Geometry of Simplex Lab

Objectives

- Understand the geometry of a linear program's feasible region.
- Use isoprofit lines and planes to solve 2D and 3D LPs graphically.
- Identify the most limiting constraint in an iteration of simplex both algebraically and geometrically.
- Identify the geometric features corresponding to dictionaries.
- Describe the geometrical decision made at each iteration of simplex.

Review

Recall, linear programs (LPs) have three main components: decision variables, constraints, and an objective function. The goal of linear programming is to find a **feasible solution** (a solution satisfying every constraint) with the highest objective value. The set of feasible solutions form a **feasible region**. In lecture, we learned about isoprofit lines. For every objective value, we can define an isoprofit line. Isoprofit lines have the property that two solutions on the same line have the same objective value and all isoprofit lines are parallel.

In the first part of the lab, we will use a Python package called GILP to solve linear programs graphically. We introduce the package now.

GILP

If you are running this file in a Google Colab Notebook, uncomment the following line and run it. Otherwise, you can ignore it.

In [1]:
```
#!pip install gilp
```

This lab uses default LPs built in to GILP. We import them below.

In [2]:
```
from gilp import examples as ex
```

We access the LP examples using `ex.NAME` where `NAME` is the name of the example LP. For example, consider:

$$\begin{array}{rl} \max & 5x_1 + 3x_2 \\ \text{s.t.} & 2x_1 + 1x_2 \leq 20 \\ & 1x_1 + 1x_2 \leq 16 \\ & 1x_1 + 0x_2 \leq 7 \\ & x_1, x_2 \geq 0 \end{array}$$

This example LP is called `ALL_INTEGER_2D_LP`. We assign this LP to the variable `lp` below.

In [3]:
```
lp = ex.ALL_INTEGER_2D_LP
```

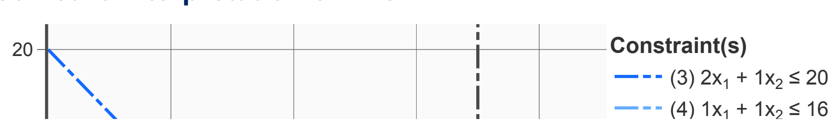
We can visualize this LP using a function called `lp_visual()`. First, we must import it.

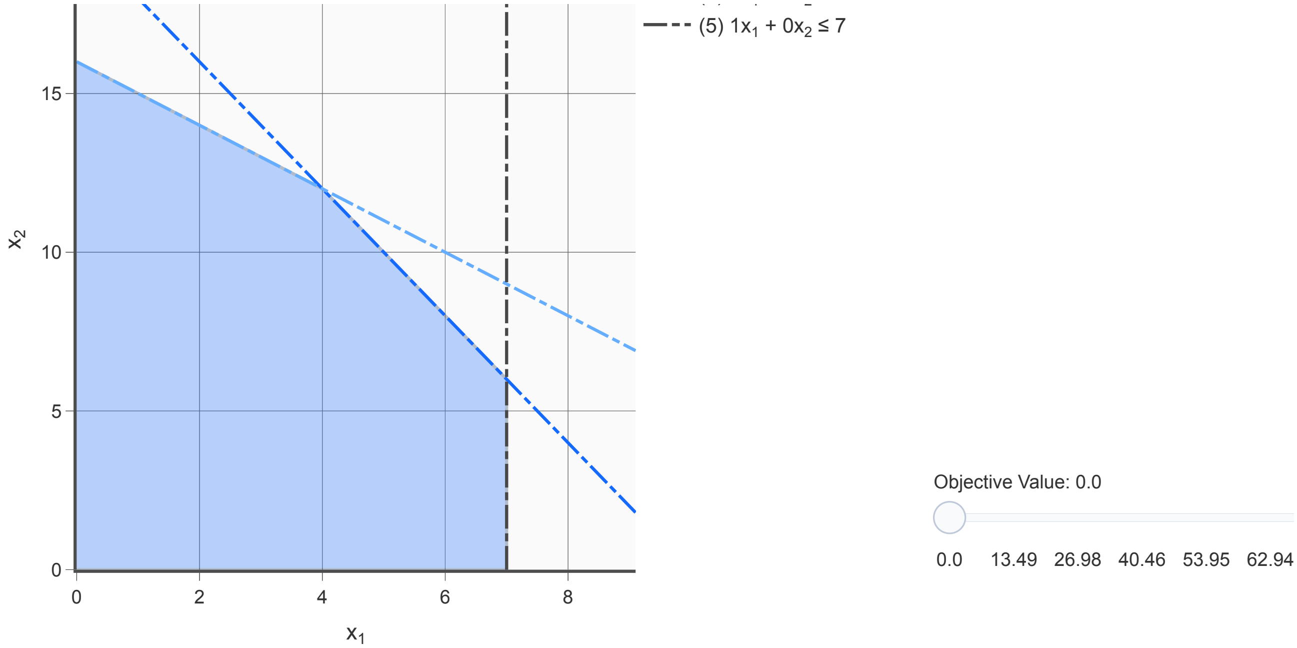
In [4]:
```
from gilp.visualize import lp_visual
```

The function `lp_visual()` takes an LP and returns a visualization. We then use the `.show()` function to display the visualiazation.

In [5]:
```
lp_visual(lp).show()
```

Geometric Interpretation of LPs





On the left, you can see a coordinate plane where the x-axis corresponds to the value of x_1 and the y-axis corresponds to the value of x_2. The region shaded blue is the feasible region. Along the perimeter of the feasible region, you can see points where two edges come to a "corner". You can hover over these **corner points** to see information about them. Only some of the information in the hover box will be relevant for Part I. The first two values of **BFS** represent the values of x_1 and x_2 respectively and **Obj** is the objective value. For example, the upper left corner point has solution $x_1 = 0$ and $x_2 = 16$ with objective value 48. The dashed lines represent the constraints. You can click on the constraints in the legend to mute and un-mute them. Note this does not alter the LP; it just changes visibility. Lastly, the objective slider allows you to see the isoprofit line for a range of objective values.

Part I: Solving Linear Programs Graphically

Let's use GILP to solve the following LP graphically:

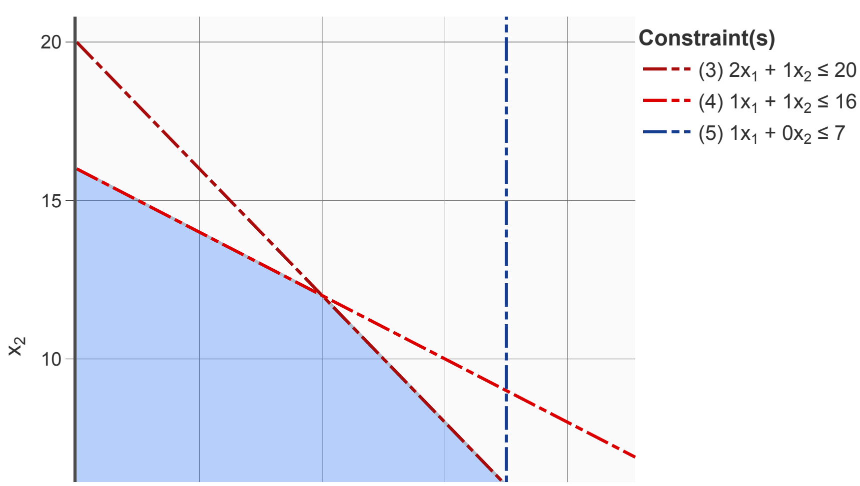
$$\begin{aligned} \max \quad & 5x_1 + 3x_2 \\ \text{s.t.} \quad & 2x_1 + 1x_2 \leq 20 \\ & 1x_1 + 1x_2 \leq 16 \\ & 1x_1 + 0x_2 \leq 7 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Recall, this LP is called `ALL_INTEGER_2D_LP`.

