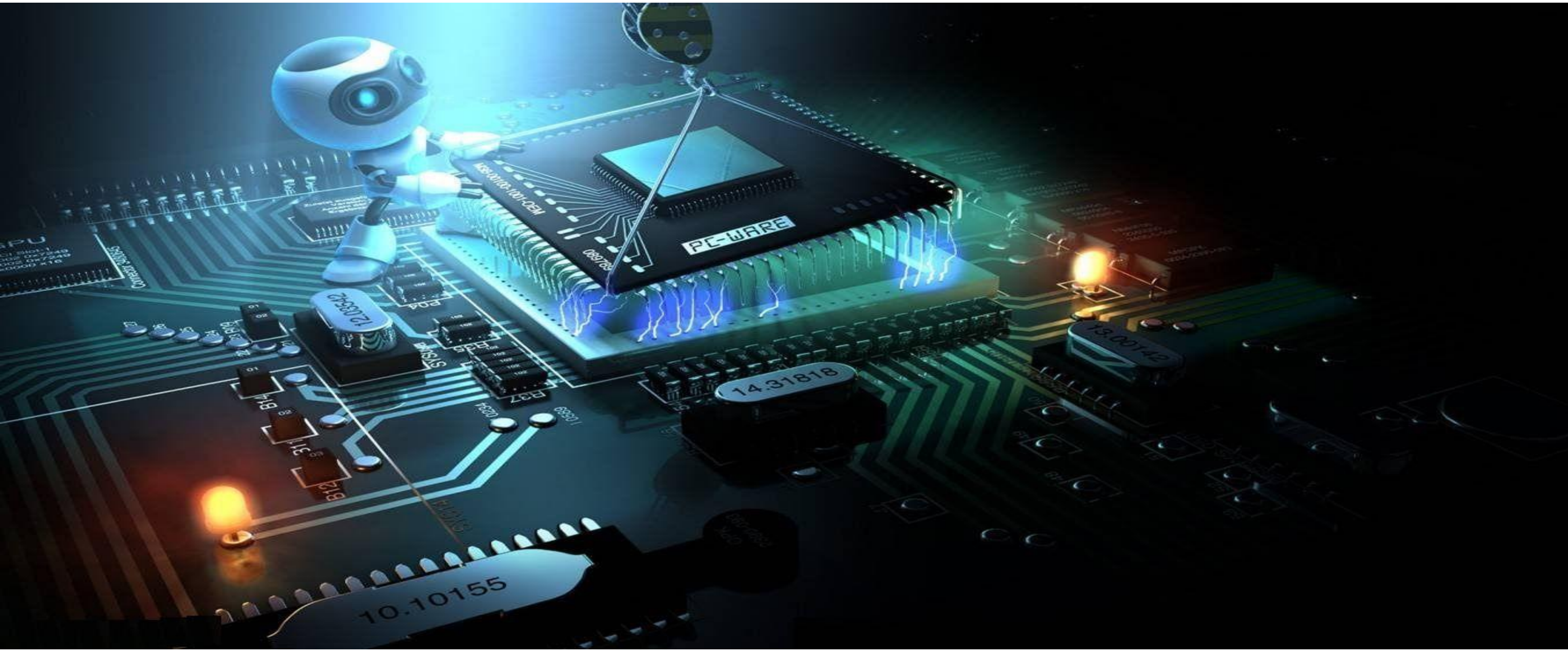
جامعة القاهرة الجديدة التكنولوجية

Fall 2022

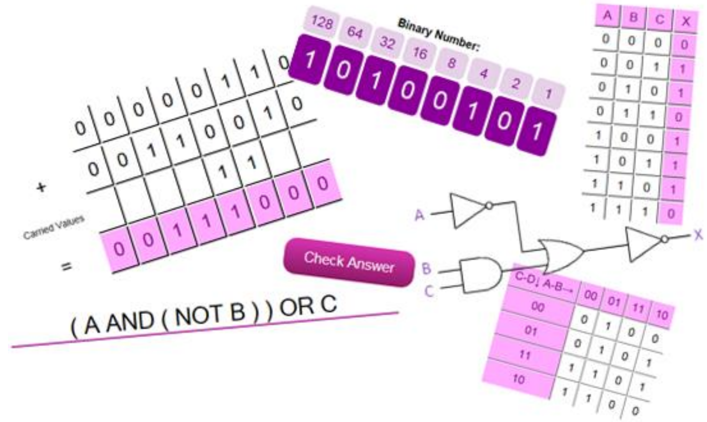
(3)



Digital Engineering

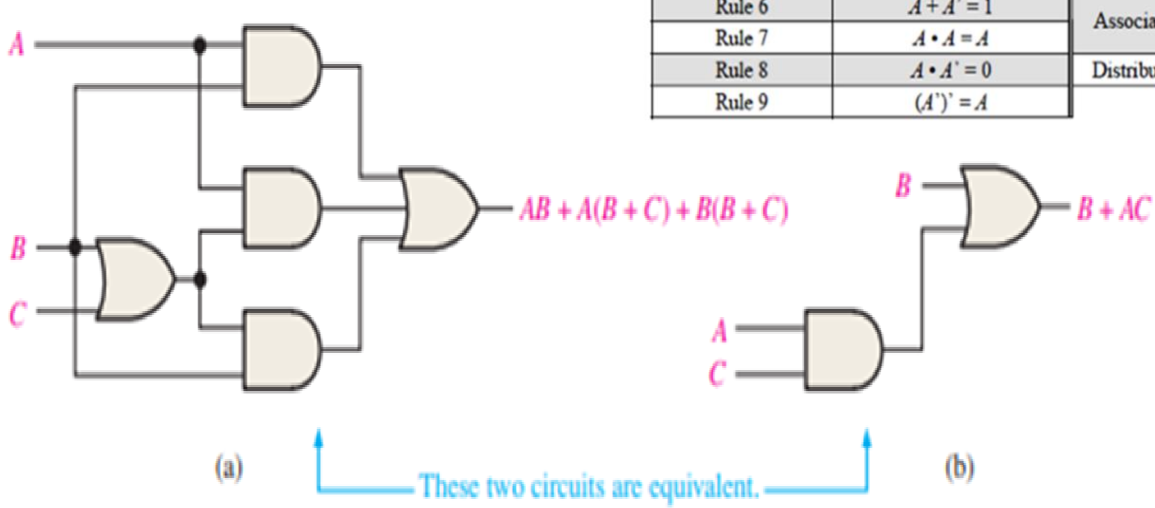
Dr. Hatem Yousry

Agenda



- **Basic Logic Theorems.**
- **DeMorgan's Theorems.**
- **Algebraic Manipulation.**
- **Logic Simplification.**

