Electrical grid simulation in the COLMENA framework

Pablo de Juan Vela [1]
[1]eRoots, Barcelona, Spain

April 24, 2025

1 MPC Formulation

The decision variables are:

- f_t the area's frequency.
- $\delta_{i,t}$ the angle of area i.
- $P_t^{\text{gen}_i}$ the generator's i power output in pu.
- $P_t^{i,j}$ the power exchanged between area i and area j in pu.

The parameters are:

- $B_{i,j}$ the susceptance between area i and area j in p.u.
- $\hat{\delta}_{i,j,t}$ is the previous optimal solution of the MPC of area i for the area angles $\delta^*_{j,t}$.
- $\hat{\delta}_{i,0}$ the initial value of the area i in radians.
- $\hat{f}_0$ the current area initial frequency value.
- $u^{min}_{gen_i}, u^{max}_{gen_i}$ the minimum and maximum ramp up speed of generator i in pu/s.
- M_i the area wise inertia constant defined as $\frac{\sum_{gen_k \in AreaGenerators} Sn^{gen_k} M_{gen_k}}{\sum_{gen_k \in AreaGenerators} Sn^{gen_k}}$
- D_i the area wise damping coefficient defined as $\frac{\sum_{gen_k \in AreaGenerators} Sn^{gen_k} D_{gen_k}}{\sum_{gen_k \in AreaGenerators} Sn^{gen_k}}$

The dynamics of the MPC model can be expressed as the following differential equations:

$$\dot{\delta}_i = 2\pi(f_t - f_0) \quad \forall t = 0, \dots, T \tag{1}$$

$$M(\dot{f}) = -D(f - f_0) + \sum_{gen \in area} P_t^{gen} + \sum_{j \in \text{other area}} P_{i,j} - P_t^{demand} \tag{2}$$

The MPC formulation for area i is.

$$\begin{aligned}
\min_{f_t, \delta_{i,t}, P_t^{gen}, P_t^{i,j}} \quad & \|f - f_0\|^2 + \sum_{j \in \text{NeighboringAreas}} \lambda_{j,i,t}(\delta_{j,t} - \hat{\delta}_{i,j,t}) \cdot \text{sign}(i - j) && (3)\\
& + \sum_{j \in \text{NeighboringAreas}} \lambda_{i,j,t}(\delta_{i,t} - \hat{\delta}_{j,i,t}) \cdot \text{sign}(j - i) \\
& + \sum_{j \in \text{NeighboringAreas}} (\delta_{j,t} - \hat{\delta}_{i,j,t})^2 \\
\text{s.t.} \quad & \delta_{t+1} - \delta_t = 2\Delta t\pi(f_t - f_0) \quad \forall t = 0, \dots, T && (4)\\
& M(f_{t+1} - f_t) = -\Delta t \left(D(f - f_0) + \sum_{\text{gen} \in \text{area}} P_t^{gen} + \sum_{j \in \text{other area}} P_{i,j} - P_t^{demand} \right) && (5)\\
& \delta_{j,0} = \hat{\delta}_{j,0} \forall j \in \text{Areas} && (6)\\
& f_0 = \hat{f}_0 && (7)\\
& P_{i,j} = \sum_{j \in \text{Areas}} B_{i,j}(\delta_{i,t} - \delta_{j,t}) && (8)\\
& u_{\min} \le P_{t+1}^{gen_k} - P_t^{gen_k} \le u_{\max} \quad \forall gen_k \in AreaGenerators \forall t = 0, ..., T - 1 && (9)\\
& P_{\min}^{gen_k} \le P_{i,t}^{gen} \le P_{\max}^{gen_k} \quad \forall gen_k \in AreaGenerators && (10)
\end{aligned}$$

Where we have that this expression is the cost in the objective function associated to the error of the agent's data about other agents angles,

$$\sum_{j \in NeighboringAreas} \lambda_{j,i,t}(\delta_{j,t} - \hat{\delta}_{i,j,t})(sign(i - j)) \tag{11}$$

whereas this other expression is the error of the other agent's data tells about the value they have.

$$\sum_{j \in NeighboringAreas} \lambda_{i,j,t}(\delta_{i,t} - \hat{\delta}_{j,i,t})(sign(j - i)) \tag{12}$$

The objective of the extended formulation is to expand the current problem where we consider the areas as a simple aggregation of devices to one where the topology inside each grid is also considered. For this purpose, we add new constraints related to the power balance in each individual bus and not just in the whole area.

$$P^{gen}_{\text{bus}_i} - P^{d}_{\text{bus}_i} - \sum_{j \in area} P_{\text{bus}_i,\text{bus}_j} = 0 \quad \forall \text{bus}_i \in \text{Area} \tag{13}$$

$$P_{\text{bus}_i,\text{bus}_j} = B_{i,j}(\theta_i - \theta_j) \tag{14}$$

Algorithm 1 Distributed MPC ADMM algorithm

Initialize grid states x to x_0 using Power Flow results.
while error > tolerance **do**
  for each agent in area_agents **do**
    Agent controlling area i
    Get initial values $f_0^{\text{area}_i}$, $\delta_{\text{area},0}$ $\forall$agent from other agents via `@Data(horizon)`
    Get state horizon of the other $\hat{\delta}_{\text{area},0}$ $\forall$agent from other agents via `@Data(horizon)`
    Solve local area MPC
    Publish state horizon solution $\delta^*_{\text{area}_i,t} \forall t = 0...T$ to `@Data(horizon)`
  end for
  $error \leftarrow \delta^*_i - \hat{\delta}_i$
  $\lambda_i \leftarrow \lambda_i + \alpha \cdot error$
end while
for gen in generators **do**
  Consider the area's MPC the generator belongs to.
  Update generator power setpoint $P_{ref,gen} \leftarrow (P^g_{gen,0})^*$
end for

1.1 Distributed MPC with role dynamics