Egg Dropping

You are given n Eggs and a building with m floors, *find out the* ***minimum number of drops*** *that is required* to know the floor from which If an egg is dropped, the egg will break.

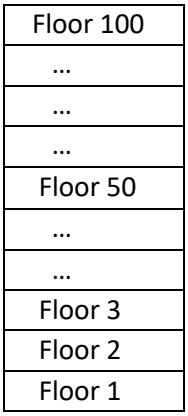
Conditions :

1. ***If the egg doesn't break at a certain floor x, it will definitely not break at any floor below x.***
2. ***If the egg break at a certain floor x, it will definitely break at any floor above x.***
3. ***The egg may break at the first floor.***
4. ***The egg may not break at the last floor.***
5. ***If an egg does not break it can be reused***

Critical Floor – Last safe floor from which we can drop an egg without breaking it, so If we go even one floor above critical floor, the egg will break.

Note : Our aim is not to find the critical floor, **but to find out in worst case, at minimum how many drops (attempts) we will require** to find the critical floor.

One Egg



If we randomly drop the egg from any floor and if it breaks, we will lose our egg without finding the critical floor.

If we have just one egg, the only possible way is to, try dropping the egg from 1st floor, then 2nd floor, then 3rd floor and so on. In worst case the egg will break on dropping from the last floor of the building. So in worst case we will have to make m drops (attempts) to find the critical floor.

Note : **When we have just one egg**, we can not drop the egg randomly from any floor, because if it breaks, we can't guarantee which floor is the critical floor.

Two Egg

- ➔ **Given 2 eggs and a building with 100 floors, find out the minimum number of drops required to know the critical floor.**

Using Binary search approach, drop the first egg from 50th floor.

If egg doesn't break after dropping from 50th floor, try dropping from 75, 88, 94, 97, 99, 100. In the best case scenario we will be able to cover all floors with 7 drops.

But what if the first egg break when dropping from the 50th floor, now since we only have one egg remaining we will have to try all floors one by one from 1st floor to 49th floor. So in total it will take us 50 drops (attempts).

Can we do better?

10, 20, 30, 40, 50, 60, 70, 80, 90, 100

With this approach we will be able to cover all 100 floors of the building with 19 drops, which is better than our first approach.

Can we do better?

14, 27, 39, 50, 60, 69, 77, 84, 90, 95, 99, 100

1^{st} egg	If the first egg breaks $\rightarrow$ 2^{nd} egg	Drops
14	$1\rightarrow 2\rightarrow 3\rightarrow 4\rightarrow 5\rightarrow 6\rightarrow 7\rightarrow 8\rightarrow 9\rightarrow 10\rightarrow 11\rightarrow 12\rightarrow 13$	1+13=14
27	$15\rightarrow 16\rightarrow 17\rightarrow 18\rightarrow 19\rightarrow 20\rightarrow 21\rightarrow 22\rightarrow 23\rightarrow 24\rightarrow 25\rightarrow 26$	2+12=14
39	$28\rightarrow 29\rightarrow 30\rightarrow 31\rightarrow 32\rightarrow 33\rightarrow 34\rightarrow 35\rightarrow 36\rightarrow 37\rightarrow 38$	3+11=14
50	$40\rightarrow 41\rightarrow 42\rightarrow 43\rightarrow 44\rightarrow 45\rightarrow 46\rightarrow 47\rightarrow 48\rightarrow 49$	4+10=14
60	$51\rightarrow 52\rightarrow 53\rightarrow 54\rightarrow 55\rightarrow 56\rightarrow 57\rightarrow 58\rightarrow 59$	5+9=14
69	$61\rightarrow 62\rightarrow 63\rightarrow 64\rightarrow 65\rightarrow 66\rightarrow 67\rightarrow 68$	6+8=14
77	$70\rightarrow 71\rightarrow 72\rightarrow 73\rightarrow 74\rightarrow 75\rightarrow 76$	7+7=14
84	$78\rightarrow 79\rightarrow 80\rightarrow 81\rightarrow 82\rightarrow 83$	8+6=14
90	$85\rightarrow 86\rightarrow 87\rightarrow 88\rightarrow 89$	9+5=14
95	$91\rightarrow 92\rightarrow 93\rightarrow 94$	10+4=14
99	100	11+1=12*

*Altough the number of drops here is 12, this is not the worst case.

