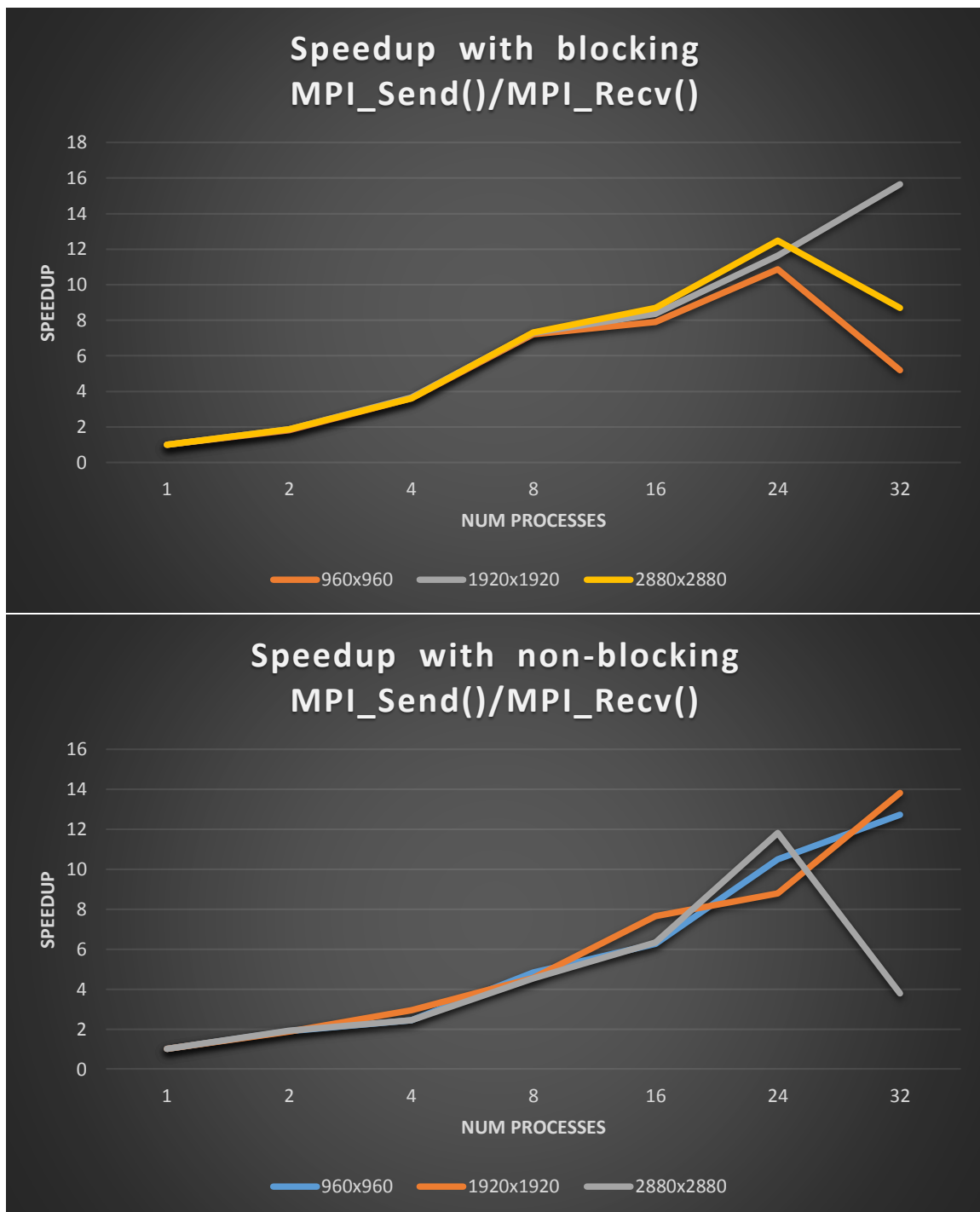