In [6]:

```
lp = ex.ALL_INTEGER_2D_LP # get LP example
lp_visual(lp).show() # visualize it
```

Geometric Interpretation of LPs



Q1: How can you use isoprofit lines to solve LPs graphically?

A: we can move isoprofit line to see the maximum value it can get when it's inside the feasible region

Q2: Use the objective slider to solve this LP graphically. Give an optimal solution and objective value. Argue why it is optimal. (Hint: The objective slider shows the isoprofit line (in red) for some objective value.)

A: maximum value is 56, x1=4, x2=12. when we move isoprofit line a little bit upward, it will not in feasible region.

Q3: Plug your solution from **Q2** back into the LP and verify that each constraint is satisfied (don't forget non-negativity constraints!) and the objective value is as expected. Show your work.

A:
$x1 >= 0, x2 >= 0$

$2x1 + 1x2 = 20 <= 20$

$x1 + x2 = 16 <= 16$

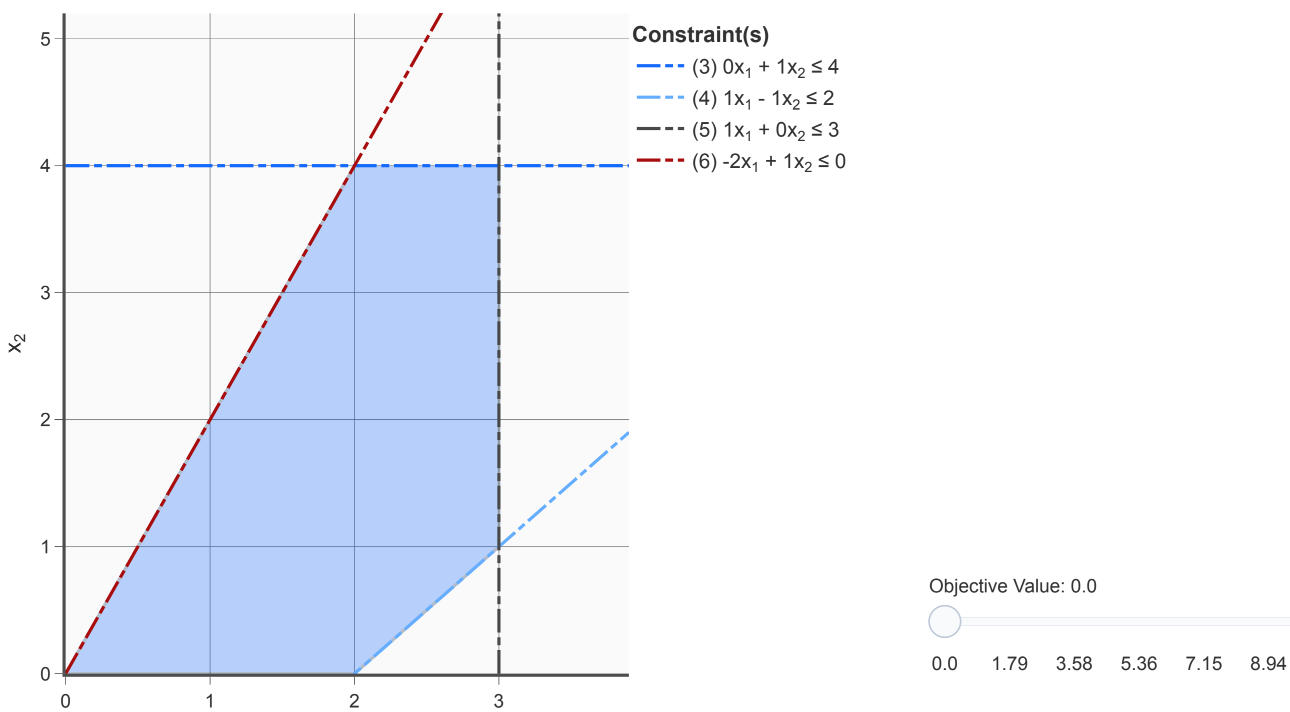
$1x1 + 0x2 = 4 <= 7$

Let's try another! This LP is called `DEGENERATE_FIN_2D_LP` .

$$\begin{aligned} \max \quad & 1x_1 + 2x_2 \\ \text{s.t.} \quad & 0x_1 + 1x_2 \leq 4 \\ & 1x_1 - 1x_2 \leq 2 \\ & 1x_1 + 0x_2 \leq 3 \\ & -2x_1 + 1x_2 \leq 0 \\ & x_1, x_2 \geq 0 \end{aligned}$$

In [7]:
```
lp = ex.DEGENERATE_FIN_2D_LP # get LP example
lp_visual(lp).show() # visualize it
```

Geometric Interpretation of LPs



x_1

Q4: Use the objective slider to solve the `DEGENERATE_FIN_2D_LP` LP graphically. Give an optimal solution and objective value. (Hint: The objective slider shows the isoprofit line (in red) for some objective value.)

A: optimal value is 11, x1=3, x2=4

You should now be comfortable solving linear programs with two decision variables graphically. In this case, each constraint is a line representing an inequality. These inequalites define a shaded region in the coordinate plane which is our feasible region. Lastly, the isoprofits are parallel lines. To find an optimal solution, we just increase the objective value while the corresponding isoprofit line still intersects the 2D feasible region.

Now, we will try to wrap our head around an LP with three decision variables! Similar to before, we can plot solutions to a 3D LP on a plot with 3 axes. Here, the x-axis corresponds to the value of x_1 and the y-axis corresponds to the value of x_2 as before. Furthermore, the z-axis corresponds to the value of x_3. Now, constraints are *planes* representing an inequality. These inequality planes define a 3D shaded region which is our feasible region. The isoprofits are isoprofit *planes* which are parallel. To find an optimal solution, we just increase the objective value while the corresponding isoprofit plane still intersects the 3D feasible region. Let us look at an example.

