

Binary Sessions + DbC

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Binary Sessions + DbC

- ▶ An extension of FuSe with dynamically checked contracts that states properties¹
 - ▶ about exchanged messages
 - ▶ the structure of the protocol

¹M., Luca Padovani: Chaperone contracts for higher-order sessions. PACMPL 1(ICFP).

FuSe + Service channels (shared channels)

```
module type Service = sig
  type α t
  val register : ((β, α) st → unit) → (α, β) st t
  val connect   : (α, β) st t → (α, β) st
end
```

- ▶ α is the session type from the client's viewpoint
- ▶ **register f** creates a new shared channel and registers the service f to it.
 - ▶ Each connection spawns a new thread running f
 - ▶ returns the shared channel
- ▶ **connect ch** connects with the service on the shared channel ch
 - ▶ return the client endpoint of the established session.

FuSe + Service channels (shared channels)

Roots of a polynomial

```
let server ep =
  let p, ep = receive ep in
  let root = ... in
  let ep = send root ep in
  close ep

let math_service = register server
```

```
val server : ?poly.!float.end → unit
val math_service : !poly.?float.end Service.t
```

```
let user () =
  let ep = connect math_service in
  let ep = send (from_list [2.0; -3.0; 1.0]) ep in
  let _, ep = receive ep in
  close ep
```

A simple FuSe program + Contracts

Roots of a polynomial

```
let server ep =
  let p, ep = receive ep in
  let root = ... in (* assumes p is a linear equation *)
  let ep = send root ep in
  close ep

let math_service = register server contract "Server"
  (*service with a contract and a blame label*)

let user () =
  let ep = connect math_service "Client" in
  let ep = send (from_list [2.0; -3.0; 1.0]) ep in
  let _, ep = receive ep in
  close ep
```

Language for Contracts

Constructors

`flat_c` : $(t \rightarrow \text{bool}) \rightarrow \text{con}(t)$ $t :: \omega$

`send_c` : $\text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(!t.T)$

`receive_c` : $\text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(?t.T)$

`end_c` : $\text{con}(\text{end})$

Dependent Contracts

Roots of a polynomial

```
let degree p = ... (* computes the degree of a polynomial *)  
  
let contract = send_c (flat_c (fun p → degree p == 1)) @@  
                     ... (* contract for the continuation *)
```

Contracts

Roots of a polynomial

```
let contract = send_c  (flat_c (fun p → degree p == 1)) @@  
                      receive_c (flat_c (fun _ → true)) @@  
                      end_c
```

- ▶ The continuation does not impose any restriction to the communication protocol
- ▶ ... but tedious to write

any_c

Constructors

```
flat_c : (t → bool) → con(t)           t :: ω

send_c : con(t) → con(T) → con(!t. T)
receive_c : con(t) → con(T) → con(?t. T)

end_c : con(end)

any_c : con(α)
```

Roots of a polynomial

```
let contract = send_c (flat_c (fun p → degree p == 1)) @@  
any_c (* trivial contract *)
```

- ▶ Can we give some guarantee about the response?
- ▶ We would like to specify that the response is a root of the polynomial

Dependent Contracts

Constructors

`flat_c` : $(t \rightarrow \text{bool}) \rightarrow \text{con}(t)$ $t :: \omega$

`send_c` : $\text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(!t.T)$

`receive_c` : $\text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(?t.T)$

`end_c` : $\text{con}(\text{end})$

`any_c` : $\text{con}(\alpha)$

`send_d` : $\text{con}(t) \rightarrow (t \rightarrow \text{con}(T)) \rightarrow \text{con}(!t.T)$ $t :: \omega$

`receive_d` : $\text{con}(t) \rightarrow (t \rightarrow \text{con}(T)) \rightarrow \text{con}(?t.T)$ $t :: \omega$

Contracts

Roots of a polynomial

```
let root_of p r = ... (* check if r is a root of p *)  
  
let contract = send_d (flat_c (fun p → degree p == 1)) @@  
                     fun p → receive_c (flat_c (root_of p)) @@  
                     end_c
```