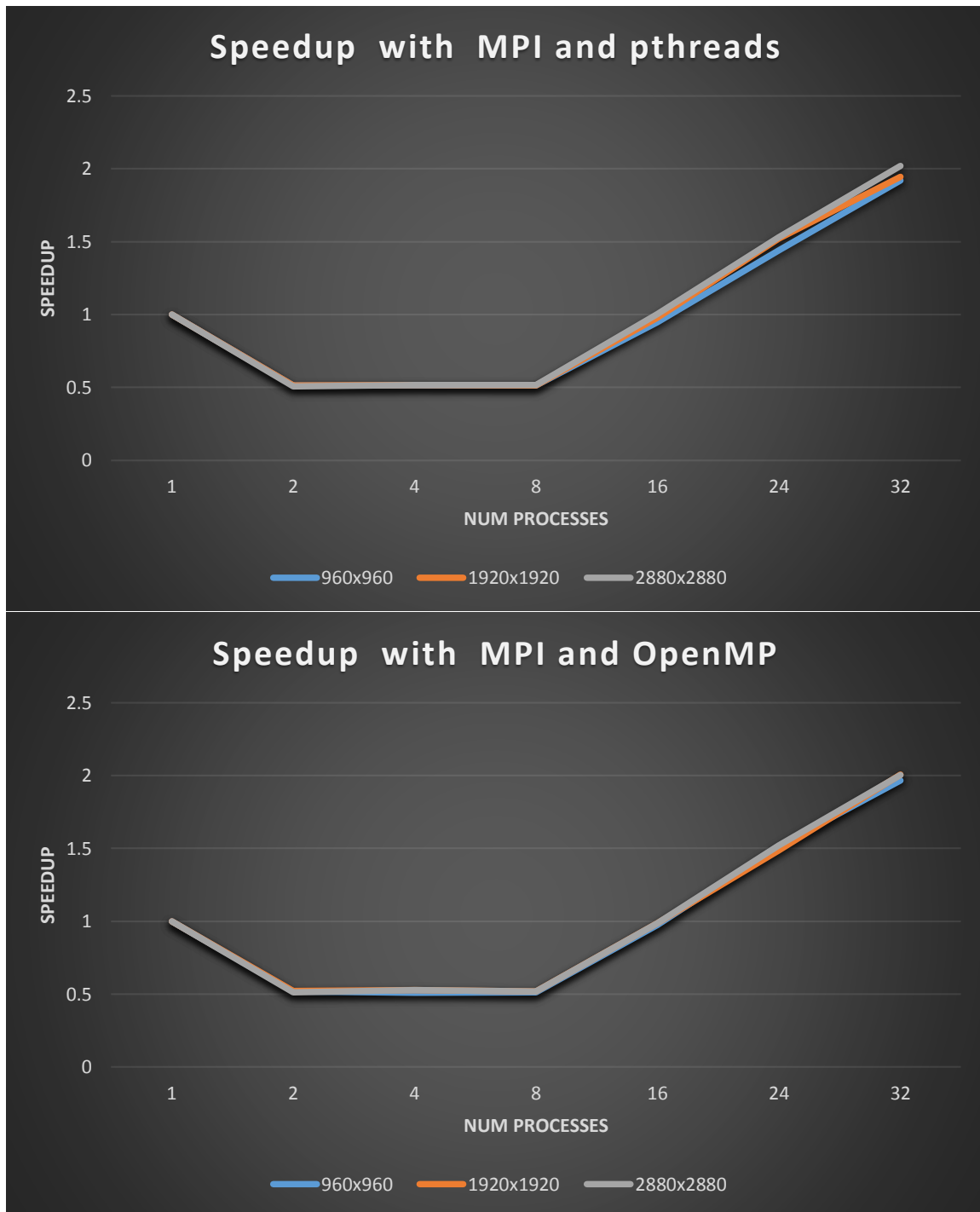