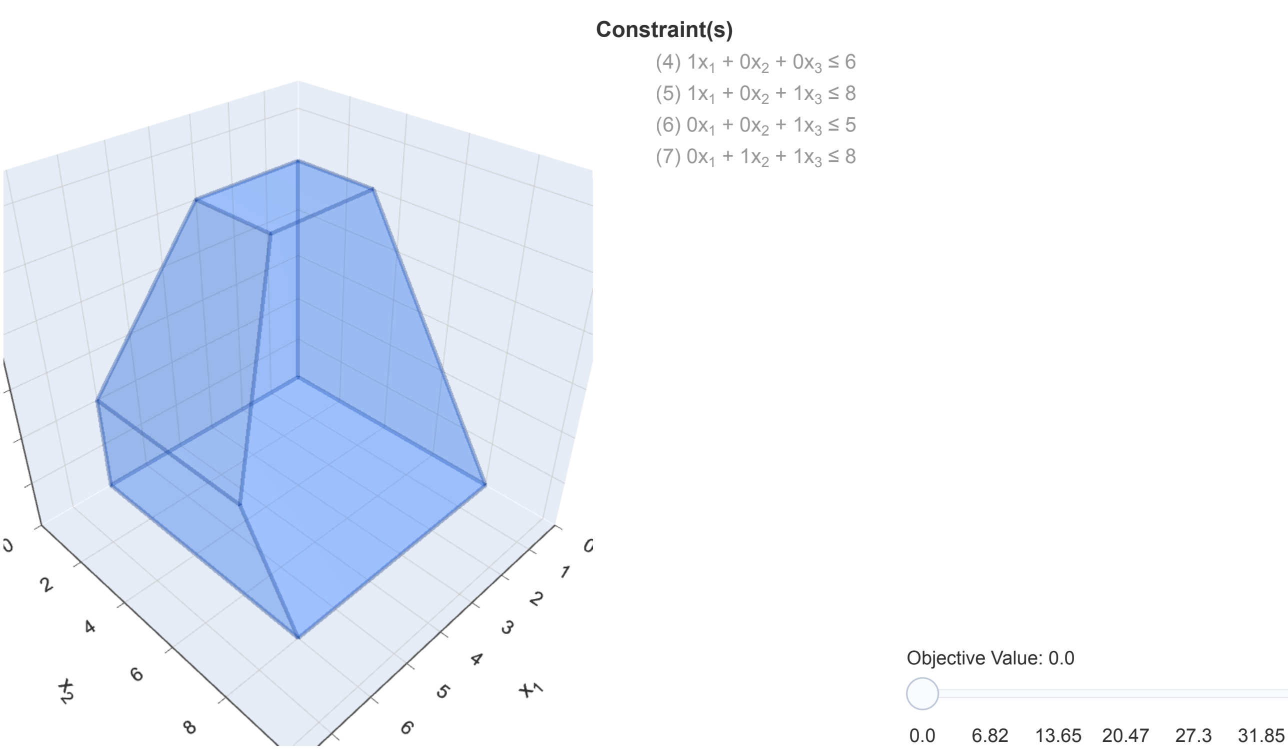
This LP is called `ALL_INTEGER_3D_LP` :

$$
\begin{aligned}
\max \quad & 1x_1 + 2x_2 + 4x_3 \\
\text{s.t.} \quad & 1x_1 + 0x_2 + 0x_3 \leq 6 \\
& 1x_1 + 0x_2 + 1x_3 \leq 8 \\
& 0x_1 + 0x_2 + 1x_3 \leq 5 \\
& 0x_1 + 1x_2 + 1x_3 \leq 8 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

In [8]:

```
lp = ex.ALL_INTEGER_3D_LP # get LP example
lp_visual(lp).show() # visualize it
```

Geometric Interpretation of LPs



The 3D feasible region is shown on the left. Hold and drag the mouse to examine it from different angles. Next, click on a constraint to un-mute it. Each constraint is a gray plane in 3D space. Un-mute the constraints one by one to see how they define the 3D feasible region. Move the objective slider to see the isoprofit planes. The isoprofit plane is light gray and the intersection with the feasible region is shown in red. Like the 2D visualization, you can hover over corner points to see information about that point.

Q5: Use the objective slider to solve this LP graphically. Give an optimal solution and objective value. (Hint: The objective slider shows the isoprofit plane for some objective value in light gray and the intersection with the feasible region in red.)

A: objective value =29,x1=3,x2=3,x3=5

When it comes to LPs with 4 or more decision variables, our graphical approaches fail. We need to find a different way to solve linear programs of this size.

Part II: The Simplex Algorithm for Solving LPs

Dictionary Form LP

First, let's answer some guiding questions that will help to motivate the simplex algorithm.

Q6: Does there exist a unique way to write any given inequality constraint? If so, explain why each constraint can only be written one way. Otherwise, give 2 ways of writing the same inequality constraint.

A: yes. you can add constant in both side, or do some operations.

Q7: Consider the following two constraints: $2x_1 + 1x_2 \leq 20$ and $2x_1 + 1x_2 + x_3 = 20$ where all x are nonnegative. Are these the same constraint? Why? (This question is tricky!)

A: they are the same constraints

Q8: Based on your answers to **Q6** and **Q7**, do you think there exists a unique way to write any given LP?

A: yes

You should have found that there are many ways to write some LP. This begs a new question: are some ways of writing an LP harder or easier to solve than others? Consider the following LP:

$$\begin{aligned} \max \quad & 56 - 2x_3 - 1x_4 \\ \text{s.t.} \quad & x_1 = 4 - 1x_3 + 1x_4 \\ & x_2 = 12 + 1x_3 - 2x_4 \\ & x_5 = 3 + 1x_3 - 1x_4 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

Q9: Just by looking at this LP, can you give an optimal solution and its objective value. If so, explain what property of the LP allows you to do this. (Hint: Look at the objective function)

A: 56. the coefficient of parameter in objective function is negative. and all x value>=0, all the constant number in the constraints is positive

The LP above is the same as `ALL_INTEGER_2D_LP` just rewritten in a different way! This rewitten form (which we found is easier to solve) was found using the simplex algorithm. At its core, the simplex algorithm strategically rewrites an LP until it is in a form that is "easy" to solve.

The simplex algorithm relies on an LP being in **dictionary form**. Recall the following properties of an LP in dictionary form:

- All constraints are equality constraints
- All variables are constrained to be nonnegative
- Each variable only appears on the left-hand side (LHS) or the right-hand side (RHS) of the constraints (not both)
- Each constraint has a unique variable on the LHS
- The objective function is in terms of the variables that appear on the RHS of the constraints only.
- All constants on the RHS of the constraints are nonnegative

Q10: Rewrite the example LP `ALL_INTEGER_2D_LP` in dictionary form. Show your steps!

$$\begin{aligned} \max \quad & 5x_1 + 3x_2 \\ \text{s.t.} \quad & 2x_1 + 1x_2 \leq 20 \\ & 1x_1 + 1x_2 \leq 16 \\ & 1x_1 + 0x_2 \leq 7 \\ & x_1, x_2 \geq 0 \end{aligned}$$

A:

$$x_3 = 20 - 2x_1 - 1x_2$$
$$x_4 = 16 - 1x_1 - 1x_2$$
$$x_5 = 7 - 1x_1 - 0x_2$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Most Limiting Constraint

Once our LP is in dictionary form, we can run the simplex algorithm! In every iteration of the simplex algorithm, we will take an LP in dictionary form and strategically rewrite it in a new dictionary form. Note: it is important to realize that rewriting the LP **does not** change the LP's feasible region. Let us examine an iteration of simplex on a new LP.

$$\begin{array}{rl} \max & 5x_1 + 3x_2 \\ \text{s.t.} & 1x_1 + 0x_2 \leq 4 \\ & 0x_1 + 1x_2 \leq 6 \\ & 2x_1 + 1x_2 \leq 9 \\ & 3x_1 + 2x_2 \leq 15 \\ & x_1, x_2 \geq 0 \end{array}$$

Q11: Is this LP in dictionary form? If not, rewrite this LP in dictionary form.

