Modelling and Validation of Concurrent System

Computational Tree Logic

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Limitations of HML

Origins of Logic in Computer Science

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  TLA is a temporal Logic.

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Instead of aiming at stating and proving properties of programs, it aims at providing a logical account of the operational equivalence notion of process languages (observational congruence).

- Strong Bisimulation coincides with logical equivalence, considering strong modalities.
- Observational congruence coincides with logical equivalence for finite branch processes, considering weak modalities.

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There is no way of talking about something holding eventually or always in the future — formulæ in HML talk about a finite number of steps, the concrete number given by the depth of a formula.

Computational Tree Logic, CTL

A simple logic to express temporal properties of processes

In the early 1980s, Clarke, Emerson, Sistla and others developed a logic with a *next state* modality and built-in temporal connectives

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It features:

- propositional connectives
- a one-step possibility modality
- a temporal operator: *until*:
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The key idea

- Quantify over *paths* on the LTS
  (instead of only over actions)
- Talk about the future

The *until* temporal operator

Like in HML, CTL formulæ are interpreted over LTSs.

A *path* on an LTS S

is a sequence of states, linked to each other by transitions.

If P is an initial state of S, then

$$P \xrightarrow{a_1} P_1 \xrightarrow{a_2} P_2 \cdots$$

is a path of S if each triple $P_{i-1} \xrightarrow{a_i} P_i$ is in S's transition relation

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Computational Tree

The acyclic unfolding of an LTS

Syntax of CTL

Consider $\mathcal{K} \subseteq \mathsf{Act}$

The set $\mathcal{F}$ of formulæ is inductively defined by the grammar

$$\varphi ::= \bot \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \langle\mathcal{K}\rangle\varphi \mid \mathtt{E}(\varphi\,\mathtt{U}\,\psi) \mid \mathtt{A}(\varphi\,\mathtt{U}\,\psi)$$

where the temporal operators:

- $\mathtt{E}$ means *there Exists* a path in the LTS
  path *p* satisfies $\varphi\,\mathtt{U}\,\psi$ if it has a state that satisfies ψ and all previous states satisfy φ
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Furthermore, some derived temporal operators are used often

- $\mathtt{F}$ means *sometime* in the Future
  $\mathtt{F}\,\varphi = \top\,\mathtt{U}\,\varphi$
- $\mathtt{G}$ means *always* in the future
  $\mathtt{G}\,\varphi = \neg(\top\,\mathtt{U}\,\neg\varphi)$

Semantics of CTL

The *satisfaction relation*:

$\models \subseteq \mathrm{Proc} \times \mathcal{F}$ **is defined as for HML**

$P_0 \models \mathtt{E}(\varphi \,\mathtt{U}\, \psi)$ if for SOME path $P_0 \xrightarrow{a_1} P_1 \xrightarrow{a_2} \cdots$ it is the case that $\exists\, i\,.\,(i \geq 0 \land P_i \models \psi \land \forall\, j\,.\,(0 \leq j < i \land P_j \models \varphi)$

$P_0 \models \mathtt{A}(\varphi \,\mathtt{U}\, \psi)$ if for ALL paths $P_0 \xrightarrow{a_1} P_1 \xrightarrow{a_2} \cdots$ it is the case that $\exists\, i\,.\,(i \geq 0 \land P_i \models \psi \land \forall\, j\,.\,(0 \leq j < i \land P_j \models \varphi)$

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In CTL temporal operators are always prefixed by path quantifiers.

Properties of Distributed Systems in CTL

Strong Safety $\mathtt{A}\,\mathtt{G}\,\neg \mathrm{BAD}$

Strong Liveness $\mathtt{A}\,\mathtt{F}\,\mathrm{GOOD}$

Weak Safety $\mathtt{E}\,\mathtt{G}\,\neg \mathrm{BAD}$

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Absence of Deadlock

Every path is infinite: $\mathtt{A}\,\mathtt{G}\,\langle - \rangle \top$

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Mutual exclusion

In no path are two processes simultaneously in the critical region $\mathtt{A}\,\mathtt{G}\,([\mathit{rel}1]\bot \vee [\mathit{rel}2]\bot)$

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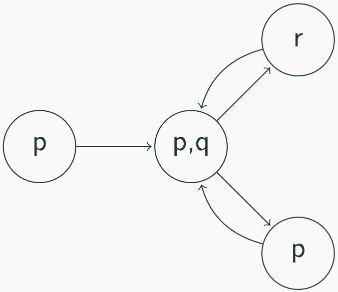

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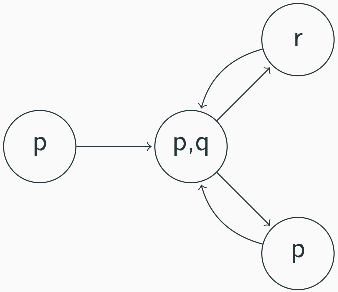

- $P \models \mathtt{A}(p\,\mathtt{U}\,q)$ does not hold.
- $P \models \mathtt{E}\,\mathtt{F}\,r$ holds
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- $P \models \mathtt{E}\,\mathtt{F}\,p$ holds
- $P \models \mathtt{A}\,\mathtt{F}\,(\mathtt{E}\,\mathtt{G}\,r \vee \mathtt{E}\,\mathtt{G}\,p)$ does not hold.

Beyond CCS

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Beyond the π-calculus

- Higher-order π allows to pass processes
- SPI adds encryption and decryption primitives
- ψ-calculi allows to have terms of an algebra instead of names

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Solutions

- CTL* lifts the alternation limitation
- The modal μ-calculus substitutes the temporal operators by fix-points (smallest and greatest)
  It turns out it includes CTL*