

Introduction to Formal Methods

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Grammars

A grammar is a 4-tuple $G = \langle T, N, S, P \rangle$ where

- T is a finite set of terminals
- N is a finite set of non-terminals ($N \cap T = \emptyset$)

- $S \in N$ starting symbol

- $P \subseteq (T \cup N)^* \times (T \cup N)^*$ s.t.

$$(u, v) \in P \Rightarrow \exists X \in N, l, z \in (T \cup N)^* : u = lXz$$

$$(u, v_1, \dots, (l, v_n)) \in P$$

$$u ::= v_1 \dots v_n$$

$$L(G) = \{ w \in T^* \mid S \Rightarrow^* w \} \text{ where } \Rightarrow^* = \{ (luz, l'vz) \mid l, z \in (T \cup N)^* \wedge (u, v) \in P \}$$

Exercise 4

Find two derivations of the regular expression $1 + a \cdot b$ using the grammar of regular expressions on page 16.

Regular expressions

BNF-like syntax A , finite alphabet

$E ::= \emptyset \mid 1 \mid a \mid E + E \mid E \cdot E \mid E^*$

\nearrow end
 \nearrow skip
 \nearrow atomic instruct.
 \nearrow if-then-else
 \nearrow iteration

Denotational semantics: $\mathcal{L}: E \rightarrow \mathcal{L}^{A^*}$

$$\mathcal{L}(\emptyset) = \emptyset \quad \mathcal{L}(1) = \{ \epsilon \} \quad \mathcal{L}(a) = \{ a \}$$

$$\mathcal{L}(E_1 + E_2) = \mathcal{L}(E_1) \cup \mathcal{L}(E_2)$$

$$\mathcal{L}(E_1 \cdot E_2) = \mathcal{L}(E_1) \cdot \mathcal{L}(E_2) \triangleq \{ vw \mid v \in \mathcal{L}(E_1), w \in \mathcal{L}(E_2) \}$$

$$\mathcal{L}(E^*) = \mathcal{L}(E)^* = \bigcup_{n \geq 0} \mathcal{L}(E)^n$$

we'll see that grammars specify Term-algebras

Term-Algebra homomorphism

Exercise 3

Prove or disprove that $(a + b)^* = (a^* + b^*)^*$