Mathematics for Computer Science Summative - Elis Mostyn

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1 Discrete Mathematics and Linear Algrebra

1

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2^{n+5} \cdot 3^{4n} + 5^{3n+1} = 37m
Base Case: n = 1
2^6 \cdot 3^4 + 5^4 = 37m
64 \cdot 81 + 625 = 37m
5184 + 625 = 37m
5809 = 37m
So for n=1, m=157. Hence n=1 holds
Assume that n = k holds, where:
2^{k+5} \cdot 3^{4k} + 5^{3k+1} = 37m
Let n = k+1 = 2^{k+6} \cdot 3^{4k+4} + 5^{3k+4}
124 \cdot (2^{k+5} \cdot 3^{4k} + 5^{3k+1}) + 37 \cdot (2^{k+5} \cdot 3^{4k} \cdot 161) = 37c
2^{k+5} \cdot 3^{4k} \cdot 161 + 5^{3k+1} \cdot 124 = 37c
2^{k+5} \cdot 3^{4k}(2 \cdot 3^4 - 1) + 5^{3k+1}(5^3 - 1) = 37c
(2^{k+6} \cdot 3^{4k+4} - 2^{k+5} \cdot 3^{4k}) + (5^{3k+4} - 5^{3k+1}) = 37c
(2^{k+6} \cdot 3^{4k+4} + 5^{3k+4}) - (2^{k+5} \cdot 3^{4k} + 5^{3k+1}) = 37c
2^{k+6} \cdot 3^{4k+4} + 5^{3k+4} = 37m
As n=k+1 holds we can say that:
2^{n+5} \cdot 3^{4n} + 5^{3n+1}
Is divisible by 37 for all n \ge 1
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2

To visualise the number of bit strings, we will first provide example numbers.

Let n = 30 so there are 30 '0s'

Let k = 4 so there are 4 '1s'

No restrictions on position of 1s: $\binom{34}{4}$

At least one zero between each 1 (m=1) so 1s look like:

10 10 10 1

Taking each of the above as an indivdual item, there are 27 other '0s' and 4 groups of '1s'. Hence the combinations are: $\binom{31}{4}$

At least two zeroes between each 1 (m=2) so 1s look like:

100 100 100 1

Again, taking each of the above as an individual item, there are 24 other '0s' and once again 4 groups of '1s'. So the combinations are: $\binom{28}{4}$

Using this example we can see a pattern occurring: where the number of combinations of bit strings is:

$$\binom{NumOf0s+NumOf1s-NumBetween(NumOf1s-1)}{NumOf1s}$$

Which in terms of n,k and m is:

$$\binom{n+k-m\cdot(k-1)}{k}$$

3

To identify the probability that neither of the outcomes appears 3 times in a row, we will first identify the probability of that occuring.

Each of the 2^{10} outcomes can be viewed as a bit string.

Then using a program in python we can see where three of the same outcome

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OCCUTS:

def printOptions():
    counter = 0 #Count from 0
    for i in range(0,1024): #For all of the combinations
        num = str(format(i, '#012b')) #Converts the number into a 10 bit binary string
    counter = check3(num,counter,i) #Check the number for 3 ls or 0s
           print(counter)
   def check3(num,counter,i):
           cnecks(num,counter,1):
for j in range(0,len(num)-2): #So checks every possible place
    if((num[j] == num[j+1]) and (num[j] == num[j+2])): #If the 3 of the same outcome occurs
        counter += 1 #Increment counter
        print(num + "-" + str(i)) #Print the binary value + the decimal value
           return counter
print("break") #Number is not valid so print "break"
            return counter
```

Starting at 0000000000, we can count the ranges in which it occurs:

0000000000 0010010001 146 0010010111 0010011000 2 0010011100 0010100011 8 0010101111 0010101000 2 0010101110 0010101000 4 001101111 0011010001 4 001101110 0011011000 2 001101111 0011011000 2 001101110 0100100011 72 0100101110 0100100000 2 0100101110 0100100000 2 0100101110 0101010000 4 0100101110 0101010000 18 0101010111 0101010000 2 010101110 0101100001 4 010101110 0101100001 8 010101110 0101010001 8 011001111 0101010001 8 0110011110 011010001 8 0110011110 011010001 8 011001111 0110100001 4 0110101110 0110100001	Begin	End	Number of Combinations
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	1101001110	1101010001	
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	1101101110	1111111111	146

Total = 846 Combinations where 3 of either outcomes occurs. P(No outcome 3 times in a row) = 1 - $\frac{846}{1024}$

P(No outcome 3 times in a row) = $\frac{89}{512}$

4

If variable X is equal to the number of the umbrellas left on the bus. Then:
$$\begin{split} \text{ExpectedValue}(\mathbf{X}) = & 4 \cdot P(all4left) + 3 \cdot P(exactly3left) + 2 \cdot P(exactly2left) + \\ & 1 \cdot P(exactly1left) + 0 \cdot P(noneleft) \\ & P(\text{all 4 left}) = (0.4 \cdot 0.5 \cdot 0.6 \cdot 0.8) \\ & P(\text{all 4 left}) = 0.096 \\ & P(\text{Exactly 3 Left}) \end{split}$$

Combination	Probability
P1,P2,P3,¬ P4	0.024
P1,P2,P4,¬ P0	0.064
P1,P3,P4,¬P2	0.096
P2,P3,P4,¬P1	0.144
Total	0.328

P(Exactly 2 Left):

Combination	Probability
P1,P2,¬P3,¬P4	0.016
P1,¬P2,P3,¬P4	0.024
P1,¬P2,¬P3,P4	0.064
¬P1,P2,P3,¬P4	0.036
$\neg P1,P2,\neg P3,P4$	0.096
$\neg P1, \neg P2, P3, P4$	0.144
Total	0.364

P(Exactly 1 Left)