Reference	Rule or Law	Reference	Rule or Law
Rule 1	$A + 0 = A$	Rule 10	$A + AB = A$
Rule 2	$A + 1 = 1$	Rule 11	$A + A'B = A$
Rule 3	$A \bullet 0 = 0$	Rule 12	$(A + B)(A + C) = A + BC$
Rule 4	$A \bullet 1 = A$	Commutative Law	$A + B = B + A$
Rule 5	$A + A = A$		$AB = BA$
Rule 6	$A + A' = 1$	Associative Law	$A + (B + C) = (A + B) + C$
Rule 7	$A \bullet A = A$		$A(BC) = (AB)C$
Rule 8	$A \bullet A' = 0$	Distributive Law	$A(B + C) = AB + AC$
Rule 9	$(A')' = A$		



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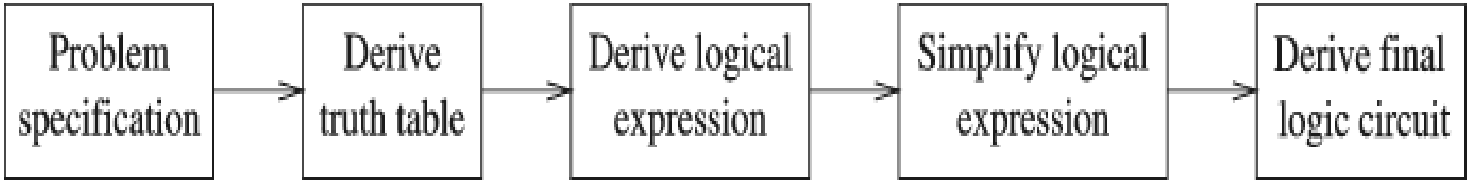
Logic Circuit Design Process

- **Problem specification.**
- **Truth table derivation.**
- **Derivation of logical expression.**
- **Simplification of logical expression.**
- **Implementation.**



Conversion from Decimal to a system with base R

A decimal number can be converted into its equivalent in base R using the following procedure:

Step 1: Perform the integer division of the decimal number by R and record the remainder.

e.g. if the number is 70 and the base is 4 then $70/4 = 17 + 2/4$

Step 2: Replace the decimal number with the result of the division in step 1. Repeat step 1, until a zero result is found.

e.g. $17/4 = 4 + 1/4$

$4/4 = 1 + 0/4$

$1/4 = 0 + 1/4$

Step 3: The number is formed by reading the remainders in reversed order.

e.g. $(70)_{10} = (1012)_4$

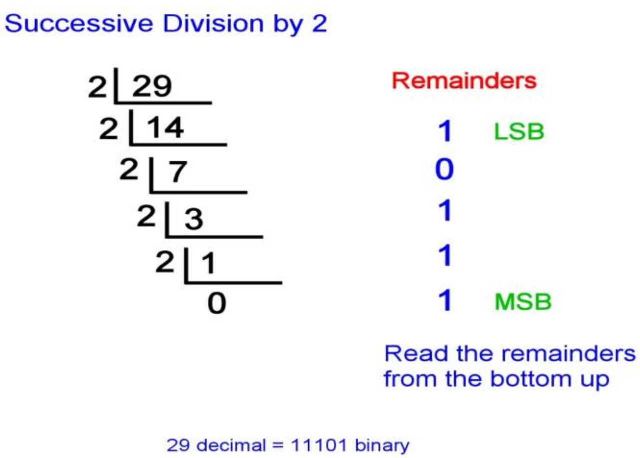
Conversion from decimal to binary

- Divide the number by 2 obtaining quotient and remainder
- Divide the new quotient by 2 obtaining **quotient and remainder**
- Repeat until quotient is 0
- The binary number digits are the remainder digits in reverse order

