Modelling with Graphs Summative

March 3, 2020

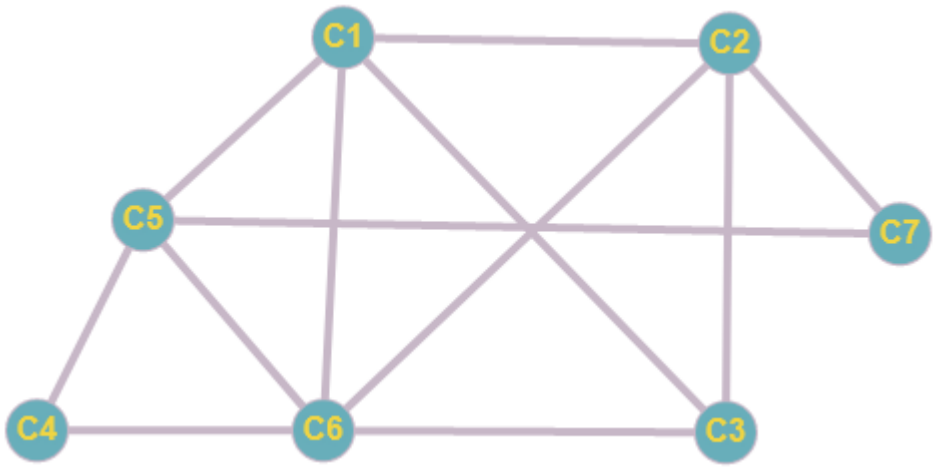
A.3

A.3.1

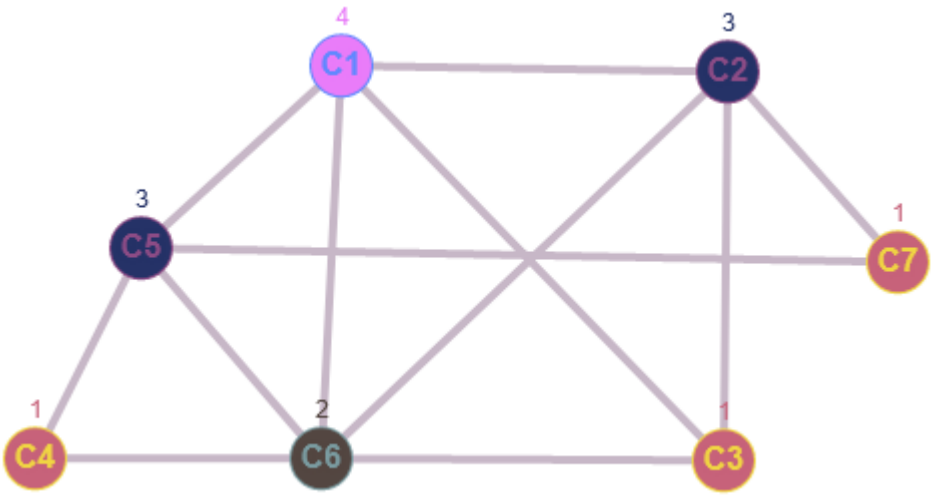
To transform problem A.3 into a graph colouring problem , the problem is represented as a graph where:
Nodes = individual classes $(C_1, ..., C_7)$
Edges = classes with at least one shared student (eg. "Amy" in C_1 and C_5)
Colour = A timeslot
A representation of the graph as an adjacency matrix is:

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

An example graph would be:



The solution to the problem is the answer to the question, can the graph be coloured in at most 4 colours?
This will solve the problem because if the graph can be coloured so that no neighbouring nodes have the same timeslot then the student can physially attend every class, and by doing it in at most 4 colours means there are sufficient time slots.
An example colouring of the above graph would be:



A.3.2

As there are currently no visited nodes, the algorithm will first visit node C_1.

Visited - C_1
Adjacent - C_2, C_3, C_5, C_6

Visit C_2

Visited - C_1, C_2
Adjacent - C_3, C_5, C_6, C_7

Visit C_3

Visited - C_1, C_2, C_3
Adjacent - C_5, C_6, C_7

Visit C_6

Visited - C_1, C_2, C_3, C_6
Adjacent - C_5, C_7

Visit C_4

Visited - C_1, C_2, C_3, C_6, C_4
Adjacent - C_5, C_7

Visit C_5

Visited - $C_1, C_2, C_3, C_6, C_4, C_5$
Adjacent - C_7

Visit C_7

Visited - $C_1, C_2, C_3, C_6, C_4, C_5, C_7$

Order in which the algorithm visits the vertices to find a proper colouring is:

$$C_1, C_2, C_3, C_6, C_4, C_5, C_7$$

A.3.3

The colours the algorithm assigns to each vertex are below:

Vertex	Colour
C_1	1
C_2	2
C_3	3
C_4	1
C_5	2
C_6	4
C_7	1

A.3.4

The chromatic number of G, χG, is 4 because it the minimum number of colours needed to properly colour G is 4.

