Semantics

$$\boxed{TS \models \Phi \iff \forall s \in I.\ s \models \Phi}$$

$s \models \text{true}$; $s \models a \iff a \in L(s)$

$s \models \neg \Phi \iff \text{not } s \models \Phi$

$s \models \Phi \wedge \Psi \iff s \models \Phi \ \& \ s \models \Psi$

$s \models \exists \varphi \iff \exists \pi \in Paths(s) : \pi \models \varphi$

$s \models \forall \varphi \iff \forall \pi \in Paths(s) : \pi \models \varphi$

$\pi \models \bigcirc \Phi \iff \pi[1] \models \Phi$

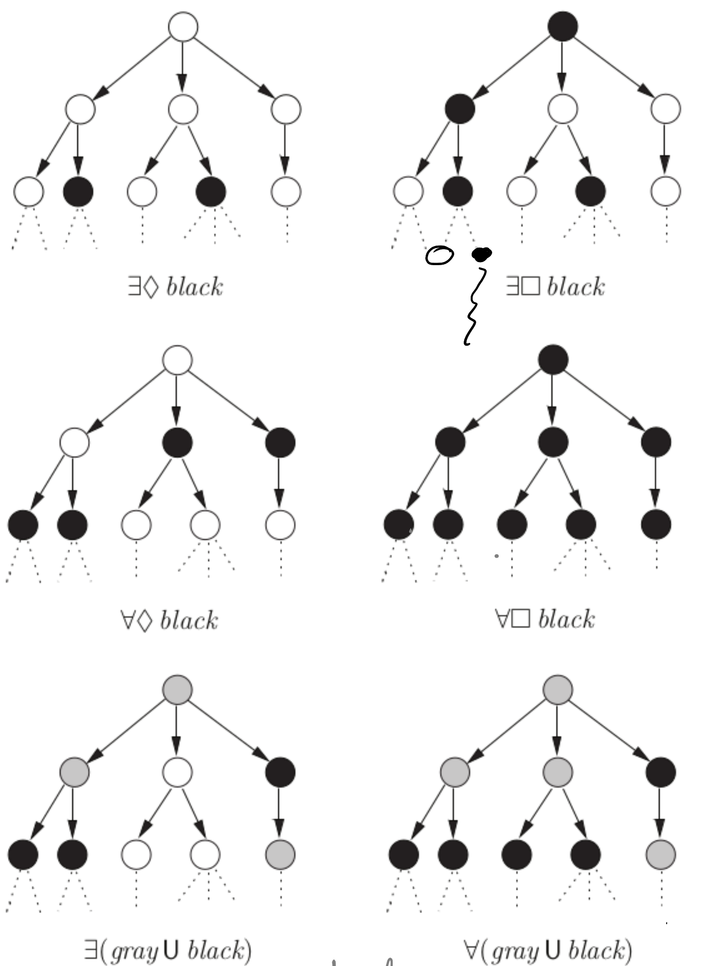
$\pi \models \Phi \, U \, \Psi \iff \exists_{j \geq 0} : \pi[j] \models \Psi \ \& \ \forall_{0 \leq k < j} : \pi[k] \models \Phi$

Eventually

$$\begin{cases} \exists \Diamond \phi \equiv \exists(\text{true } U \phi) & \text{potentially } \phi \\ \forall \Diamond \phi \equiv \forall(\text{true } U \phi) & \text{inevitably } \phi \end{cases}$$

Always

$$\begin{cases} \exists \Box \phi \equiv \neg \forall \Diamond \neg \phi & \text{potentially invariantly } \phi \\ \forall \Box \phi \equiv \neg \exists \Diamond \neg \phi & \text{invariantly } \phi \end{cases}$$



Borrowed from [1]

Figure 6.2: Visualization of semantics of some basic CTL formulae.

The syntactic restrictions of CTL forbid writing e.g.

$$\underline{\text{fairness}} \quad \bigwedge_{0\le i\le n} (\Box\Diamond w_i \to \Box\Diamond c_i) \quad \text{is } \underline{\text{not}} \text{ a CTL f.la}$$

which is not in CTL because of the consecutive temporal operators.

[Emerson & Halpern 86] propose CTL*

Syntax

$$\Phi ::= true \mid \underset{\cap\, AP}{a} \mid \neg\Phi \mid \Phi\wedge\Phi \mid \exists\varphi \qquad \text{state f.lae}$$

$$\varphi ::= \Phi \mid \varphi\wedge\varphi \mid \neg\varphi \mid \circ\varphi \mid \varphi\, U\, \varphi \qquad \text{path f.lae}$$

<u>Semantics</u> $\boxed{TS \models \Phi \iff \forall s\in I.\ s\models\Phi}$

$$\left.\begin{array}{lll}
\forall s\in S\ s\models true & ; & s\models a \iff a\in L(s) \\
s\models\neg\Phi & \iff & \text{not } s\models\Phi \\
s\models\Phi\wedge\Psi & \iff & s\models\Phi \ \&\ s\models\Psi \\
s\models\exists\varphi & \iff & \exists\pi\in Paths(s):\ \pi\models\varphi
\end{array}\right\} \text{ as for CTL}$$

$$\begin{array}{lll}
\pi\models\Phi & \iff & \pi[0]\models\Phi \\
\pi\models\varphi_1\wedge\varphi_2 & \iff & \pi\models\varphi_1 \ \&\ \pi\models\varphi_2 \\
\pi\models\neg\varphi & \iff & \pi\not\models\varphi \\
\pi\models\circ\varphi & \iff & \pi_{\geq 1}\models\varphi \\
\pi\models\varphi_1 U\varphi_2 & \iff & \exists j\geq 0:\ \pi_{\geq j}\models\varphi_2 \ \&\ (\forall 0\leq h<j:\ \pi_{\geq h}\models\varphi_1)
\end{array}$$