$(24)_{10} = (?)_2$

	q	r
24/2	12	0
12/2	6	0
6/2	3	0
3/2	1	1
1/2	0	1

$(24)_{10} = (11000)_2$



2^4	2^3	2^2	2^1	2^0	
16	8	4	2	1	
0	0	0	0	0	0 0

Conversion from decimal to binary

Binary **Decimal**

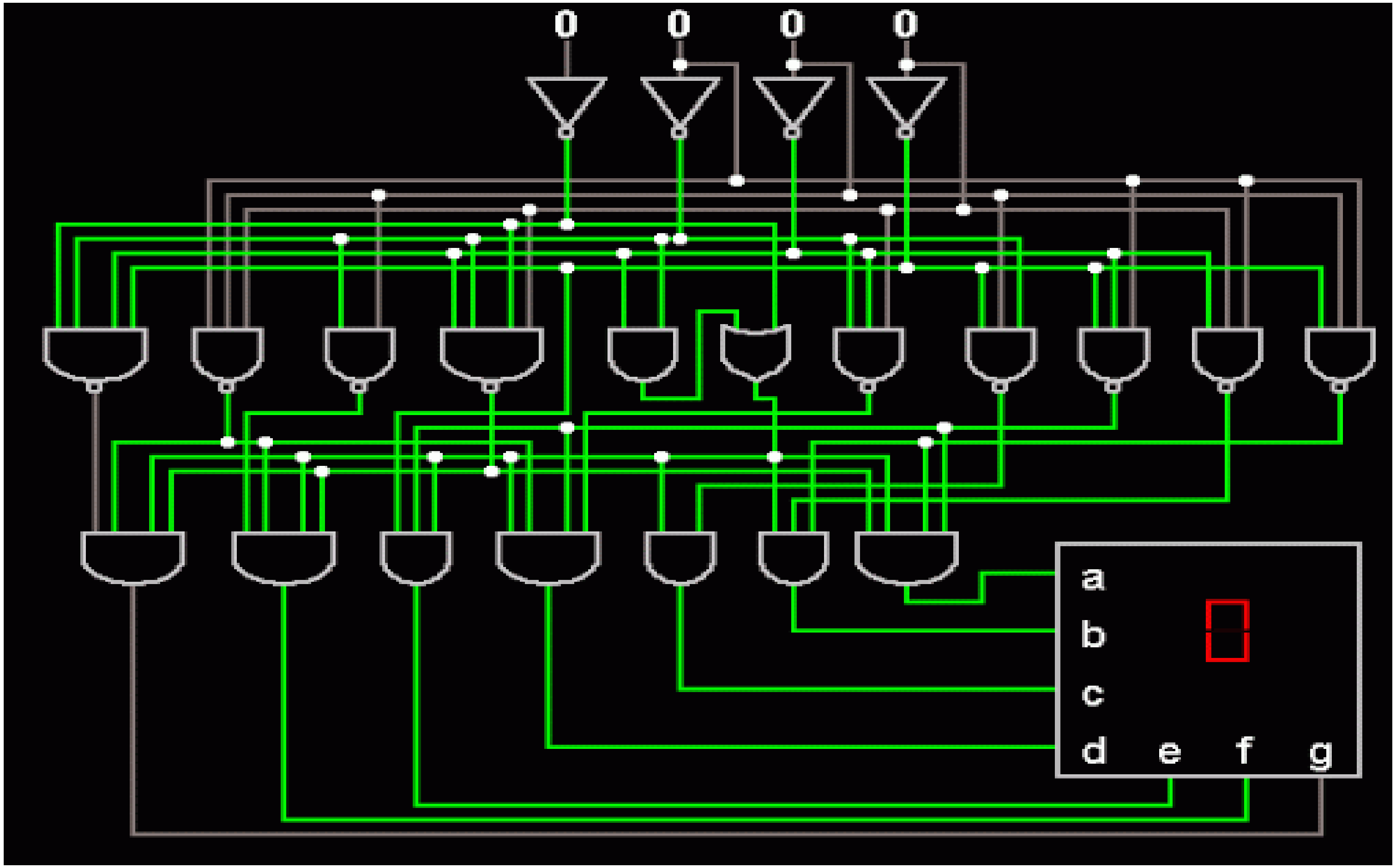
MSB **LSB**

00000000 = 000

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0 \qquad 10^2\ 10^1\ 10^0$

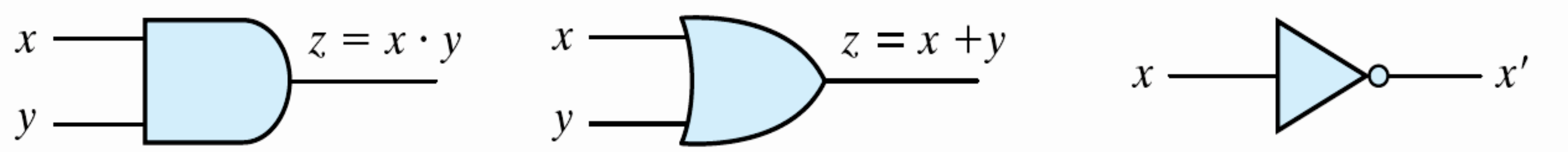
$0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0 + 0 + 0$

Conversion from decimal to binary

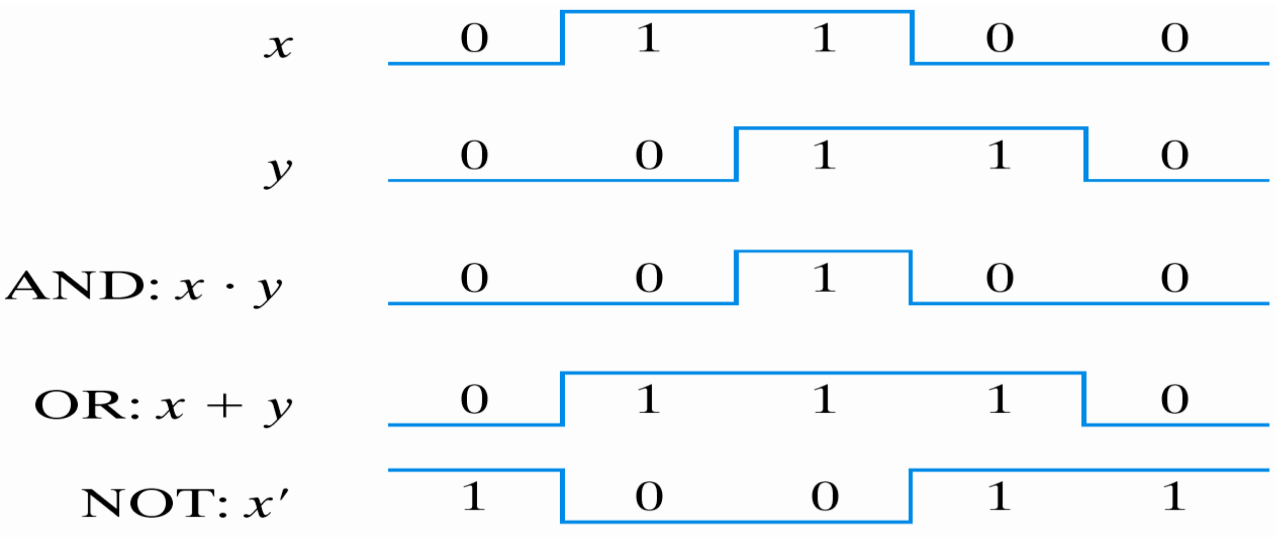


Binary Logic

- Logic gates
  - Graphic Symbols and Input-Output Signals for Logic gates:

