جامعة القاهرة الجديدة التكنولوجية

Fall 2023

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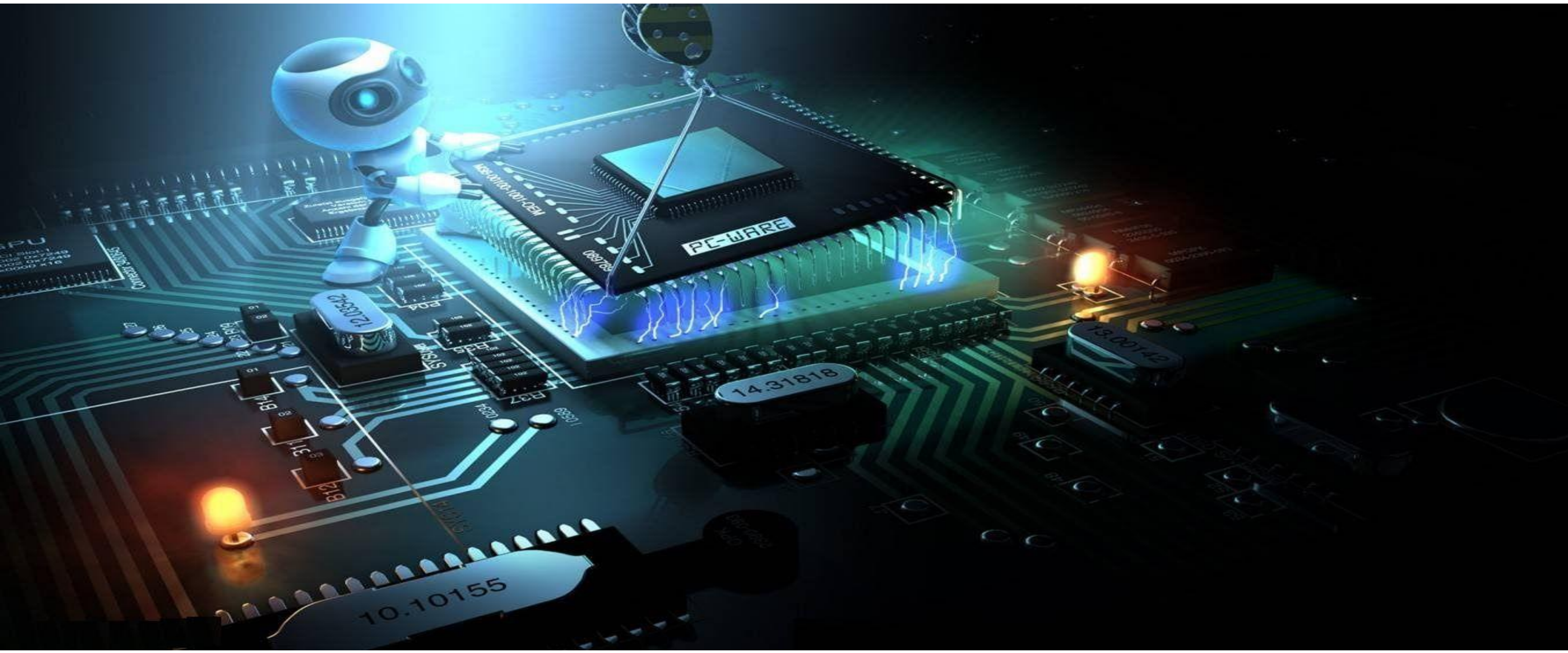
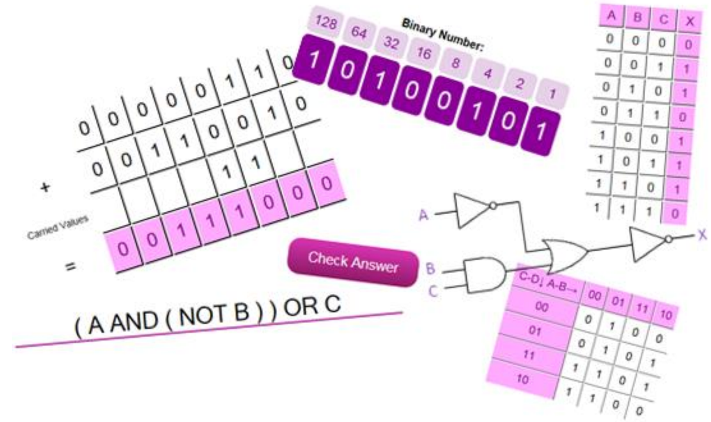


Digital Engineering

Dr. Hatem Yousry

Agenda

- **Karnaugh map.**
- **Three-variable K-Map**
- **Multisim.**



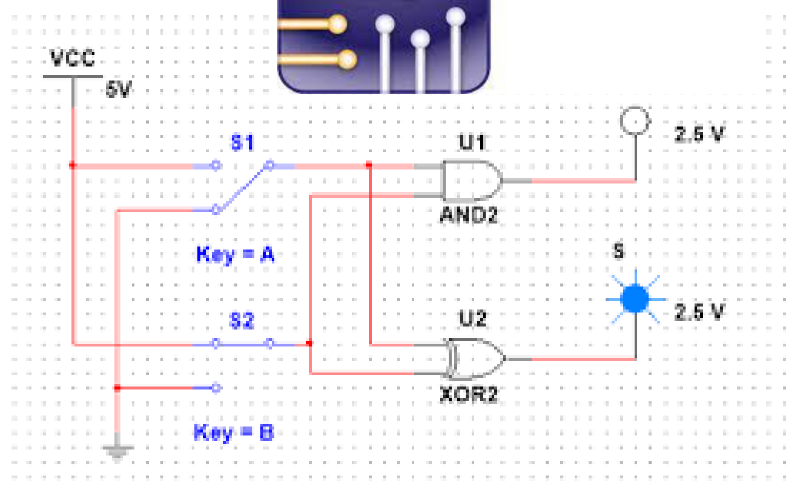
	y'	y'	y	y
x'				
x				
	z'	z	z	z'

x	y	z	F
0	0	0	m0
0	0	1	m1
0	1	0	m2
0	1	1	m3
1	0	0	m4
1	0	1	m5
1	1	0	m6
1	1	1	m7

3-variable Truth table of F

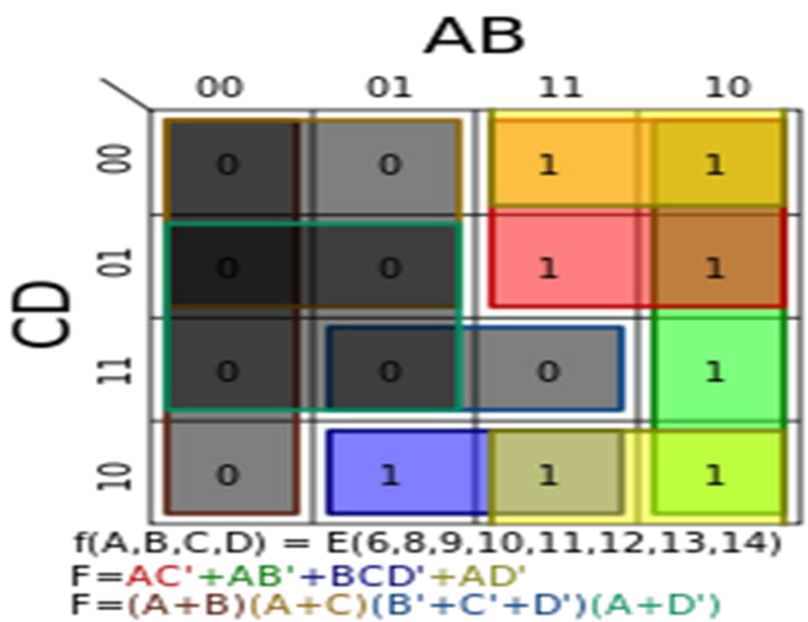
x \ yz	y'z'	y'z	yz	yz'
x'	m0 x'y'z' 0	m1 x'y'z 1	m3 x'yz 3	m2 x'yz' 2
y	m4 xy'z' 4	m5 xy'z 5	m7 xyz 7	m6 xyz' 6

3-Variable K-Map, minterm and cell position



Karnaugh Maps

- Karnaugh maps (K-maps) are ***graphical*** representations of boolean functions.
- One ***map cell*** corresponds to a row in the truth table.
- Also, one map cell corresponds to a minterm or a maxterm in the boolean expression
- **Multiple-cell areas of the map correspond to standard terms.**



The Karnaugh map (KM or K-map)

- **Karnaugh maps** are used **to simplify real-world logic** requirements so that they can be implemented using a minimum number of physical logic gates.
- A **sum-of-products** expression can always be implemented using **AND gates feeding into an OR gate**, and a p**roduct-of-sums** expression leads to **OR gates feeding an AND gate**.
- Karnaugh maps can also be used to simplify logic expressions in **software design**. Boolean conditions can get very complicated, which makes the code difficult to read and to maintain. Once **minimized, canonical sum-of-products and product-of-sums expressions can be implemented directly using AND and OR logic operators.**

What are Karnaugh maps?

- **Karnaugh maps provide an alternative way of simplifying logic circuits.**
- Instead of using Boolean algebra simplification techniques, you can **transfer logic values from a Boolean statement or a truth table into a Karnaugh map.**
- The arrangement of 0's and 1's within the map helps you to visualize the logic relationships between the variables and leads directly to a simplified Boolean statement.

