Nonlinear Time-Delay Systems: A Polynomial Approach Using Ore Algebras

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Summary. In this work Ore algebra defined over the field of meromorphic functions is introduced to provide a polynomial approach to nonlinear time-delay systems. In comparison to the systems without delays the polynomials related to the nonlinear time-delay systems belong now to the multivariate skew polynomial ring and have the sense of differential time-delay operators. Then the condition for accessibility of nonlinear time-delay systems is presented in terms of common left factors of such polynomials. Transfer functions of nonlinear time-delay systems are introduced as well and their basic properties are shown.

1 Introduction

Algebraic approach of differential forms, originally developed for nonlinear systems without delays, [5] and [1], was recently extended to the case of time-delay systems [13, 14, 15, 18] and was shown to be effective in solving control problems like accessibility and observability, disturbance decoupling, etc.

On the other side, in the case of systems without delays, there exists, in comparison to the machinery of one-forms, an alternative approach in which the system properties are described by skew polynomials from non-commutative polynomial rings. Such polynomials act as differential [19, 20] or shift [11, 12] operators on the differentials of the system inputs and outputs. This approach allows us, for instance, to introduce the accessibility condition expressed in terms of common left factors of skew polynomials derived from the input-output equation. Moreover, the polynomial approach, after defining quotients of skew polynomials, provides also possibility to introduce transfer functions of nonlinear systems, both continuous [6, 7] and discrete-time [8, 9]. Such transfer functions show many properties we expect from transfer functions, like the invariance to state transformations, transfer function algebra and others and was, for instance, already used in [10] to recast and solve the nonlinear model matching problem.

However, a similar polynomial approach is not yet available for nonlinear time-delay systems and in what follows, it is, therefore, extended also to this case.

J.J. Loiseau et al. (Eds.): Topics in Time Delay Systems, LNCIS 388, pp. 109–119.
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2 Algebraic setting

The mathematical setting, to be used in this paper for dealing with nonlinear time-delay systems , was recently introduced in [13, 14, 15, 18] and will be now briefly reviewed. In order to avoid technicalities, we use slightly abbreviated notations and the reader is referred to those works for detailed technical constructions which are not found here.

The nonlinear time-delay systems considered in this paper are objects of the form

$$\begin{aligned} \dot{x}(t) &= f(\{x(t-i), u(t-j); i, j \geq 0\}) \\ y(t) &= g(\{x(t-i), u(t-j); i, j \geq 0\}) \end{aligned} \tag{1}$$

where the entries of f and g are meromorphic functions and $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$ and $y \in \mathbf{R}^p$ denote state, input and output to the system.

Note that it is not restrictive to assume $i, j \in \mathbf{N}$ since all the delays can be considered as multiples of an elementary delay h [14].

Let $\mathcal{K}$ be the field of meromorphic functions of $\{x(t-i), u^{(k)}(t-j); i, j, k \geq 0\}$ and let $\mathcal{E}$ be the formal vector space over $\mathcal{K}$ given by

$$\mathcal{E} = \mathrm{span}_{\mathcal{K}}\{\mathrm{d}\xi; \xi \in \mathcal{K}\}$$

The delay operator δ is defined on $\mathcal{K}$ and $\mathcal{E}$ as

$$\begin{aligned} \delta(\xi(t)) &= \xi(t-1) \\ \delta(\alpha(t)\mathrm{d}\xi(t)) &= \alpha(t-1)\mathrm{d}\xi(t-1) \end{aligned} \tag{2}$$

for any $\xi(t) \in \mathcal{K}$ and $\alpha(t)\mathrm{d}\xi(t) \in \mathcal{E}$. The delay operator (2) induces the non-commutative polynomial ring $\mathcal{K}[\delta]$ with the multiplication given by the commutation rule

$$\delta a(t) = a(t-1)\delta$$

for any $a(t) \in \mathcal{K}$. The ring $\mathcal{K}[\delta]$ thus represents the ring of linear shift (delay) operators.

Properties of the system (1) can be now analyzed by introducing the machinery of one-forms known from systems without delays [5, 1]. This time, rather than vector spaces we introduce modules over $\mathcal{K}[\delta]$, generally

$$\mathcal{M} = \mathrm{span}_{\mathcal{K}[\delta]}\{\mathrm{d}\xi; \xi \in \mathcal{K}\}$$

Such an approach was shown to be effective in solving a number of control problems, like accessibility and observability, disturbance decoupling, to name a few. See for instance [13, 14, 15, 18].

2.1 Pseudo-linear algebra

To handle different types of linear operators, as for instance differential, shift, difference, q-shift, we, to advantage, introduce univariate skew polynomial rings, or Ore rings [2]. Such structures allow us to handle those types of operators from a uniform standpoint.

Definition 1. *Let K be a field and $\sigma : K \to K$ an automorphism of K. A map $\delta : K \to K$ which satisfies*

$$\begin{aligned} \delta(a+b) &= \delta(a) + \delta(b) \\ \delta(ab) &= \sigma(a)\delta(b) + \delta(a)b \end{aligned} \tag{3}$$

is called a pseudo-derivation (or a σ-derivation).