Contracts for choices

Simplified version of choices

```
left : T ⊕ S → T
right : T ⊕ S → S
branch : T & S → T + S
```

```
type α + β = [ Left of α | Right of β ]
val left : (Ø, (ρ₁, σ₁) st + (ρ₂, σ₂) st) → (σ₁, ρ₁) st
val right : (Ø, (ρ₁, σ₁) st + (ρ₂, σ₂) st) → (σ₂, ρ₂) st
val branch : ((ρ₁, σ₁) st + (ρ₂, σ₂) st, Ø)
                    → (ρ₁, σ₁) st + (ρ₂, σ₂) st
```

```
let left ep = send true ep
let right ep = send false ep
let branch ep =
  use ep;
  if UnsafeChannel.receive ep.channel
  then Left (fresh ep)
  else Right (fresh ep)
```

Contracts for choices

Constructors

`flat_c` : $(t \rightarrow \text{bool}) \rightarrow \text{con}(t)$ $t :: \omega$

`send_c` : $\text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(!t.T)$

`receive_c` : $\text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(?t.T)$

`end_c` : $\text{con}(\text{end})$

`any_c` : $\text{con}(\alpha)$

`send_d` : $\text{con}(t) \rightarrow (t \rightarrow \text{con}(T)) \rightarrow \text{con}(!t.T)$ $t :: \omega$

`receive_d` : $\text{con}(t) \rightarrow (t \rightarrow \text{con}(T)) \rightarrow \text{con}(?t.T)$ $t :: \omega$

`choice_c` : $\text{con}(\text{bool}) \rightarrow \text{con}(T) \rightarrow \text{con}(S) \rightarrow \text{con}(T \oplus S)$

`branch_c` : $\text{con}(\text{bool}) \rightarrow \text{con}(T) \rightarrow \text{con}(S) \rightarrow \text{con}(T \& S)$

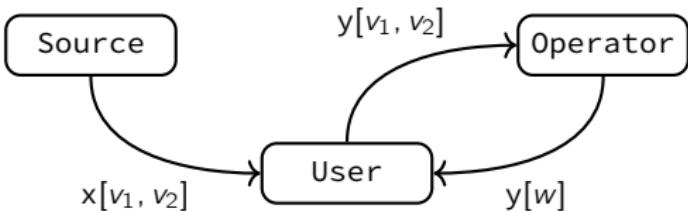
Contracts for choices

Roots of a polynomial

```
let server ep =
  let p, ep = receive ep in
    (* it sends as many messages as the real roots of p *)
  ...
val server : ?poly.rec A.(!float.A ⊕ end) -> unit

let contract =
  send_d (flat_c (fun p → degree p > 0)) @@
  fun p →
    let rec missing_roots n =
      if n > 0 then
        branch_c
        any_c
        (receive_c (flat_c (root_of p)) @@
         missing_roots (n - 1))
        end_c
      else
        branch_c (flat_c not) any_c end_c
    in missing_roots (degree p)
```

First order interaction and blame



$x : ?int.?int.end$

$src_c = \text{any_c}$

$y : !int.!int.?int.end$

$op_c = \text{send_c } \text{any_c} @@$
 $\text{send_c } (\text{flat_c } ((<>) \ 0)) @@$
 $\text{receive_c } (\text{flat_c } (>= \ 0)) @@ \text{end_c}$

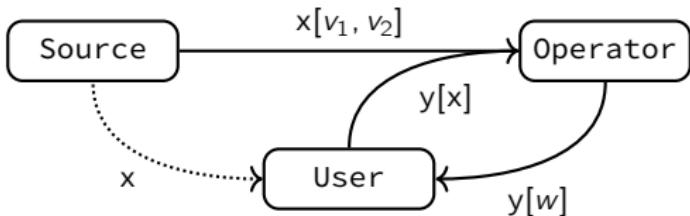
First order interaction and blame

First order user

```
let user () =
  let x = connect source_chan "User" in
  let y = connect operator_chan "User" in
  let v1, x = receive x in
  let v2, x = receive x in
  let y = send v1 y in
  let y = send v2 y in
  let w, y = receive y in
  print_int w; close x; close y
```