A. SOP: -

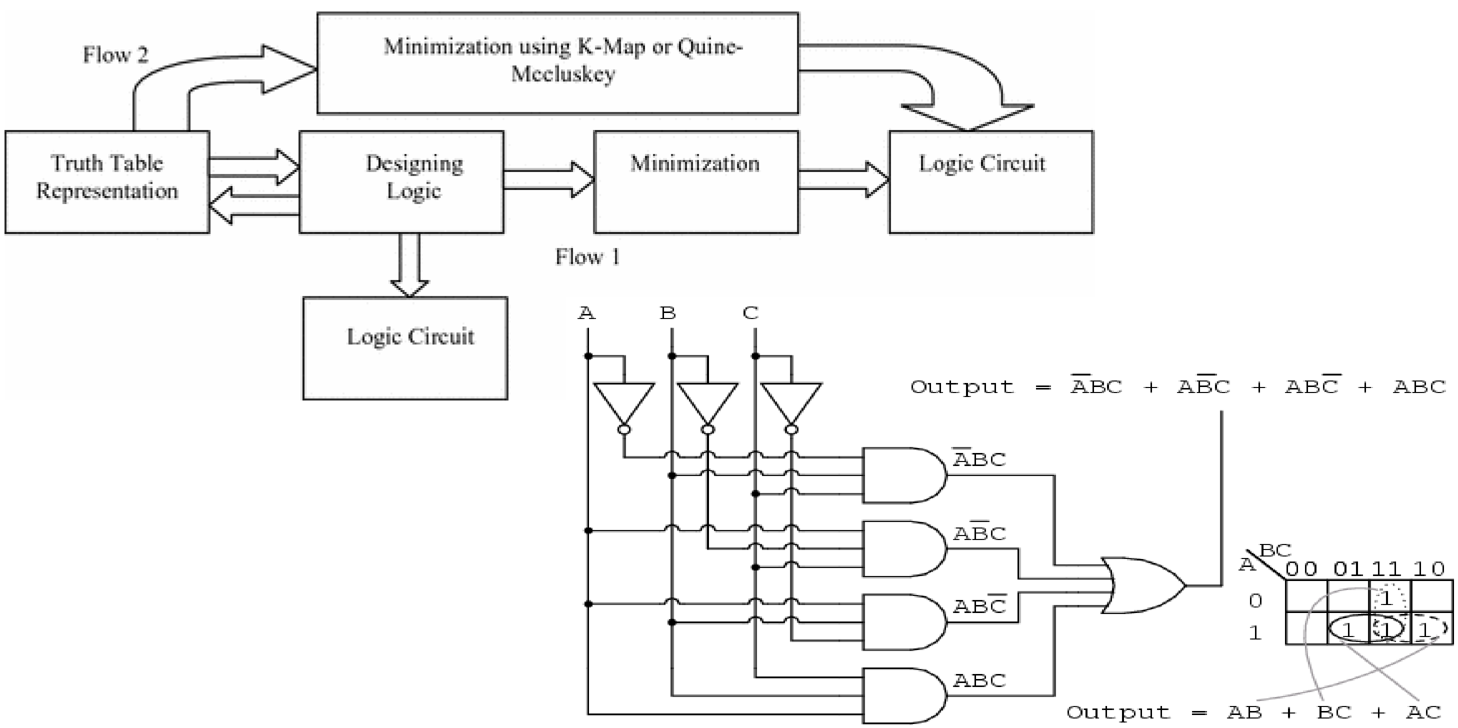
A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0	$\overline{A}.\overline{B}$	$\overline{A}.B$
A 1	$A.\overline{B}$	$A.B$

B. POS: -

A \ B	B 0	$\overline{B}$ 1
A 0	$A+B$	$A+\overline{B}$
$\overline{A}$ 1	$\overline{A}+B$	$\overline{A}+\overline{B}$

Mathematical Study for Reduction of Variables in Karnaugh Map

- For designing any logic circuit, the major important stages may be truth table representation, framing logical statements, reduction using minimizing technique, and the most important and the desired part is design.



Two-Variable K-Map

- Simplifying Boolean Expressions Using K-Map Method

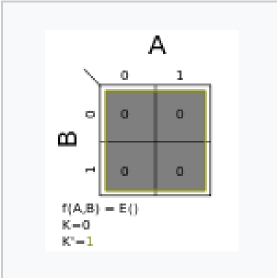
Y \ X	0	1
0	00	10
1	01	11

Y \ X	0	1
0	X’Y’	XY’
1	X’Y	XY

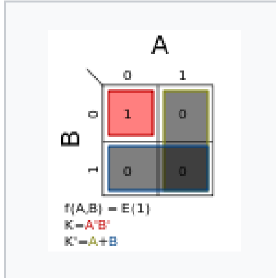
Y \ X	0	1
0	m0	m2
1	m1	m3

Note:

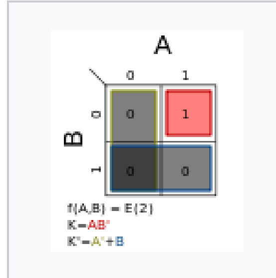
- **Logically adjacent cells are physically adjacent in the k-map**
- **Each cells has two adjacent cells**



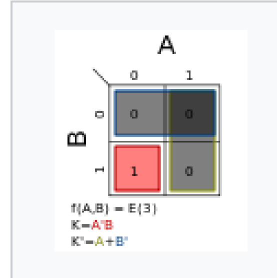
$\Sigma m(0)$; $K = 0$



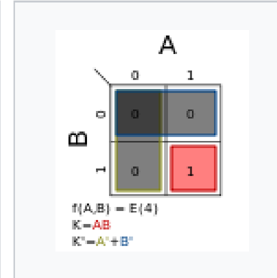
$\Sigma m(1)$; $K = A'B'$



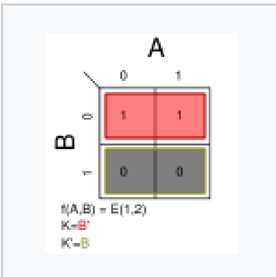
$\Sigma m(2)$; $K = AB'$



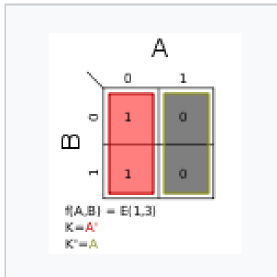
$\Sigma m(3)$; $K = A'B$



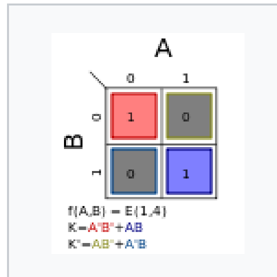
$\Sigma m(4)$; $K = AB$



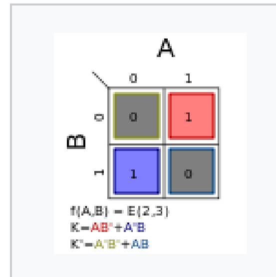
$\Sigma m(1,2)$; $K = B'$



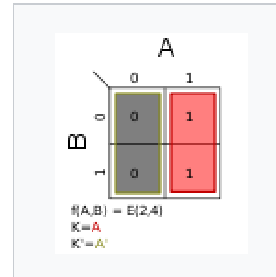
$\Sigma m(1,3)$; $K = A'$



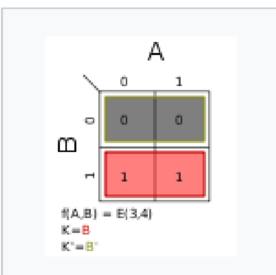
$\Sigma m(1,4)$; $K = A'B' + AB$



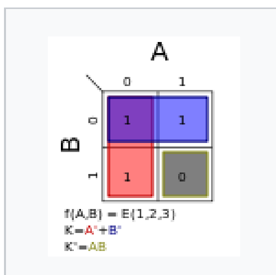
$\Sigma m(2,3)$; $K = AB' + A'B$



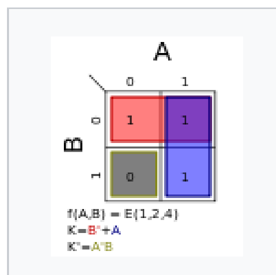
$\Sigma m(2,4)$; $K = A$



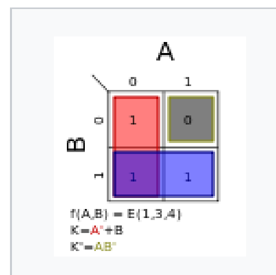
$\Sigma m(3,4)$; $K = B$



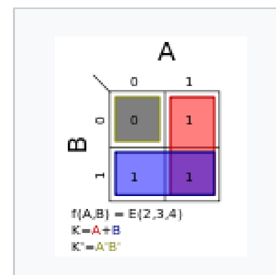
$\Sigma m(1,2,3)$; $K = A' + B'$



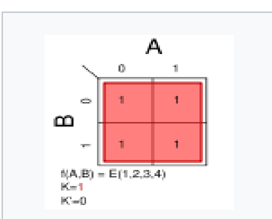
$\Sigma m(1,2,4)$; $K = A + B'$



$\Sigma m(1,3,4)$; $K = A' + B$



$\Sigma m(2,3,4)$; $K = A + B$



$\Sigma m(1,2,3,4)$; $K = 1$

Three-variable K-Map

- 3 Variables K-map solver, table & work with steps to find the **Sum of Products (SOP)** or to minimize the given logical (Boolean) expressions formed by A, B & C or x, y & z based on the laws & theorems of AND, OR & NOT gates in digital electronics.
- In a three-variable map it is **possible to combine cells to produce product terms that correspond to a single cell**, two adjacent cells, or a group of four adjacent cells.