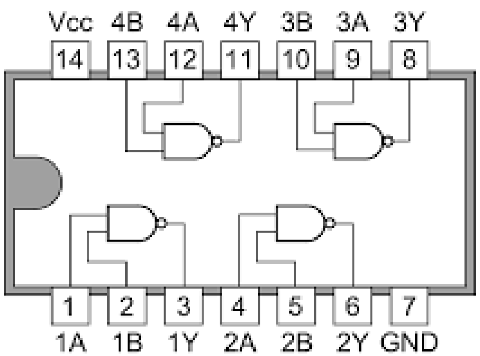


(a) Two-input AND gate (b) Two-input OR gate (c) NOT gate or inverter

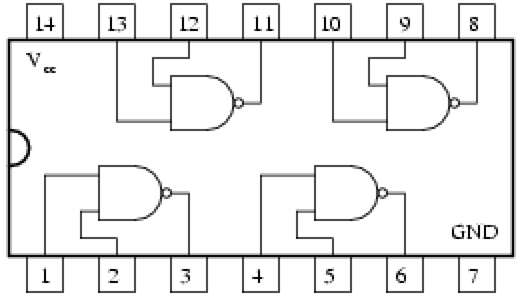


Logic Chips

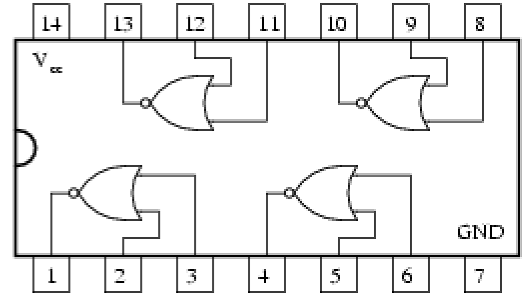
7400 Quad 2-input NAND Gates



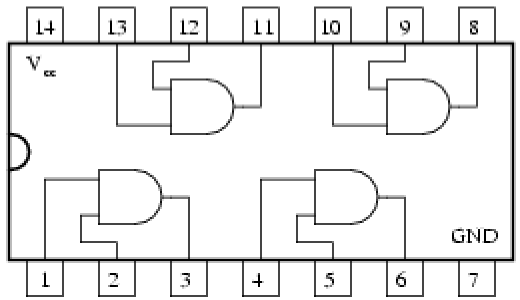
5400/7400
Quad NAND gate



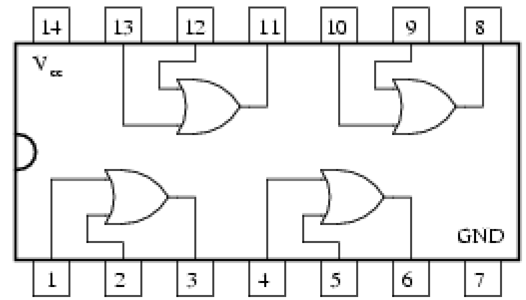
5402/7402
Quad NOR gate



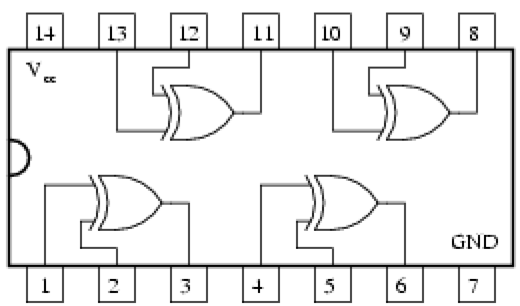
5408/7408
Quad AND gate



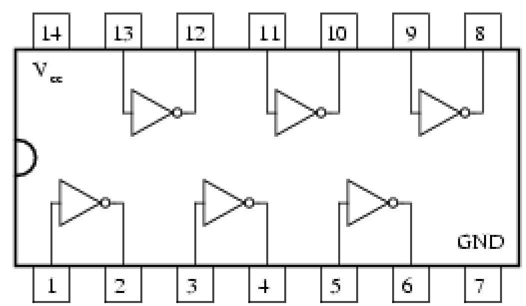
5432/7432
Quad OR gate



5486/7486
Quad XOR gate



5404/7404
Hex inverter



Boolean Algebra

- Complement
  - $x+x'=1 \rightarrow 0+0'=0+1=1;\ 1+1'=1+0=1$
  - $x.x'=0 \rightarrow 0.\ 0'=0.\ 1=0;\ 1.\ 1'=1.\ 0=0$
- Duality
  - Form the dual of the expression
  - $a + (bc) = (a + b)(a + c)$
  - Following the replacement rules…
  - $a(b + c) = ab + ac$
- Absorption Property (Covering)
  - $x + xy = x$

x	y	xy	$x+xy$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

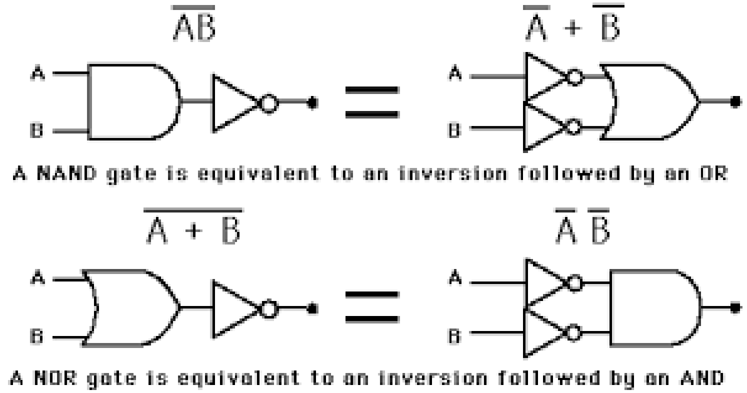
Basic Logic Theorems

Reference	Rule or Law
Rule 1	$A + 0 = A$
Rule 2	$A + 1 = 1$
Rule 3	$A \bullet 0 = 0$
Rule 4	$A \bullet 1 = A$
Rule 5	$A + A = A$
Rule 6	$A + A' = 1$
Rule 7	$A \bullet A = A$
Rule 8	$A \bullet A' = 0$
Rule 9	$(A')' = A$

Reference	Rule or Law
Rule 10	$A + AB = A$
Rule 11	$A + A'B = A$
Rule 12	$(A + B)(A + C) = A + BC$
Commutative Law	$A + B = B + A$ $AB = BA$
Associative Law	$A + (B + C) = (A + B) + C$ $A(BC) = (AB)C$
Distributive Law	$A(B + C) = AB + AC$