Which party should be blamed if $v2 < 0$? User

Higher-order communication and blame



$x : ?int.?int.end$

`src_c = any_c`

$y : !(?int.?int.end).?int.end$

```
op_c = send_c d_c @@  
       receive_c (flat_c ((<=) 0)) @@  
       end_c  
d_c = receive_c any_c @@  
      receive_c (flat_c ((>) 0)) @@  
      end_c
```

Higher-order communication and blame

Delegating user

```
let user_deleg () =
  let x = connect source_chan "User" in
  let y = connect operator_deleg_chan "User" in
  let y = send x y in
  let res, y = receive y in
  print_int res; close y
```

Which party should be blamed if the second value generated by source_chan is negative? **User** (despite it is not involved in the communication)

Syntax

Expression	$e ::= v$	value
	x	variable
	$e_1 e_2$	application
	$\text{let } x, y = e_1 \text{ in } e_2$	pair splitting
	$\text{case } e \text{ of } e_1 \mid e_2$	case analysis
	$\text{mon}^{k,l}(e_2, e_1)$	monitor
	$v \triangleleft^k e$	busy monitor
	$\text{blame } k$	blame
Value	$v, w, \kappa_1, \kappa_2 ::= c^n v_1 \dots v_n$	applied constant
	$\lambda x. e$	abstraction
	ε	endpoint
Process	$P, Q ::= \langle e \rangle_k$	thread
	$P \parallel Q$	composition
	$a \Leftarrow_k^{\kappa_1} v$	service
	$(\nu a)P$	session
Endpoint	$\varepsilon ::= a^p$	lone endpoint
	$\text{mon}^{k,l}(\kappa_1, \varepsilon)$	monitored endpoint

Constants

c^n	n	max	Sugared	Description
<code>()</code>		0		unit
<code>true, false</code>		0		boolean values
<code>pair</code>		2	(v, w)	pair creation
<code>inl, inr</code>		1		left/right injection
<code>fix</code>		0		fixpoint combinator
<code>connect</code>		0		initiate session
<code>close</code>		0		terminate session
<code>receive</code>		0		input
<code>send</code>		1		output
<code>branch</code>		0		offer choice
<code>left</code>		0		choose left
<code>right</code>		0		choose right
<code>flat_c</code>		1		flat contract
<code>end_c</code>		0		closed endpoint
<code>receive_c</code>	2		$? \kappa_1 . \kappa_2$	non-dependent input
<code>send_c</code>	2		$! \kappa_1 . \kappa_2$	non-dependent output
<code>receive_d</code>	2		$? \kappa_1 \mapsto w$	dependent input
<code>send_d</code>	2		$! \kappa_1 \mapsto w$	dependent output
<code>branch_c</code>	3		$? \kappa_1 \mapsto \kappa_2 : \kappa_3$	external choice
<code>choice_c</code>	3		$! \kappa_1 \mapsto \kappa_2 : \kappa_3$	internal choice
<code>dual</code>		0		compute dual contract

Typing of λ CoS

Types

Session Type $T, S ::= \text{end} \mid !t.T \mid ?t.T \mid T \oplus S \mid T \& S$

Type $t, s ::= \text{unit} \mid \text{bool} \mid t \rightarrow^{\iota} s \mid t + s \mid T \mid \text{con}(t) \mid t \times s \mid \#T$