A	B	C	Minterm
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

A \ BC	00	01	11	10
0	m0	m1	m3	m2
1	m4	m5	m7	m6

Three-variable K-Map

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

- The number of cells in 3 variable K-map is eight, since the number of variables is three. The following figure shows 3 variable K-Map.
  - **There is only one possibility of grouping 8 adjacent min-terms.**
  - **The possible combinations of grouping 4 adjacent min terms are {(m0, m1, m3, m2), (m4, m5, m7, m6), (m0, m1, m4, m5), (m1, m3, m5, m7), (m3, m2, m7, m6) and (m2, m0, m6, m4)}.**
  - **The possible combinations of grouping 2 adjacent min terms are {(m0, m1), (m1, m3), (m3, m2), (m2, m0), (m4, m5), (m5, m7), (m7, m6), (m6, m4), (m0, m4), (m1, m5), (m3, m7) and (m2, m6)}.**
  - **If x=0, then 3 variable K-map becomes 2 variable K-map.**

x	y	z	F
0	0	0	m0
0	0	1	m1
0	1	0	m2
0	1	1	m3
1	0	0	m4
1	0	1	m5
1	1	0	m6
1	1	1	m7

3-variable Truth table of F

x \ yz	y'z'	y'z	yz	yz'
x'	m0 x'y'z' 0	m1 x'y'z 1	m3 x'yz 3	m2 x'yz' 2
y	m4 xy'z' 4	m5 xy'z 5	m7 xyz 7	m6 xyz' 6

3-Variable K-Map, minterm and cell position

A	B	C	Minterm
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

Example

A \ BC	00	01	11	10
0	m0	m1	m3	m2
1	m4	m5	m7	m6

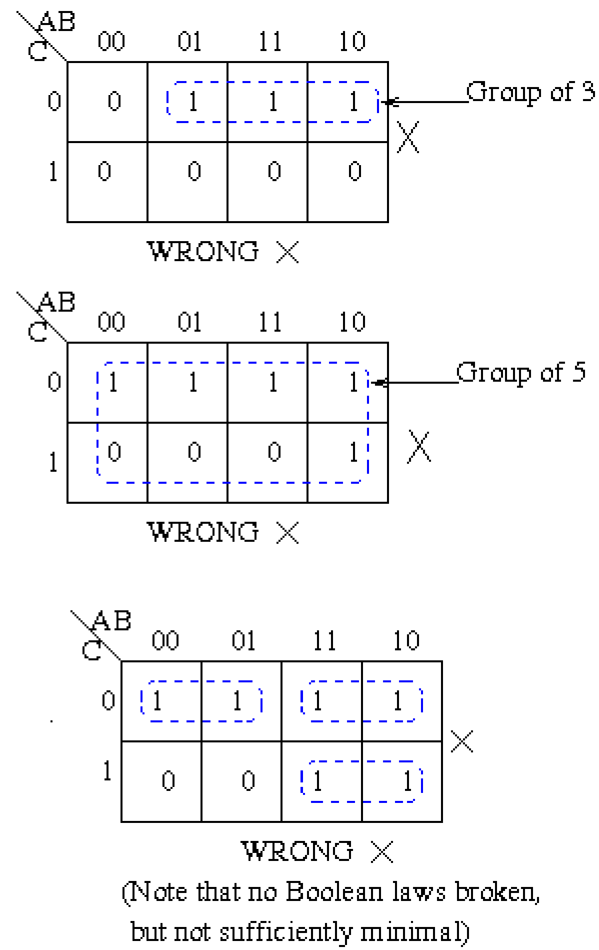
- **Solve 3 Variables K-Map for ∑(0, 2, 5, 7)**

x	y	z	F
0	0	0	m0
0	0	1	m1
0	1	0	m2
0	1	1	m3
1	0	0	m4
1	0	1	m5
1	1	0	m6
1	1	1	m7

	y'	*y'*	*y*	*y*
x'				
x				
	z'	*z*	*z*	*z'*

What are the rules of K-Map?

- Groups may **not include** any cell containing a **Zero.**
- Groups may be **horizontal or vertical,** but not diagonal.
- Groups must contain **1, 2, 4, 8,** or in general $\mathbf{2^n}$ **cells. ...**
- Each group should be **as large as** possible.
- Each cell containing a **One** must be in at least one group.
- Groups may **overlap.**
- Groups may **wrap around** the table.



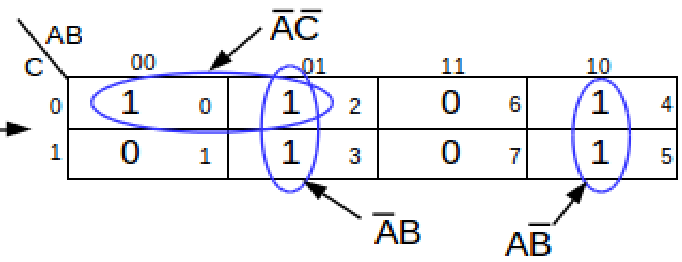
Example

Resolve the logic function

3 Variable Truth Table

	A	B	C	F
0	0	0	0	1
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

K-Map



Note : $\bar{A}B + A\bar{B} = A \oplus B$ (Exclusive OR)

$F = \bar{A}\bar{C} + \bar{A}B + A\bar{B}$

$F = \bar{A}\bar{C} + A \oplus B$

Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	***y***	***z***	**Term**	**Designation**	**Term**	**Designation**
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Example

1 $_{0}$	0 $_{1}$	0 $_{3}$	1 $_{2}$
0 $_{4}$	1 $_{5}$	1 $_{7}$	0 $_{6}$

- **Step 1**
  - input = $\sum(0, 2, 5, 7)$
  - Place the values in the corresponding places of Karnaugh's Map Table
- **Step 2**
  - 2 cell Grouping
  - **Group 1:**
    - Positions = {0, 2}
    - Simplified Expression = x'z'
  - **Group 2:**
    - Positions = {5, 7}
    - Simplified Expression = xz
- **Step 3**
  - Form Output expression from mapped and unmapped variables
  - output = sum(unmapped & mapped cells)

1	0	0	1
0	0	0	0

0	0	0	0
0	1	1	0

$\boxed{y = x'z' + xz}$

Three-variable K-Map

	y'	y'	y	y
x'				
x				
	z'	z	z	z'

- **A three-variable map**
  - **Eight minterms**
  - Any two adjacent squares in the map differ by only on variable
    - Primed in one square and unprimed in the other
    - e.g., m_5 and m_7 can be simplified
    - $\boldsymbol{m_5 + m_7 = xy'z + xyz = xz\,(y'+y) = xz}$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

(a)

$x \backslash yz$	00	01	11	10
0	m_0 $x'y'z'$	m_1 $x'y'z$	m_3 $x'yz$	m_2 $x'yz'$
1	m_4 $xy'z'$	m_5 $xy'z$	m_7 xyz	m_6 xyz'

(b)

Three-variable Map

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

- m_0 and m_2 (m_4 and m_6) **are adjacent**
- $m_0 + m_2 = x'y'z' + x'yz' = x'z'(y'+y) = x'z'$
- $m_4 + m_6 = xy'z' + xyz' = xz'(y'+y) = xz'$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

(a)

			y	y
x \ yz	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
x 1	$xy'z'$	$xy'z$	xyz	xyz'
		z	z	

(b)

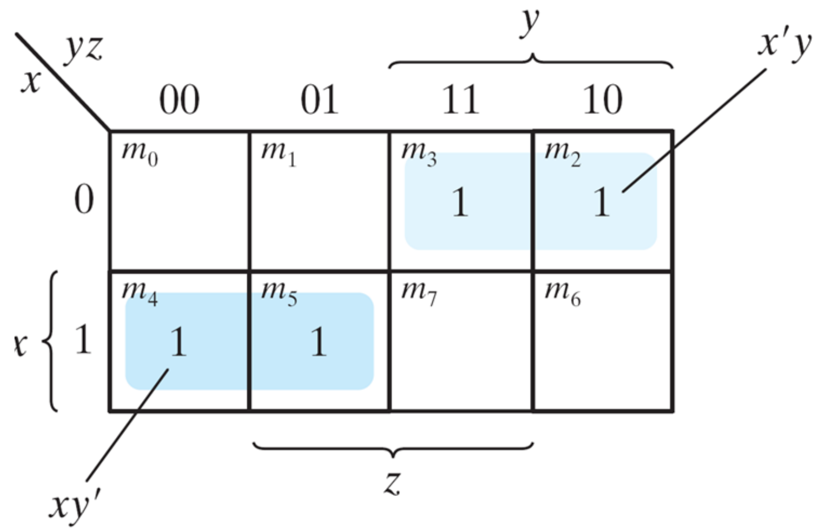
Example

- simplify the Boolean function F(*x*, *y*, *z*) = ∑(2, 3, 4, 5)
  - ***F*(*x*, *y*, *z*) = ∑(2, 3, 4, 5) = *x'y* + *xy'***

x	y	z	F
0	0	0	m0
0	0	1	m1
0	1	0	m2
0	1	1	m3
1	0	0	m4
1	0	1	m5
1	1	0	m6
1	1	1	m7

0	0	1	1
0	0	0	0

0	0	0	0
1	1	0	0



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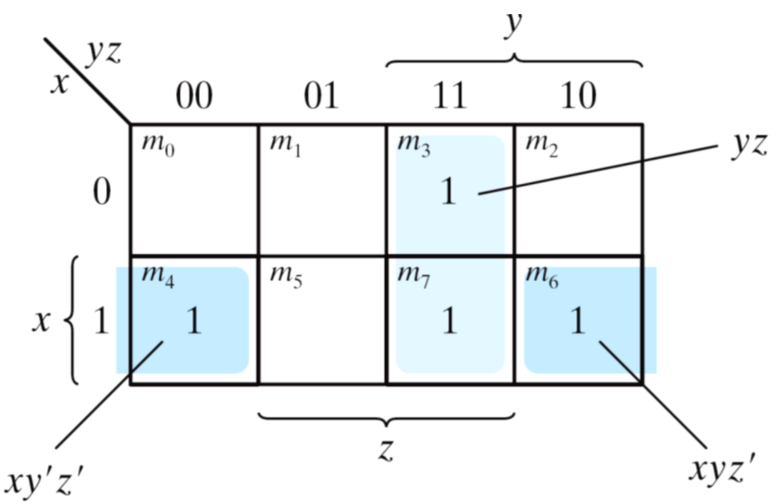
Example

