| ENGRI 1101 | Engineering Applications of OR | Fall 2020 | Prelab 9 |
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# IP, Branch-and-Bound, & the Knapsack Problem

## Name: ______________________________

Objectives:

- Introduce students to solving a small integer program by branch-and-bound.
- Give students an appreciation of the difficulty of solving integer programming problems exactly.

Key Ideas:

- linear programming relaxation
- branching
- branch-and-bound tree

**Reading Assignment:**

- Handout 10 from the course-pack

**Brief description:** In this lab we will explore solving integer programs by branch-and-bound, including an example of the knapsack problem. We will also see how adding a cutting place helps in reducing the computation time and effort of the algorithm.

**Prelab exercise:** The knapsack problem is often motivated by the following (not very realistic) story. A hiker discovers a cave filled with valuable metal ingots. Each ingot $i$ has a specified weight $w_i$, and a specified value $v_i$. The hiker has the possibility of carrying out of the cave a knapsack containing a subset of the ingots, provided that the total weight is no greater than a specified weight $W$. Suppose that we consider the integer programming formulation of this problem: that is, maximize

$$\sum_{i=1}^{n} v_i x_i,$$

subject to

$$\sum_{i=1}^{n} w_i x_i \leq W,$$

$$0 \leq x_i \leq 1, \text{ integer}, i = 1, \ldots, n,$$

where we are given the input data $W = 13$ and

| $i$ | $v_i$ | $w_i$ |
|---|---|---|
| 1 | 14 | 3 |
| 2 | 33 | 7 |
| 3 | 11 | 2 |
| 4 | 24 | 5 |
| 5 | 5 | 1 |
| 6 | 29 | 6 |
| 7 | 20 | 4 |

For this data, it turns out that if you solve the linear program where you ignore the constraint that each $x_i$ must be integer, you obtain the following optimal solution:

$$x_1 = x_2 = x_4 = 0 \text{ and } x_3 = x_5 = x_6 = x_7 = 1.$$

What can you conclude about this input to the knapsack problem, i.e., the corresponding integer program?

Fortunately, your knapsack can hold more; $W = 16$. Now the optimal solution to the corresponding linear program is:

$$x_1 = x_2 = 0 \text{ and } x_3 = x_5 = x_6 = x_7 = 1 \text{ and } x_4 = .6.$$

What can you conclude about the optimal value for this input to the knapsack problem?

Suppose that I tell you that, if you are allowed to select from among just the first 6 items, then the optimal solution is take items 2,3,5 and 6, which has total value 78. Suppose that I also tell you that if I change $W$ to be 12, and then solve the knapsack problem again restricted to taking items from among the first 6, the optimal solution is to take pieces 1,3,5 and 6, which has total value 59. Use this information to deduce the value of the original data for the knapsack problem (that is, selecting from among all 7 items).