Strong Scaling:





From the graphs above, it can be seen that pure MPI with both synchronous and asynchronous communication scale strongly quite well. For each increase in processes, the speedup almost doubles. This trend occurs until 32 processes for both types of communication. With synchronous communication, speedup drops at the 960x960 and 1920x1920 matrix sizes but keeps increasing for a matrix size of 2880x2880. This likely stems from the fact that the

communication overhead for the smaller matrix sizes begins to take more effect compared to the larger matrices. In asynchronous communication the large (2880x2880) matrix drops in speedup compared to the smaller ones. Both synchronous and asynchronous had very similar runtimes and speedups. See full results in Appendix A.

When threading inside the nodes was introduced with pthreads and OpenMPI the speedup was not remotely as good as pure MPI. This was because of our methodology in threading inside the nodes. Each time through the loop that iterated through the number of nodes the desired number of pthreads or OpenMP threads was created. For example, if there were four MPI nodes and eight threads, then eight threads were being created three times for a total of 24 threads being created. This caused a large amount of overhead that quickly overwhelmed any advantage of multithreading. However, when the number of total threads began to increase past 8 (1 MPI node with 8 threads) the speedup began to increase to reasonable levels. Unfortunately, pure runtime was much slower than just MPI. Theoretically, with a better implementation, the hybrid cases should match, if not better, the speedup of pure MPI.

Overall, the pure MPI versions had much better strong scaling, with synchronous communication MPI being the best, than the hybrid implementations.

In order to test strong scaling with pure MPI we ran the following commands:

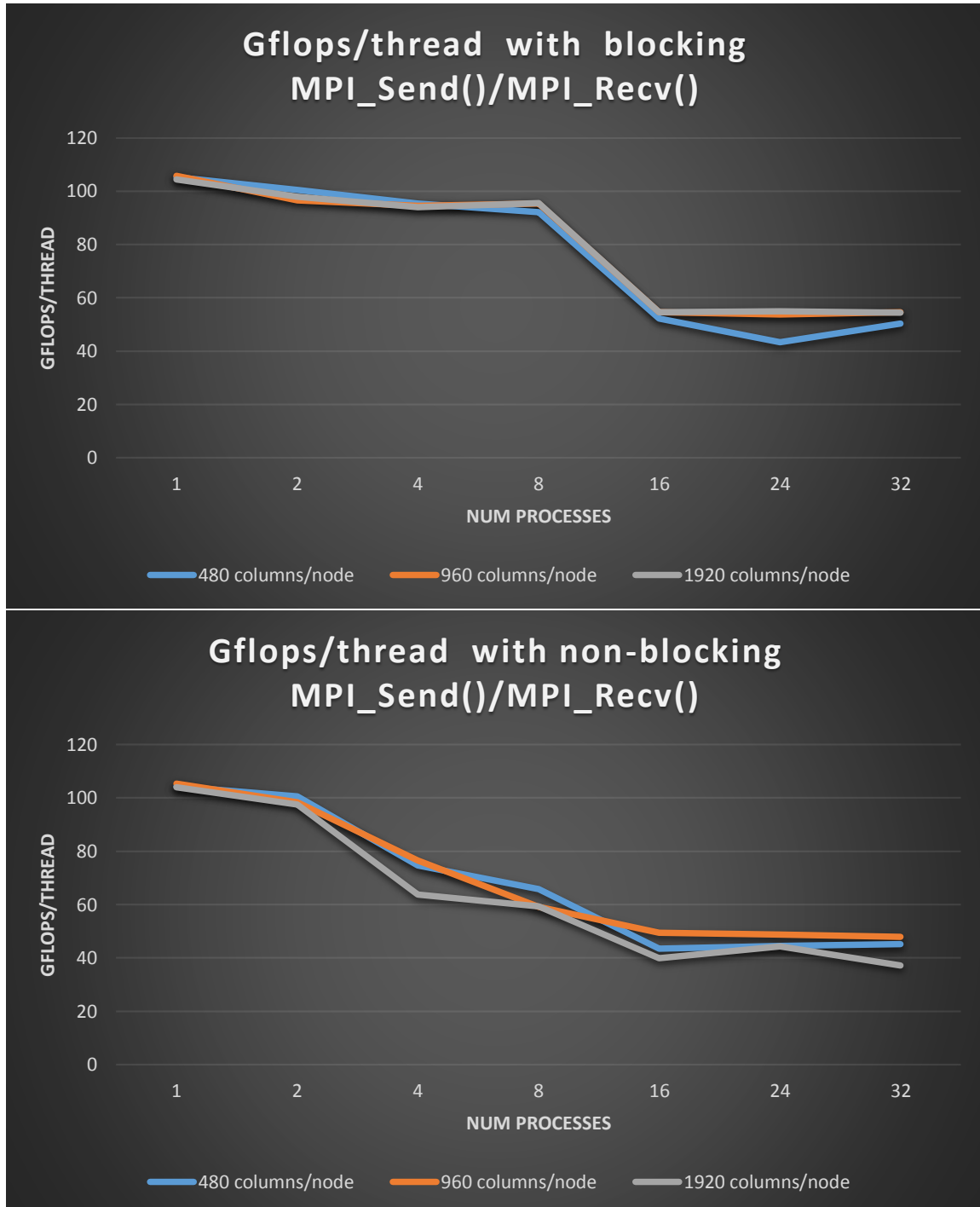
<code>mpirun -np 1 -N 1 -hostfile nodes</code>	(1 node with 1 process: 1 total)
<code>mpirun -np 2 -N 2 -hostfile nodes</code>	(1 node with 2 processes 2 total)
<code>mpirun -np 4 -N 4 -hostfile nodes</code>	(1 node with 4 processes: 4 total)
<code>mpirun -np 8 -N 8 -hostfile nodes</code>	(1 node with 8 processes: 8 total)
<code>mpirun -np 16 -N 8 -hostfile nodes</code>	(2 nodes with 8 processes 16 total)
<code>mpirun -np 24 -N 8 -hostfile nodes</code>	(3 nodes with 8 processes 24 total)
<code>mpirun -np 32 -N 8 -hostfile nodes</code>	(4 nodes with 8 processes: 32 total)