- Simplify $F(x, y, z) = \Sigma(3, 4, 6, 7)$
  - $F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$

x	y	z	F
0	0	0	m0
0	0	1	m1
0	1	0	m2
0	1	1	m3
1	0	0	m4
1	0	1	m5
1	1	0	m6
1	1	1	m7



Note: $xy'z' + xyz' = xz'$

Four Adjacent Squares

- Consider four adjacent squares
  - 2, 4, and 8 squares
  - $m_0+m_2+m_4+m_6$ = $x'y'z'+x'yz'+xy'z'+xyz'$ = $x'z'(y'+y)$ $+xz'(y'+y)$ = $x'z'$ + xz' = z'
  - $m_1+m_3+m_5+m_7$ = $x'y'z+x'yz+xy'z+xyz$ =$x'z(y'+y)$ + $xz(y'+y)$ =$x'z$ + xz = z

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

(a)

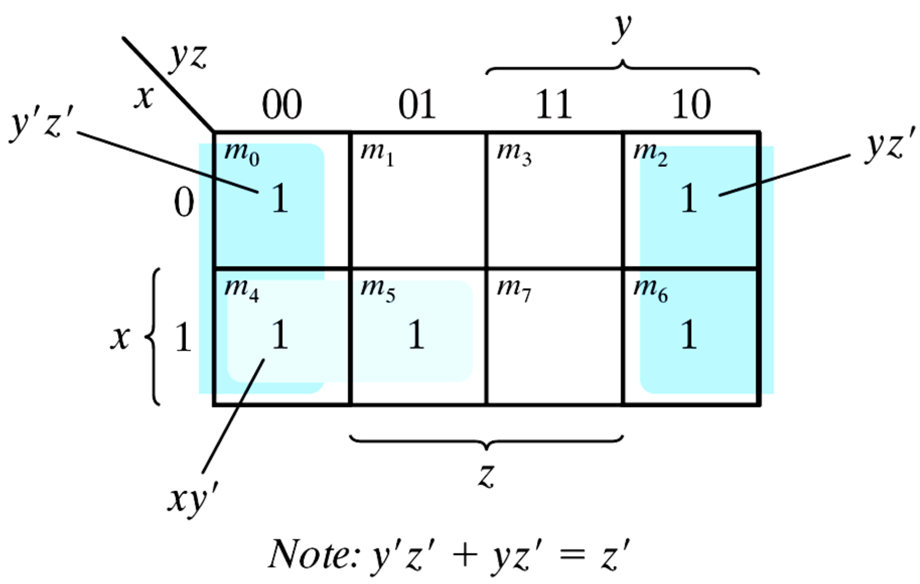
$x \backslash yz$	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
x 1	$xy'z'$	$xy'z$	xyz	xyz'

y (columns 11, 10); z (columns 01, 11)

(b)

Example

- Simplify $F(x, y, z) = \Sigma\,(0, 2, 4, 5, 6)$
- $F(x, y, z) = \Sigma\,(0, 2, 4, 5, 6) = z' + xy'$

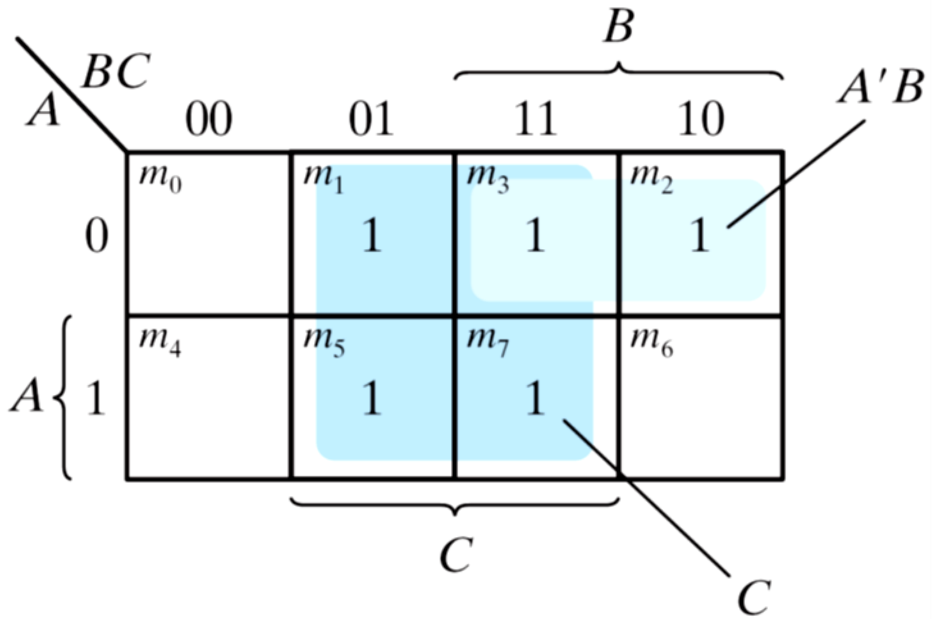


Example

- Let $F = A'C + A'B + AB'C + BC$
  a) Express it in sum of minterms.
  b) Find the minimal sum of products expression.

  Ans:

  $$F(A, B, C) = \Sigma(1, 2, 3, 5, 7) = C + A'B$$



Name:

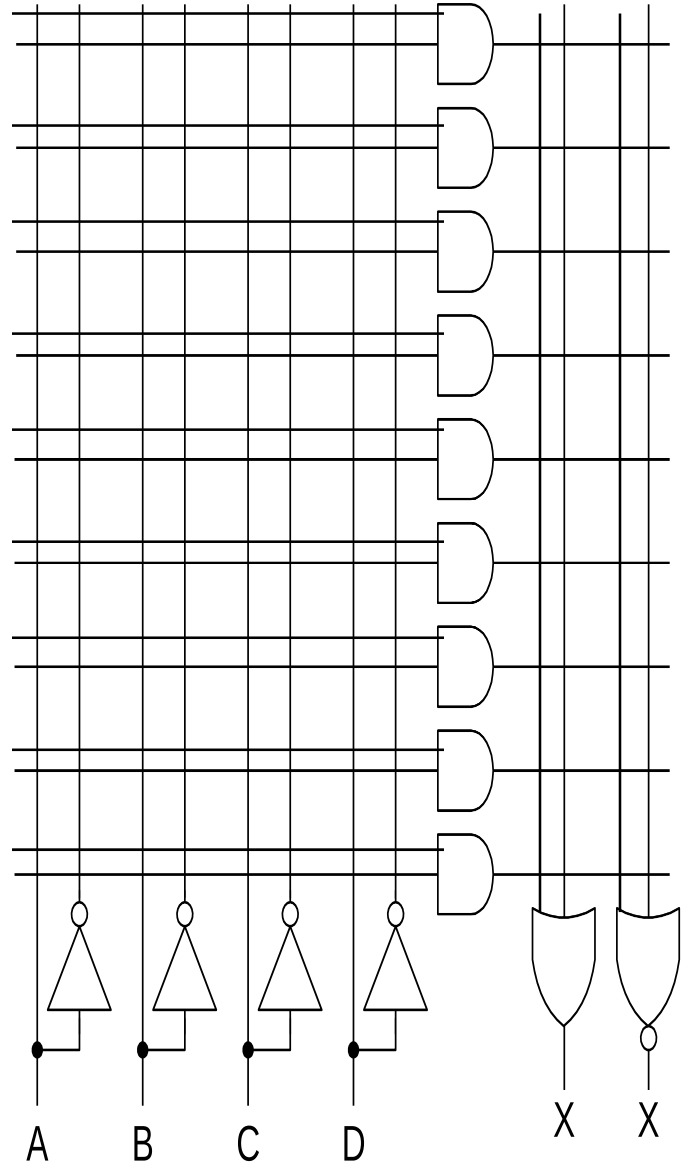
	Inputs				Output		
0	0	0	0	0			
1	0	0	0	1			
2	0	0	1	0			
3	0	0	1	1			
4	0	1	0	0			
5	0	1	0	1			
6	0	1	1	0			
7	0	1	1	1			
8	1	0	0	0			
9	1	0	0	1			
10	1	0	1	0			
11	1	0	1	1			
12	1	1	0	0			
13	1	1	0	1			
14	1	1	1	0			
15	1	1	1	1			

X =

K- Map

	y'	*y'*	*y*	*y*
x'				
x				
	z'	*z*	*z*	*z'*

X =



Three-Variable K-Maps

$f = \sum(0,4) = \overline{B}\,\overline{C}$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**0**
1	**1**	**0**	**0**	**0**

$f = \sum(4,5) = A\,\overline{B}$

A \ BC	00	01	11	10
0	**0**	**0**	**0**	**0**
1	**1**	**1**	**0**	**0**

$f = \sum(0,1,4,5) = \overline{B}$

A \ BC	00	01	11	10
0	**1**	**1**	**0**	**0**
1	**1**	**1**	**0**	**0**

$f = \sum(0,1,2,3) = \overline{A}$

A \ BC	00	01	11	10
0	**1**	**1**	**1**	**1**
1	**0**	**0**	**0**	**0**

$f = \sum(0,4) = \overline{A}\,C$

A \ BC	00	01	11	10
0	**0**	**1**	**1**	**0**
1	**0**	**0**	**0**	**0**

$f = \sum(4,6) = A\,\overline{C}$

A \ BC	00	01	11	10
0	**0**	**0**	**0**	**0**
1	**1**	**0**	**0**	**1**

$f = \sum(0,2) = \overline{A}\,\overline{C}$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**1**
1	**0**	**0**	**0**	**0**