Kind $\iota ::= 1 \mid \omega$

Type schemes of λ CoS constants

$()$	$: \text{unit}$	
$\text{true}, \text{false}$	$: \text{bool}$	
pair	$: t \rightarrow s \rightarrow^{\iota} t \times s$	$t :: \iota$
inl	$: t \rightarrow t + s$	
inr	$: s \rightarrow t + s$	
close	$: \text{end} \rightarrow \text{unit}$	
send	$: t \rightarrow !t. T \rightarrow^{\iota} T$	$t :: \iota$
receive	$: ?t. T \rightarrow t \times T$	
left	$: T \oplus S \rightarrow T$	
right	$: T \oplus S \rightarrow S$	
branch	$: T \& S \rightarrow T + S$	
connect	$: \#T \rightarrow T$	
flat_c	$: (t \rightarrow \text{bool}) \rightarrow \text{con}(t)$	$t :: \omega$
end_c	$: \text{con}(\text{end})$	
send_c	$: \text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(!t. T)$	
receive_c	$: \text{con}(t) \rightarrow \text{con}(T) \rightarrow \text{con}(?t. T)$	
send_d	$: \text{con}(t) \rightarrow (t \rightarrow \text{con}(T)) \rightarrow \text{con}(!t. T)$	$t :: \omega$
receive_d	$: \text{con}(t) \rightarrow (t \rightarrow \text{con}(T)) \rightarrow \text{con}(?t. T)$	$t :: \omega$
choice_c	$: \text{con}(\text{bool}) \rightarrow \text{con}(T) \rightarrow \text{con}(S) \rightarrow \text{con}(T \oplus S)$	
branch_c	$: \text{con}(\text{bool}) \rightarrow \text{con}(T) \rightarrow \text{con}(S) \rightarrow \text{con}(T \& S)$	
dual	$: \text{con}(T) \rightarrow \text{con}(\overline{T})$	

Typing

Typing rules for expressions

 $\boxed{\Gamma \vdash e : t}$

[t-const]

$$\frac{}{t \in \text{typeof}(c)} \quad \Gamma :: \omega$$

$$\Gamma \vdash c : t$$

[t-name]

$$\frac{}{\Gamma :: \omega}$$

$$\Gamma, u : t \vdash u : t$$

[t-fun]

$$\frac{\Gamma, x : t \vdash e : s \quad \Gamma :: \iota}{\Gamma \vdash \lambda x. e : t \rightarrow^{\iota} s}$$

[t-app]

$$\frac{\Gamma_1 \vdash e_1 : t \rightarrow^{\iota} s \quad \Gamma_2 \vdash e_2 : t}{\Gamma_1 + \Gamma_2 \vdash e_1 e_2 : s}$$

[t-split]

$$\frac{\Gamma_1 \vdash e_1 : t_1 \times t_2 \quad \Gamma_2, x : t_1, y : t_2 \vdash e_2 : t}{\Gamma_1 + \Gamma_2 \vdash \text{let } x, y = e_1 \text{ in } e_2 : t}$$

[t-case]

$$\frac{\Gamma_1 \vdash e : t_1 + t_2 \quad \Gamma_2 \vdash e_i : t_i \rightarrow^{\iota_i} t \ (i=1,2)}{\Gamma_1 + \Gamma_2 \vdash \text{case } e \text{ of } e_1 \mid e_2 : t}$$

[t-blame]

$$\Gamma \vdash \text{blame } k : t$$

[t-monitor]

$$\frac{\Gamma_1 \vdash e_1 : t \quad \Gamma_2 \vdash e_2 : \text{con}(t)}{\Gamma_1 + \Gamma_2 \vdash \text{mon}^{k,l}(e_2, e_1) : t}$$

[t-busy-monitor]

$$\frac{\Gamma_1 \vdash e : \text{bool} \quad \Gamma_2 \vdash v : t}{\Gamma_1 + \Gamma_2 \vdash v \triangleleft^k e : t}$$

Typing

Typing rules for processes

 $\boxed{\Gamma \vdash P}$

[t-thread]

$$\frac{\Gamma \vdash e : \text{unit}}{\Gamma \vdash \langle e \rangle_k}$$

[t-par]

$$\frac{\Gamma_i \vdash P_i \ (i=1,2)}{\Gamma_1 + \Gamma_2 \vdash P_1 \parallel P_2}$$

[t-session]

$$\frac{\Gamma, a^+ : T, a^- : \overline{T} \vdash P}{\Gamma \vdash (\nu a)P}$$

[t-service]

$$\frac{\emptyset \vdash \kappa_1 : \text{con}(T) \quad \Gamma \vdash v : \overline{T} \rightarrow \text{unit}}{\Gamma + a : \# T \vdash a \Leftarrow_k^{\kappa_1} v}$$