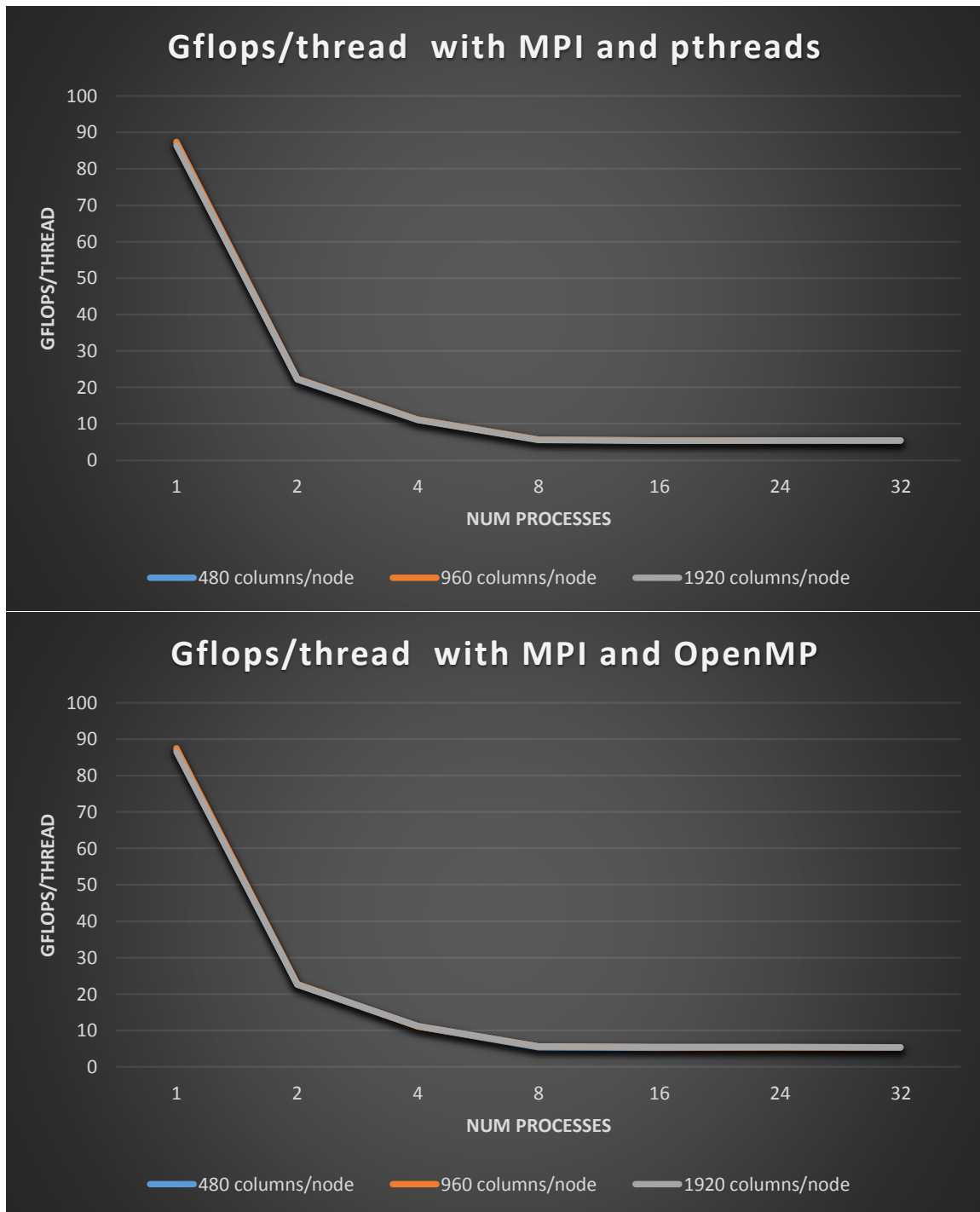
This ensured that each node was fully utilized and filled up before launching a new node.