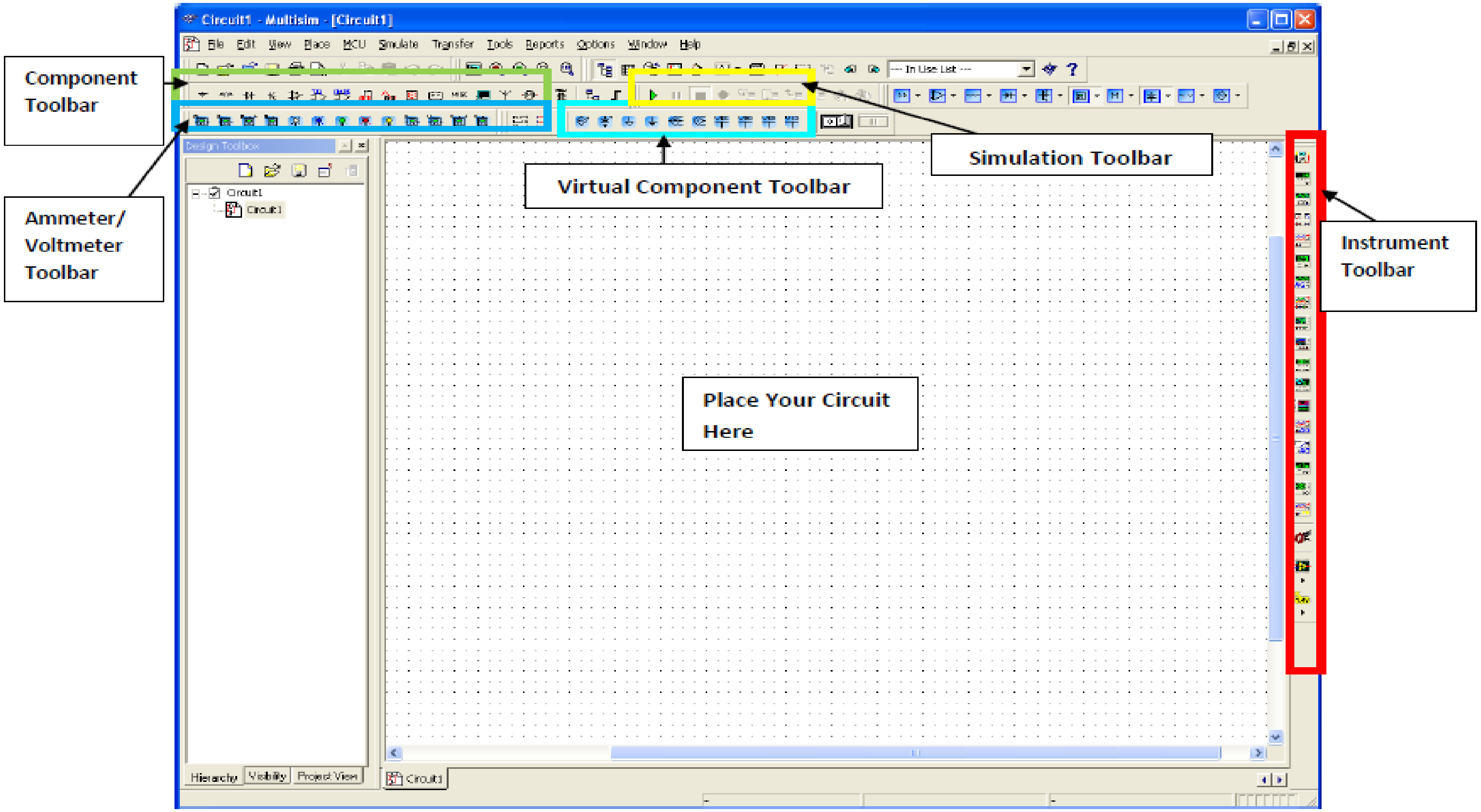
$f = \sum(0,2,4,6) = \overline{C}$

A \ BC	00	01	11	10
0	**1**	**0**	**0**	**1**
1	**1**	**0**	**0**	**1**

Multisim

Start

Click on Start ⟶ All Programs ⟶ National Instruments ⟶ Circuit Design Suite 10.0 ⟶ Multisim.

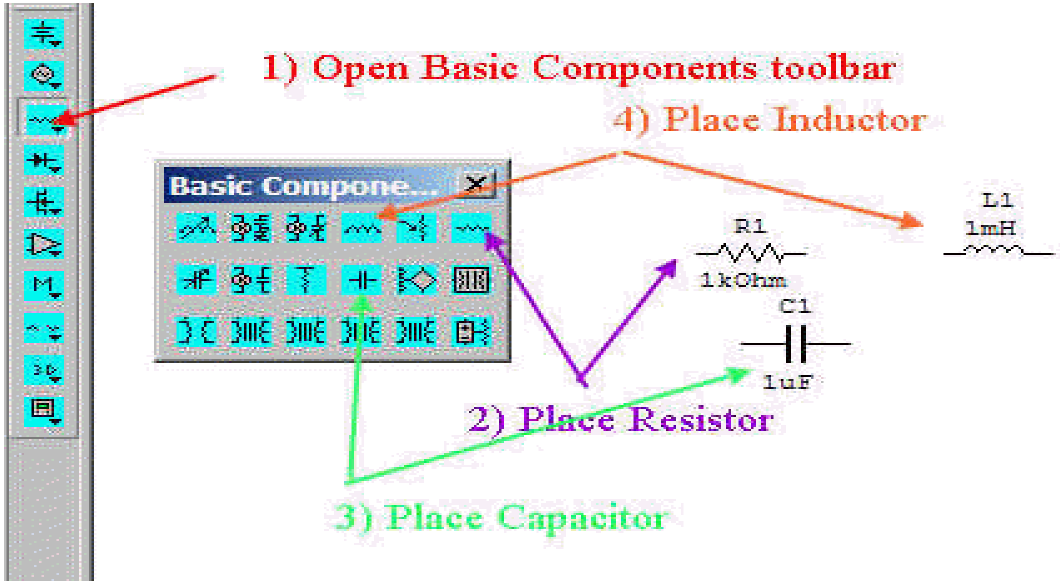


MultiSim

- **1. Open/Create Schematic**
- A blank schematic Circuit 1 is automatically created. To create a new schematic click on File – New – Schematic Capture. To save the schematic click on File /Save As.
- To open an existing file click on File/ Open in the toolbar.
- **2. Place Components**
- To Place Components click on Place/Components. On the Select Component Window click on Group to select the components needed for the circuit.
- Click OK to place the component on the schematic

MultiSim - Components

- There are two kinds of component models used in MultiSim: Those modeled after **actual components** and those modeled after “ideal” components. Those modeled after ideal components are referred to as **“virtual” components**. There is a broad selection of virtual components available, as shown in Figure



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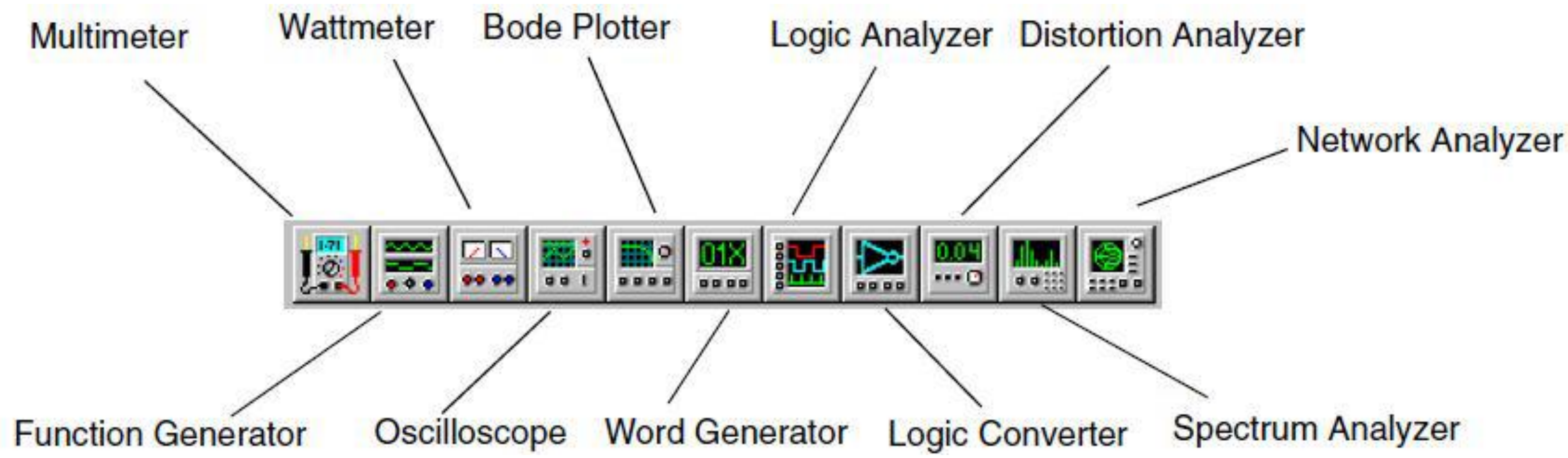
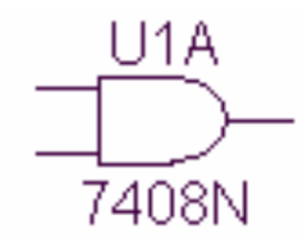
Measurement Devices