A:

$$x_3 = 4 - 1x_1 - 0x_2$$
$$x_4 = 6 - 0x_1 - 1x_2$$
$$x_5 = 9 - 2x_1 - 1x_2$$
$$x_6 = 15 - 3x_1 - 2x_2$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Q12: Recall from **Q9** how you found a feasible solution (which we argued to be optimal) just by looking at the LP. Using this same stratagy, look at the LP above and give a feasible solution and its objective value for this LP. Describe how you found this feasible solution. Is it optimal? Why?

A: feasible solution x1=0,x2=0, objective function is 0. this is not optimal sulution because each of x1 and x2 value can increase.

From **Q12** we see that every dictionary form LP has a corresponding feasible solution. Furthermore, there are positive coefficents in the objective function. Hence, we can increase the objective value by increaseing the corresponding variable. In our example, both x_1 and x_2 have positive coefficents in the objective function. Let us choose to increase x_1.

Q13: What do we have to be careful about when increasing x_1?

A: there are constraints of how much x_1 can increase

Q14: After choosing a variable to increase, we must determine the most limiting constraint. Let us look at the first constraint $x_3 = 4 - 1x_1 - 0x_2$. How much can x_1 increase? (Hint: what does a dictionary form LP require about the constant on the RHS of constraints?)

A: 4

Q15: Like in **Q14**, determine how much each constraint limits the increase in x_1 and identify the most limiting constraint.

A: first constraint is the most limiting one, value is 4

If we increase x_1 to 4, note that x_3 will become zero. Earlier, we identified that each dictionary form has a corresponding feasible solution acheived by setting variables on the RHS (and in the objective function) to zero. Hence, since x_3 will become zero, we want to rewrite our LP such that x_3 appears on the RHS. Furthermore, since x_1 is no longer zero, it should now appear on the LHS.

Q16: Rewrite the most limiting constraint $x_3 = 4 - 1x_1 - 0x_2$ such that x_1 appears on the left and x_3 appears on the right.

A: $x_1 = 4 - 1x_3 - 0x_2$

Q17: Using substitution, rewrite the LP such that x_3 appears on the RHS and x_1 appears on the LHS. (Hint: Don't forget the rule about which variables can appear in the objective function)

A:

$$
\begin{aligned}
\max \quad & 20 - 5x_3 + 3x_2 \\
& x_1 = 4 - 1x_3 - 0x_2 \\
& x_4 = 6 - 0x_3 - 1x_2 \\
& x_5 = 1 + 2x_3 - 1x_2 \\
& x_6 = 3 + 3x_3 - 2x_2 \\
& x_1, x_2, x_3, x_4, x_5, x_6 \geq 0
\end{aligned}
$$

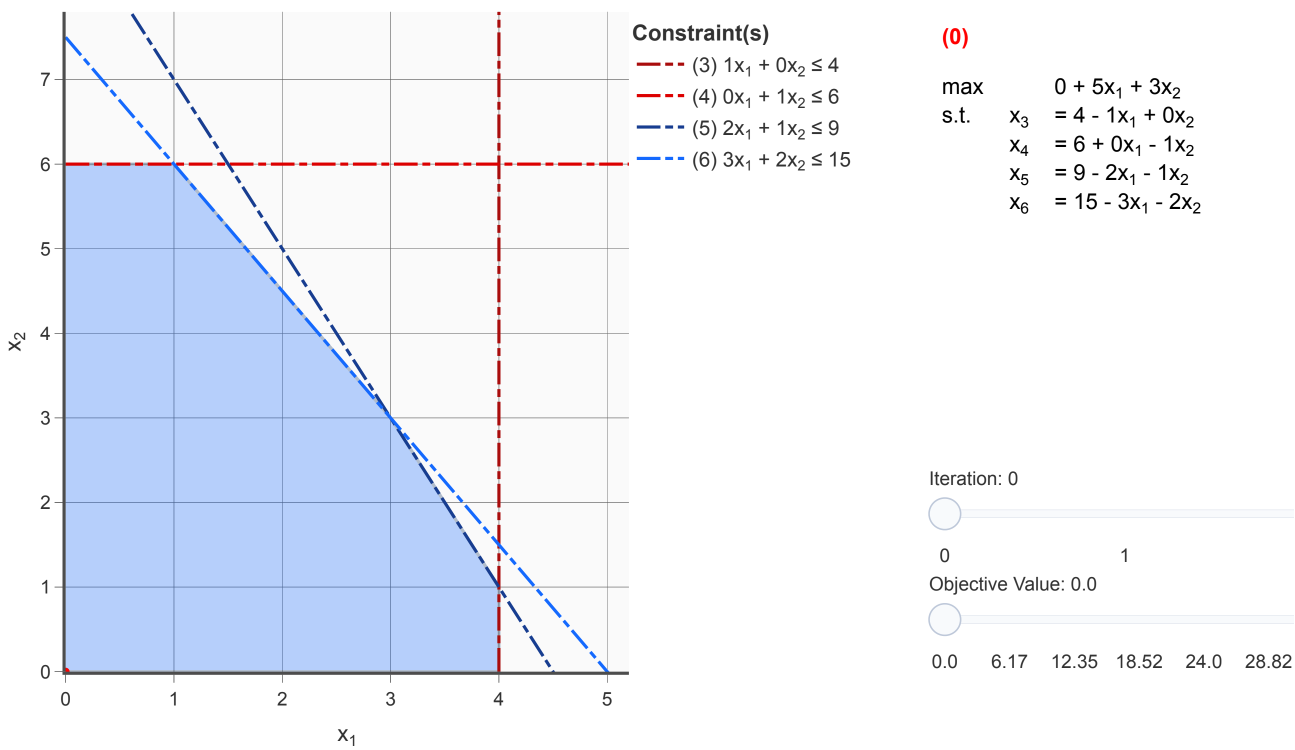
Q18: We have now completed an iteration of simplex! What is the corresponding feasible solution of the new LP?

A: (4,0,0,6,1,3), objective function = 20

Now that we have seen an iteration of simplex algebraically, let's use GILP to visualize it! The LP example we have been using is called `LIMITING_CONSTRAINT_2D_LP`. To visualize simplex, we must import a function called `simplex_visual()`.