Reduction of expressions (1)

[r - beta]	$(\lambda x.e)v \rightarrow e\{v/x\}$
[r - split]	$\text{let } x,y = (v,w) \text{ in } e \rightarrow e\{v,w/x,y\}$
[r - inl]	$\text{case inl } v \text{ of } e_1 \mid e_2 \rightarrow e_1 v$
[r - inr]	$\text{case inr } v \text{ of } e_1 \mid e_2 \rightarrow e_2 v$
[r - flat]	$\text{mon}^{k,!}(\text{flat_c } w, v) \rightarrow v \triangleleft^k w v$
[r - true]	$v \triangleleft^k \text{true} \rightarrow v$
[r - false]	$v \triangleleft^k \text{false} \rightarrow \text{blame } k$
[r - context]	$\mathcal{E}[e] \rightarrow \mathcal{E}[e'] \quad \text{if } e \rightarrow e'$

$$\mathcal{E} ::= [] \mid \mathcal{E}e \mid v\mathcal{E} \mid \text{mon}^\sigma(e, \mathcal{E}) \mid v \triangleleft^k \mathcal{E} \mid \text{let } x, y = \mathcal{E} \text{ in } e \mid \text{case } \mathcal{E} \text{ of } e_1 \mid e_2 \mid \text{mon}^\sigma(\mathcal{E}, v)$$

Semantics

Session establishment

[r - connect]

$$\left(\langle \mathcal{E}[\text{connect } a] \rangle_k \right) \rightarrow (\nu b) \left(\langle \mathcal{E}[\text{mon}^{l,k}(\kappa_1, b^+)] \rangle_k \parallel \langle \nu \text{ mon}^{k,l}(\text{dual } \kappa_1, b^-) \rangle_l \right) \parallel a \Leftarrow_I^{\kappa_1} \nu \quad b \text{ fresh}$$

$\mathcal{E} ::= [] \mid \mathcal{E}e \mid \nu\mathcal{E} \mid \text{mon}^\sigma(e, \mathcal{E}) \mid \nu \triangleleft^k \mathcal{E} \mid \text{let } x, y = \mathcal{E} \text{ in } e \mid \text{case } \mathcal{E} \text{ of } e_1 \mid e_2 \mid \text{mon}^\sigma(\mathcal{E}, \nu)$

Reduction of expressions (2)

[d – end]	dual end_c	\rightarrow	end_c
[d – send – c]	$\text{dual } !\kappa_1.\kappa_2$	\rightarrow	$?!\kappa_1.(\text{dual } \kappa_2)$
[d – receive – c]	$\text{dual } ?\kappa_1.\kappa_2$	\rightarrow	$!?\kappa_1.(\text{dual } \kappa_2)$
[d – send – d]	$\text{dual } !\kappa_1 \mapsto w$	\rightarrow	$?!\kappa_1 \mapsto (\lambda x. \text{dual } (wx))$
[d – receive – d]	$\text{dual } ?\kappa_1 \mapsto w$	\rightarrow	$!?\kappa_1 \mapsto (\lambda x. \text{dual } (wx))$
[d – choice]	$\text{dual } !\kappa_1 \mapsto \kappa_2 : \kappa_3$	\rightarrow	$?!\kappa_1 \mapsto (\text{dual } \kappa_2) : (\text{dual } \kappa_3)$
[d – branch]	$\text{dual } ?\kappa_1 \mapsto \kappa_2 : \kappa_3$	\rightarrow	$!?\kappa_1 \mapsto (\text{dual } \kappa_2) : (\text{dual } \kappa_3)$

$$\mathcal{E} ::= [] \mid \mathcal{E}e \mid v\mathcal{E} \mid \text{mon}^\sigma(e, \mathcal{E}) \mid v \triangleleft^k \mathcal{E} \mid \text{let } x, y = \mathcal{E} \text{ in } e \mid \text{case } \mathcal{E} \text{ of } e_1 \mid e_2 \mid \text{mon}^\sigma(\mathcal{E}, v)$$