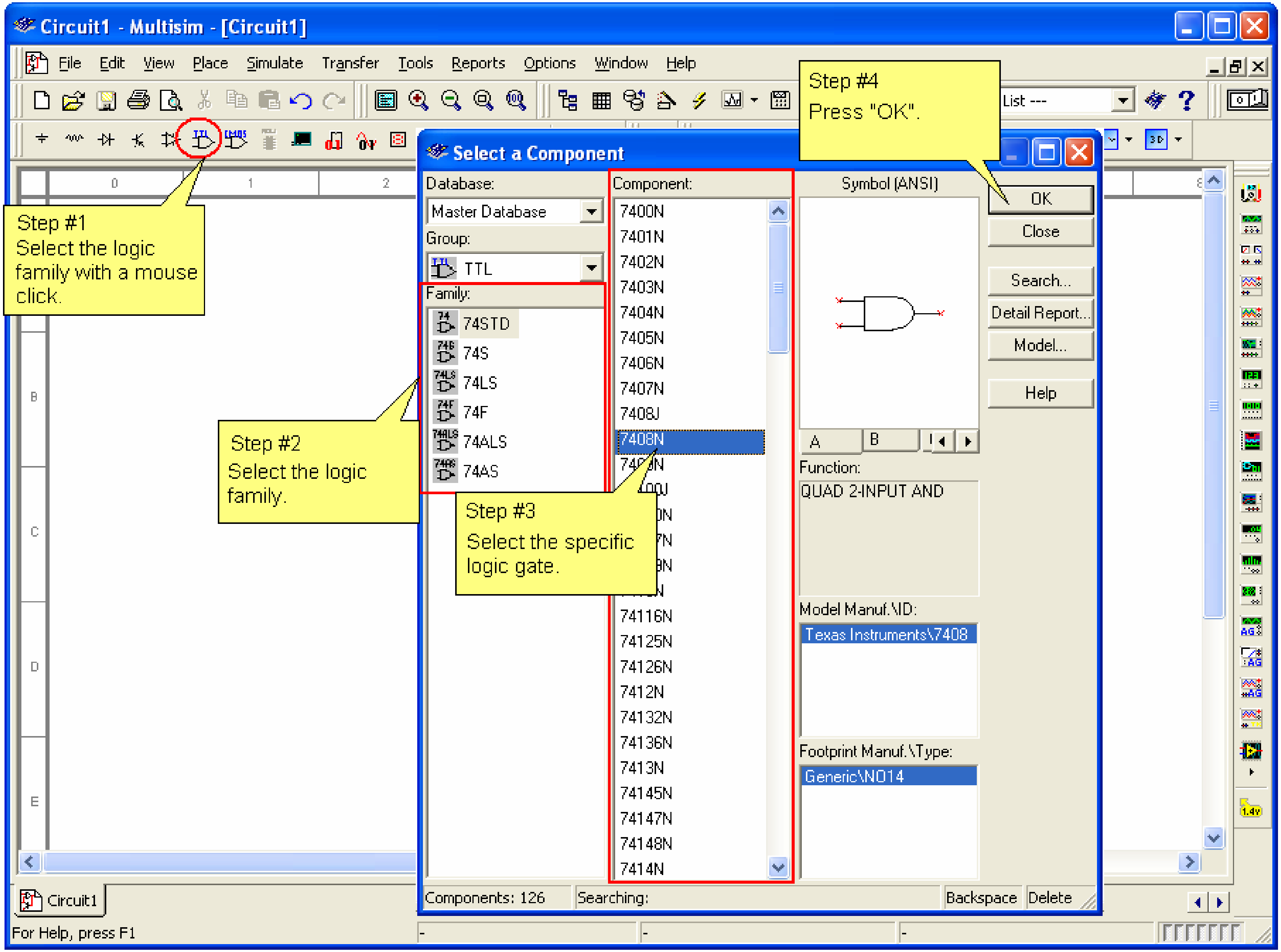
- Multimeter
- Function Generator
- Oscilloscope and etc..



MultiSim - Logic Gates

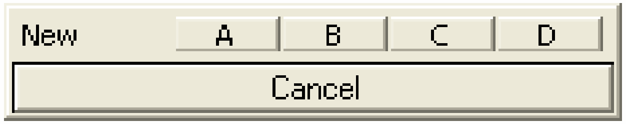
- The logic families are represented for both TTL and CMOS integrated circuits (IC).
- The logic gates are shown as a single gate, not the entire IC. If an IC contains more that one logic gate, the schematic symbol for the gate will contain a letter designation.
- For example, the **7408N is a quad 2-input AND gate.**
- The logic gates located inside the 7408N are designated A, B, C, and D.
- The user selects either the TTL or CMOS family of integrated circuits. Then the IC family is selected and finally the specific IC.



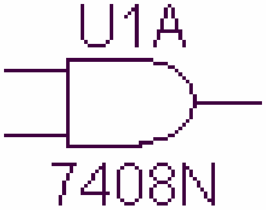

Circuit1 - Multisim - [Circuit1]
File Edit View Place Simulate Transfer Tools Reports Options Window Help
Select a Component
Database:
Master Database
Group:
TTL
Family:
74STD
74S
74LS
74F
74ALS
74AS
Component:
7400N
7401N
7402N
7403N
7404N
7405N
7406N
7407N
7408J
7408N
74116N
74125N
74126N
7412N
74132N
74136N
7413N
74145N
74147N
74148N
7414N
Symbol (ANSI)
A
B
Function:
QUAD 2-INPUT AND
Model Manuf.\ID:
Texas Instruments\7408
Footprint Manuf.\Type:
Generic\N014
OK
Close
Search...
Detail Report...
Model...
Help
Components: 126
Searching:
Backspace
Delete
Circuit1
For Help, press F1
Step #1
Select the logic family with a mouse click.
Step #2
Select the logic family.
Step #3
Select the specific logic gate.
Step #4
Press "OK".

MultiSim - Logic Gates

- Once the IC is selected, MultiSim will prompt the user as to which logic gate inside the IC is to be used.
- The 7408N was selected. The 7408N is a quad 2-input AND gate. Therefore the user will have to choose from gates A through D as shown in Figure

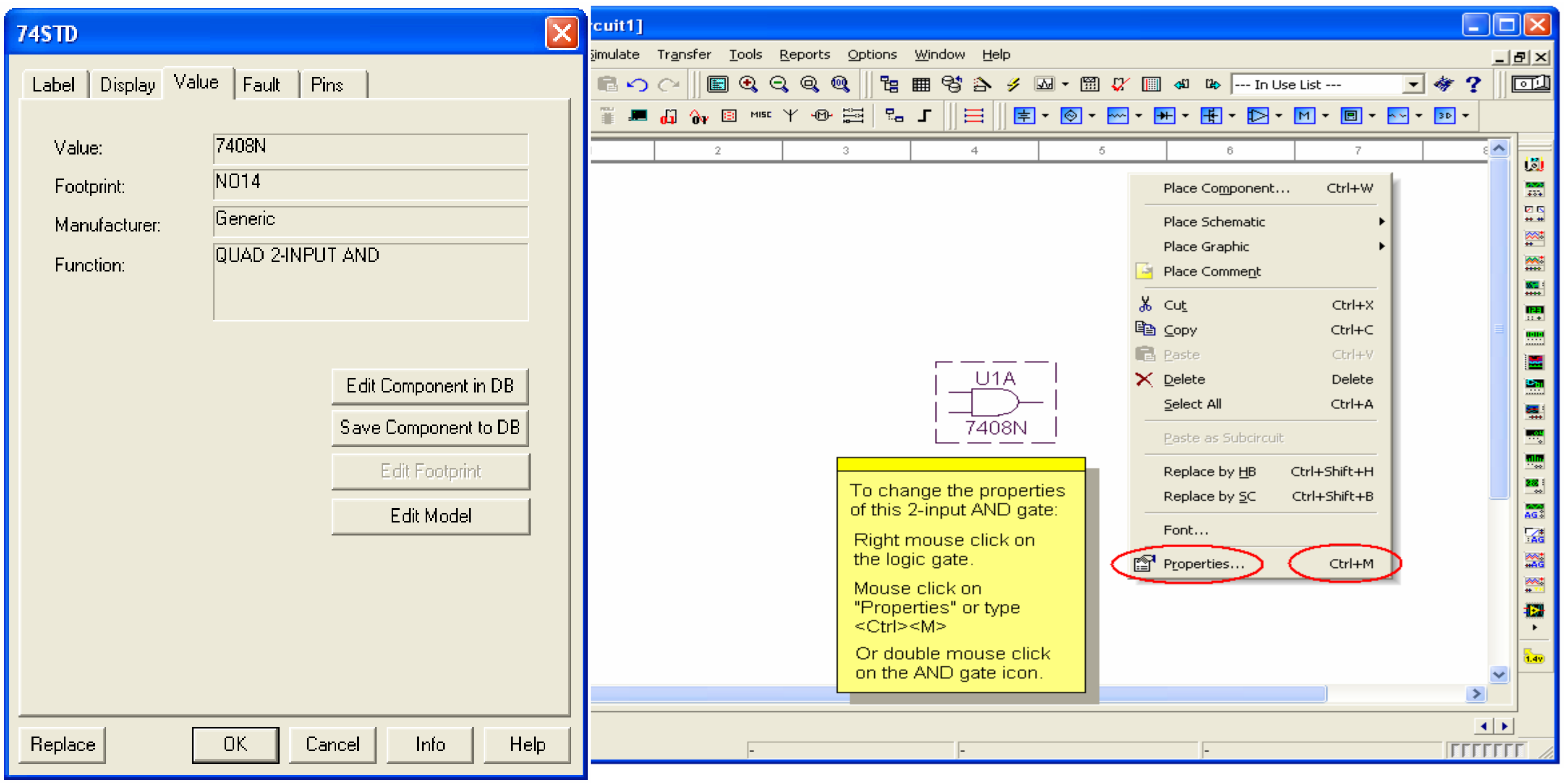


- The schematic symbol for the 2-input AND gate will contain two important pieces of information. The IC number will have a “U” prefix. The IC number will follow, with an A through D suffix designating the specific gate within the IC.



