

Note: experimenting with flipped lecture

## INTRODUCTION

- Bugs = loss of lives and/or of money

Example Therac-25 ≥ 6 deaths between 1985-1987

Ariane-5 exploded 36 sec after launch

Pentium bug 4.85M USD

Baggage handling system @ Denver airport 1.1M USD × day × 9 months

It is fair to state, that in this digital era correct systems for information processing are more valuable than gold.  
(H. Barendregt, 'The quest for correctness', in Images of SMC Research 1996)

<https://www.reuters.com/business/autos-transportation/us-probing-fatal-tesla-crash-that-killed-pedestrian-2021-09-03>  
<https://www.tesladeaths.com/>

- SW ubiquitous ⇒ sw "correctness" valuable

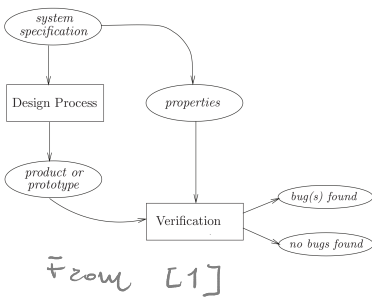
always relative to specs!

Simulation / testing

- + concrete artefacts are checked
- + "simple"
- partial (when should we stop?)

Deductive reasoning

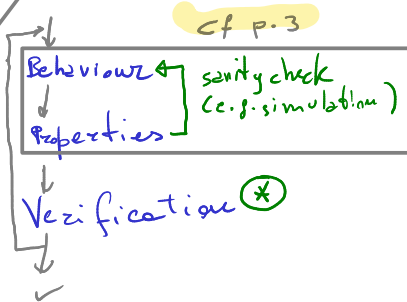
- + infinite state systems
- 'hard' & time consuming
- interactive



Model checking

- state explosion prob
- finite state spaces
- + "easier"
- + "automatic" (the verification phase +/-)
- + → 'yes' = no bugs c.f. with testing
- + → 'no' & C.E.

the methodology of MC



MODELING PHASE

design → machine processable models

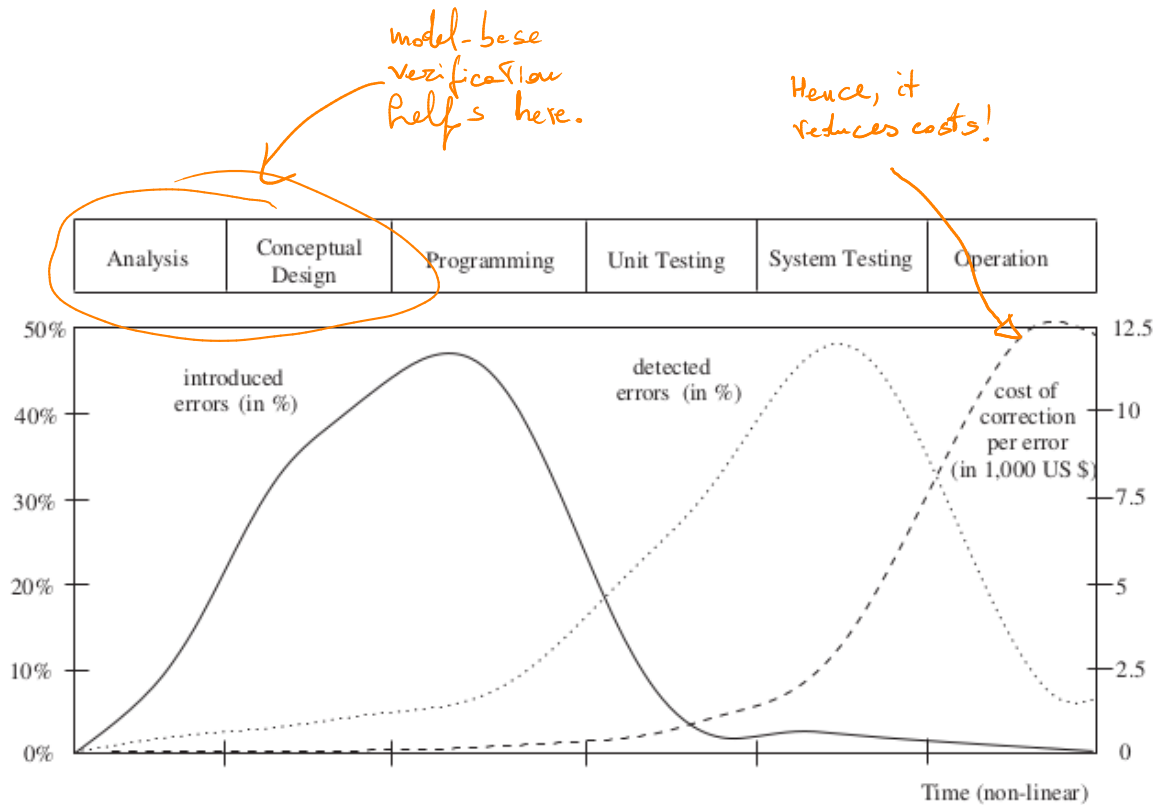
properties → typically some (temporal) logics

ideally "push-button" in practice analysis of results

Note: Modelling could be partly 'automatic' (e.g. compiling from design)  
Verification is mainly automatic