Combination	Probability
$P1, \neg P2, \neg P3, \neg P4$	0.016
$\neg P1,P2,\neg P3,\neg P4$	0.024
$\neg P1, \neg P2, P3, \neg P4$	0.036
$\neg P1, \neg P2, \neg P3, P4$	0.096
Total	0.172

 $P(None left) = 0.6 \cdot 0.5 \cdot 0.4 \cdot 0.2$

P(None left) = 0.024

Expected Value = 4(0.096) + 3(0.328) + 2(0.364) + 1(0.172) + 0(0.024)

Expected Value = 0.384 + 0.984 + 0.728 + 0.172 + 0

Expected Value = 2.268

 $Variance(x) = E(x^2) - (E(x))^2$

 $Variance(x) = 16 \cdot (0.096) + 9 \cdot (0.328) + 4 \cdot (0.364) + 1 \cdot (0.172) + 0 \cdot (0.024) - (2.0268)^{2}$

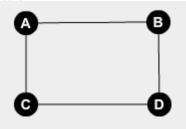
 $Variance(x) = 6.116 - (2.268)^2$

Variance(x) = 0.972

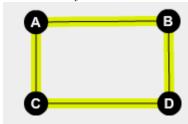
5

For n=4, there exists a graph with diameter 2:

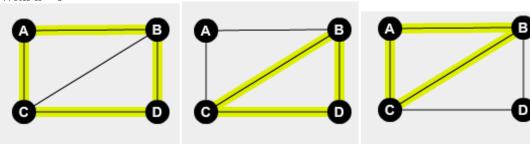
With k = 1



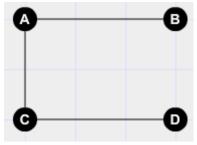
Where the cycle is as follows:



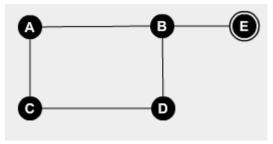
With k = 3



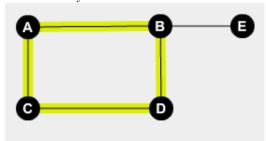
k=0 Doesn't exist for diameter 2 as removing an edge makes the diameter 3.



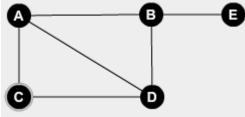
For n = 5, there exists a graph with diameter 3 With k =1



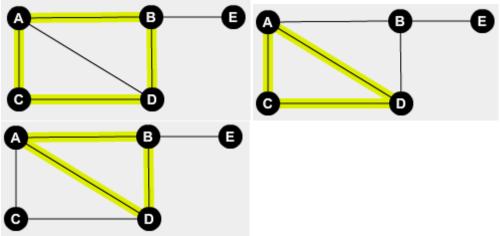
Where the cycle is as follows



With k=3

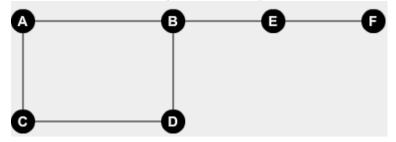


With cycles as follows:



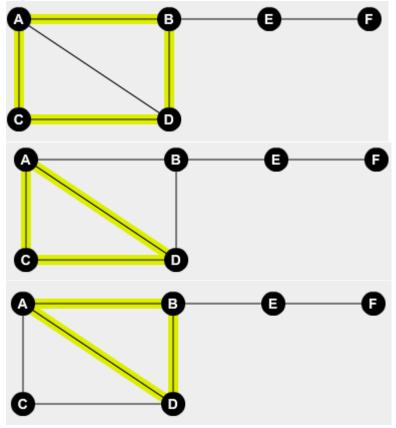
For any $n \ge 4$ a graph can be drawn which has a diameter of n-2 and k=1 and k=3 by adding an extra vertex in a chain from "B".

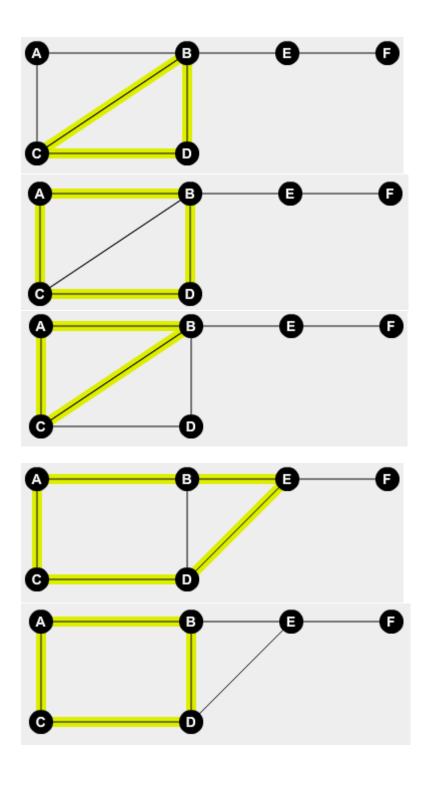
And either having an edge between A & D for k=3 or not for k=1 k=2 cannot exist in a graph because adding another edge to a graph where k=1 holds will add an extra 2 cycles. For example n=6:

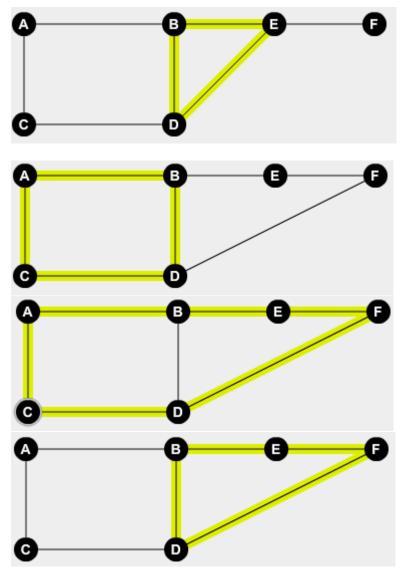


Which holds for k=1

No edge can be added which adds only 1 cycle, even ignoring the n-2 diameter requirement:







Therefore $k\neq 2$ Furthermore, for n-2 diameter, $1\geq k\leq 3$ as adding more edges to increase cycles will decrease the diameter.

2 Logic and Discrete Structures

6a

To convert a formula to disjunctive normal form using truth tables,

a	b	c	$\neg a \wedge \neg b$	$(a \wedge b) \implies c$	$\neg(a \land b \implies c) \lor \neg a \land \neg b$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	0	1	0
0	1	1	0	1	0
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	0	0	1
1	1	1	0	1	0