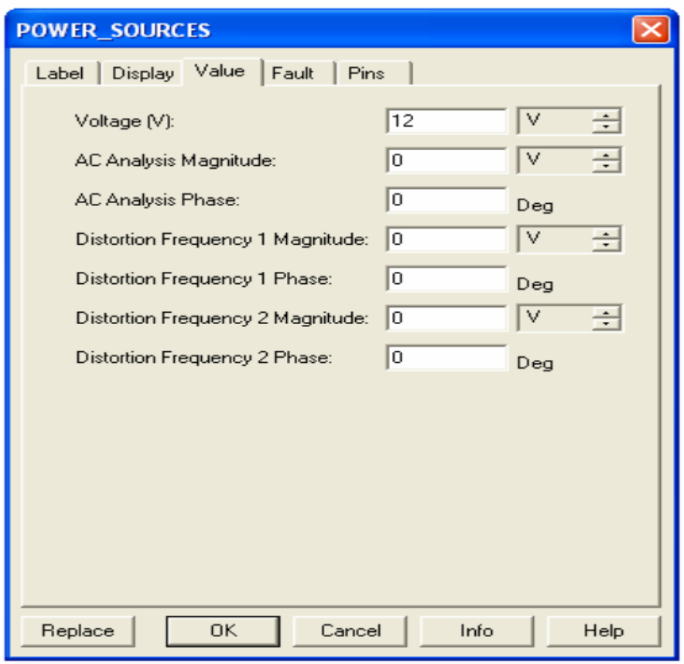
MultiSim - Logic Gates

- To change the properties of the logic gate, right mouse click on the gate and either select “Properties” from the pop-up menu or type <Ctrl><M> or simply double mouse click on the logic gate. This will cause the configuration screen to pop up.



Sources

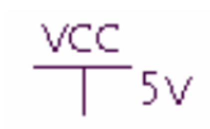
- In the study of Digital Electronics, all of the circuits contain a DC voltage source to supply power to the integrated circuits.
- A significant number of circuits also require a clock signal.
- The DC voltage source can be represented two ways: As a battery and as a voltage supply.

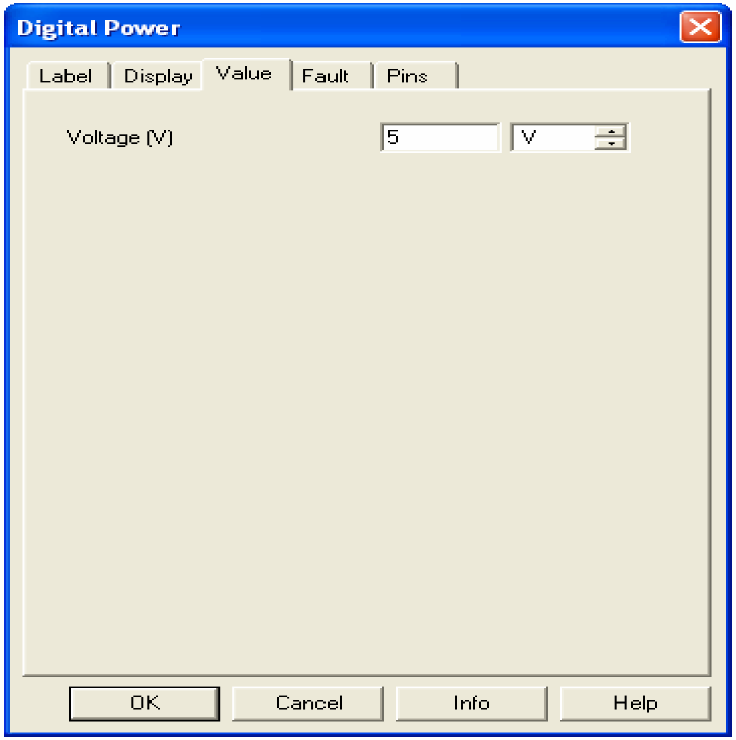


The voltage rating is fully adjustable.
The default value is 12 VDC.
If the component is double clicked, the configuration screen will pop up and the voltage value can be changed.

Sources

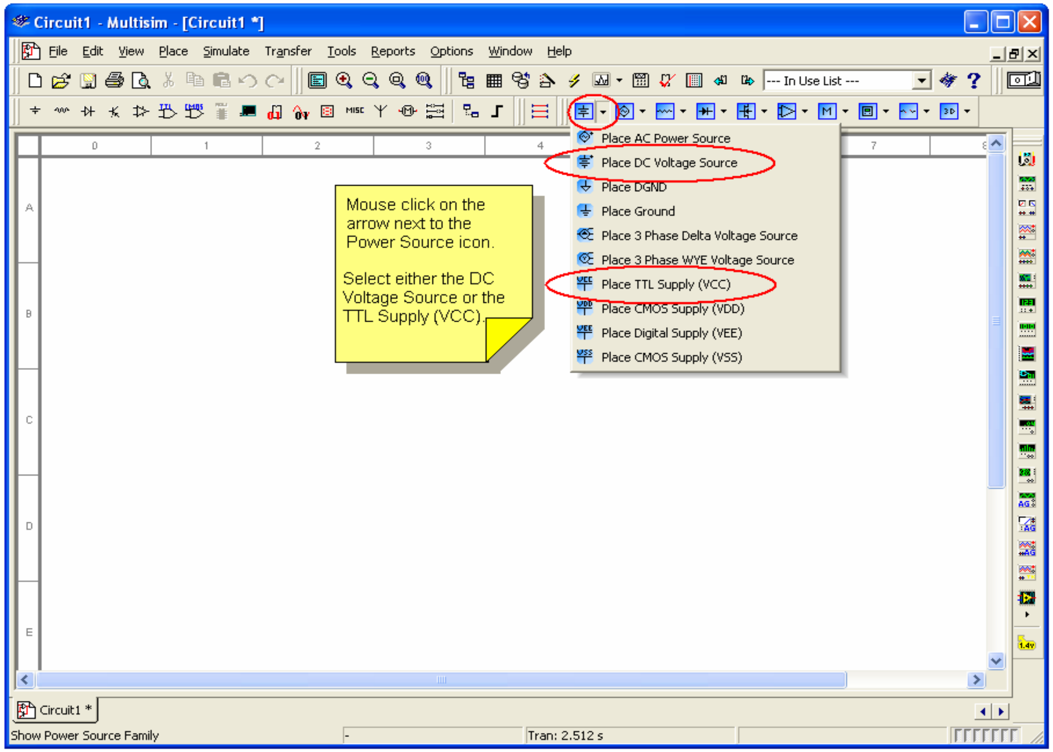
- The voltage rating is fully adjustable for the VCC voltage supply. The default value is +5 VDC.
- If the voltage supply is double clicked, the configuration screen will pop up and the voltage value can be changed.





Sources

- The DC voltage sources are located on the drop down menu under the Power Source icon. To view the Power Source drop down menu, mouse click on the arrow next to the Power Source icon. The DC voltage sources can then be selected as shown in Figure



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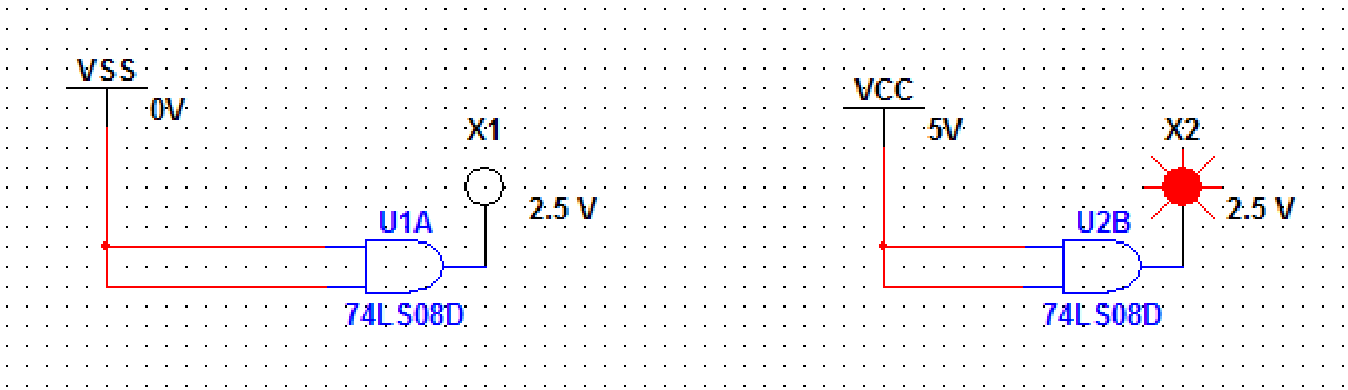
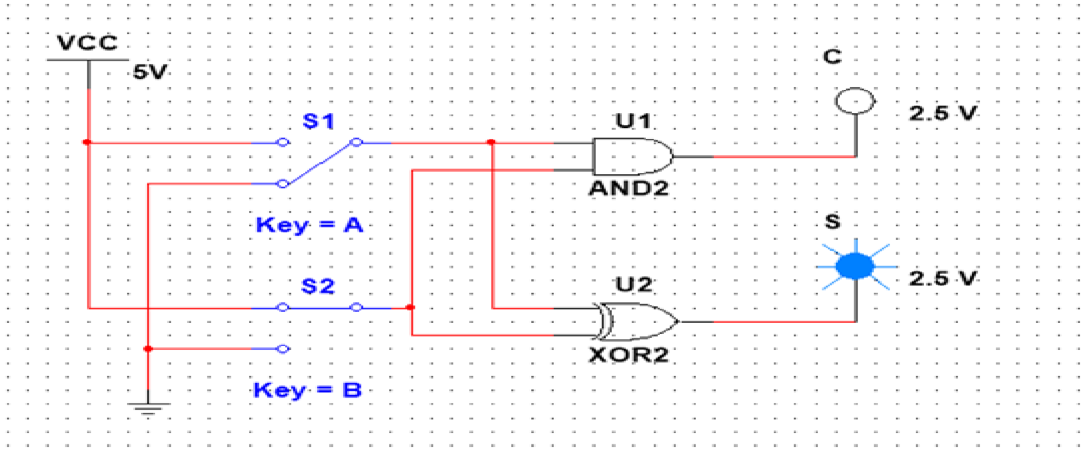
Example