In [6]:
```
from gilp.visualize import simplex_visual # import the function
import numpy as np
lp = ex.LIMITING_CONSTRAINT_2D_LP # get the LP example
simplex_visual(lp, initial_solution=np.array([[0],[0]])).show() # show the simplex visualization
```

Geometric Interpretation of LPs



This visualization is much the same as the previous one but we now have an addtional slider which allows you to toggle through iterations of simplex. Furthermore, the corresponding dictionary at every iteration of simplex is shown in the top right. If you toggle between two iterations, you can see the dictionary form for both the previous and next LP at the same time.

Q19: Starting from point (0,0), by how much can you increase x_1 before the point is no longer feasible? Which constraint do you *hit* first? Does this match what you found algebraically?

A: 4, constaint:$x_1 <= 4$ it matches with previous result

Q20: Which variable will be the next increasing variable and why? (Hint: Look at the dictionary form LP at iteration 1)

A: x2 value, it's coefficient is positive in objective function

Q21: Visually, which constraint do you think is the most limiting constraint? How much can x_2 increase? Give the corresponding feasible solution and its objective value of the next dictionary form LP. (Hint: hover over the feasible points to see information about them.)

A: $2x_1 + x_2 <= 9$ can increase value 1 corresponding feasible solution (4,1,0,5,0,1) obj = 23

Q22: Move the slider to see the next iteration of simplex. Was your guess from **Q21** correct? If not, describe how your guess was wrong.

A: yes, this is correct

Q23: Look at the dictionary form LP after the second iteration of simplex. What is the increasing variable? Identify the most limiting constraint graphically and algebraically. Show your work and verify they are the same constraint. In addition, give the next feasible solution and its objective value.

A: x_3 constraint:$3x_1 + 2x_2 <= 15$, x_3 can increase at most one. next feaasible solution:(3,3,1,3,0,0) objective function 24

Q24: Is the new feasible solution you found in **Q23** optimal? (Hint: Look at the dictionary form LP)

A: yes, this is optimal

Q25: In **Q21** and **Q23**, how did you determine the most limiting constraint graphically?

A: increase along the line, and see which line truncate the line.

(BONUS): In 2D, we can increase a variable until we hit a 2D line representing the most limiting constraint. What would be the analagous situation in 3D?

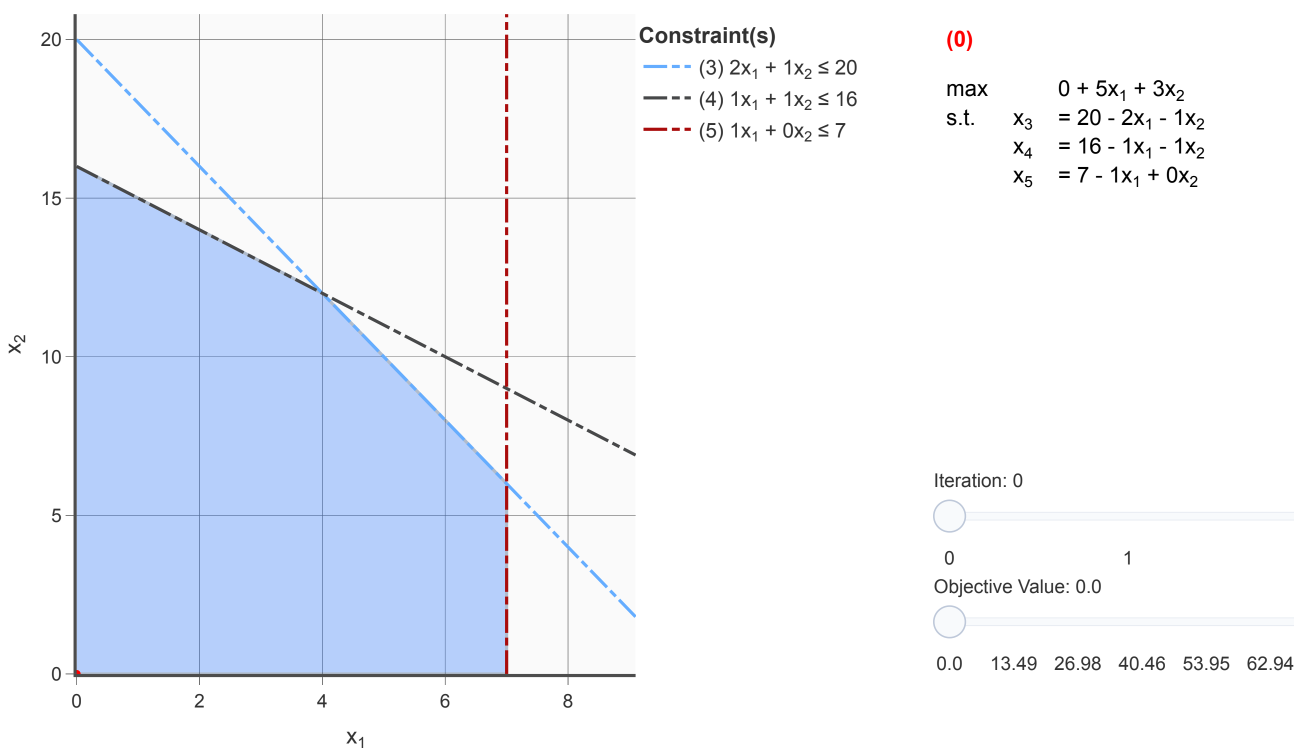
A: we increase along a line until hit the surface.

Part III: Geometrical Interpretation of the Dictionary

We have seen how the simplex algorithm transforms an LP from one dictionary form to another. Each dictionary form has a corresponding dictionary defined by the variables on the LHS of the constraints. Furthermore, each dictionary form has a corresponding feasible solution obtained by setting all non-dictionary variables to 0 and the dictionary variables to the constants on the RHS. In this section, we will explore the geometric interpretation of a dictionary.

In [7]:
```
lp = ex.ALL_INTEGER_2D_LP # get LP example
simplex_visual(lp, initial_solution=np.array([[0],[0]])).show() # visualize it
```

Geometric Interpretation of LPs



Recall, we can hover over the corner points of the feasible region. **BFS** indicates the feasible solution corresponding to that point. For example, (7,0,6,9,0) means $x_1 = 7, x_2 = 0, x_3 = 6, x_4 = 9$, and $x_5 = 0$. **B** gives the indices of the variables in the dictionary. For example, (1,3,4) means that $x_1, x_3,$ and x_4 are in the dictionary. Lastly, the objective value at that point is given.

Q26: Hover over the point (7,6) where $x_1 = 7$ and $x_2 = 6$. What is the feasible solution at that point ?

A: (7,6,0,3,0) obj =53

We have a notion of *slack* for an inequality constraint. Consider the constraint $x_1 \geq 0$. A feasible solution where $x_1 = 7$ has a slack of 7 in this constraint. Consider the constraint $2x_1 + 1x_2 \leq 20$. The feasible solution with $x_1 = 7$ and $x_2 = 6$ has a slack of 0 in this constraint.

Q27: What is the slack in constraint $1x_1 + 1x_2 \leq 16$ when $x_1 = 7$ and $x_2 = 6$?

A: 3

Q28: Look at the constraint $2x_1 + 1x_2 \leq 20$. After rewriting in dictionary form, the constraint is $x_3 = 20 - 2x_1 - 1x_2$. What does x_3 represent?