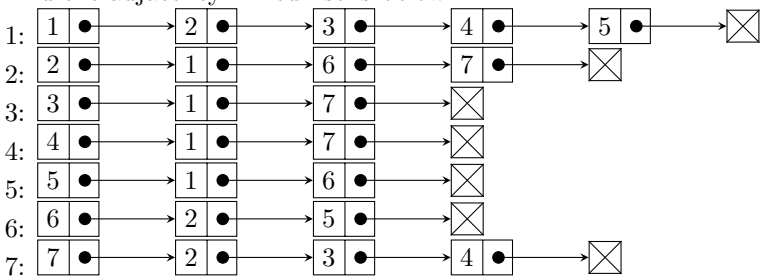
B.2

B.2.1

The adjacency matrix for the graph is below:

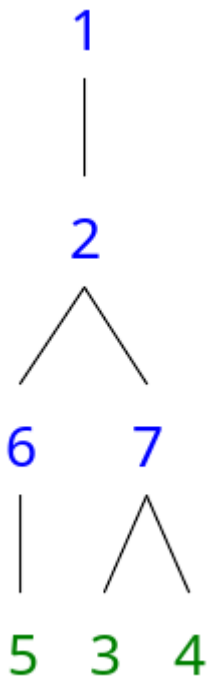
$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

And the adjacency linked list is below:



B.2.2

The depth-first search tree for the graph in question B.2.1 is below:

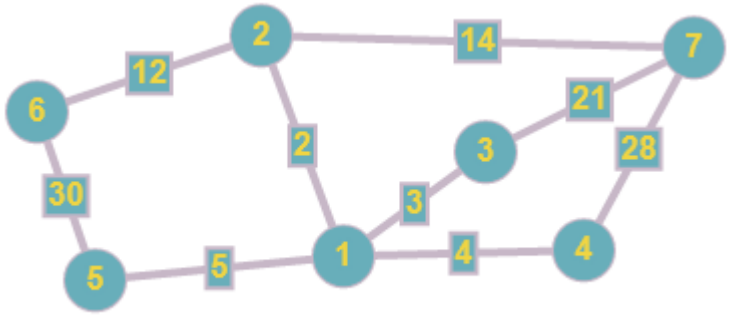


The table for when the in which the vertices are reached for the first time and he order in which the vertices become dead ends is below:

-	1	2	3	4	5	6	7
On	1	2	6	7	4	3	5
Off	7	6	3	4	1	2	5

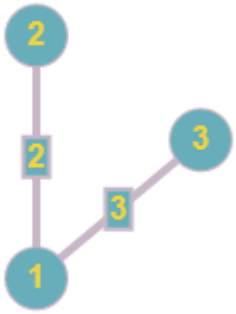
2.3

The graph with the addition of the weights is below:

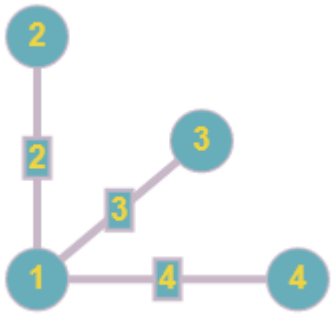


As Kruskal's algorithm runs it firsts adds the edge with the smallest weight to the tree, which is the edge between vertices '1' and '2' with weight 2.

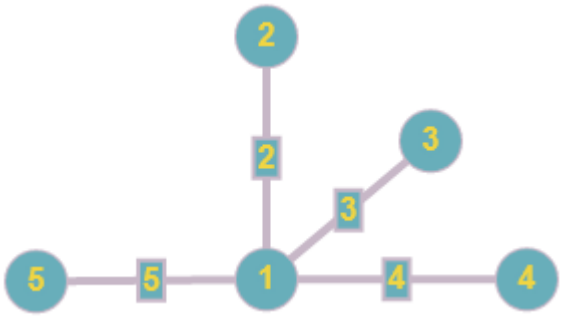
Secondly, it adds the edge between '1' and '3' with weight 3.