- Construct the circuits as shown in the figure below and observe the response using the simulator.
- Replace the 74LS08D gate with a 74LS03D in both the above circuits and observe the response.



SPST switch

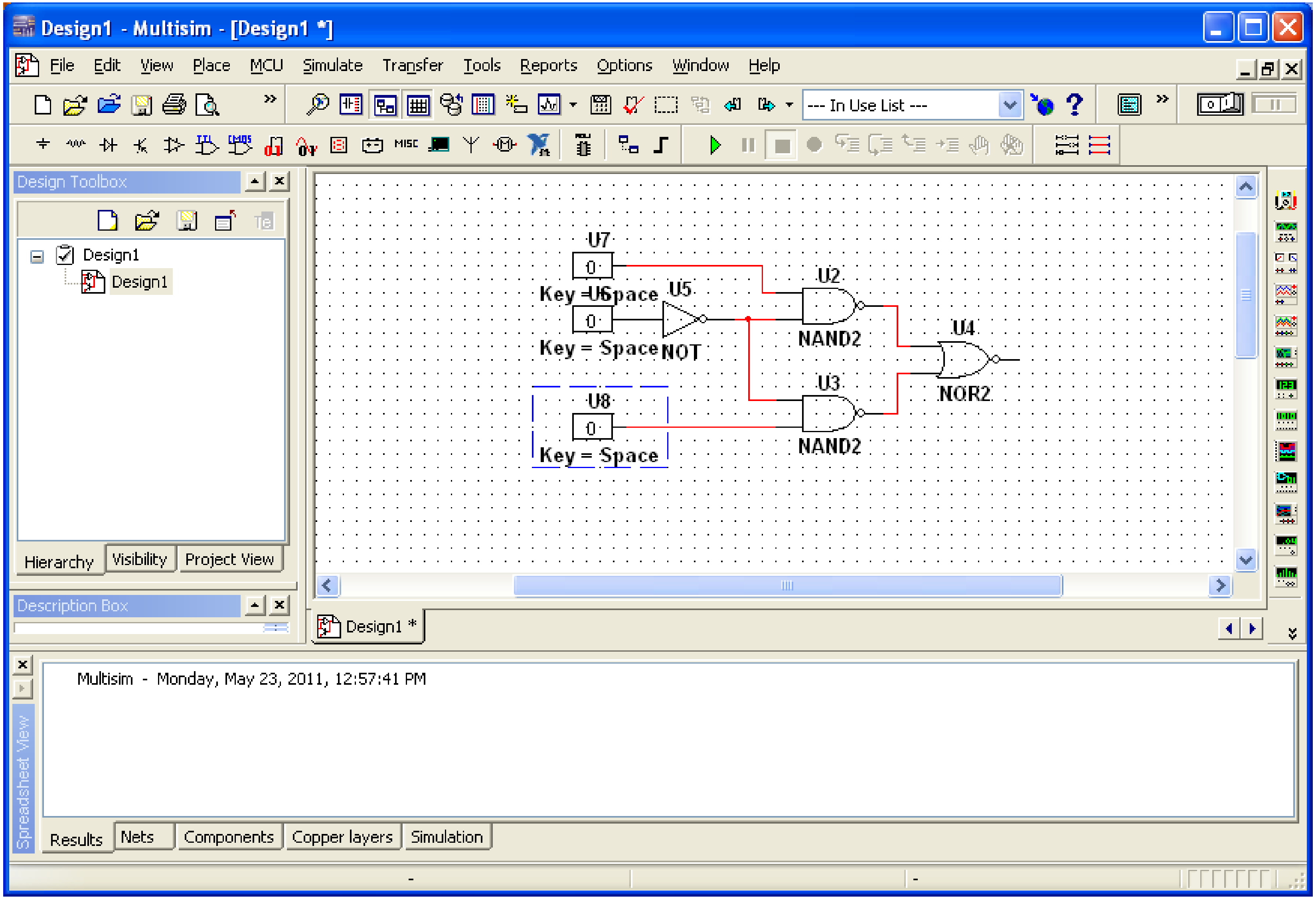
- This **single-pole double-throw switch** can be toggled on and off as described in Interactive components.
- To toggle the switch open or closed using the **keyboard, press the key that you entered in Key for toggle.** To toggle the switch open or closed using the mouse, hover the cursor over the switch's push button and click when the push button takes on a thickened appearance.



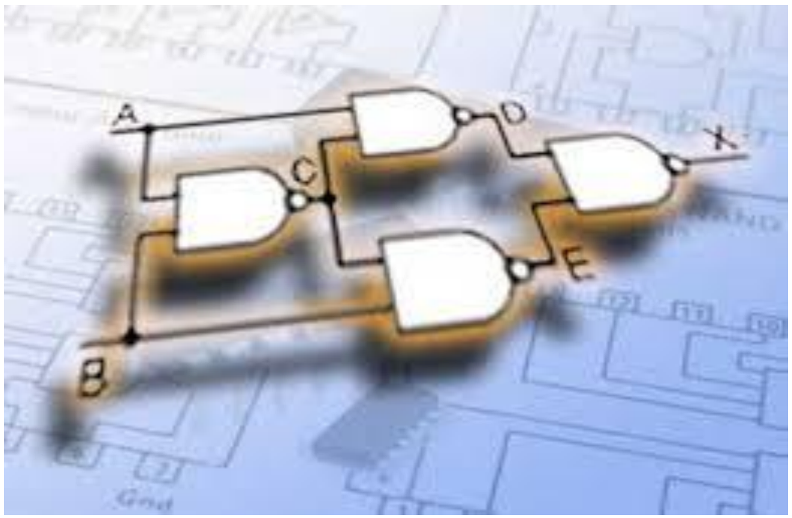
Digital Sources

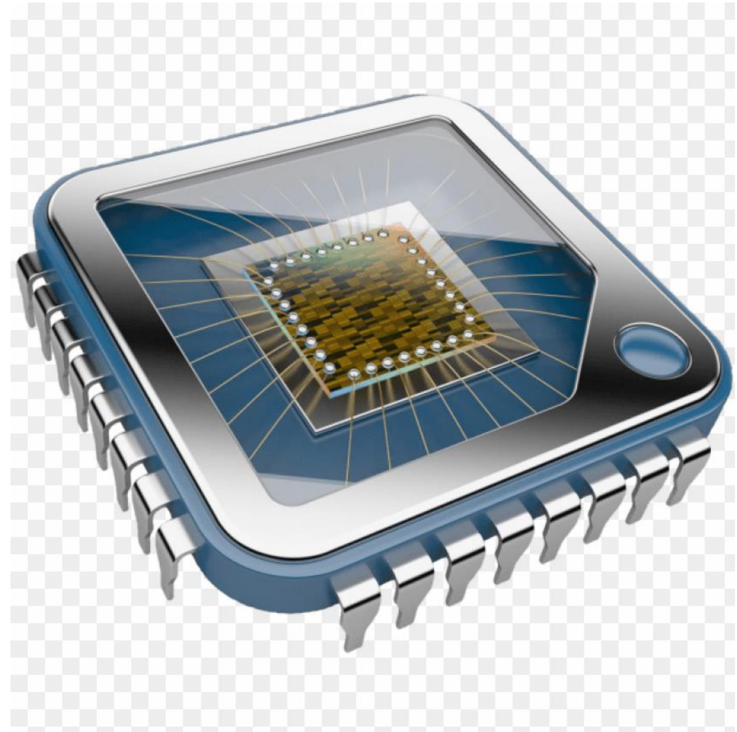
- There are three basic digital sources:
- **1. DIGITAL_CONSTANT** – this is a box with a **constant logic 1 or 0 output,** and would be used where the logic values is not to be changed during simulation. To change the output value, right click on the box, select Properties, select the desired value on the Value tab, and click the OK button.
- **2. INTERACTIVE_DIGITAL_CONSTANT** – this is a clickable box that can be connected to a circuit input. Clicking on the box toggles its output between 0 and 1. This can be used to interactively change a circuit input during simulation.
- **3. DIGITAL_CLOCK –** this is a box that produces a **repeating pulse train (square waveform), oscillating between 0 and 1 at a specified frequency.** To set the frequency and duty cycle, right click on the box, select Properties, select the desired frequency and duty cycle value on the Value tab, and click the OK button.

Digital Sources



Thank You





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