DeMorgan’s Theorem

- $(x + y)' = x'y'$
- $(xy)' = x' + y'$
- By means of truth table



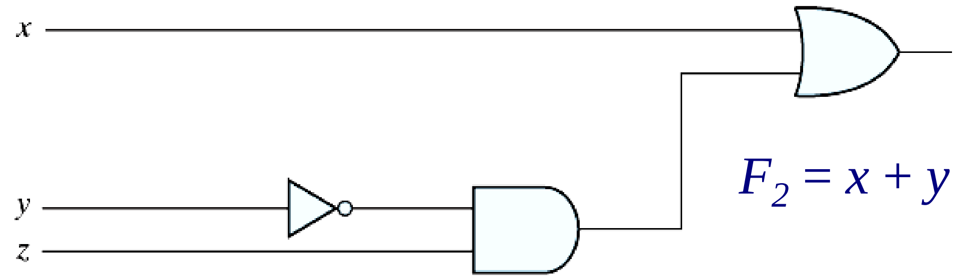
x	y	x'	y'	$x+y$	$(x+y)'$	$x'y'$	xy	$x'+y'$	$(xy)'$
0	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	0	0	1	1
1	0	0	1	1	0	0	0	1	1
1	1	0	0	1	0	0	1	0	0

Examples

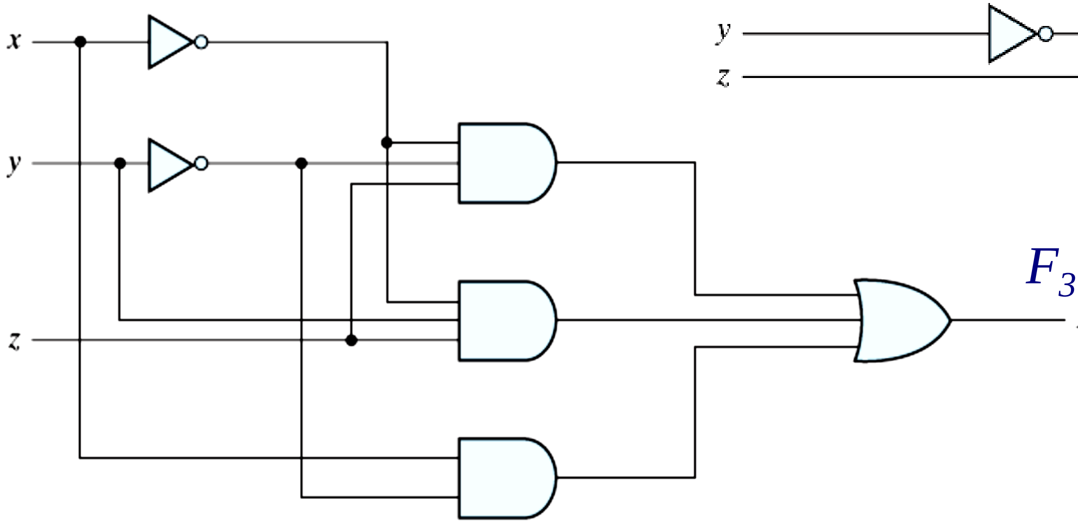
- $F_1 = x\,y\,z'$
- $F_2 = x + y'z$
- $F_3 = x'\,y'\,z + x'\,y\,z + x\,y'$
- $F_4 = x\,y' + x'\,z$

x	y	z	F_1	F_2	F_3	F_4
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0

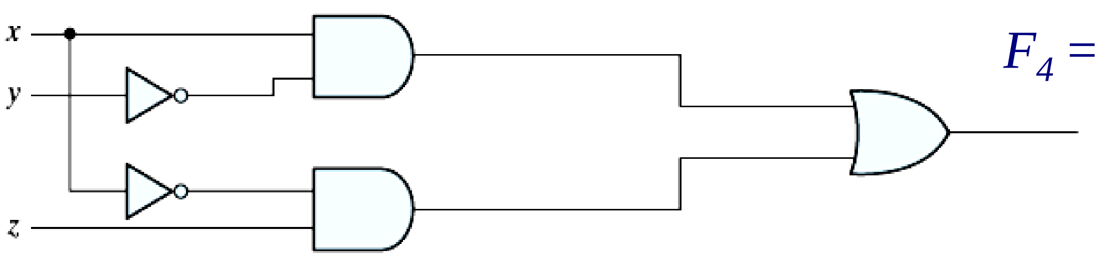
Boolean Functions



$$F_2 = x + y'z$$



$$F_3 = x'y'z + x'yz + xy'$$



$$F_4 = xy' + x'z$$

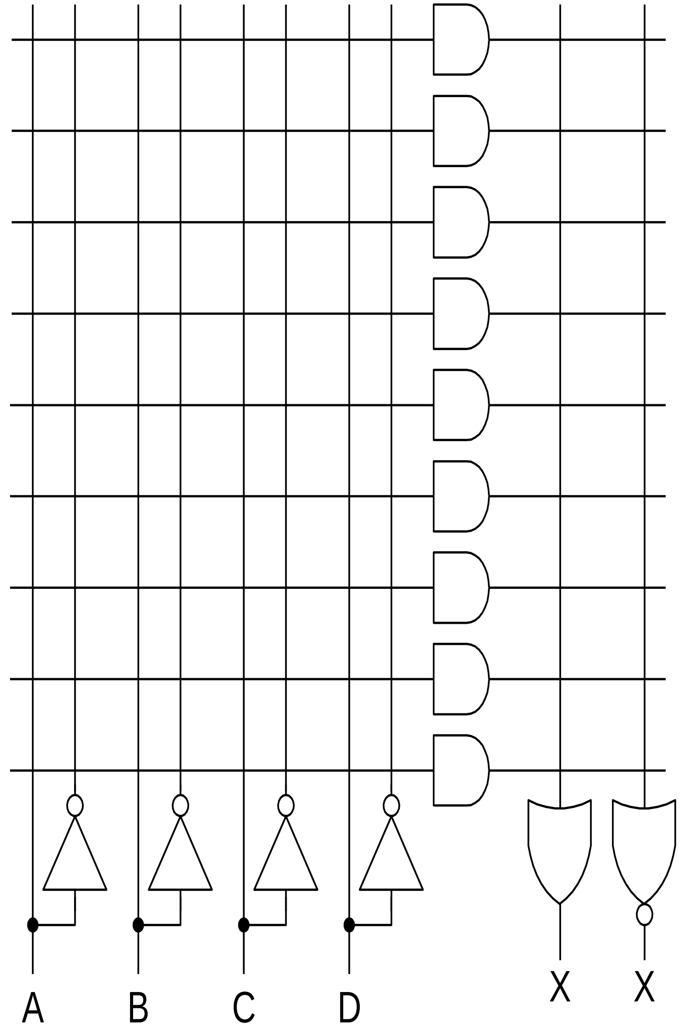
Name:

	Inputs				Output		
	A	B	C	D			
0	0	0	0	0			
1	0	0	0	1			
2	0	0	1	0			
3	0	0	1	1			
4	0	1	0	0			
5	0	1	0	1			
6	0	1	1	0			
7	0	1	1	1			
8	1	0	0	0			
9	1	0	0	1			
10	1	0	1	0			
11	1	0	1	1			
12	1	1	0	0			
13	1	1	0	1			
14	1	1	1	0			
15	1	1	1	1			

X =

Logic Diagram

X =



Boolean Expressions

Boolean Expressions for the 16 Functions of Two Variables

Boolean Functions	Operator Symbol	Name	Comments
$F_0 = 0$		Null	Binary constant 0
$F_1 = xy$	$x \cdot y$	AND	x and y
$F_2 = xy'$	x/y	Inhibition	x, but not y
$F_3 = x$		Transfer	x
$F_4 = x'y$	y/x	Inhibition	y, but not x
$F_5 = y$		Transfer	y
$F_6 = xy' + x'y$	$x \oplus y$	Exclusive-OR	x or y, but not both
$F_7 = x + y$	$x + y$	OR	x or y
$F_8 = (x + y)'$	$x \downarrow y$	NOR	Not-OR
$F_9 = xy + x'y'$	$(x \oplus y)'$	Equivalence	x equals y
$F_{10} = y'$	y'	Complement	Not y
$F_{11} = x + y'$	$x \subset y$	Implication	If y, then x
$F_{12} = x'$	x'	Complement	Not x
$F_{13} = x' + y$	$x \supset y$	Implication	If x, then y
$F_{14} = (xy)'$	$x \uparrow y$	NAND	Not-AND
$F_{15} = 1$		Identity	Binary constant 1

Algebraic Manipulation

- **To minimize Boolean expressions**
  - Literal: A primed or unprimed variable (an input to a gate).
  - Term: An implementation with a gate.
  - The minimization of the number of literals and the number of terms → a circuit with less equipment

$$A + \overline{A}B$$

$\downarrow$ Applying the previous rule to expand A term $A + AB = A$

$$A + AB + \overline{A}B$$

$\downarrow$ Factoring B out of 2nd and 3rd terms

$$A + B(A + \overline{A})$$

$\downarrow$ Applying identity $A + \overline{A} = 1$

$$A + B(1)$$

$\downarrow$ Applying identity $1A = A$

$$A + B$$

Example

1. $x(x'+y) = xx' + xy = 0+xy = xy$

2. $x+x'y = (x+x')(x+y) = 1\ (x+y) = x+y$

3. $(x+y)(x+y') = x+xy+xy'+yy' = x(1+y+y') = x$

4. $xy + x'z + yz = xy + x'z + yz(x+x') = xy + x'z + yzx + yzx' = xy(1+z) + x'z(1+y) = xy + x'z$

5. $(x+y)(x'+z)(y+z) = (x+y)(x'+z)$, **by duality from function 4. (*consensus theorem* with duality)**

Rules of Boolean Algebra

- **12 basic rules** that are useful in manipulating and simplifying Boolean expressions. Rules 1 through 9 will be viewed in terms of their application to logic gates.
- Rules 10 through 12 will be derived in terms of the simpler rules

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

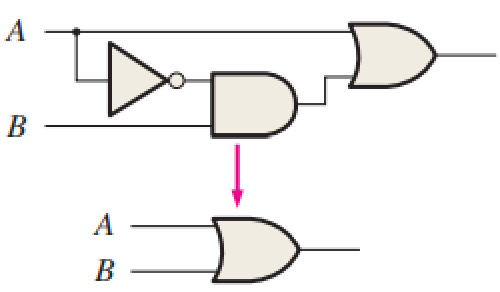
Rule 11

Rule 11: $A + \overline{A}B = A + B$ This rule can be proved as follows:

$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B && \text{Rule 10: } A = A + AB \\
&= (AA + AB) + \overline{A}B && \text{Rule 7: } A = AA \\
&= AA + AB + A\overline{A} + \overline{A}B && \text{Rule 8: adding } A\overline{A} = 0 \\
&= (A + \overline{A})(A + B) && \text{Factoring} \\
&= 1 \cdot (A + B) && \text{Rule 6: } A + \overline{A} = 1 \\
&= A + B && \text{Rule 4: drop the 1}
\end{aligned}
$$

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



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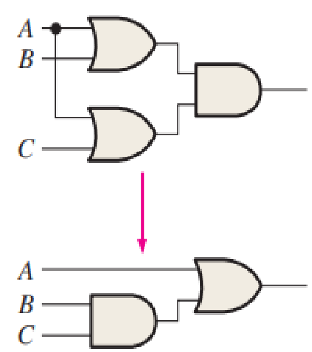
Rule 12

Rule 12: $(A + B)(A + C) = A + BC$ This rule can be proved as follows:

$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

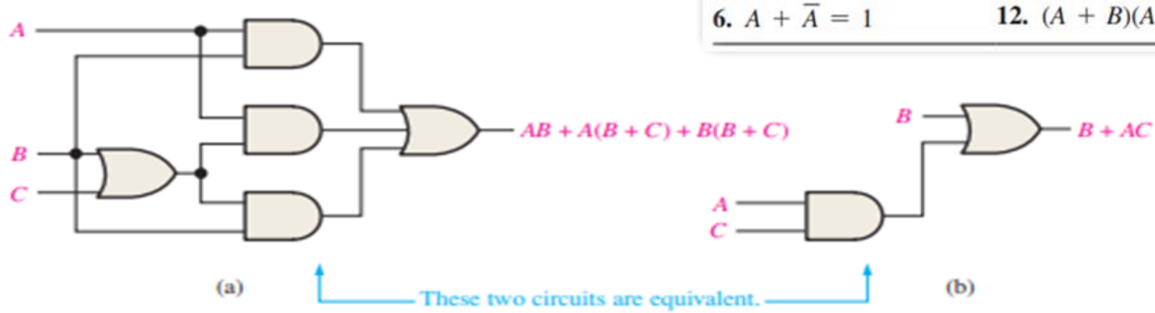
equal



Logic Simplification

Basic rules of Boolean algebra.