A: the slack for an inequality constraint

Q29: What do you notice about the feasible solution at point (7,6) and the slack in each constraint?

A: slack variable is non-zero, decision variables is 0

It turns out that each decision variable is really a measure of slack in some corresponding constraint!

Q30: If the slack between a constraint and a feasible solution is 0, what does that tell you about the relationship between the feasible solution and constraint geometrically?

A: point is on the constraint line.

Q31: For (7,6), which variables are **not** in the dictionary? For which constraints do they represent the slack? (Hint: The **B** in the hover box gives the indices of the variables in the dicitonary)

A: x_3, x_5 slack is x_1, x_2, x_4

Q32: For (7,6), what are the values of the non-dictionary variables? Using what you learned from **Q30**, what does their value tell you about the feasible solution at (7,6)?

A: 0, feasible solution = constant in the objective function. slack variable = 0 means where two constraint lines intersect.

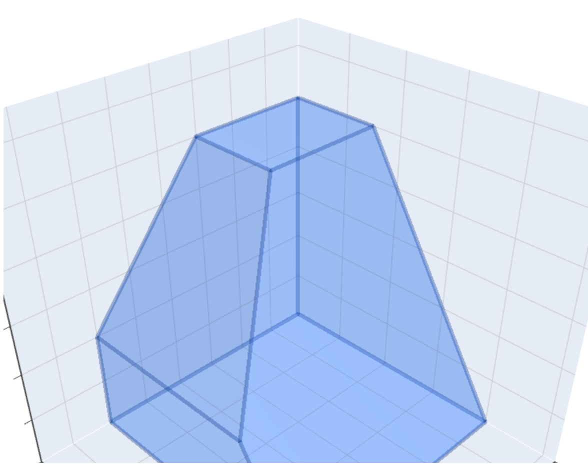
Q33: Look at some other corner points with this in mind. What do you find?

A: they are the intersections of two constraints, have two non-dictionary variables.

Now, let's look at a 3 dimensional LP!

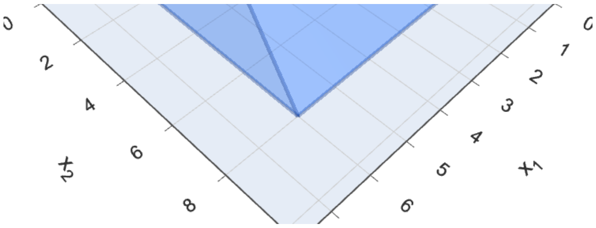
In [8]:
```
lp = ex.ALL_INTEGER_3D_LP # get LP example
lp_visual(lp).show() # visualize it
```

Geometric Interpretation of LPs



Constraint(s)

(4) $1x_1 + 0x_2 + 0x_3 \leq 6$
(5) $1x_1 + 0x_2 + 1x_3 \leq 8$
(6) $0x_1 + 0x_2 + 1x_3 \leq 5$
(7) $0x_1 + 1x_2 + 1x_3 \leq 8$



Objective Value: 0.0

0.0 6.82 13.65 20.47 27.3 31.85

Q34: Hover over the point (6,6,2) where $x_1 = 6, x_2 = 6,$ and $x_3 = 2.$ Note which variables are not in the dictionary. Toggle the corresponding constraints on. What do you notice?

A: 4,5,7. intersection of three constraints.

Q35: Look at some other corner points and do as you did in Q34. Do you see a similar pattern? Combining what you learned in Q33, what can you say about the relationship between the variables not in the dictionary at some corner point, and the corresponding constraints?

A: variables not in dictionary have reach it's contraints.

Q36: What geometric feature do feasible solutions for a dictionary correspond to?

A: corner point in the graph

Part IV: Pivot Rules

The first step in an iteration of simplex is to choose an increasing variable. Sometimes, there are multiple options since multiple variables have a positive coefficent in the objective function. Here, we will explore what this decison translates to geometrically.

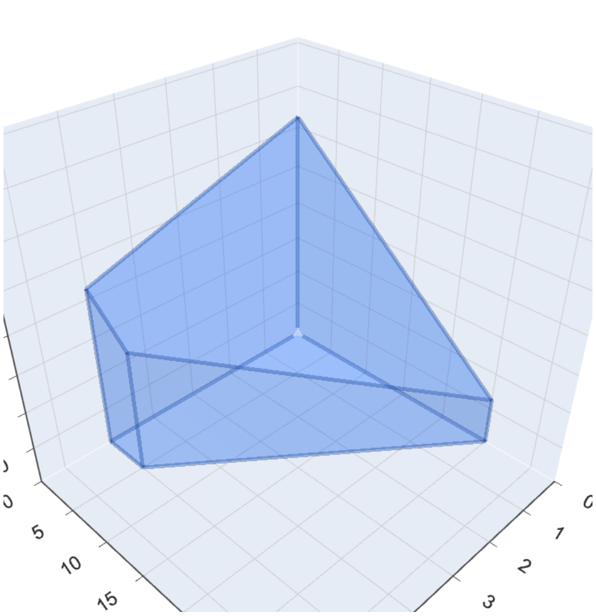
In this section, we will use a special LP commonly referred to as the Klee-Minty Cube.

$$\begin{array}{ll} \max & 4x_1 + 2x_2 + x_3 \\ \text{s.t.} & x_1 \leq 5 \\ & 4x_1 + x_2 \leq 25 \\ & 8x_1 + 4x_2 + x_3 \leq 125 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

Furthermore, we will use an optional parameter called `rule` for the `simplex_visual()` function. This rule tells simplex which varaible to choose as an increasing variable when there are multiple options.

In [9]:
```
simplex_visual(ex.KLEE_MINTY_3D_LP, rule='dantzig', initial_solution=np.array([[0],[0],[0]])).show()
```

Geometric Interpretation of LPs



Constraint(s)

(4) $1x_1 + 0x_2 + 0x_3 \leq 5$
(5) $4x_1 + 1x_2 + 0x_3 \leq 25$
(6) $8x_1 + 4x_2 + 1x_3 \leq 125$

(0)

$$\begin{array}{lll} \max & & 0 + 4x_1 + 2x_2 + 1x_3 \\ \text{s.t.} & x_4 & = 5 - 1x_1 + 0x_2 + 0x_3 \\ & x_5 & = 25 - 4x_1 - 1x_2 + 0x_3 \\ & x_6 & = 125 - 8x_1 - 4x_2 - 1x_3 \end{array}$$

Iteration: 0

0 1 2 3 4

Q37: Use the iteration slider to examine the path of simplex on this LP. What do you notice?