Communication (simplified)

[$r - \text{comm}$]

$$\left(\langle \mathcal{E}[\text{send} \vee \text{mon}^\sigma(!\kappa_1 \cdot \kappa_2, a^P)] \rangle_k \right) \\ \left(\langle \mathcal{E}'[\text{receive } \text{mon}^\varrho(? \kappa_3 \cdot \kappa_4, a^{\bar{P}})] \rangle_l \right) \rightarrow$$

$$\left(\langle \mathcal{E}[\text{mon}^\sigma(\kappa_2, a^P)] \rangle_k \right) \\ \left(\langle \mathcal{E}'[(\text{mon}^\varrho(\kappa_3, \text{mon}^{-\sigma}(\kappa_1, v)), \text{mon}^\varrho(\kappa_4, a^{\bar{P}}))] \rangle_l \right)$$

where $\neg(k, l) = l, k$

- ▶ Note that v can be of a non basic type, hence the monitor cannot be evaluated.
- ▶ Endpoints have a stack of monitors

$$\text{mon}^{\vec{\sigma}}(\vec{\kappa}, e) \quad \text{for} \quad \text{mon}^{\sigma_n}(\kappa_n, \dots \text{mon}^{\sigma_1}(\kappa_1, e) \dots)$$

$$\text{mon}^{\vec{\tau}}(\vec{\kappa}, e) \quad \text{for} \quad \text{mon}^{\sigma_1}(\kappa_1, \dots \text{mon}^{\sigma_n}(\kappa_n, e) \dots)$$

Semantics

Communication

[r - comm]

$$\left(\begin{array}{l} \langle \mathcal{E}[\text{send} \vee \text{mon}^{\vec{\sigma}}(\overrightarrow{! \kappa_1 . \kappa_2}, a^p)] \rangle_k \\ \langle \mathcal{E}'[\text{receive } \text{mon}^{\vec{\varrho}}(\overrightarrow{? \kappa_3 . \kappa_4}, a^{\bar{p}})] \rangle_l \end{array} \right) \rightarrow \left(\begin{array}{l} \langle \mathcal{E}[\text{mon}^{\vec{\sigma}}(\overrightarrow{\kappa_2}, a^p)] \rangle_k \\ \langle \mathcal{E}'[(\text{mon}^{\vec{\varrho}}(\overrightarrow{\kappa_3}, \text{mon}^{\overleftarrow{\sigma}}(\overleftarrow{\kappa_1}, v)), \text{mon}^{\vec{\varrho}}(\overrightarrow{\kappa_4}, a^{\bar{p}}))] \rangle_l \end{array} \right)$$

Dependent communication

$$\begin{aligned}
 & [r - \text{comm} - d] \\
 & \left(\langle \mathcal{E}[\text{send} \vee \text{mon}^{\vec{\sigma}}(\overrightarrow{! \kappa_1 \mapsto w_1}, a^p)] \rangle_k \right. \\
 & \quad \left. \langle \mathcal{E}'[\text{receive } \text{mon}^{\vec{\varrho}}(\overrightarrow{? \kappa_2 \mapsto w_2}, a^{\bar{p}})] \rangle_l \right) \rightarrow \\
 & \qquad \left(\langle \mathcal{E}[\text{mon}^{\vec{\sigma}}(\overrightarrow{w_1 v}, a^p)] \rangle_k \right. \\
 & \qquad \left. \langle \mathcal{E}'[(\text{mon}^{\vec{\varrho}}(\overrightarrow{\kappa_2}, \text{mon}^{\overleftarrow{\sigma}}(\overleftarrow{\kappa_1}, v)), \text{mon}^{\vec{\varrho}}(\overrightarrow{w_2 v}, a^{\bar{p}}))] \rangle_l \right)
 \end{aligned}$$