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$



- Using Boolean algebra techniques, simplify this expression:
- **AB + A(B + C) + B(B + C)**
- Solution; The following is not necessarily the only approach.
- **Step 1:** Apply the **distributive law** to the second and third terms in the expression, as follows: AB + AB + AC + BB + BC
- **Step 2:** Apply rule 7 (BB = B) to the fourth term. AB + AB + AC + B + BC.
- **Step 3:** Apply rule 5 (AB + AB = AB) to the first two terms. AB + AC + B + BC.
- **Step 4:** Apply rule 10 (B + BC = B) to the last two terms. AB + AC + B.
- **Step 5:** Apply rule 10 (AB + B = B) to the first and third terms. B + AC At this point the expression is simplified as much as possible.
- Once you gain experience in applying Boolean algebra, you can often combine many individual steps.

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

Solution

Step 1: Apply the distributive law to the terms within the brackets.

$$(A\overline{B}C + A\overline{B}BD + \overline{A}\,\overline{B})C$$

Step 2: Apply rule 8 ($\overline{B}B = 0$) to the second term within the parentheses.

$$(A\overline{B}C + A \cdot 0 \cdot D + \overline{A}\,\overline{B})C$$

Step 3: Apply rule 3 ($A \cdot 0 \cdot D = 0$) to the second term within the parentheses.

$$(A\overline{B}C + 0 + \overline{A}\,\overline{B})C$$

Step 4: Apply rule 1 (drop the 0) within the parentheses.

$$(A\overline{B}C + \overline{A}\,\overline{B})C$$

Step 5: Apply the distributive law.

$$A\overline{B}CC + \overline{A}\,\overline{B}C$$

Step 6: Apply rule 7 ($CC = C$) to the first term.

$$A\overline{B}C + \overline{A}\,\overline{B}C$$

Step 7: Factor out $\overline{B}C$.

$$\overline{B}C(A + \overline{A})$$

Step 8: Apply rule 6 ($A + \overline{A} = 1$).

$$\overline{B}C \cdot 1$$

Step 9: Apply rule 4 (drop the 1).

$$\overline{B}C$$

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Simplify the following Boolean expression:

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

Solution

Step 1: Factor BC out of the first and last terms.

$$BC(\overline{A} + A) + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C$$

Step 2: Apply rule 6 ($\overline{A} + A = 1$) to the term in parentheses, and factor $A\overline{B}$ from the second and last terms.

$$BC \cdot 1 + A\overline{B}(\overline{C} + C) + \overline{A}\,\overline{B}\,\overline{C}$$

Step 3: Apply rule 4 (drop the 1) to the first term and rule 6 ($\overline{C} + C = 1$) to the term in parentheses.

$$BC + A\overline{B} \cdot 1 + \overline{A}\,\overline{B}\,\overline{C}$$

Step 4: Apply rule 4 (drop the 1) to the second term.

$$BC + A\overline{B} + \overline{A}\,\overline{B}\,\overline{C}$$

Step 5: Factor $\overline{B}$ from the second and third terms.

$$BC + \overline{B}(A + \overline{A}\,\overline{C})$$

Step 6: Apply rule 11 ($A + \overline{A}\,\overline{C} = A + \overline{C}$) to the term in parentheses.

$$BC + \overline{B}(A + \overline{C})$$

Step 7: Use the distributive and commutative laws to get the following expression:

$$BC + A\overline{B} + \overline{B}\,\overline{C}$$

Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

Applying DeMorgan's Theorems

- The following procedure illustrates the application of DeMorgan's theorems and Boolean algebra to the specific expression

$$\overline{\overline{A + B\overline{C}} + \overline{D(E + \overline{F})}}$$

Step 1: Identify the terms to which you can apply DeMorgan's theorems, and think of each term as a single variable. Let $\overline{A + B\overline{C}} = X$ and $\overline{D(E + \overline{F})} = Y$.

Step 2: Since $\overline{X + Y} = \overline{X}\,\overline{Y}$,

$$\overline{(\overline{A + B\overline{C}}) + (\overline{D(E + \overline{F})})} = (\overline{\overline{A + B\overline{C}}})(\overline{\overline{D(E + \overline{F})}})$$

Step 3: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the left term (this is not part of DeMorgan's theorem).

$$(\overline{\overline{A + B\overline{C}}})(\overline{\overline{D(E + \overline{F})}}) = (A + B\overline{C})(\overline{\overline{D(E + \overline{F})}})$$

Step 4: Apply DeMorgan's theorem to the second term.

$$(A + B\overline{C})(\overline{D(E + \overline{F})}) = (A + B\overline{C})(\overline{D} + (\overline{\overline{E + \overline{F}}}))$$

Step 5: Use rule 9 ($\overline{\overline{A}} = A$) to cancel the double bars over the $E + \overline{F}$ part of the term.

$$(A + B\overline{C})(\overline{D} + \overline{\overline{E + \overline{F}}}) = (A + B\overline{C})(\overline{D} + E + \overline{F})$$

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Applying DeMorgan's Theorems

- Apply DeMorgan's theorems to each of the following expressions:

  (a) $\overline{(A + B + C)D}$

  (b) $\overline{ABC + DEF}$

  (c) $\overline{A\overline{B} + \overline{C}D + EF}$

(a) Let $A + B + C = X$ and $D = Y$. The expression $\overline{(A + B + C)D}$ is of the form $\overline{XY} = \overline{X} + \overline{Y}$ and can be rewritten as

$$\overline{(A + B + C)D} = \overline{A + B + C} + \overline{D}$$

Next, apply DeMorgan's theorem to the term $\overline{A + B + C}$.

