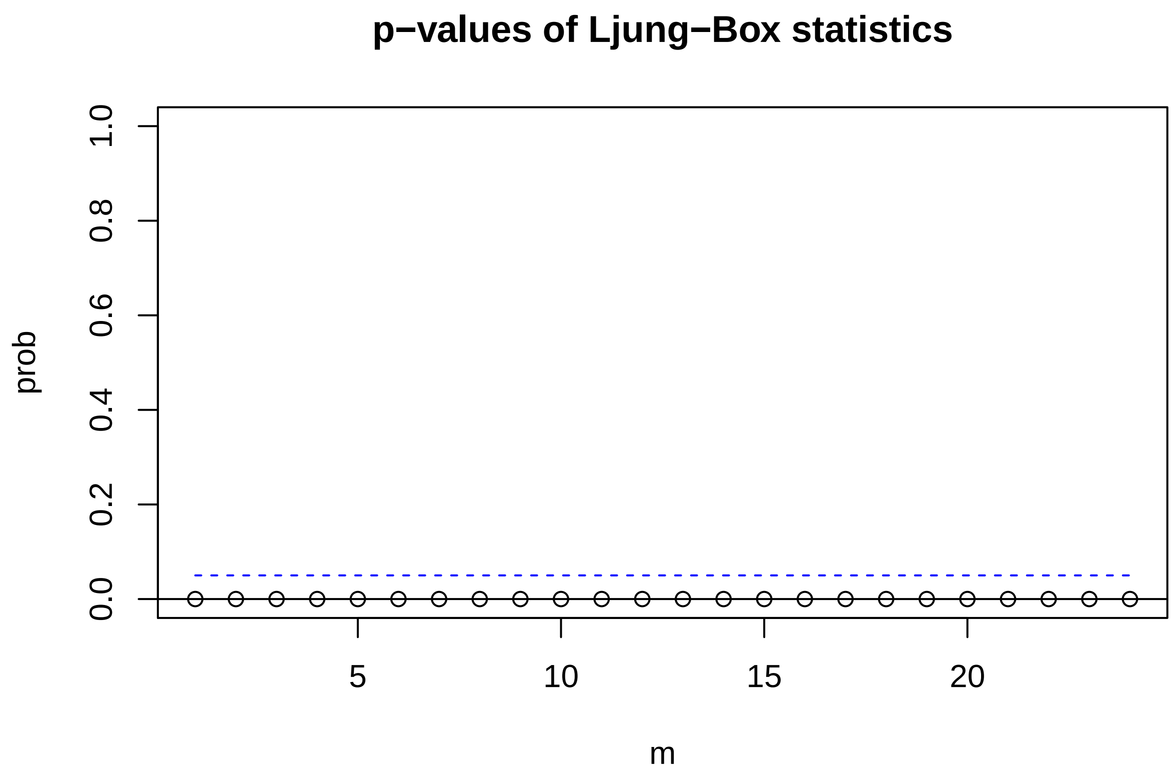



```
## Hit Enter for p-value plot of individual ccm:
```

