

Introduction to Formal Methods

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Grammars

A grammar is a 4-tuple $G = \langle T, N, S, P \rangle$ where

- T is a finite set of terminals
- N is a finite set of non-terminals ($N \cap T = \emptyset$)

$S \in N$ starting symbol

$$P \subseteq (T \cup N)^* \times (T \cup N)^*$$

$$(w, r) \in P \Rightarrow \exists X \in N, \ell, r \in (T \cup N)^*: w = \ell X r$$

$$\begin{matrix} (u_1, v_1), \dots, (u_n, v_n) \in P \\ u_i := v_{i+1} \cdots v_n \end{matrix}$$



$$L(G) = \{ w \in T^* \mid S \Rightarrow^* w \} \text{ where } \Rightarrow^* = \{ (\ell u r, l o r) \mid \ell, r \in (T \cup N)^*, u \in (u, v) \in P \}$$

Exercise 4
Find two derivations of the regular expression $1 + a \cdot b$ using the grammar of regular expressions on page 16.

Regular expressions

BNF-like syntax \mathcal{L} , finite alphabet A

$$E ::= 0 \mid 1 \mid a \mid \bar{E} + E \mid E \cdot E \mid E^*$$

and $\begin{cases} \text{skip} \\ \text{atomic} \\ \text{if-then-else} \\ \text{iteration} \end{cases}$
instruction

Deduced semantics: $\mathcal{L}: E \rightarrow 2^{A^*}$

$$\mathcal{L}(0) = \emptyset \quad \mathcal{L}(1) = \{e\} \quad \mathcal{L}(a) = \{a\}$$

$$\mathcal{L}(\bar{E}_1 + E_2) = \mathcal{L}(\bar{E}_1) \cup \mathcal{L}(E_2)$$

$$\mathcal{L}(E_1 \cdot E_2) = \mathcal{L}(E_1) \cdot \mathcal{L}(E_2) \triangleq \{v w \in A^* \mid v \in \mathcal{L}(E_1), w \in \mathcal{L}(E_2)\}$$

$$\mathcal{L}(E^*) = \mathcal{L}(E)^* = \bigcup_{n \geq 0} \mathcal{L}(E)^n$$

Term-Algebra homomorphism

we'll see that grammars specify Term-algebras

Exercice 3

Prove or disprove that $(a+b)^* = (a^*+b^*)^*$