Context Free Grammars

(infinite)

$G = (N, \Sigma, P, s \in N)$

$P \subseteq N \times (N \cup \Sigma)^*$ $\quad$ s start symbol

(fixed grammar G)
the corresponding TS is

$S = (N \cup \Sigma)^*$

$$\rightarrow_G = \bigcup_{(X,w)\in P} \{(uXv, uwv) \mid u, v \in (N \cup \Sigma)^*\}$$

<u>Variants</u>

terminal TS $\quad$ $Sys = (S, \rightarrow, T)$ where

$(S, \rightarrow)$ is a TS &

$T \subseteq S$ s.t. $\forall s \in T \; \forall s' \in S \;\; s \not\rightarrow s'$

Exercise 4 Give a terminal TS for each of the examples above

An important variant

A labelled transition system is a triple $(S, A, \rightarrow)$ where

- S is a set of states
- A is a set of labels (or actions, or operations, or events, ...)
- $\rightarrow \subseteq S \times A \times S$ $\quad (\rightarrow : S \longrightarrow 2^{A \times S})$ Transition relation

observable information about what happens during the transition
particularly handy to model communication / concurrency / distribution

$$s \xrightarrow{\alpha} s'$$

Example An FSA, $M = (Q, \Sigma, q_0, \delta, F)$ is an LTS:

$$S_M = (S \cup \{\bullet\}, \Sigma \cup \{\checkmark\}, \rightarrow) \text{ where } s \xrightarrow{a} s' \iff \begin{array}{l} a \in \Sigma \ \& \ s' \in \delta(s,a) \\ \text{or} \\ a = \checkmark \ \& \ s' = \bullet \ \& \ s \in F \end{array}$$

$$\mathcal{L}_M = \{a_1 \ldots a_n \in \Sigma^* \mid \exists q_1, \ldots, q_n \mid q_0 \xrightarrow{a_1} \ldots q_{n-1} \xrightarrow{a_n} q_n \xrightarrow{\checkmark} \bullet\}$$

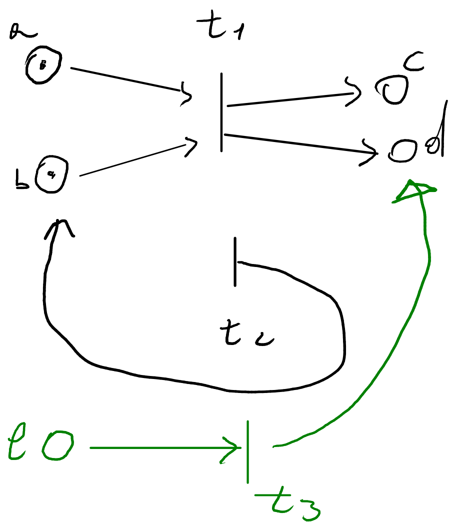
A more sophisticated example

Petri Nets

A Petri net (aka place-transition) net is a 4-tuple

$N = (P, T, F, m)$ where

- P is a finite set (of places)
- T is a finite set (of transitions)
- $F \subseteq (P \times T) \cup (T \times P)$ is a (flow) relation
- $m \subseteq P$ is the initial marking

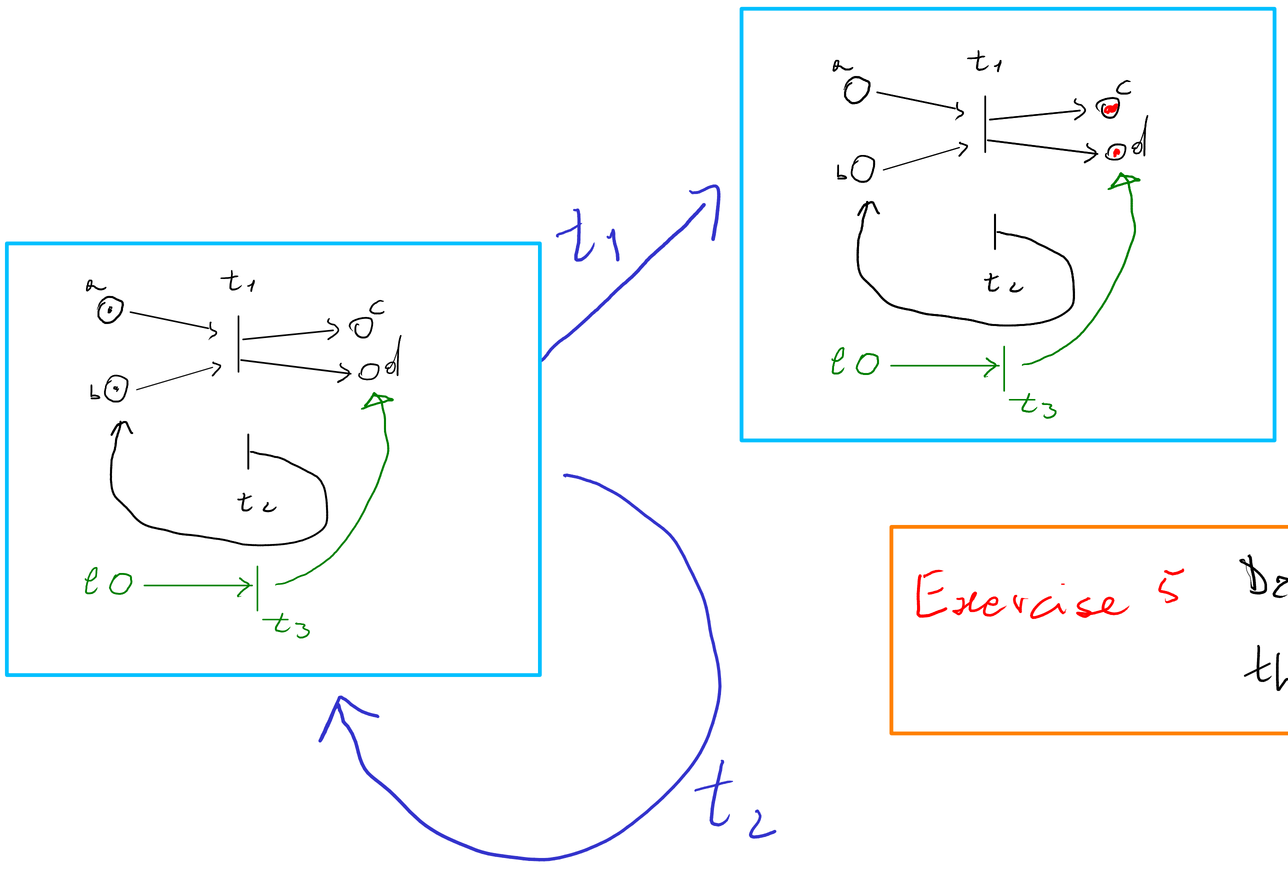


with marking $m = \{a, b\}$

$$t_1 \# t_2 \iff {}^\bullet t_1 \cap {}^\bullet t_2 \neq \emptyset \wedge t_1 \neq t_2$$

$$A = \{ X \subseteq T \mid \forall t,t' \in X : \text{not } t \# t' \}$$

$$m \xrightarrow{X} m' \quad \text{if} \quad \overbrace{{}^\bullet X}^{=\bigcup_{t \in X} {}^\bullet t} \subseteq m \;\&\; m' = m \setminus {}^\bullet X \cup \overbrace{X^\bullet}^{=\bigcup_{t \in X} t^\bullet}$$



Exercise 5 Draw the transition system from the initial marking a, b, e