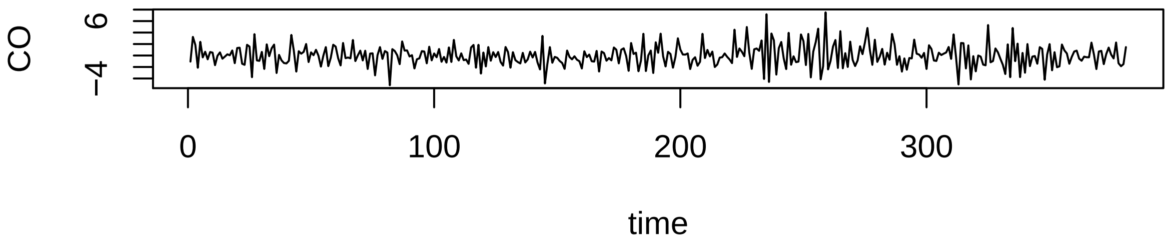


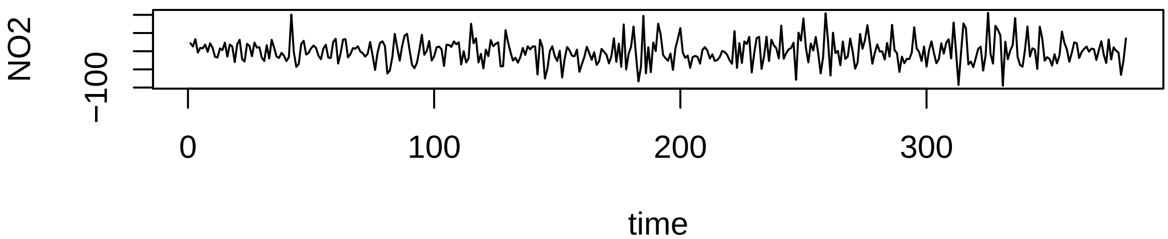

```
## Hit Enter to compute MQ-statistics:
##
## Ljung-Box Statistics:
##          m       Q(m)     df    p-value
##  [1,]     1       147       4         0
##  [2,]     2       230       8         0
##  [3,]     3       288      12         0
##  [4,]     4       322      16         0
##  [5,]     5       325      20         0
##  [6,]     6       330      24         0
##  [7,]     7       334      28         0
##  [8,]     8       342      32         0
##  [9,]     9       347      36         0
## [10,]    10       360      40         0
## [11,]    11       371      44         0
## [12,]    12       397      48         0
## [13,]    13       403      52         0
## [14,]    14       427      56         0
## [15,]    15       435      60         0
## [16,]    16       447      64         0
## [17,]    17       448      68         0
## [18,]    18       453      72         0
## [19,]    19       457      76         0
## [20,]    20       462      80         0
## [21,]    21       469      84         0
## [22,]    22       473      88         0
```

```
## [23,]     23        490       92         0
## [24,]     24        503       96         0
```




```
## Hit Enter to obtain residual plots:
```



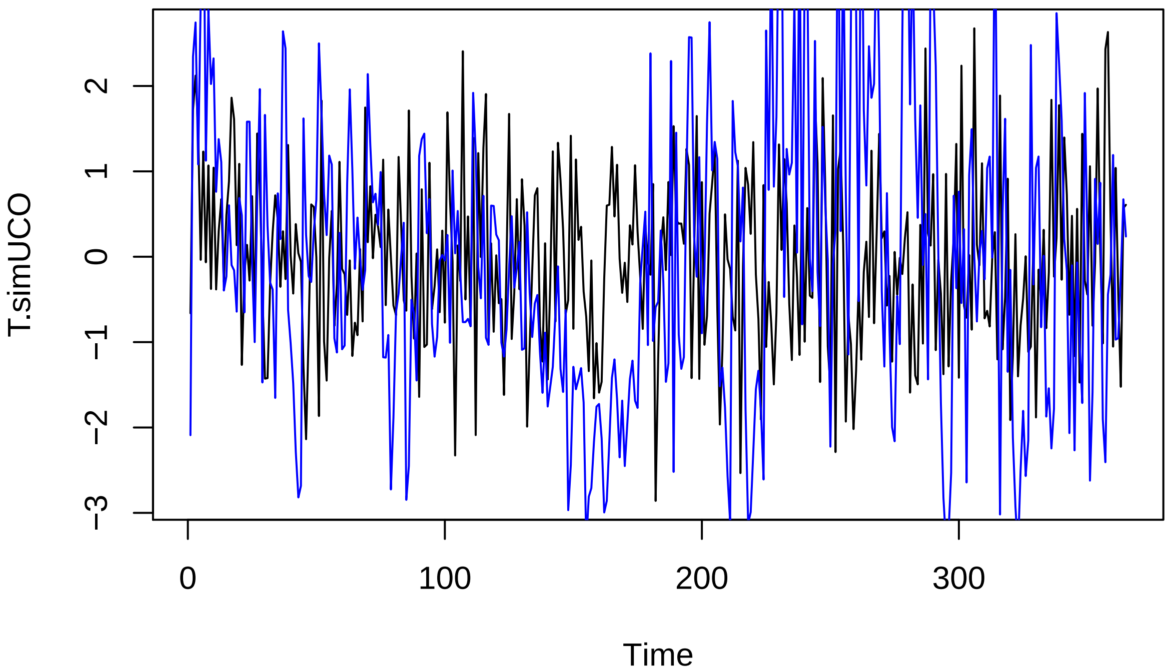


The significance plot of CCM shows that … The plot of p-values for Ljung-Box statistics shows that … The residual plot for CO shows that … The residual plot for NO2 shows that …