\* Question: If you get an error trace, what do you do?

The sooner, the better!



ref.

P. Liggesmeyer and M. Rothfelder and M. Rettelbach and T. Ackermann. Qualitätssicherung Software-basierter technischer Systeme. Informatik Spektrum, 21(5):249–258, 1998.

Quoting [1]

"In software and hardware design of complex systems, more time and effort are spent on verification than on construction. Techniques are sought to reduce and ease the verification efforts while increasing their coverage. Formal methods offer a large potential to obtain an early integration of verification in the design process, to provide more effective verification techniques, and to reduce the verification time."

# Glancing at temporal logics

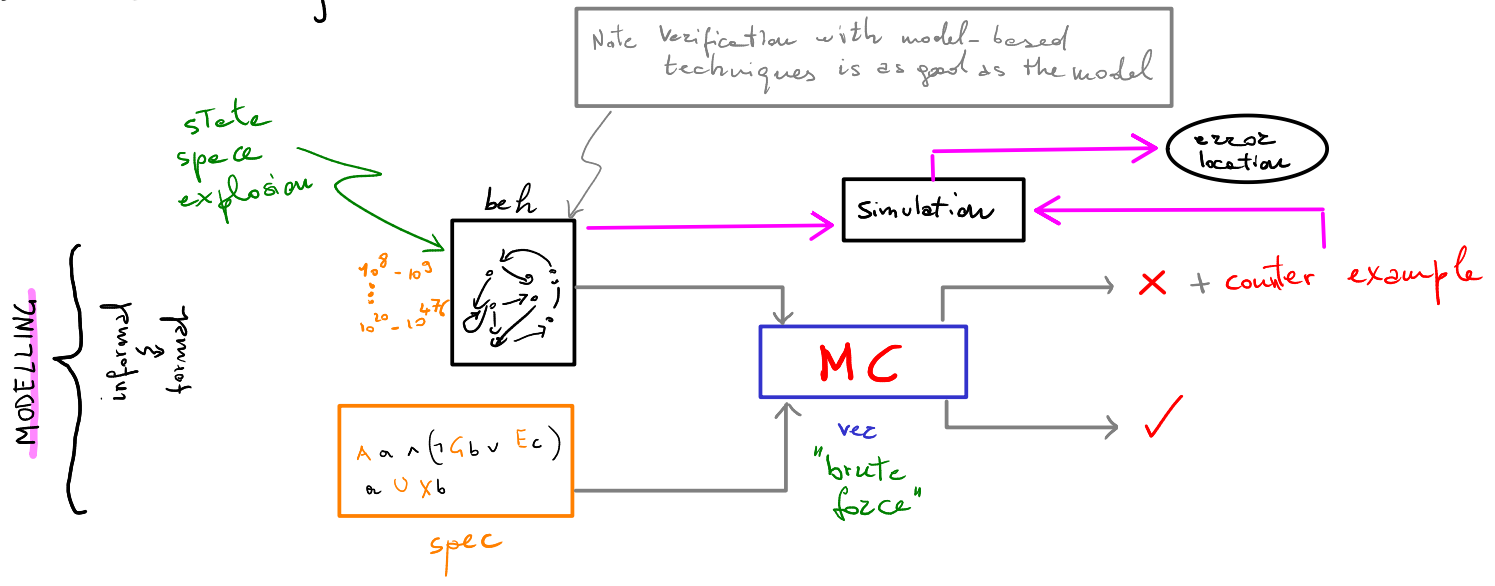
Note Temporal logics stem from philosophy: modal logics to reason about time in natural language!

- Designed to predicate on concurrent events
- ordered in time
- but without (explicit) time!

modality  $\Box \phi$

eg  $\Box(\neg(a \wedge b))$  = it will never happen that events a and b occur "at the same time".  
 = thread a writes x  
 = thread b reads x

Schematically cf Fig 1.4 [1]



Example 1.1 [1] Three parallel processes

```
def inc(): while true: if x < 200: x := x + 1
def dec(): while true: if x > 0: x := x - 1
def reset(): while true: if x == 200: x := 0
```

$\varphi$  = Always  $0 \leq x \leq 200$

Exercise: Does  $\varphi$  hold of the parallel execution of the three processes in Example 1.1?