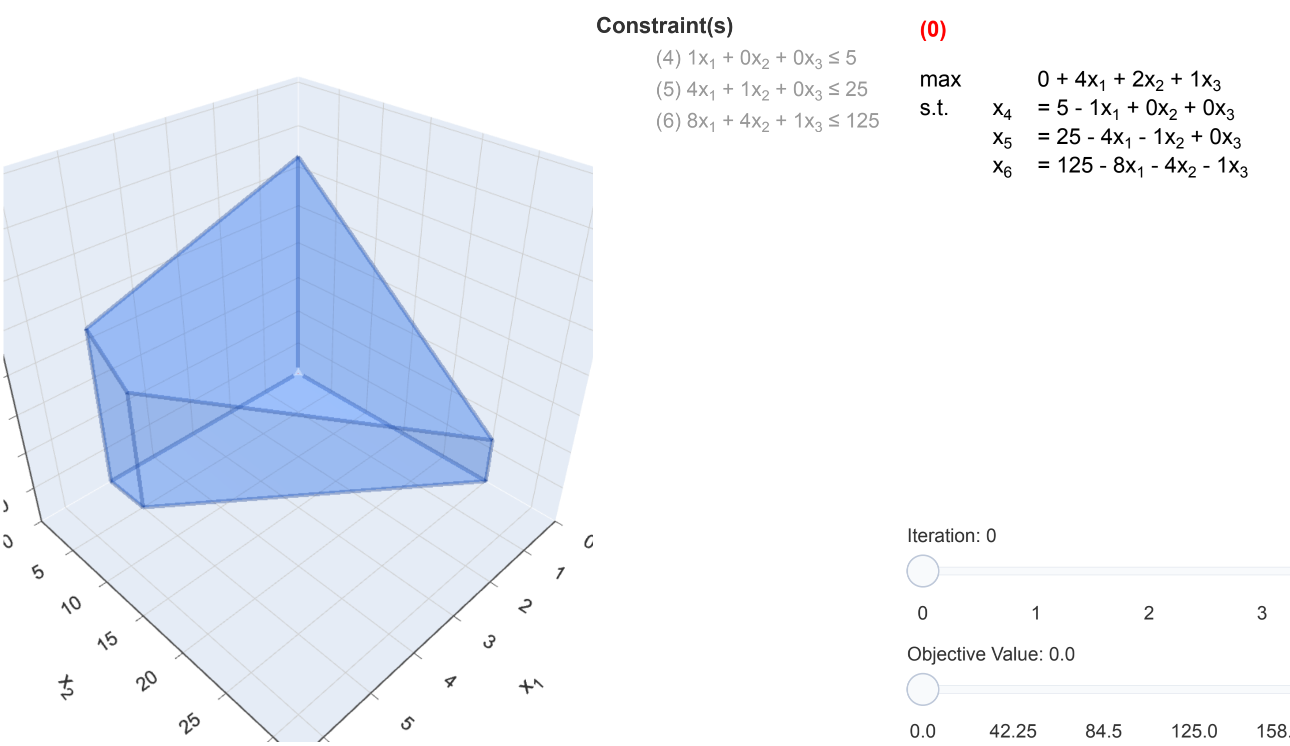
A: it detours to get to the final place. if the initial increase variable is x3, it can get to the optimal point quickly.

Above, we used a pivot rule proposed by Dantzig. In this rule, the variable with the largest positive coefficient in the objective function enters the dictionary. Go through the iterations again to verify this.

Let us consider another pivot rule proposed by Bland, a professor here at Cornell. In his rule, of the variables with positive coefficents in the objective function, the one with the smallest index enters. Let us examine the path of simplex using this pivot rule! Again, look at the dictionary form LP at every iteration.

In [10]:
```
simplex_visual(ex.KLEE_MINTY_3D_LP,rule='bland', initial_solution=np.array([[0],[0],[0]])).show()
```

Geometric Interpretation of LPs



Q38: What is the difference between the path of simplex using Dantzig's rule and Bland's rule?

A: the current path takes less egdes to get to the optimal.

Can you do any better? By setting `rule='manual_select'`, you can choose the entering variable explicitly at each simplex iteration.

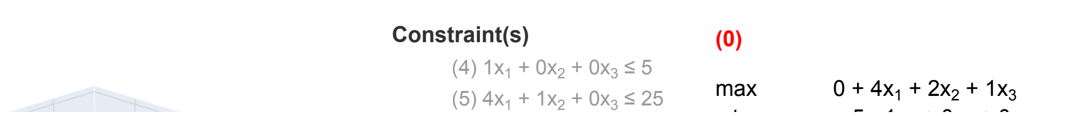
Q39: Can you do better than 5 iterations? How many paths can you find? (By my count, there are 7)

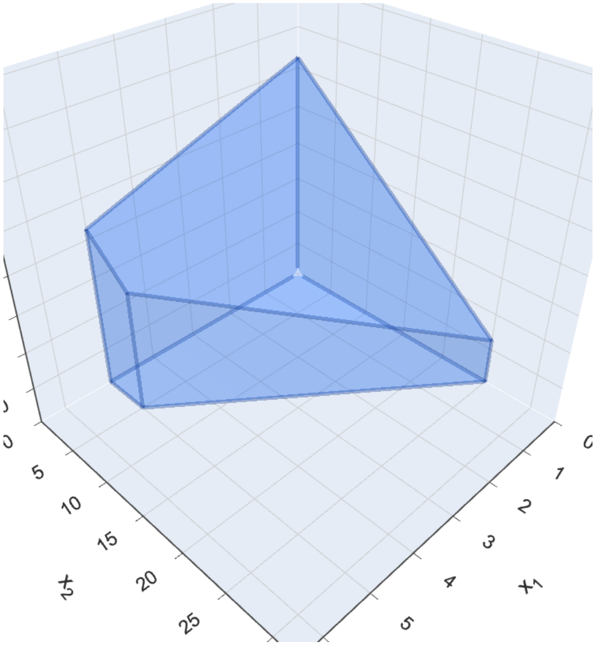
A: yes. 7

In [13]:
```
simplex_visual(ex.KLEE_MINTY_3D_LP,rule='manual_select', initial_solution=np.array([[0],[0],[0]])).show()
```

```
Pick one of [1, 2, 3]2
Pick one of [3]3
Pick one of [1, 5]1
Pick one of [5]5
Pick one of [4]4
```

Geometric Interpretation of LPs





(6) $8x_1 + 4x_2 + 1x_3 \leq 125$

s.t. $x_4 = 5 - 1x_1 + 0x_2 + 0x_3$
$x_5 = 25 - 4x_1 - 1x_2 + 0x_3$
$x_6 = 125 - 8x_1 - 4x_2 - 1x_3$

Iteration: 0

0 1 2 3

Objective Value: 0.0

0.0 42.25 84.5 125.0 158.4

Q40: What does the choice of increasing variable correspond to geometrically?

A: increase along an edge

Q41: Are there any paths you could visualize taking to the optimal solution that `rule='manual_select'` prevented you from taking? If yes, give an example and explain why it is not a valid path for simplex to take. (Hint: Look at the objective value after each simplex iteration.)

A: after inccrase x_2, I can't incrase x5, because the coeffiient is negative.

Part V: Creating LPs in GILP (Optional)

We can also create our own LPs! First, we must import the `LP` class.