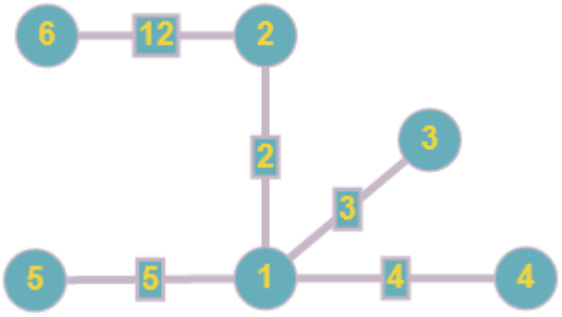
Then, the algorithm will add the next smallest edge, the edge between '1' and '4' with weight 4.



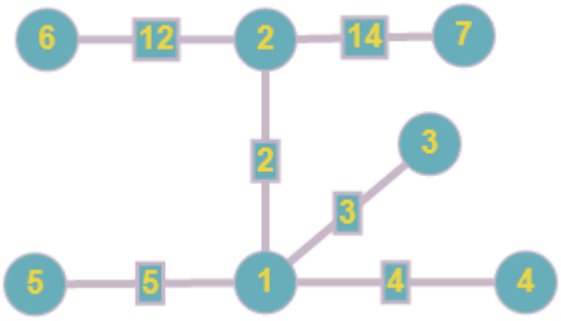
The next edge added is the node between '1' and '5' with weight 5.



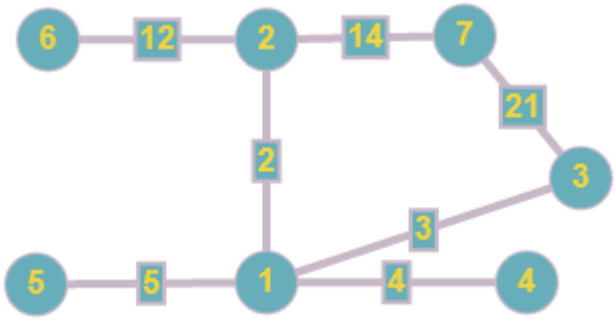
Then, the algorithm will add the edge between '2' and '6' with weight 12.



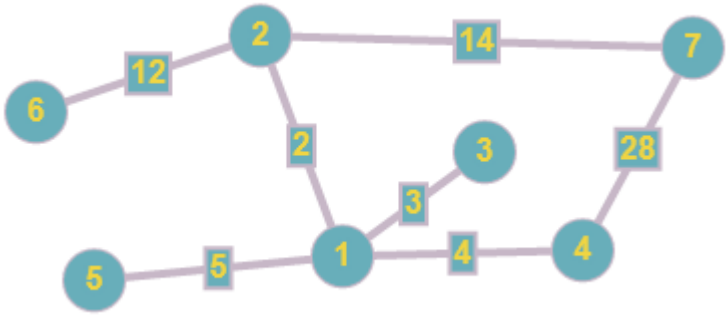
The algorithm then finds the next smallest edge, the edge between '2' and '7' with weight 14 and adds it.



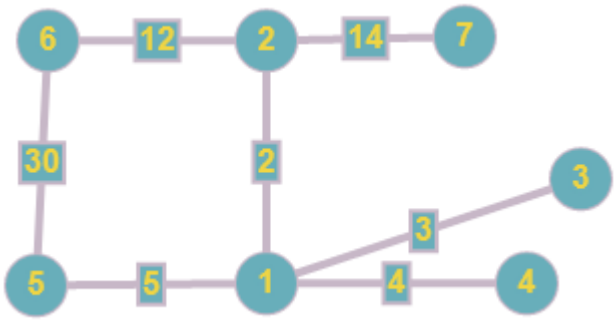
The algorithm then looks at the next smallest edge, which is the edge between '3' and '7' which has a weight of 21. However this will not be added as it would create a cycle, '1,2,7,3,1'.



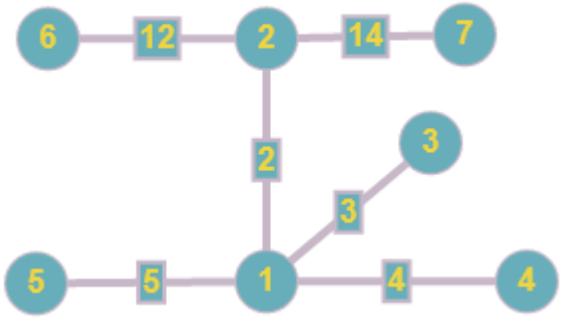
The next edge the algorithm looks at is the edge between '4' and '7' which has weight 28. This will not be added as it would create a cycle '1,2,7,4,1'.



Finally the algorithm looks at the edge between '5' and '6' which has a weight of 30. However this is not added as it would create a cycle, '1,2,6,5,1'.



So the final order minimum spanning tree is:



With total weight (cost) is 40.