$$\overline{A + B + C} + \overline{D} = \overline{A}\,\overline{B}\,\overline{C} + \overline{D}$$

Applying DeMorgan's Theorems

(a) $\overline{(A + B + C)D}$

(b) $\overline{ABC + DEF}$

(c) $\overline{A\overline{B} + \overline{C}D + EF}$

(b) Let $ABC = X$ and $DEF = Y$. The expression $\overline{ABC + DEF}$ is of the form $\overline{X + Y} = \overline{X}\,\overline{Y}$ and can be rewritten as

$$\overline{ABC + DEF} = (\overline{ABC})(\overline{DEF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{ABC}$ and $\overline{DEF}$.

$$(\overline{ABC})(\overline{DEF}) = (\overline{A} + \overline{B} + \overline{C})(\overline{D} + \overline{E} + \overline{F})$$

(c) Let $A\overline{B} = X$, $\overline{C}D = Y$, and $EF = Z$. The expression $\overline{A\overline{B} + \overline{C}D + EF}$ is of the form $\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$ and can be rewritten as

$$\overline{A\overline{B} + \overline{C}D + EF} = (\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF})$$

Next, apply DeMorgan's theorem to each of the terms $\overline{A\overline{B}}$, $\overline{\overline{C}D}$, and $\overline{EF}$.

$$(\overline{A\overline{B}})(\overline{\overline{C}D})(\overline{EF}) = (\overline{A} + B)(C + \overline{D})(\overline{E} + \overline{F})$$

Complement of a Function

- An interchange of 0's for 1's and 1's for 0's in the value of F
  - By DeMorgan's theorem
  - $(A+B+C)' = (A+X)'$ **let** $B+C = X$

    $= A'X'$ **by (DeMorgan's)**

    $= A'(B+C)'$ **substitute** $B+C = X$

    $= A'(B'C')$ **by (DeMorgan's)**

    $= A'B'C'$ **by (associative)**
- *Generalizations*: a function is obtained by interchanging AND and OR operators and complementing each literal.
  - $(A+B+C+D+ \ldots +F)' = A'B'C'D' \ldots F'$
  - $(ABCD \ldots F)' = A'+ B'+C'+D' \ldots +F'$

Examples

- Example $F_1' = (x'yz' + x'y'z)' = (x'yz')' (x'y'z)' = (x+y'+z)(x+y+z')$
  - $F_2' = [x(y'z'+yz)]' = x' + (y'z'+yz)' = x' + (y'z')' (yz)'$
    $= x' + (y+z) (y'+z')$
    $= x' + yz'+y'z$
- Example : a simpler procedure
  - Take the dual of the function and complement each literal

1. $F_1 = x'yz' + x'y'z$.

The dual of F_1 is $(x'+y+z') (x'+y'+z)$.

Complement each literal: $(x+y'+z)(x+y+z') = F_1'$

2. $F_2 = x(y' z' + yz)$.

The dual of F_2 is $x+(y'+z') (y+z)$.

Complement each literal: $x'+(y+z)(y' +z') = F_2'$

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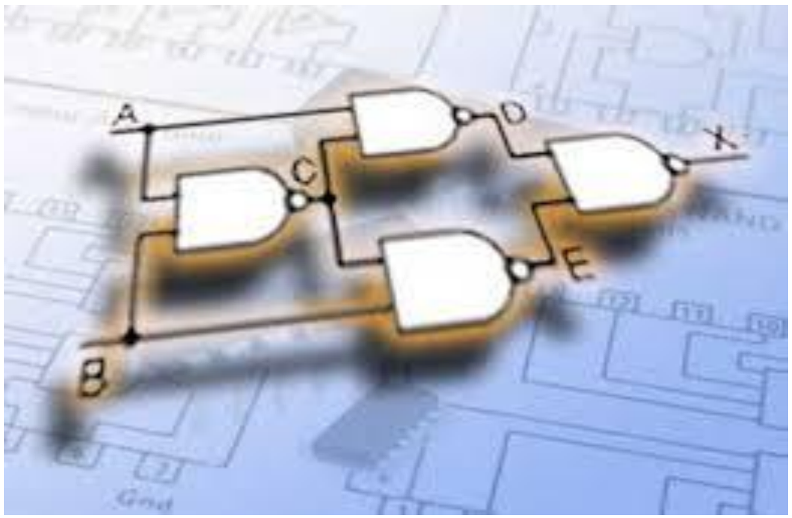
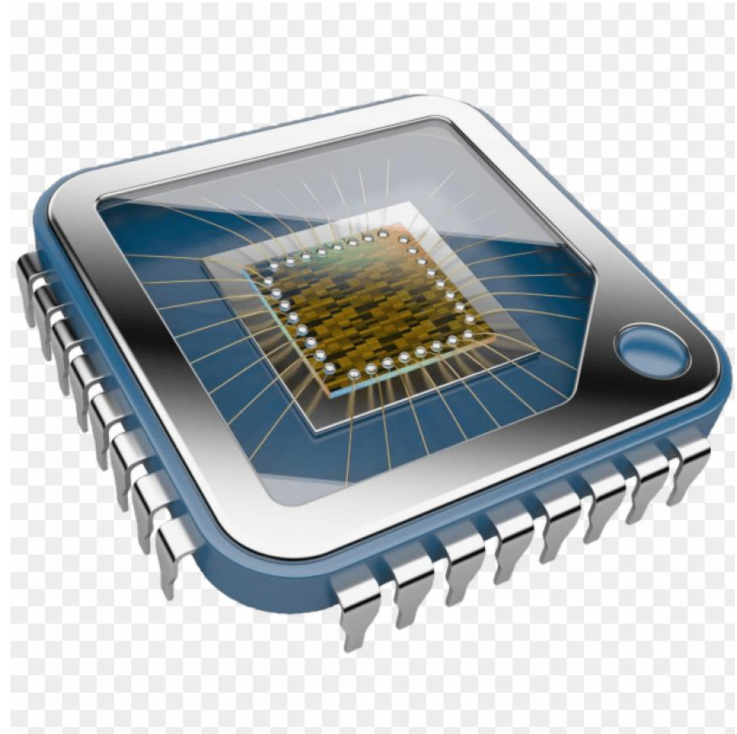
Thank You





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