

# Binary Session Types

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## Type Judgement

$$\Gamma \vdash P$$

$P$  uses channels as specified by  $\Gamma$

## Environments $\Gamma$

- ▶ Partial function from polarized names to types
- ▶ Written  $x_1^{p_1} : t_1, x_2^{p_2} : t_2, \dots, x_n^{p_n} : t_n$
- ▶ Its satisfies one of the following conditions
  - ▶  $x^+, x^-, x \notin \text{dom}(\Gamma)$
  - ▶  $x \in \text{dom}(\Gamma)$  and  $x^+, x^- \notin \text{dom}(\Gamma)$
  - ▶  $x^p \in \text{dom}(\Gamma)$  and  $p \in \{+, -\}$  and  $x^{\bar{p}}, x \notin \text{dom}(\Gamma)$
  - ▶  $x^+, x^- \in \text{dom}(\Gamma)$  and  $x \notin \text{dom}(\Gamma)$

# Typing

$x^+ : ?\text{int}.\text{!bool}.\text{end} \vdash x^+?(y:\text{int}).x^+!\text{true}.0$

$x^+ : ?\text{int}.\text{!bool}.\text{end} \not\vdash x^+?(y:\text{int}).x^+!y.0$

$x^+ : ?\text{int}.\text{end}, y^- : \text{!int}.\text{end} \vdash x^+?(z:\text{int}).y^-!z.0$

$x^+ : ?\text{int}.\text{end}, y^- : \text{!bool}.\text{end} \not\vdash 0$

$\vdash (\nu x:?\text{int}.\text{end})(x^+?(z:\text{int}).0 \mid x^-!1.0)$

$\not\vdash (\nu x:?\text{int}.\text{end})(x^+?(z:\text{int}).0)$

$\not\vdash (\nu x:?\text{int}.\text{end})(x^+?(z:\text{int}).0 \mid x^-!1.0 \mid x^-!2.0)$

# Typing

$\nVdash (\nu x: ?\text{int}. ?\text{int}. \text{end})(x^+?(z:\text{int}). x^+?(z:\text{int}). 0 \mid x^-!1. 0 \mid x^-!2. 0)$

Think about

$(\nu x: ?\text{int}. !\text{int}. ?\text{int}. !\text{int}. \text{end})($   
 $x^+?(z:\text{int}). x^+!(z+1). 0 \mid$   
 $x^+?(z:\text{int}). x^+!(z+1). 0 \mid$   
 $x^-!1. x^-?(z:\text{int}). Q_1 \mid$   
 $x^-!2. x^-?(z:\text{int}). Q_2 \quad )$

## Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} [\text{T-Par}]$$

$$x^+ : \text{Tester}, x^- : \overline{\text{Tester}} \vdash P_{\text{server}} \mid P_{\text{client}}$$

where

`Tester` = `?int.!bool.end`

$P_{\text{server}} = x^+?(y:\text{int}).x^+!\text{true}.0$

$P_{\text{client}} = x^-!1.x^-(z:\text{bool}).Q$

## Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} [\text{T-Par}]$$

$$x^+ : \text{Tester}, x^- : \overline{\text{Tester}} \not\vdash P_{\text{server}} \mid P_{\text{client}} \mid P_{\text{client}}$$

where

`Tester = ?int.!bool.end`

$P_{\text{server}} = x^+?(y:\text{int}).x^+!\text{true}.0$

$P_{\text{client}} = x^-!1.x^-(z:\text{bool}).Q$

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} [\text{T-Par}]$$

## Context split

$$\begin{aligned} \Gamma + x^+ : t &= \Gamma, x^+ : t && \text{if } x, x^+ \notin \text{dom}(\Gamma) \\ \Gamma + x^- : t &= \Gamma, x^- : t && \text{if } x, x^- \notin \text{dom}(\Gamma) \\ \Gamma + x : t &= \Gamma, x : t && \text{if } x, x^+, x^- \notin \text{dom}(\Gamma) \\ (\Gamma, x : t) + x : t &= \Gamma, x : t && \text{if } t \text{ is not a session type} \end{aligned}$$

Extended on context as

$$\begin{aligned} \Gamma + \emptyset &= \Gamma \\ \Gamma + (x^p : t, \Delta) &= (\Gamma + x^p : t) + \Delta \end{aligned}$$

Linear usage of session endpoints

# Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} [\text{T-Par}]$$

$$\frac{\Gamma, x^+ : S, x^- : \bar{S} \vdash P}{\Gamma \vdash (\nu x : S) P} [\text{T-Res}]$$

$$(\nu x : \text{Tester})(P_{\text{server}} \mid P_{\text{client}})$$

where

$\text{Tester} = ?\text{int} . !\text{bool} . \text{end}$

$P_{\text{server}} = x^+ ? (y : \text{int}) . x^+ ! \text{true} . 0 \quad (\text{faulty})$

$P_{\text{client}} = x^- ! 1 . x^- ? (z : \text{bool}) . Q$

# Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} \text{[T-Par]}$$

$$\frac{\Gamma, x^p : S, y : t \vdash P}{\Gamma, x^p : ?t.S \vdash x^p?(y:t).P} \text{[T-In]}$$

$$\frac{\Gamma, x^+ : S, x^- : \bar{S} \vdash P}{\Gamma \vdash (\nu x:S)P} \text{[T-Res]}$$

$$\frac{\Gamma_1 \vdash v : t \quad \Gamma_2, x^p : S \vdash P}{\Gamma_1 + (\Gamma_2, x^p : !t.S) \vdash x^p!v.P} \text{[T-Out]}$$

Auxiliary Typing on expressions  $\Gamma \vdash v : t$

$$\begin{array}{ll} \emptyset \vdash \text{true} : \text{bool} & \emptyset \vdash \text{false} : \text{bool} \\ \emptyset \vdash () : \text{unit} & x^p : t \vdash x^p : t \end{array}$$

# Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} [\text{T-Par}]$$

$$\frac{\Gamma, x^+ : S, x^- : \bar{S} \vdash P}{\Gamma \vdash (\nu x : S) P} [\text{T-Res}]$$

$$\frac{\Gamma, x^p : S, y : t \vdash P}{\Gamma, x^p : ?t.S \vdash x^p ?(y : t).P} [\text{T-In}]$$

$$\frac{\Gamma_1 \vdash v : t \quad \Gamma_2, x^p : S \vdash P}{\Gamma_1 + (\Gamma_2, x^p : !t.S) \vdash x^p !v.P} [\text{T-Out}]$$

$$\frac{\Gamma, x^p : S_j \vdash P \quad j \in I}{\Gamma, x^p : \oplus [\mathfrak{l}_i : S_i]_{i \in I} \vdash x^p \triangleleft \mathfrak{l}_j.P} [\text{T-Choice}]$$

$$\frac{\Gamma, x^p : S_i \vdash P_i \quad \forall i \in I}{\Gamma, x^p : \& [\mathfrak{l}_i : S_i]_{i \in I} \vdash x^p \triangleright [\mathfrak{l}_i : P_i]_{i \in I}} [\text{T-Branch}]$$

$$\frac{\Gamma \text{ completed}}{\Gamma \vdash 0} [\text{T-Nil}]$$

$\Gamma$  completed if  $\Gamma(x^p) = S$  implies  $S = \text{end}$