Fairness

Usually liveness property do not hold, unless fairness assumptions are made

$A \subseteq Act$

An execution (fragment) $\rho = s_0 \xrightarrow{\alpha_1} s_2 \xrightarrow{\alpha_2} \cdots$ is

there is a wealth of fairness conditions

- unconditionally A-fair: $\overset{\infty}{\exists} j \geq 0 : \alpha_j \in A$
- strongly A-fair: $\overset{\infty}{\exists} j \geq 0 : A \cap Act(s_j) \neq \emptyset \Rightarrow \overset{\infty}{\exists} j \geq 0 : \alpha_j \in A$
- weakly A-fair: $\overset{\infty}{\forall} j \geq 0 : A \cap Act(s_j) \neq \emptyset \Rightarrow \overset{\infty}{\exists} j \geq 0 : \alpha_j \in A$

$Act(s_j) := \{\alpha \in Act \mid \exists s' \in S : s_j \xrightarrow{\alpha} s'\}$

Checking liveness property is often made by restricting to fair executions: $\boxed{TS \models_F P \iff FairTraces(TS) \subseteq P}$

Linear Temporal Logic (propositional)

Syntax $\varphi ::= \text{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi$ logical connectives

$\mid \circ\varphi \mid \varphi_1 \, U \, \varphi_2$ temporal modalities

redundant if $AP \neq \emptyset$; $a \in AP$; U right associative

Obs false, $\vee$, $\rightarrow$, $\leftrightarrow$, $\oplus$ obtained as usual eg

$$\varphi_1 \oplus \varphi_2 \stackrel{def}{=} (\varphi_1 \wedge \neg\varphi_2) \vee (\neg\varphi_1 \wedge \varphi_2)$$

Intuitive semantics

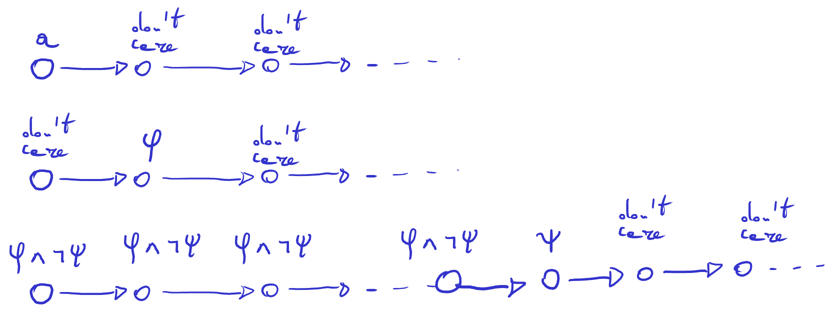
An LTL f.la expresses a property of an infinite "path"

(i.e. the models of an LTL f.la are infinite sequences of 2^{AP} (= states))

$a \rightsquigarrow$

$\circ\varphi \rightsquigarrow$

$\varphi \, U \, \psi \rightsquigarrow$



Formal Semantics

Let $\sigma \in (2^{AP})^\omega$ and $\sigma = A_0 \ldots A_i A_{i+1} \ldots$ then $\begin{cases} \sigma_{\geq i} = A_i A_{i+1} \ldots \\ \sigma[i] = A_i \end{cases}$

$\sigma \in (2^{AP})^\omega$ models $\varphi \in LTL$ if $\sigma \models \varphi$ can be derived from the following statements

$\sigma \models \text{true}$

$\sigma \models a$ iff $a \in \sigma[0]$ $(\equiv \sigma[0] \models a)$

$\sigma \models \varphi \wedge \psi$ iff $\sigma \models \varphi$ and $\sigma \models \psi$

$\sigma \models \neg\varphi$ iff $\sigma \not\models \varphi$

$\sigma \models \circ\varphi$ iff $\sigma_{\geq 1} \models \varphi$

$\sigma \models \varphi \, U \, \psi$ iff $\exists_{j \geq 0} : \sigma_{\geq j} \models \psi$ and $\forall_{0 \leq i < j} : \sigma[i] \models \varphi$

$$\text{Words}(\varphi) = \{ \sigma \in (2^{AP})^\omega \mid \sigma \models \varphi \}$$

Some important derived modalities

"eventually" $\Diamond$

$$\Diamond \varphi \equiv \text{true} \; U \; \varphi$$

$$\equiv \neg \Box \neg \varphi$$

"always" $\Box$

$$\Box \varphi \equiv \varphi \; U \; \text{false}$$

$$\equiv \neg \Diamond \neg \varphi$$

Exercise Define "infinitely often". A: $\Box \Diamond \varphi$

"eventually forever" $\Diamond \Box \varphi$

Exercise Which of the following equivalences are correct:

a) $\Box(\varphi \to \Diamond \psi) \equiv \varphi \; U \; (\varphi \wedge \neg \varphi)$

b) $\circ \Diamond \varphi \equiv \Diamond \circ \varphi$

c) $\Box(\varphi \wedge \circ \Diamond \varphi) \equiv \Box \varphi$

d) $\Diamond(\varphi \wedge \psi) \equiv \Diamond \varphi \wedge \Diamond \psi$

e) $\Box(\varphi \wedge \psi) \equiv \Box \varphi \wedge \Box \psi$

f) $\Box \Box (\psi \to \varphi) \equiv \neg \Diamond (\neg \varphi \wedge \psi)$

Exercise Give an LTL f.la expressing safety & liveness of the mutual exclusion problem