Part 3: Simulating from Univariate and Multivariate Time Series Models

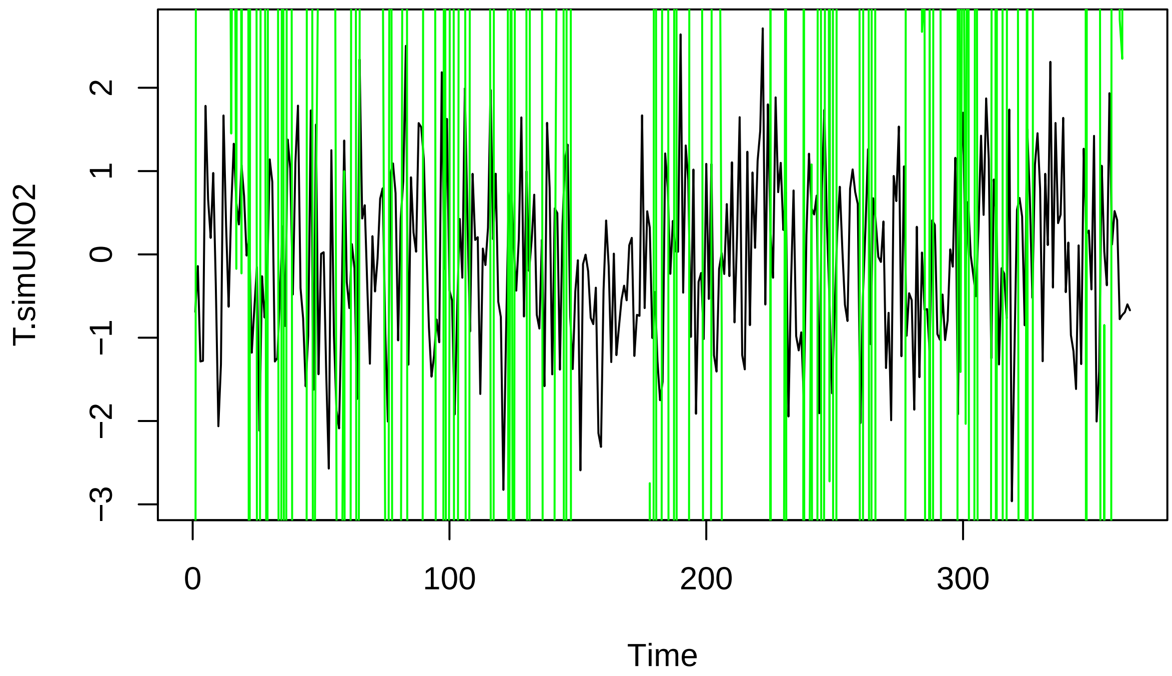
```
set.seed(14)
T.simUCO = arima.sim(CO.ar1$model,365)
T.simUNO2 = arima.sim(NO2.ma3$model,365)
```

```
{plot(T.simUCO)
lines(e.ts.CO[1:365],col="blue")}
```



Above is a plot of ? which shows _____

```
{plot(T.simUNO2)
lines(e.ts.NO2[1:365],col="green")}
```



Above is a plot of ? which shows _____

```r
T.sim = VARMAsim(365,phi=varma.model$Phi,theta=varma.model$Theta,sigma=varma.model$Sigma)
```

Part A: Ability to reproduce appearance

Compare correlation of simulated residuals to actual residuals

```r
cor(T.sim$series)
```

```
##           [,1]      [,2]
## [1,] 1.0000000 0.4726089
## [2,] 0.4726089 1.0000000
```

```r
cor(allResiduals)
```

```
##            CO       NO2
## CO  1.0000000 0.5891419
## NO2 0.5891419 1.0000000
```

Interpret these results ..

Plot observations and simulations

Part B: Ability to reproduce observed trends

Part C: Ability to reproduce seasonality

Part D: Ability to reproduce observed mean and variance

Part E: Ability to reproduce auto-correlation

Part F: Ability to reproduce observed cross-correlation