$N_n = 2N_{n-1} + N_{n-2} \quad N_0 = 1 \quad N_1 = 3$
$f(x) = \sum_{n=0}^{\infty} N_n x^n$
$= 1 + \sum_{n=1}^{\infty} N_n x^n$
$= 1 + 3x + \sum_{n=2}^{\infty} N_n x^n$
$= 1 + 3x + \sum_{n=2}^{\infty} (2N_{n-1} + N_{n-2}) x^n$
$= 1 + 3x + \sum_{n=2}^{\infty} 2N_{n-1} x^n + \sum_{n=2}^{\infty} N_{n-2} x^n$
$\sum_{n=2}^{\infty} 2N_{n-1} x^n = x \sum_{n=2}^{\infty} N_{n-1} x^{n-1}$
$= 2x \sum_{n=1}^{\infty} N_n x^n$
$= 2x(f(x) - 1)$
$\sum_{n=2}^{\infty} N_{n-2} x^n = x^2 \sum_{n=2}^{\infty} N_{n-2} x^{n-2}$
$= x^2 \sum_{n=0}^{\infty} x^n$
$= x^2 f(x)$
$f(x) = 1 + 3x + 2x(f(x) - 1) + x^2 f(x)$
$= 1 + x + 2xf(x) + x^2 f(x)$
$f(x) - 2xf(x) - x^2 f(x) = x + 1$
$1 - 2x - x^2 = \frac{x+1}{f(x)}$
$-\frac{x^2+2x-1}{x+1} = \frac{1}{f(x)}$
$f(x) = -\frac{x+1}{x^2+2x-1}$
$= -\frac{x+1}{(x+1+\sqrt{2})(x+1-\sqrt{2})}$
$= \frac{A}{x+1+\sqrt{2}} + \frac{B}{x+1-\sqrt{2}}$
$-x - 1 = A(x + 1 - \sqrt{2}) + B(x + 1 + \sqrt{2})$
$(1 - \sqrt{2})A + (1 + \sqrt{2})B = -1$
$Ax + Bx = -x$
$A = -\frac{1}{2}$
$B = -\frac{1}{2}$
$f(x) = -\frac{1}{2}(\frac{1}{x+1+\sqrt{2}} + \frac{1}{x+1-\sqrt{2}})$
$\frac{1}{x+1+\sqrt{2}} = \frac{1}{(1+\sqrt{2})(\frac{x}{1+\sqrt{2}}+1)}$
$= (\sqrt{2} - 1) \sum_{n=0}^{\infty} (\frac{-x}{1+\sqrt{2}})^n$
$= (\sqrt{2} - 1) \sum_{n=0}^{\infty} (1 - \sqrt{2})^n x^n$
$\frac{1}{2(x+1-\sqrt{2})} = \frac{1}{(1-\sqrt{2})(\frac{x}{1-\sqrt{2}}+1)}$
$= -(\sqrt{2} + 1) \sum_{n=0}^{\infty} (\frac{-x}{1-\sqrt{2}})^n$
$= -(\sqrt{2} + 1) \sum_{n=0}^{\infty} ((1 + \sqrt{2})^n x^n)$
$f(x) = -\frac{1}{2}((\sqrt{2} - 1) \sum_{n=0}^{\infty} (1 - \sqrt{2})^n x^n - (\sqrt{2} + 1) \sum_{n=0}^{\infty} ((1 + \sqrt{2})^n x^n))$
$f(x) = \sum_{n=0}^{\infty} -\frac{1}{2}((\sqrt{2} - 1)(1 - \sqrt{2})^n - (\sqrt{2} + 1)(1 + \sqrt{2})^n) x^n$
$N_n = \frac{1}{2}((1 - \sqrt{2})^{n+1} + (1 + \sqrt{2})^{n+1})$

$D_n = 2D_{n-1} + D_{n-2} \quad D_0 = 1 \quad D_1 = 2$
$g(x) = \sum_{n=0}^{\infty} D_n x^n$
$= 1 + \sum_{n=1}^{\infty} D_n x^n$
$= 1 + 2x + \sum_{n=2}^{\infty} D_n x^n$
$= 1 + 2x + \sum_{n=2}^{\infty}(2D_{n-1} + D_{n-2})x^n$
$= 1 + 2x + \sum_{n=2}^{\infty} 2D_{n-1}x^n + \sum_{n=2}^{\infty} D_{n-2}x^n$