When testing strong scaling with pthreads and OpenMPI, a similar pattern was followed:

<code>mpirun -np 1 -hostfile nodes a.out 1</code>	(1 node with 1 threads: 1 total)
<code>mpirun -np 1 -hostfile nodes a.out 2</code>	(1 node with 2 threads 2 total)
<code>mpirun -np 1 -hostfile nodes a.out 4</code>	(1 node with 4 threads: 4 total)
<code>mpirun -np 1 -hostfile nodes a.out 8</code>	(1 node with 8 threads: 8 total)
<code>mpirun -np 2 -hostfile nodes a.out 8</code>	(2 nodes with 8 threads: 16 total)
<code>mpirun -np 3 -hostfile nodes a.out 8</code>	(3 nodes with 8 threads: 24 total)
<code>mpirun -np 4 -hostfile nodes a.out 8</code>	(4 nodes with 8 threads: 32 total)

Weak Scaling:





From the graphs above, pure MPI has decent weak scaling properties. Gflops/thread (gigaflops/thread) remain relatively constant at about 100 until 16 processes. This occurs because after 8 processes (1 node with 8 processes) a new node is introduced (2 nodes with 8 processes each) and internode communication overhead starts to add up. After 16 processes Gflops/thread remains pretty constant despite adding more nodes. However, this obvious drop

in Gflops/thread only occurs with synchronous communication. With asynchronous communication, Gflops/thread slowly decreases as more and more processes are introduced. This probably stems from the fact that each process must wait on the rest to ensure that the correct data has been received before continuing on. From these observations, it is reasonable to conclude that synchronous communication has better weak scaling properties, especially since synchronous communication levels out at higher Gflops/thread.

