A formal model of concurrency [Bergstra et al]

Extending regular expressions to model concurrency

$A_\tau = A \cup \{\tau\} \qquad \tau \notin A$

Note the different yet equivalent definition wrt [Bergstra et al.]

$E ::= \ldots \mid E \parallel E$

(Act) $$\frac{a \in A_\tau}{a \xrightarrow{a} 1}$$

(Cho1) $$\frac{x \xrightarrow{a} x' \quad x' \neq 1}{x + y \xrightarrow{a} x'}$$

(Cho2) $$\frac{x \xrightarrow{a} 1}{x + y \xrightarrow{a} 1}$$

(Cho3) $$\frac{y \xrightarrow{a} y' \quad y' \neq 1}{x + y \xrightarrow{a} y'}$$

(Cho4) $$\frac{y \xrightarrow{a} 1}{x + y \xrightarrow{a} 1}$$

(Seq1) $$\frac{x \xrightarrow{a} x' \quad x' \neq 1}{x \cdot y \xrightarrow{a} x' \cdot y}$$

(Seq1) $$\frac{x \xrightarrow{a} 1}{x \cdot y \xrightarrow{a} y}$$

(per1) $$\frac{x \xrightarrow{a} x' \quad x' \neq 1}{x \parallel y \xrightarrow{a} x' \parallel y}$$

(per2) $$\frac{y \xrightarrow{a} y' \quad y' \neq 1}{x \parallel y \xrightarrow{a} x \parallel y'}$$

(per3) $$\frac{x \xrightarrow{a} 1}{x \parallel y \xrightarrow{a} y}$$

(per4) $$\frac{y \xrightarrow{a} 1}{x \parallel y \xrightarrow{a} x}$$

Interleaving semantics

What about communication?

Let $A_\perp = A \cup \{\perp\}$ $\quad \perp \notin A$ and fix a communication function

$$\_\circ\_ : A_\perp \times A_\perp \longrightarrow A_\perp \left\{ \begin{array}{l} \circ \text{ commutative} \\ \circ \text{ associative} \\ \forall a \in A_\perp : a \circ \perp = \perp \circ a = \perp \end{array} \right.$$

$$(com_1)\ \frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} y' \quad a \circ b \in A}{x \parallel y \xrightarrow{a \circ b} x' \parallel y'} \qquad (com_2)\ \frac{x \xrightarrow{a} 1 \quad y \xrightarrow{b} y' \quad a \circ b \in A}{x \parallel y \xrightarrow{a \circ b} y'}$$

$$(com_3)\ \frac{x \xrightarrow{a} x' \quad y \xrightarrow{b} 1 \quad a \circ b \in A}{x \parallel y \xrightarrow{a \circ b} x'} \qquad (com_4)\ \frac{x \xrightarrow{a} 1 \quad y \xrightarrow{b} 1 \quad a \circ b \in A}{x \parallel y \xrightarrow{a \circ b} 1}$$

Example

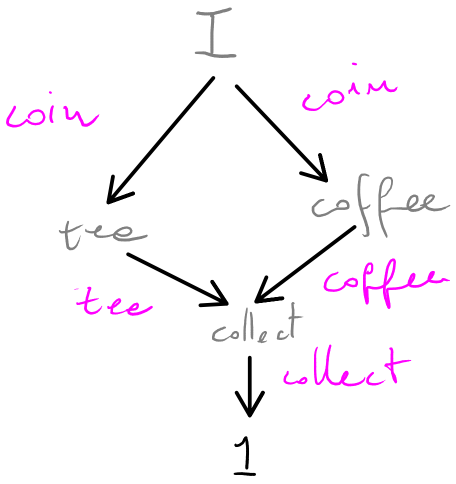
Show that $a x + b y \parallel c z \xrightarrow{b} x \parallel z$ if $a \circ b = b$, $x \neq 1$, and $y \neq 1$

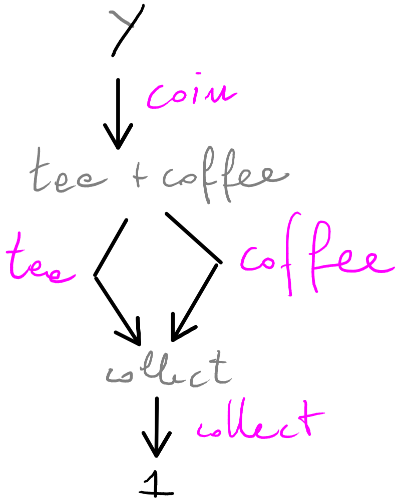
$$
\dfrac{
\dfrac{\dfrac{\dfrac{a \in A}{a \xrightarrow{a} 1}\ \text{Act}}{a x \xrightarrow{a} x}\ \text{Seq}_1}{a x + b y \xrightarrow{a} x}\ \text{Cho}_1
\qquad\qquad
\dfrac{\dfrac{c \in A}{c \xrightarrow{c} 1}\ \text{Act}}{c z \xrightarrow{c} z}\ \text{Seq}_2
}{a x + b y \parallel c z \xrightarrow{b} x \parallel z}\ \text{Com}_1
$$

A glimpse of reactive systems

$I = (coin \cdot tee + coin \cdot coffee) \cdot collect$

$Y = coin \cdot (tee + coffee)$





- I & Y exhibit the same traces (words)
- But they differ to an external observer!

Summary

- FM : what for & basic (fundamental questions)

- Brief overview of concurrency
  - Problems
  - Shared-memory vs communication

- Operational semantics
  - Transition Systems
  - Structural operational semantics
  - Reg Exp
  - BPAs