$$\begin{array}{l} \varphi = (\neg a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge c) \vee (a \wedge b \wedge \neg c) \\ \text{Although, due to } \neg c \vee c \\ \varphi = (\neg a \wedge \neg b) \vee (a \wedge b \wedge \neg c) \end{array}$$

6b.

$$\neg \varphi = \neg((\neg a \wedge \neg b) \vee (a \wedge b \wedge \neg c))$$

$$\neg \varphi = (a \vee b) \wedge (\neg a \vee \neg b \vee c)$$

7

To prove a set functionally complete we must prove it can represent all the logical operators of the set:

$$\{\vee,\wedge,\neg\}$$

As that is a set we know to be functionally complete:

If
$$P \bowtie Q \equiv (P \iff Q) \lor P$$

Then the truth table is as follows

P	Q	$P \iff Q$	$(P \iff Q) \wedge P$
0	0	1	0
0	1	0	0
1	0	0	0
1	1	1	1

From this truth table we can see that $P \bowtie Q \equiv P \land Q$

As a result we can say that the logical operator \wedge can be produced from the set $\{\vee,\bowtie\}$

Providing truth premises to the set produces this:

Р	Q	Operator	Output
1	1	V	1
1	1	\bowtie	1

This is significant because it means that the set is truth preserving. Meaning that it does not produce a false output given a true input. As a result of this, the set cannot represent the \neg operator.

Therefore, the set is not functionally complete.

8a

8b

$$\begin{array}{llll} 1)a \wedge b & Premise \\ 2) \ a \implies (\neg c \wedge \neg b) & Premise \\ 3)a & \wedge e(1) \\ 4) \ (\neg c \wedge \neg d) & \implies e(3), (2) \\ 5) \ \neg c & \wedge e(4) \\ 6) \ \neg d & \wedge e(4) \\ 7) & c \vee d & Assumption \\ 8) & \bot & \neg e(5), (6), (7) \\ 9) \ \neg (c \vee d) & \neg i(7-8) \end{array}$$

9

To identify whether φ is a theorem, where:

 $\varphi = \neg((\neg t \lor \neg r) \land (p \lor p) \land (\neg p \lor \neg q) \land (r \lor p \lor q \lor \neg r) \land (t \lor s) \land (\neg r \lor \neg s \lor t) \land (q \lor r))$ We must negate φ to find $\neg \varphi$ and apply resolution continually until we have inferred the empty clause or reach a point where we cannot infer any new clauses.

$$\neg \varphi = ((\neg t \vee \neg r) \wedge (p \vee p) \wedge (\neg p \vee \neg q) \wedge (r \vee p \vee q \vee \neg r) \wedge (t \vee s) \wedge (\neg r \vee \neg s \vee t) \wedge (q \vee r))$$

- 1) $\neg t \vee \neg r$
- 2) $p \lor p$
- 3) $\neg p \lor \neg q$
- $4)\ r \vee p \vee q \vee \neg r$
- 5) $t \vee s$
- $6) \neg r \lor \neg s \lor t$

As $\neg \varphi$ Produces the empty clause, φ must be always true, hence it is a theorem.