$\sum_{n=2}^{\infty} 2D_{n-1}x^n = x\sum_{n=2}^{\infty} D_{n-1}x^{n-1}$
$= 2x\sum_{n=1}^{\infty} D_n x^n$
$= 2x(g(x) - 1)$
$\sum_{n=2}^{\infty} D_{n-2}x^n = x^2\sum_{n=2}^{\infty} D_{n-2}x^{n-2}$
$= x^2\sum_{n=0}^{\infty} x^n$
$= x^2 g(x)$
$g(x) = 1 + 2x + 2x(g(x) - 1) + x^2 g(x)$
$= 1 + 2xg(x) + x^2 g(x)$
$g(x) - 2xg(x) - x^2 g(x) = 1$
$1 - 2x - x^2 = \frac{1}{g(x)}$
$g(x) = -\frac{1}{x^2+2x-1}$
$= -\frac{1}{(x+1+\sqrt{2})(x+1-\sqrt{2})}$
$= \frac{C}{x+1+\sqrt{2}} + \frac{D}{x+1-\sqrt{2}}$
$-1 = C(x + 1 - \sqrt{2}) + D(x + 1 + \sqrt{2})$
$(1 - \sqrt{2})C + (1 + \sqrt{2})D = -1$
$C + D = 0$
$C = \frac{\sqrt{2}}{4}$
$D = -\frac{\sqrt{2}}{4}$
$g(x) = \frac{\sqrt{2}}{4}(\frac{1}{x+1+\sqrt{2}} - \frac{1}{x+1-\sqrt{2}})$
$\frac{1}{x+1+\sqrt{2}} = \frac{1}{(1+\sqrt{2})(\frac{x}{1+\sqrt{2}}+1)}$
$= (\sqrt{2} - 1)\sum_{n=0}^{\infty}(\frac{-x}{1+\sqrt{2}})^n$
$= (\sqrt{2} - 1)\sum_{n=0}^{\infty}(1 - \sqrt{2})^n x^n$
$\frac{1}{2(x+1-\sqrt{2})} = \frac{1}{(1-\sqrt{2})(\frac{x}{1-\sqrt{2}}+1)}$
$= -(\sqrt{2} + 1)\sum_{n=0}^{\infty}(\frac{-x}{1-\sqrt{2}})^n$
$= -(\sqrt{2} + 1)\sum_{n=0}^{\infty}((1 + \sqrt{2})^n x^n)$
$g(x) = \frac{\sqrt{2}}{4}((\sqrt{2} - 1)\sum_{n=0}^{\infty}(1 - \sqrt{2})^n x^n + (\sqrt{2} + 1)\sum_{n=0}^{\infty}((1 + \sqrt{2})^n x^n))$
$g(x) = \sum_{n=0}^{\infty} \frac{\sqrt{2}}{4}((\sqrt{2} - 1)(1 - \sqrt{2})^n + (\sqrt{2} + 1)(1 + \sqrt{2})^n)x^n$
$D_n = -\frac{\sqrt{2}}{4}((1 - \sqrt{2})^{n+1} - (1 + \sqrt{2})^{n+1})$

$\lim_{n\to\infty} \frac{N_n}{D_n} = \lim_{n\to\infty} \frac{\frac{1}{2}((1-\sqrt{2})^{n+1}+(1+\sqrt{2})^{n+1})}{-\frac{\sqrt{2}}{4}((1-\sqrt{2})^{n+1}-(1+\sqrt{2})^{n+1})}$
$|1-\sqrt{2}| < 1$
$\lim_{n\to\infty}(\pm(1-\sqrt{2}))^n = 0$
$\lim_{n\to\infty} \frac{N_n}{D_n} = \lim_{n\to\infty} \frac{\frac{1}{2}(0+(1+\sqrt{2})^{n+1})}{-\frac{\sqrt{2}}{4}(0-(1+\sqrt{2})^{n+1})}$
$= \lim_{n\to\infty} \frac{\frac{1}{2}(1+\sqrt{2})^{n+1}}{\frac{\sqrt{2}}{4}(1+\sqrt{2})^{n+1}}$
$= \lim_{n\to\infty} \frac{\frac{1}{2}}{\frac{\sqrt{2}}{4}}$
$= \frac{1}{2} \times \frac{4}{\sqrt{2}}$
$= \frac{1}{2} \times \frac{4\sqrt{2}}{2}$
$= \frac{2\sqrt{2}}{2}$
$= \sqrt{2}$