Choices

[r - left]

$$\left(\begin{array}{l} \langle \mathcal{E}[\text{left } \text{mon}^{\vec{\sigma}}(\overrightarrow{! \kappa_1 \mapsto \kappa_2 : \kappa_3}, a^P)] \rangle_k \\ \langle \mathcal{E}'[\text{branch } \text{mon}^{\vec{\theta}}(\overrightarrow{? \kappa_4 \mapsto \kappa_5 : \kappa_6}, a^{\bar{P}})] \rangle_I \end{array} \right) \rightarrow$$

$$\left(\begin{array}{l} \langle \mathcal{E}[\text{mon}^{\vec{\sigma}}(\overrightarrow{\kappa_2}, a^P)] \rangle_k \\ \langle \mathcal{E}'[(\lambda_. \text{inl } \text{mon}^{\vec{\theta}}(\overrightarrow{\kappa_5}, a^{\bar{P}})) \text{ mon}^{\vec{\theta}}(\overrightarrow{\kappa_4}, \text{mon}^{-\sigma}(\overleftarrow{\kappa_1}, \text{true}))] \rangle_I \end{array} \right)$$

[r - right]

$$\left(\begin{array}{l} \langle \mathcal{E}[\text{right } \text{mon}^{\vec{\sigma}}(\overrightarrow{! \kappa_1 \mapsto \kappa_2 : \kappa_3}, a^P)] \rangle_k \\ \langle \mathcal{E}'[\text{branch } \text{mon}^{\vec{\theta}}(\overrightarrow{? \kappa_4 \mapsto \kappa_5 : \kappa_6}, a^{\bar{P}})] \rangle_I \end{array} \right) \rightarrow$$

$$\left(\begin{array}{l} \langle \mathcal{E}[\text{mon}^{\vec{\sigma}}(\overrightarrow{\kappa_2}, a^P)] \rangle_k \\ \langle \mathcal{E}'[(\lambda_. \text{inr } \text{mon}^{\vec{\theta}}(\overrightarrow{\kappa_5}, a^{\bar{P}})) \text{ mon}^{\vec{\theta}}(\overrightarrow{\kappa_4}, \text{mon}^{-\sigma}(\overleftarrow{\kappa_1}, \text{false}))] \rangle_I \end{array} \right)$$

Session termination

[r - close]

$$(\nu a) \begin{pmatrix} \langle \mathcal{E}[\text{close} \text{ mon}^{\vec{\sigma}}(\overrightarrow{\text{end_c}}, a^+)] \rangle_k \\ \langle \mathcal{E}'[\text{close} \text{ mon}^{\vec{\varrho}}(\overrightarrow{\text{end_c}}, a^-)] \rangle_l \end{pmatrix} \rightarrow \langle \mathcal{E}[(\cdot)] \rangle_k \parallel \langle \mathcal{E}'[(\cdot)] \rangle_l$$

Properties

Subject reduction

- ▶ If $\Gamma :: \omega$ and $\Gamma \vdash P$ and $P \rightarrow Q$, then $\Gamma \vdash Q$.
- ▶ If Γ is balanced and $\Gamma \vdash P$ and $P \rightarrow Q$, then there exists Γ' such that $\Gamma \rightarrow^* \Gamma'$ and $\Gamma' \vdash Q$.

Blame safety

Goal

- ▶ to ensure that a process that *honours its contracts* cannot be blamed
 - ▶ Roughly, if a process sends a value, it is one accepted by the contracts of the monitored channel.
 - ▶ ... the formal definition is involved because of dependent contracts and delegation

Contract entailment

$\kappa_1 \leqslant \kappa_2$ if each value that satisfies κ_1 also satisfies κ_2

`flat_c (≥ 3) \leqslant flat_c (≥ 0)`

Contract entailment

$e_1 \leqslant e_2$ implies either:

1. $e_1 \Downarrow \text{flat_c } w_1$ and $e_2 \Downarrow \text{flat_c } w_2$ and for every $v \in w_1$ we have $v \in w_2$, or
2. $e_1 \Downarrow \text{end_c}$ and $e_2 \Downarrow \text{end_c}$, or
3. $e_1 \Downarrow !\kappa_1.\kappa_2$ and $e_2 \Downarrow !\kappa_3.\kappa_4$ and $\kappa_3 \leqslant \kappa_1$ and $\kappa_2 \leqslant \kappa_4$, or
4. $e_1 \Downarrow ?\kappa_1.\kappa_2$ and $e_2 \Downarrow ?\kappa_3.\kappa_4$ and $\kappa_1 \leqslant \kappa_3$ and $\kappa_2 \leqslant \kappa_4$, or
5. ...