2 eggs and k floors

Suppose the minimum number of attempts in the worst case, while using the best strategy, is x. In our attempt, we will drop the egg at the x^{th} floor, covering x floors, then we will drop it at the $\big(x+(x-1)\big)^{\text{th}}$ floor, covering $x-1$ floors, and the third drop would be at the $\big(x+(x-1)+(x-2)\big)^{\text{th}}$ floor, covering $x-2$ floors. We can see that using this strategy we would cover

$$x+(x-1)+(x-2)+(x-3)+\cdots+2+1=\frac{x(x+1)}{2}$$

floors.

Now since we have to cover all k floors of the building, we just have to find the minimum value of x, such that

$$\frac{x(x+1)}{2} \geq k$$

$$x^2+x-2k=0 \implies x=\frac{-1+\sqrt{1+8k}}{2}$$

Now since x represents a floor, it must be a whole number (e.g. we can not drop an egg from 5.4 floor)

$$x=\left\lceil \frac{-1+\sqrt{1+8k}}{2} \right\rceil$$

For a 100 floor building, solving with k=100 will result in

x = $\lceil 13.65 \rceil = 14$

2 eggs and 36 floors

8, 15, 21, 26, 30, 33, 35, 36

We will be able to cover all 36 floors with just 8 egg drop attempts.

3 eggs and 100 floors

37, 66, 88, 100

We will be able to cover all 100 floors with just 9 egg drop attempts.

	Eggs									
Building Height	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1 floor	1	1	1	1	1	1	1	1	1	1
2 floors	2	2	2	2	2	2	2	2	2	2
3 floors	3	2	2	2	2	2	2	2	2	2
4 floors	4	3	3	3	3	3	3	3	3	3
5 floors	5	3	3	3	3	3	3	3	3	3
6 floors	6	3	3	3	3	3	3	3	3	3
7 floors	7	4	3	3	3	3	3	3	3	3
8 floors	8	4	4	4	4	4	4	4	4	4
9 floors	9	4	4	4	4	4	4	4	4	4
10 floors	10	4	4	4	4	4	4	4	4	4
11 floors	11	5	4	4	4	4	4	4	4	4
12 floors	12	5	4	4	4	4	4	4	4	4
13 floors	13	5	4	4	4	4	4	4	4	4
14 floors	14	5	4	4	4	4	4	4	4	4
15 floors	15	5	5	4	4	4	4	4	4	4
16 floors	16	6	5	5	5	5	5	5	5	5
17 floors	17	6	5	5	5	5	5	5	5	5
18 floors	18	6	5	5	5	5	5	5	5	5
19 floors	19	6	5	5	5	5	5	5	5	5
20 floors	20	6	5	5	5	5	5	5	5	5
21 floors	21	6	5	5	5	5	5	5	5	5
22 floors	22	7	5	5	5	5	5	5	5	5
23 floors	23	7	5	5	5	5	5	5	5	5
24 floors	24	7	5	5	5	5	5	5	5	5
25 floors	25	7	5	5	5	5	5	5	5	5
30 floors	30	8	6	5	5	5	5	5	5	5
35 floors	35	8	6	6	6	6	6	6	6	6
40 floors	40	9	6	6	6	6	6	6	6	6
45 floors	45	9	7	6	6	6	6	6	6	6
50 floors	50	10	7	6	6	6	6	6	6	6
100 floors	100	14	9	8	7	7	7	7	7	7
200 floors	200	20	11	9	8	8	8	8	8	8
300 floors	300	24	13	10	9	9	9	9	9	9
400 floors	400	28	14	11	10	9	9	9	9	9
500 floors	500	32	15	11	10	10	9	9	9	9
1,000 floors	1000	45	19	13	11	11	11	10	10	10