In [14]:
```
from gilp.simplex import LP
```

Let us create the following LP.

$$\begin{aligned} \max \quad & 3x_1 + 2x_2 \\ \text{s.t.} \quad & 2x_1 + 1x_2 \leq 6 \\ & 0x_1 + 1x_2 \leq 2 \\ & x_1, x_2 \geq 0 \end{aligned}$$

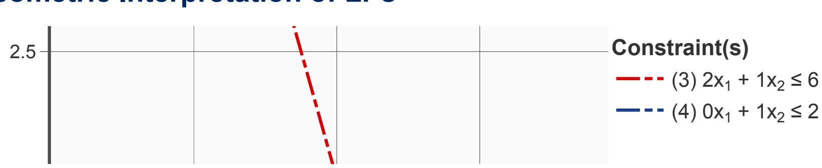
We will create this LP by specifying 3 arrays of coefficents. We define the NumPy arrays `A`, `b`, and `c` and then pass them to the `LP` class to create the LP.

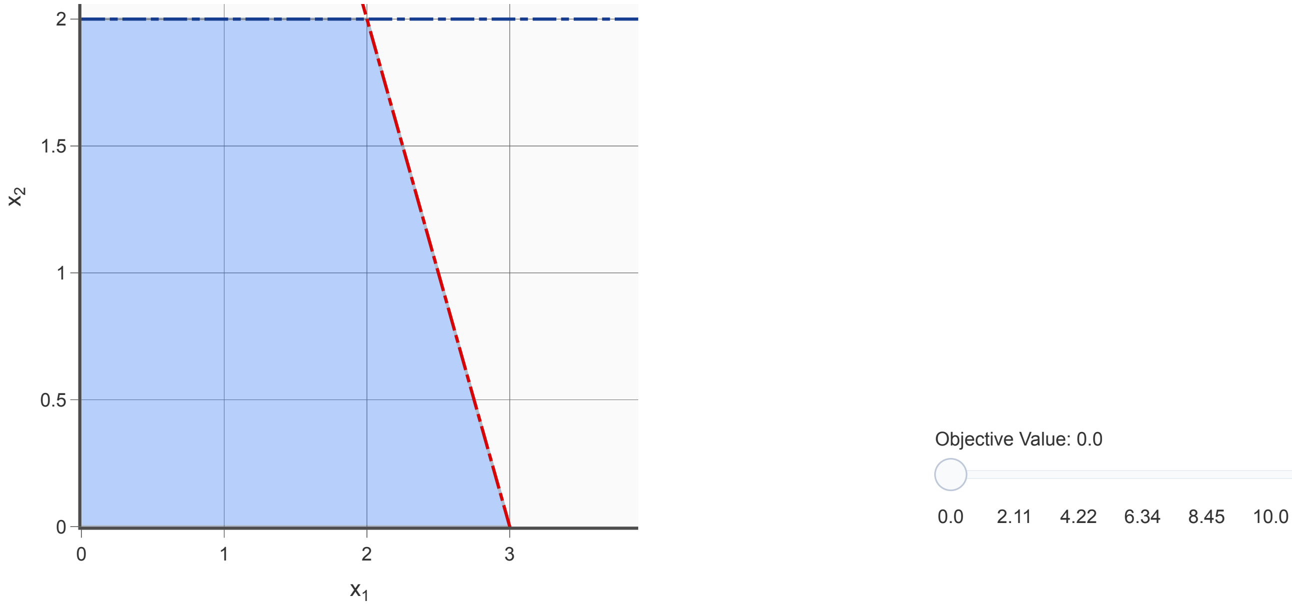
In [15]:
```
A = np.array([[2,1],  # LHS constraint coefficents
              [0,1]])
b = np.array([6,2])  # RHS constraint coefficents
c = np.array([3,2])  # objective function coefficents
lp = LP(A,b,c)
```

Let's visualize it!

In [16]:
```
lp_visual(lp).show()
```

Geometric Interpretation of LPs

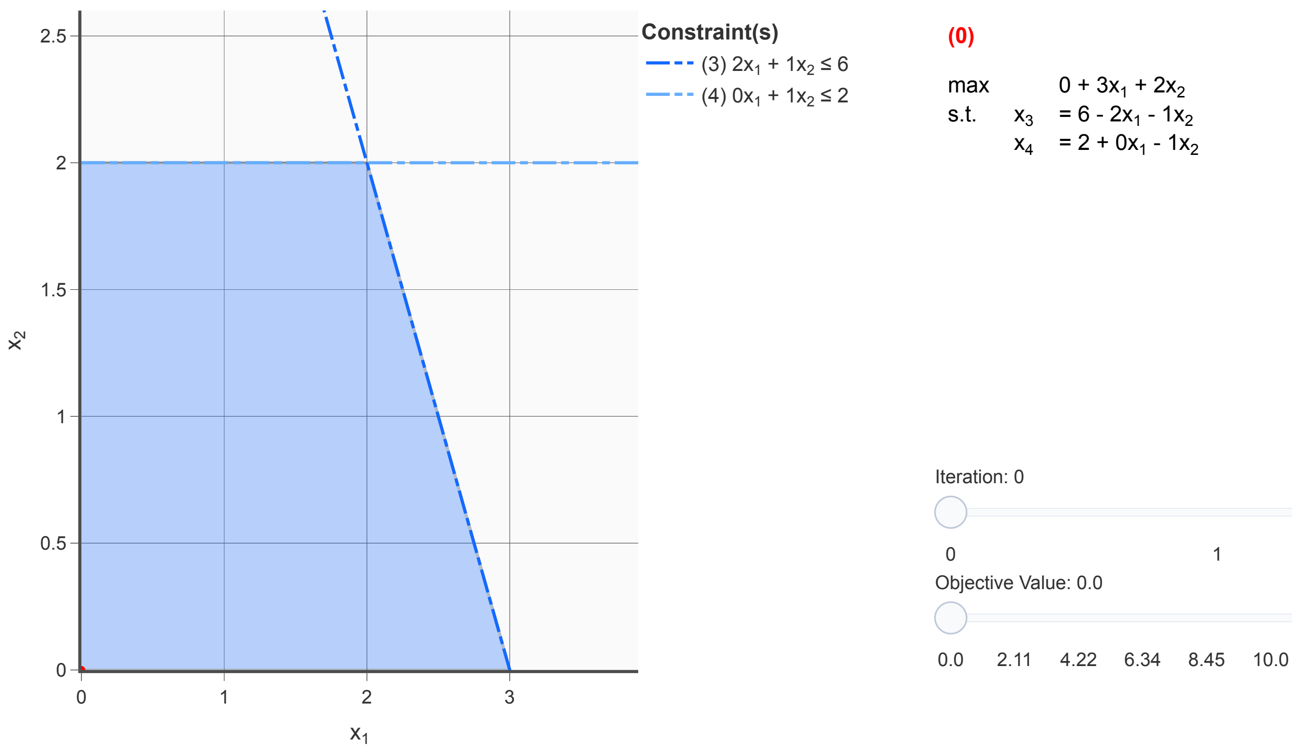




... and solve it!

In [17]:
```
simplex_visual(lp, initial_solution=np.array([[0],[0]])).show()
```

Geometric Interpretation of LPs



In [ ]:
```

```