# Modelling

4

what?

VALIDATION

vs

VERIFICATION

Are we building the right thing?

Are we building the thing right?

is the design faithfully "capturing" the reqs?

does the design satisfy the properties?

- Going formal
  - Right level of abstraction
- = precise, but not "cumbersome"

Reactive Systems

(Mazur, Pnueli 1995)

- Concurrent
  - Interact with an environment ("open")
  - possibly non terminating
- NOT FUNCTIONS!

STATE

snapshot of the system "at a given time"

&

TRANSITION

evolution of the system "in time"

LTS

$S \xrightarrow{\alpha} S'$

KRIPKE Structures

Transition system  $TS = (S, Act, \rightarrow, I, AP, L)$

where  $S$  is a set of states

Actions come handy to model interactions

- $Act$  is a set of actions; in kripke structures  $Act$  is a singleton
- $\rightarrow \subseteq S \times Act \times S$  transition relation
- $I \subseteq S$  are the initial states
- $AP$  is a set of atomic propositions
- $L: S \rightarrow 2^{AP}$

$TS$  is finite if  $S$ ,  $Act$ , and  $AP$  are finite

$S$  &  $L(S)$  are finite

WLOG we consider transition systems where  $I \neq \emptyset$

if  $I = \emptyset \Rightarrow$  no behaviour

# Example A (simplified) slot machine

$S = \{0, \dots, n+1\}$  &  $I = \{0\}$

$Act = \{bet, win, loose, pull, release\}$

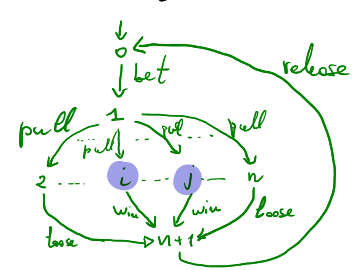
$\rightarrow = \{(0, bet, 1)\} \cup \bigcup_{1 \leq h \leq n} \{(1, pull, h)\} \cup \{(h, \alpha, n+1) \mid \alpha = \begin{cases} loose & \text{if } \neg w(h) \\ win & \text{if } w(h) \end{cases}\} \cup \{(n+1, release, 0)\}$

$AP = \{w_i = \{1 \leq i \leq 3 \text{ \& } f \in F_{fruits}\} \cup \{price = n \mid n \in \omega\}\}$

where  $F_{fruits} = \{apple, pear, banana, \dots\}$ . Let  $W = \bigcup_{h \in \{1, \dots, n\}} \{(h, f_{w_1}, f_{w_2}, f_{w_3})\}$

$L: h \mapsto \{price = h, w_1 = f_1, w_2 = f_2, w_3 = f_3\}$  if  $h \in \{1, \dots, n\}$  &  $(h, f_1, f_2, f_3) \in W$

we can get rid of  $n+1$  and those trans.



Exercise: Define  $L$  on  $R \neq \{i, \dots, j\}$

## Non-determinism

- crucial modelling mechanism
- under-specification

Deterministic TS  $|I| \leq 1$

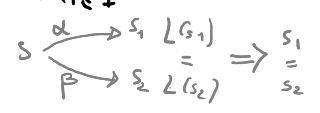
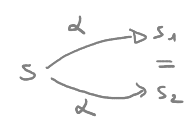
• action-deterministic

$\forall s \in S, a \in Act: |Post(s, a)| \leq 1$

• AP-deterministic

$\forall A \in 2^{AP} \quad \forall s \in S: |\{s' \in Post(s) \mid L(s') = A\}| \leq 1$

$\therefore \exists s \in S: s \rightarrow s' \rightarrow s''$



## Executions / Traces

Execution fragment  $p \in$

finite  $S(Act S)^*$

infinite  $\cup S(Act S)^\omega$

s.t.  $p = s_0 \alpha_1 s_1 \alpha_2 s_2 \dots \alpha_n s_n \dots \Rightarrow s_i \xrightarrow{\alpha_{i+1}} s_{i+1}$  for all  $i$

$p$  maximal if  $p$  infinite or

$p = s_0 \alpha_1 s_1 \alpha_2 s_2 \dots \alpha_n s_n \wedge Post(s_n) = \emptyset$

$p$  initial if  $s_0 \in I$

Execution initial maximal execution fragment.

## Reachable states

$Reach(TS) = \{s \mid \exists p \text{ initial execution fragment ending in } s\}$