Robot Mapping

Assignment: Sheet 3, Extended Kalman Filter

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1 Exercise 1: Bayes Filter and Extended Kalman Filter

- Q1 : Describe briefly the two main steps of the Bayes filter in your own words
  - Prediction step: use the motion model to predict the next state x_t given that the robot(system) is at state x_{t-1} and execute the commands u_t, this occurs by convolving the motion model $p(x_t|x_{t-1}, u_t)$ with the prior belief of state x_{t-1}
  - Update step : exploit the information from the sensor/s reading to re-weight our belief about the possible state x_t value
- Q2 : Describe briefly the meaning of the following probability density functions
  - $p(x_t|x_{t-1}, u_t)$: the probability of ending up at state x_t given that the robot(system) is at state x_{t-1} and execute the command u_t; this probability distribution is over the state x_t random variable
  - $p(z_t|x_t)$: the probability that the sensor would produce a reading of z_t at the state x_t; this probability distribution is over the measurement z_t random variable
  - $bel(x_t)$: the probability (internal belief) that the robot is at pose x_t; given all the history (poses,measurements and applied commands) up to time t.
- Q3 : Specify the distributions that correspond to the above mentioned three terms in the EKF
  - $p(x_t|x_{t-1}, u_t)$: the mean of this distribution in case of EKF is the system dynamics $g(x_{t-1}, u_t)$, while the system motion noise still $Q \in \mathcal{R}^{nXn}$, due to the assumption of additive noise.
  - $p(z_t|x_t)$: the mean of this distribution in case of EKF is the sensor mapping nonlinear function $h(\bar{\mu})$,while the system motion noise still $R \in \mathcal{R}^{mXm}$, due to the assumption of additive noise.
  - $bel(x_t)$: the mean and covarience of this distribution is is calculated using the Jacobians matrices of the system dynamic model and the measurement model.
- Q3 : Explain in a few sentences all of the components of the **EKF** algorithm, and why they are needed. Specify the dimensionality of these components.
  - μ_t : state estimate at time t, $(n, 1)$.
  - Σ_t : covariance matrix at time t, (n, n).
  - $\bar{\mu_t}$: state estimate propagated through time using the process model, $(n, 1)$.
  - $\bar{\Sigma_t}$: covariance matrix at time t updated with Jacobian **G**, and process noise **Q**, (n, n).
  - g : function to update state space based on system model, $(n, 1)$.
  - G_t^x : Jacobian of the motion matrix, (n, n).
  - R_t : measurement noise, (m, m).
  - h : measurement function, $(m, 1)$.
  - H_t^x : Jacobian of h, (m, n).
  - Q_t : process noise, (n, n).
  - K_t : Kalman gain, (n,m)

2 Exercise 2: Jacobians

- Q1 : Derive the Jacobian matrix G_t^x of the noise-free motion function g with respect to the pose of the robot. Use the odometry motion model as in exercise sheet 1:

$$\begin{pmatrix} \mathrm{x_t} \\ \mathrm{y_t} \\ \theta_\mathrm{t} \end{pmatrix} = \begin{pmatrix} \mathrm{x_{t-1}} \\ \mathrm{y_{t-1}} \\ \theta_\mathrm{t-1} \end{pmatrix} + \begin{pmatrix} \delta_\mathrm{trans} \cos(\delta_\mathrm{rot1} + \theta_\mathrm{t-1}) \\ \delta_\mathrm{trans} \sin(\delta_\mathrm{rot1} + \theta_\mathrm{t-1}) \\ \delta_\mathrm{rot1} + \delta_\mathrm{rot2} \end{pmatrix} \tag{1}$$

If we denote the control vector u_t by $(\delta_{r1}, \delta_t, \delta_{r2})$, this particular motion model is defined as:

$$\begin{pmatrix} \mathrm{x_t} \\ \mathrm{y_t} \\ \theta_\mathrm{t} \end{pmatrix} = g(x_{t-1}, u_t) = \begin{pmatrix} \mathrm{x_{t-1}} + \delta_\mathrm{t}\ \cos(\delta_\mathrm{r1} + \theta_\mathrm{t-1}) \\ \mathrm{y_{t-1}} + \delta_\mathrm{t}\ \sin(\delta_\mathrm{r1} + \theta_\mathrm{t-1}) \\ \delta_\mathrm{r1} + \delta_\mathrm{r2} + \theta_\mathrm{t-1} \end{pmatrix} \tag{2}$$

The Jacobin G_t is obtained by differentiating every row with respect to $x_{t-1}, y_{t-1}, \theta_{t-1}$, then evaluate it at the best current estimate $\mu_{x,t-1}, \mu_{y,t-1}, \mu_{\theta,t-1}$:

$$G_t = \begin{pmatrix} \nabla_X g(x_{t-1}, u_t)_1 \\ \nabla_X g(x_{t-1}, u_t)_2 \\ \nabla_X g(x_{t-1}, u_t)_3 \end{pmatrix} \tag{3}$$

Where the vector $X = (x_{t-1}, y_{t-1}, \theta_{t-1})$ is the current state so far. Evaluating the gradients in equation 3 at the best state estimate $\mu_{x,t-1}, \mu_{y,t-1}, \mu_{\theta,t-1}$

$$G_t = \begin{pmatrix} 1 & 0 & -\delta_{trans} sin(\mu_{\theta,t-1} + \delta_{rot1}) \\ 0 & 1 & \delta_{trans} cos(\mu_{\theta,t-1} + \delta_{rot1}) \\ 0 & 0 & 1 \end{pmatrix} \tag{4}$$

- Q2 : Derive the Jacobian matrix H_i^i of the noise-free sensor function h corresponding to the i^{th} landmark.

$$h(\bar{\mu}_t, i) = z_t^i = \begin{pmatrix} r_t^i \\ \phi_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(\bar{\mu}_\mathrm{i,x} - \bar{\mu}_\mathrm{t,x})^2 + (\bar{\mu}_\mathrm{i,y} - \bar{\mu}_\mathrm{t,y})^2} \\ \mathrm{atan2}(\bar{\mu}_\mathrm{i,y} - \bar{\mu}_\mathrm{t,y}, \bar{\mu}_\mathrm{i,x} - \bar{\mu}_\mathrm{t,x}) - \bar{\mu}_{\mathrm{t},\theta}) \end{pmatrix} \tag{5}$$

where $(\bar{\mu}_{i,x}, \bar{\mu}_{i,x})^T$ is the position of the landmark,$(\bar{\mu}_{i,x}, \bar{\mu}_{i,x}, \bar{\mu}_{t,\theta})^T$ is the pose of the robot at time t, and r_t^i and ϕ_t^i are the observed range and bearing of the landmark, respectively.

The sensor jacobian matrix is obtained by differentiating the sensor mapping function $h(.)$ with respect to the current best prediction of the robot pose.

$$H_t^i = \begin{pmatrix} \frac{\partial r_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t^i}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t^i}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix} \tag{6}$$

$$H_t^i = \begin{pmatrix} \frac{-\bar{\mu}_{i,x} + \bar{\mu}_{t,x}}{\sqrt{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2}} & \frac{-\bar{\mu}_{i,y} + \bar{\mu}_{t,y}}{\sqrt{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2}} & 0 \\ \frac{\bar{\mu}_{i,y} - \bar{\mu}_{t,y}}{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2} & \frac{-\bar{\mu}_{i,x} + \bar{\mu}_{t,x}}{(\bar{\mu}_{i,x} - \bar{\mu}_{t,x})^2 + (\bar{\mu}_{i,y} - \bar{\mu}_{t,y})^2} & -1 \end{pmatrix} \tag{7}$$