

# Binary Session Types

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# Typing

## Type Judgement

$$\Gamma \vdash P$$

$P$  uses channels as specified by  $\Gamma$

## Environments $\Gamma$

- ▶ Partial function from polarized names to types
- ▶ Written  $x_1^{p_1} : t_1, x_2^{p_2} : t_2, \dots, x_n^{p_n} : t_n$
- ▶ It satisfies one of the following conditions
  - ▶  $x^+, x^-, x \notin \text{dom}(\Gamma)$
  - ▶  $x \in \text{dom}(\Gamma)$  and  $x^+, x^- \notin \text{dom}(\Gamma)$
  - ▶  $x^p \in \text{dom}(\Gamma)$  and  $p \in \{+, -\}$  and  $x^{\bar{p}}, x \notin \text{dom}(\Gamma)$
  - ▶  $x^+, x^- \in \text{dom}(\Gamma)$  and  $x \notin \text{dom}(\Gamma)$

## Typing

$$x^+ : ?\text{int}.\text{!bool.end} \vdash x^+?(y:\text{int}).x^+\text{!true}.0$$
$$x^+ : ?\text{int}.\text{!bool.end} \not\vdash x^+?(y:\text{int}).x^+\text{!y}.0$$
$$x^+ : ?\text{int.end}, y^- : !\text{int.end} \vdash x^+?(z:\text{int}).y^-!\text{z}.0$$
$$x^+ : ?\text{int.end}, y^- : !\text{bool.end} \not\vdash 0$$
$$\vdash (\nu x:\text{?int.end})(x^+?(z:\text{int}).0 \mid x^-\text{!}1.0)$$
$$\not\vdash (\nu x:\text{?int.end})(x^+?(z:\text{int}).0)$$
$$\not\vdash (\nu x:\text{?int.end})(x^+?(z:\text{int}).0 \mid x^-\text{!}1.0 \mid x^-\text{!}2.0)$$

# Typing

$$\nexists (\nu x:\text{?int}.\text{?int.end})(x^+?(z:\text{int}).x^+?(z:\text{int}).0 \mid x^-!1.0 \mid x^-!2.0)$$

Think about

$$(\nu x:\text{?int}.\text{!int}.\text{?int}.\text{!int.end})(  
x^+?(z:\text{int}).x^+!(z+1).0 \mid  
x^+?(z:\text{int}).x^+!(z+1).0 \mid  
x^-!1.x^-?(z:\text{int}).Q_1 \mid  
x^-!2.x^-?(z:\text{int}).Q_2 \quad )$$

## Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} \text{[T-Par]}$$

$x^+ : \text{Tester}, x^- : \overline{\text{Tester}} \vdash P_{\text{server}} \mid P_{\text{client}}$

where

$\text{Tester} = ?\text{int}.!\text{bool}.\text{end}$

$P_{\text{server}} = x^+?(y:\text{int}).x^+!\text{true}.0$

$P_{\text{client}} = x^-!1.x^-?(z:\text{bool}).Q$

## Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} \text{[T-Par]}$$

$x^+ : \text{Tester}, x^- : \overline{\text{Tester}} \not\vdash P_{\text{server}} \mid P_{\text{client}} \mid P_{\text{client}}$

where

$\text{Tester} = ?\text{int}.!\text{bool}.\text{end}$

$P_{\text{server}} = x^+?(y:\text{int}).x^+!\text{true}.0$

$P_{\text{client}} = x^-!1.x^-?(z:\text{bool}).Q$

# Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} [\text{T-Par}]$$

## Context split

$$\begin{aligned}\Gamma + x^+ : t &= \Gamma, x^+ : t && \text{if } x, x^+ \notin \text{dom}(\Gamma) \\ \Gamma + x^- : t &= \Gamma, x^- : t && \text{if } x, x^- \notin \text{dom}(\Gamma) \\ \Gamma + x : t &= \Gamma, x : t && \text{if } x, x^+, x^- \notin \text{dom}(\Gamma) \\ (\Gamma, x : t) + x : t &= \Gamma, x : t && \text{if } t \text{ is not a session type}\end{aligned}$$

Extended on context as

$$\begin{aligned}\Gamma + \emptyset &= \Gamma \\ \Gamma + (x^p : t, \Delta) &= (\Gamma + x^p : t) + \Delta\end{aligned}$$

Linear usage of session endpoints

# Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} \text{ [T-Par]}$$

$$\frac{\Gamma, x^+ : S, x^- : \overline{S} \vdash P}{\Gamma \vdash (\nu x:S)P} \text{ [T-Res]}$$

$(\nu x:\text{Tester})(P_{\text{server}} \mid P_{\text{client}})$

where

$\text{Tester} = ?\text{int}.!\text{bool}.\text{end}$

$P_{\text{server}} = x^+?(y:\text{int}).x^+!?\text{true}.0 \quad (\text{faulty})$

$P_{\text{client}} = x^-!1.x^-?(z:\text{bool}).Q$

# Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} \text{[T-Par]}$$

$$\frac{\Gamma, x^+ : S, x^- : \bar{S} \vdash P}{\Gamma \vdash (\nu x:S)P} \text{[T-Res]}$$

$$\frac{\Gamma, x^p : S, y : t \vdash P}{\Gamma, x^p : ?t.S \vdash x^p?(y:t).P} \text{[T-In]}$$

$$\frac{\Gamma_1 \vdash v : t \quad \Gamma_2, x^p : S \vdash P}{\Gamma_1 + (\Gamma_2, x^p : !t.S) \vdash x^p!v.P} \text{[T-Out]}$$

Auxiliary Typing on expressions  $\Gamma \vdash v : t$

$\emptyset \vdash \text{true} : \text{bool}$      $\emptyset \vdash \text{false} : \text{bool}$

$\emptyset \vdash () : \text{unit}$      $x^p : t \vdash x^p : t$

# Typing

$$\frac{\Gamma_1 \vdash P_1 \quad \Gamma_2 \vdash P_2}{\Gamma_1 + \Gamma_2 \vdash P_1 | P_2} \text{[T-Par]}$$

$$\frac{\Gamma, x^+ : S, x^- : \overline{S} \vdash P}{\Gamma \vdash (\nu x : S)P} \text{[T-Res]}$$

$$\frac{\Gamma, x^p : S, y : t \vdash P}{\Gamma, x^p : ?t.S \vdash x^p?(y:t).P} \text{[T-In]}$$

$$\frac{\Gamma_1 \vdash v : t \quad \Gamma_2, x^p : S \vdash P}{\Gamma_1 + (\Gamma_2, x^p : !t.S) \vdash x^p!v.P} \text{[T-Out]}$$

$$\frac{\Gamma, x^p : S_j \vdash P \quad j \in I}{\Gamma, x^p : \oplus[\mathbf{l}_i : S_i]_{i \in I} \vdash x^p \triangleleft \mathbf{l}_j.P} \text{[T-Choice]}$$

$$\frac{\Gamma, x^p : S_i \vdash P_i \quad \forall i \in I}{\Gamma, x^p : \&[\mathbf{l}_i : S_i]_{i \in I} \vdash x^p \triangleright [\mathbf{l}_i : P_i]_{i \in I}} \text{[T-Branch]}$$

$$\frac{\Gamma \text{ completed}}{\Gamma \vdash 0} \text{[T-Nil]}$$

$\Gamma$  completed if  $\Gamma(x^p) = S$  implies  $S = \text{end}$