Weak scaling in the hybrid cases with MPI/threads and MPI/OpenMP is non-existent. Gflops/thread drop drastically in both cases as the number of processes is increased and level out at about 5 Gflops/thread at 8 threads. This is due to the same issue discussed in the strong scaling section where the thread creation overhead simply overwhelms any benefit in multithreading. Weak scaling in the hybrid cases is much worse than in pure MPI.

When testing weak scaling we used similar mpirun commands as we did in strong scaling to fill up a node with processes before adding another node, except we increased the dimension of the matrix proportionally to ensure that each node always had the same number of columns. For example, we wanted 480 columns per node, so for 1, 2, 4, and 8 processes we kept the matrix size at 480x480 since there was only one node. Then when we went to 16, 24, and 32 processes (2, 3, and 4 nodes), we increased the dimension to 960x960, 1920x1920, and 2880x2880 respectively. This made sure that each node always had 480 columns to work on.

Overall, the pure MPI implementations, with synchronous communication being the best, had better weak scaling properties than did the hybrid cases, but with a better implementation the hybrid cases should have marginally better weak scaling since internode communication overhead would be minimized.

For full data and results see Appendix B.

Appendix A:

1a	Runtime in seconds				Speedup			
Threads	960x960	1920x1920	2880x2880		Threads	960x960	1920x1920	2880x2880
1	8.37	67.8	229.91		1	1	1	1
2	4.6	36.15	123.27		2	1.819565217	1.875518672	1.865092886
4	2.32	18.57	63.64		4	3.607758621	3.651050081	3.612664991
8	1.16	9.31	31.47		8	7.215517241	7.282491944	7.305687957
16	1.06	8.11	26.45		16	7.896226415	8.360049322	8.692249527
24	0.77	5.83	18.41		24	10.87012987	11.62950257	12.48832156
32	1.61	4.33	26.45		32	5.198757764	15.65819861	8.692249527
1b	Runtime in seconds				Speedup			
Threads	960x960	1920x1920	2880x2880		Threads	960x960	1920x1920	2880x2880
1	8.4	68.1	230.94		1	1	1	1
2	4.43	36.48	120.24		2	1.896162528	1.866776316	1.920658683
4	3.45	23.18	94.16		4	2.434782609	2.937877481	2.452633815
8	1.74	14.96	50.82		8	4.827586207	4.552139037	4.544273908
16	1.34	8.89	36.48		16	6.268656716	7.660292463	6.330592105
24	0.8	7.75	19.54		24	10.5	8.787096774	11.81883316
32	0.66	4.93	61		32	12.72727273	13.81338742	3.785901639
1c	Runtime in seconds				Speedup			
Threads	960x960	1920x1920	2880x2880		Threads	960x960	1920x1920	2880x2880
1	10.12	81.8	276.54		1	1	1	1
2	19.67	159.72	546.01		2	0.51448907	0.512146256	0.50647424
4	19.66	158.59	534.09		4	0.514750763	0.515795447	0.517777903
8	19.72	158.96	534.65		8	0.513184584	0.514594867	0.517235575
16	10.7	83.51	275.45		16	0.945794393	0.97952341	1.003957161
24	7.03	53.73	180.78		24	1.439544808	1.52242695	1.529704613
32	5.27	42.07	136.91		32	1.920303605	1.944378417	2.019867066
1d	Runtime in seconds				Speedup			
Threads	960x960	1920x1920	2880x2880		Threads	960x960	1920x1920	2880x2880
1	10.13	81.93	276.52		1	1	1	1
2	19.52	156.87	540.67		2	0.518954918	0.522279595	0.51143951
4	19.86	155.6	523.41		4	0.510070493	0.526542416	0.528304771
8	19.77	158.41	533.12		8	0.512392514	0.517202197	0.518682473
16	10.4	83.05	280.52		16	0.974038462	0.986514148	0.985740767
24	6.68	55.16	181.96		24	1.516467066	1.485315446	1.519674654
32	5.15	40.86	138.05		32	1.966990291	2.005139501	2.003042376

Appendix B:

1a					Num flop (n^3)	Gflops/thread	Num flop (n^3)	Gflops/thread	Num flop (n^3)	Gflops/thread
Threads	Runtime in seconds				Columns/node: 480		Columns/node: 960		Columns/node: 1920	
1	1.05	8.37	67.8		110592000	105.3257143	884736000	105.7032258	7077888000	104.3936283
2	0.55	4.58	36.12		110592000	100.5381818	884736000	96.58689956	7077888000	97.97740864
4	0.29	2.34	18.81		110592000	95.33793103	884736000	94.52307692	7077888000	94.0708134
8	0.15	1.16	9.26		110592000	92.16	884736000	95.33793103	7077888000	95.54384449
16	1.06	8.11	27.36		884736000	52.16603774	7077888000	54.5459926	23887872000	54.56842105
24	6.8	18.51	42.94		7077888000	43.36941176	23887872000	53.77244733	56623104000	54.9440149
32	14.84	32.37	260.08		23887872000	50.30296496	56623104000	54.6639481	4.52985E+11	54.42854506
1b					Num flop (n^3)		Num flop (n^3)		Num flop (n^3)	
Threads	Runtime in seconds				Columns/node: 480		Columns/node: 960		Columns/node: 1920	
1	1.06	8.4	68.09		110592000	104.3320755	884736000	105.3257143	7077888000	103.9490087
2	0.55	4.5	36.27		110592000	100.5381818	884736000	98.304	7077888000	97.57220844
4	0.37	2.89	27.73		110592000	74.72432432	884736000	76.53425606	7077888000	63.81074648
8	0.21	1.87	14.92		110592000	65.82857143	884736000	59.14010695	7077888000	59.29865952
16	1.27	8.96	37.4		884736000	43.54015748	7077888000	49.37142857	23887872000	39.91957219
24	6.62	20.42	53.22		7077888000	44.54864048	23887872000	48.74280118	56623104000	44.33100338
32	16.54	36.97	380.39		23887872000	45.13276904	56623104000	47.8623749	4.52985E+11	37.21384894
1c					Num flop (n^3)		Num flop (n^3)		Num flop (n^3)	
Threads	Runtime in seconds				Columns/node: 480		Columns/node: 960		Columns/node: 1920	
1	1.27	10.12	81.88		110592000	87.08031496	884736000	87.42450593	7077888000	86.44220811
2	2.49	19.77	158.77		110592000	22.20722892	884736000	22.37572079	7077888000	22.28975247
4	2.47	19.77	160.11		110592000	11.19352227	884736000	11.18786039	7077888000	11.05160202
8	2.45	19.66	158.7		110592000	5.64244898	884736000	5.625228891	7077888000	5.57489603
16	10.2	81.63	279.18		884736000	5.421176471	7077888000	5.419184123	23887872000	5.347775629
24	54.73	185.02	435.6		7077888000	5.388488946	23887872000	5.379569776	56623104000	5.416198347
32	138.15	328.57	2632.54		23887872000	5.403517915	56623104000	5.38537298	4.52985E+11	5.377231115
1d					Num flop (n^3)		Num flop (n^3)		Num flop (n^3)	
Threads	Runtime in seconds				Columns/node: 480		Columns/node: 960		Columns/node: 1920	
1	1.27	10.11	81.89		110592000	87.08031496	884736000	87.51097923	7077888000	86.43165222
2	2.45	19.49	156.43		110592000	22.56979592	884736000	22.69717804	7077888000	22.6231797
4	2.49	19.79	157.27		110592000	11.10361446	884736000	11.17655382	7077888000	11.25117314
8	2.56	19.76	158.71		110592000	5.4	884736000	5.596761134	7077888000	5.574544767
16	10.58	82.8	273.91		884736000	5.226465028	7077888000	5.342608696	23887872000	5.450666277
24	54.51	185.07	432.1		7077888000	5.410236654	23887872000	5.378116388	56623104000	5.460069428
32	138.26	326.62	2651.39		23887872000	5.399218863	56623104000	5.417524953	4.52985E+11	5.339001807