

Mathematics for Computer Science

Summative Assignment

Hand-in deadline: **14:00 on 9th December 2019**. Your solutions must be submitted electronically via DUO **as a pdf file**. Submissions not in the pdf format will result in the mark of 0. Credit will be given for partial answers.

As this is a summative assignment, you are expected to work on your own.

You may use Microsoft Word or any other word processing program, but you are encouraged to use LaTeX. You may also scan in hand-written pages provided your scans are legible. It is not allowed to submit photos (e.g. made with a mobile phone), and *if we find an answer illegible, a mark of 0 will be given for it*.

LaTeX (X is pronounced as -ch in Scottish loch) is *de facto* the standard typesetting system for Maths and Computer Science. This document was prepared using LaTeX. LaTeX is very easy to learn, with plenty of manuals available on the web. It can be downloaded for free from <http://www.latex-project.org/>.

Please record how many hours did you spend on this assignment and return the number with your submission.

The problems to solve are given on the next page.

Discrete Mathematics and Linear Algebra

Your answers should include the full explanation (and not just the number or yes/no).

1. Prove, *using induction*, that 37 divides $2^{n+5} \cdot 3^{4n} + 5^{3n+1}$ for all $n \geq 1$. [6 marks]
2. Find the number of bit strings with k ones and n zeroes in which any two ones have at least m zeroes between them. [8 marks]
3. A fair coin is flipped 10 times. What is the probability that no outcome (heads or tails) ever comes up three times in a row? [8 marks]
4. There are four absent-minded people, each with one umbrella, on a bus. They leave their umbrellas (independently) with probabilities $p_1 = 0.4$, $p_2 = 0.5$, $p_3 = 0.6$, and $p_4 = 0.8$. Find the expected value and the variance of the random variable X equal to the number of umbrellas left on the bus. [8 marks]
5. Let $n \geq 4$. Find *all* values $k \geq 0$ such that there exists a graph with n vertices that has diameter $n - 2$ and *exactly* k distinct cycles. Whenever such a graph exists, describe (e.g. draw) it clearly. In all other cases, prove non-existence. [20 marks]

Logic and Discrete Structures

Your answers should include the full explanation (and not just the final formula or yes/no).

6. Let φ be the formula $\neg((a \wedge b) \Rightarrow c) \vee (\neg a \wedge \neg b)$.
 - (a) Reduce φ to disjunctive normal form using truth tables. [10 marks]
 - (b) Reduce $\neg\varphi$ to conjunctive normal form (without using truth tables). [5 marks]
7. Is the set $\{\vee, \bowtie\}$, where $p \bowtie q \equiv (p \Leftrightarrow q) \wedge p$, functionally complete? [5 marks]
8. Prove the following sequents using natural deduction. In your proofs you may (only) use the rules $\wedge i$, $\wedge e1$, $\wedge e2$, $\neg i$, $\neg e$, $\Rightarrow e$, $\Rightarrow i$, $\vee i1$, $\vee i2$, $\vee e$, $\perp e$, $\neg e$, $\neg i$ and the Law of the Excluded Middle. At each step mention the rule you have applied. You may write $\vee i$ for $\vee i1$ and $\vee i2$. Similarly, you may write $\wedge e$ for $\wedge e1$ and $\wedge e2$.
 - (a) $\neg(a \vee b) \wedge c \vdash \neg a \wedge (\neg b \wedge c)$ [10 marks]
 - (b) $a \wedge b, a \Rightarrow (\neg c \wedge \neg d) \vdash \neg(c \vee d)$ [10 marks]
9. Let φ be the formula
$$\neg((\neg t \vee \neg r) \wedge (p \vee p) \wedge (\neg p \vee \neg q) \wedge (r \vee p \vee q \vee \neg r) \wedge (t \vee s) \wedge (\neg r \vee \neg s \vee t) \wedge (q \vee r)).$$
Is φ a theorem? Use Resolution to answer this question. Mention on which clauses you use resolution each time you apply it. [10 marks]