#### **Subset Sum**

The Subset Sum problem involves searching through a collection of numbers to **find a subset that sums** to a certain number.

### **Problem statement:**

- Input:
- A collection of non-negative integers A
- A non-negative integer b
- Output:
- Boolean value indicating whether some subset of the collection sums to b

e.g.

A = 
$$\{5, 2, 1, 3\}$$
 , sum = 9  
True ,  $\{5, 1, 3\} = 5 + 1 + 3 = 9$ 

Note: Order of integers in the subset doesn't matter so {5, 1, 3} and {3, 1, 5} are same.

To get to the solution to the actual problem (given a set of non-negative integers {5, 2, 1, 3}, is there a subset whose sum will result to 9), we will first solve the subproblems.

e.g.

- Given the set {5}, is there a subset whose sum will result to 2? (answer is false)
- Given the set {5}, is there a subset whose sum will result to 5? (answer is true {5})
- Given the set {5, 2}, is there a subset whose sum will result to 2? (answer is true {2})
- Given the set {5, 2, 1}, is there a subset whose sum will result to 6? (answer is true {5,1})

Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0										
1 {5}										
2 {5,2}										
3 {5,2,1}										
2 {5,2} 3 {5,2,1} 4 {5,2,1,3}										

When we don't consider any number from the set, obviously **there will be no subset**, whose subset sum will be greater than 0.

Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0		F	F	F	F	F	F	F	F	F
1 {5}										
2 {5,2}										
3 {5,2,1}										
4 {5,2,1,3}										

If we have to find a subset whose subset sum is 0, we can do that (by not including any integer in the subset), so regardless of how many integers from the set we consider, we can always find a subset whose subset sum is 0.

Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
1 {5}	Т									
2 {5,2}	T									
3 {5,2,1}	Т									
4 {5,2,1,3}	Т									

When we consider only the first integer from the set {5}

{5, 2, 1, 3}

Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0	Т	F	F	F	F	F	F	F	F	F
1 {5}	Т	F	F	F	F	Т	F	F	F	F
2 {5,2}	Т									
3 {5,2,1}	Т									
4 {5,2,1,3}	Т									

# When we consider only the first two integers from the set {5, 2}

{5, 2, 1, 3}

Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
1 {5}	Т	F	F	F	F	Т	F	F	F	F
2 {5,2}	Т	F	Т	F	F	Т	F	Т	F	F
3 {5,2,1}	Т									
4 {5,2,1,3}	Т									

## When we consider only the first three integers from the set {5, 2, 1}

{5, 2, 1, 3}

Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0	T	F	F	F	F	F	F	F	F	F
1 {5}	T	F	F	F	F	Т	F	F	F	F
2 {5,2}	Т	F	Т	F	F	Т	F	Т	F	F
3 {5,2,1}	Т	Т	Т	Т	F	Т	Т	Т	Т	F
4 {5,2,1,3}	Т									

# When we consider all integers from the set {5, 2, 1, 3}

{5, 2, 1, 3}

Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0	Т	F	F	F	F	F	F	F	F	F
1 {5}	Т	F	F	F	F	Т	F	F	F	F
2 {5,2}	Т	F	Т	F	F	Т	F	Т	F	F
3 {5,2,1}	Т	Т	Т	Т	F	Т	Т	Т	Т	F
4 {5,2,1,3}	Т	Т	Т	Т	Т	Т	Т	Т	Т	Т

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Numbers/Sum	0	1	2	3	4	5	6	7	8	9
0	Т	F	F	F	F	F	F	F	F	F
1 {5}	Т	F	F	F	F	Т	F	F	F	F
2 {5,2}	Т	F	Т	F	F	Т	F	Т	F	F
3 {5,2,1}	Т	Т	Т	Т	F	Т	Т	Т	Т	F
4 {5,2,1,3}	Т	Т	Т	Т	Т	Т	Т	Т	Т	Т

So the subset will be {3, 1, 5}

### Implementation

```
public class App {
    public static void main(String[] args) {
        int[] numbers = { 5, 2, 1, 3 };
        int sum = 9;

        SubsetSumProblem subsetSumProblem = new SubsetSumProblem(numbers, sum);
        subsetSumProblem.solveProblem();
        if (subsetSumProblem.hasSolution()) {
            subsetSumProblem.printSolution();
        }
    }
}
```

```
public class SubsetSumProblem {
      private boolean[][] dpTable;
       private int[] numbers;
      private int sum;
       public SubsetSumProblem(int[] numbers, int sum) {
             this.numbers = numbers;
             this.sum = sum;
             this.dpTable = new boolean[numbers.length + 1][sum + 1];
      }
      public void solveProblem() {
      }
       public boolean hasSolution() {
             . . .
      }
      public void printSolution() {
             . . .
      }
}
public void solveProblem() {
      for(int j=1; j <=this.sum; j++){</pre>
                    this.dpTable[0][j] = false;
             }
      for (int i = 0; i <= this.numbers.length; i++) {</pre>
                    this.dpTable[i][0] = true;
             }
      for (int rowIndex = 1; rowIndex <= numbers.length; ++rowIndex) {</pre>
           for (int columnIndex = 1; columnIndex <= sum; ++columnIndex) {</pre>
             if (columnIndex < numbers[rowIndex - 1]) {</pre>
                    this.dpTable[rowIndex][columnIndex] =
                                  this.dpTable[rowIndex - 1][columnIndex];
             } else {
                if (this.dpTable[rowIndex - 1][columnIndex]) {
                     this.dpTable[rowIndex][columnIndex] = true;
                 } else {
                    this.dpTable[rowIndex][columnIndex] =
                    this.dpTable[rowIndex - 1][columnIndex - numbers[rowIndex - 1]];
                }
             }
          }
      }
}
```

```
public boolean hasSolution() {
      if (this.dpTable[numbers.length][sum]) {
                    System.out.println("A subset with sum "+ sum + " exists.");
                    return true;
      } else {
                    System.out.println("No feasible solution");
                    return false;
      }
}
public void printSolution() {
      int columnIndex = this.sum;
      int rowIndex = this.numbers.length;
      while (columnIndex > 0 || rowIndex > 0) {
         if (this.dpTable[rowIndex][columnIndex] ==
                    this.dpTable[rowIndex - 1][columnIndex]) {
                          rowIndex = rowIndex - 1;
         } else {
             System.out.println("We include number: " + numbers[rowIndex - 1]);
             columnIndex = columnIndex - numbers[rowIndex - 1];
             rowIndex = rowIndex - 1;
          }
      }
}
```