Locally correctness

$k \mathcal{C} P$: k is (locally) correct in P

1. $P = \mathcal{P}_k[\text{send } v \text{ mon--}(!\text{flat_c } w \cdot _, _)]$ implies $v \in w$, and
2. $P = \mathcal{P}_k[\text{send } v \text{ mon--}(!\text{flat_c } w \mapsto _, _)]$ implies $v \in w$, and
3. $P = \mathcal{P}_k[\text{send } \text{mon--}(\kappa_1, \varepsilon) \text{ mon--}(!\kappa_2 \cdot _, _)]$ implies $\kappa_1 \leq \kappa_2$, and
4. $P = \mathcal{P}_k[\text{left } \text{mon--}(!\text{flat_c } w \mapsto _ : _, _)]$ implies **true** $\in w$, and
5. $P = \mathcal{P}_k[\text{right } \text{mon--}(!\text{flat_c } w \mapsto _ : _, _)]$ implies **false** $\in w$, and
6. $P \rightarrow Q$ implies $k \mathcal{C} Q$.

$$\mathcal{P}_k ::= \langle \mathcal{E} \rangle_k \quad | \quad (\mathcal{P}_k \parallel P) \quad | \quad (P \parallel \mathcal{P}_k) \quad | \quad (\nu a)\mathcal{P}_k$$

Property

Blame safety

If $\Gamma \vdash P$ where P is a user process and k is locally correct in P , then $P \rightarrow^* Q$ implies `blame k` $\not\subset Q$.

Implementation of contracts

GADT for contracts

```
type [_] =
| Flat      : ( $\alpha \rightarrow \text{bool}$ )  $\rightarrow [\alpha]$ 
| End       : [ $\text{end}$ ]
| Receive   :  $[\alpha] \times (\alpha \rightarrow [A]) \rightarrow [?\alpha.A]$ 
| Send      :  $[\alpha] \times (\alpha \rightarrow [A]) \rightarrow [!\alpha.A]$ 
| Branch    : [ $\text{bool}$ ]  $\times [A] \times [B] \rightarrow [A \& B]$ 
| Choice    : [ $\text{bool}$ ]  $\times [A] \times [B] \rightarrow [A \oplus B]$ 
```

Implementation of contracts

Contract primitives

```
let flat_c w = Flat w
let any_c = Flat (fun _ → true)
let receive_d k f = Receive (k, f)
let receive_c k1 k2 = receive_d k1 (fun _ → k2)
...
```

Implementation of contracts

Monitored endpoint

```
type A mt =
| Channel of linearity_tag_type × A st
| Monitor of [ $\langle \alpha, \beta \rangle$ ] × string × string ×  $\langle \alpha, \beta \rangle$ 
```

Implementation of contracts

Implementation of primitives

```
let rec send v =
  function
  | Channel (lin, ep) → Channel (lin, FuSe.send v ep)
  | Monitor (Send (k, w), pos, neg, ep) →
    wrap (w v) pos neg (send (wrap k neg pos v) ep)
  | Monitor (Flat _, _, _, _) → assert false (*IMPOSSIBLE*)

let wrap : type a. [a] → string → string → a → a
= fun k pos neg v →
  match k with
  | Flat w           → if unlimited v && w v
                        then v else raise (Blame pos)
  | End as k         → Monitor (k, pos, neg, v)
  | Receive _ as k   → Monitor (k, pos, neg, v)
  | Send _ as k      → Monitor (k, pos, neg, v)
  | Branch _ as k    → Monitor (k, pos, neg, v)
  | Choice _ as k    → Monitor (k, pos, neg, v)
```