Clique encoding in recurrent neural networks

Emanuele Varriale, Claudius Gros

Institute for Theoretical Physics, Goethe University, Frankfurt am Main, Germany

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Introduction

Internal brain activity is only modulated, not driven, by sensory input [1].
Therefore the semantic learning that takes place in the brain is a result of the interaction of external sensory stimuli with an autonomously active network.
Clique encoding is a form of sparse coding, that is backed up by experimental findings about real-time memory representation in the hippocampus [2].
For this reason we work with a network with competing cliques, where the activity "flows" from one clique to another, with a transient state dynamics.
We propose a learning rule that correlates such transient states with sensory inputs from the bars and stripes problem, prompted by [3].

Network architecture

The term clique comes from graph theory and it refers to maximal fully connected sub-graphs.
In a network formed by cliques with excitatory synapses within and inhibitory ones across, the dynamics is characterized by competing cliques.
An active clique inhibits every other, so that this is a stable state.
We use networks as the one shown in Fig. 1, in which activity can spread from one clique to any other.

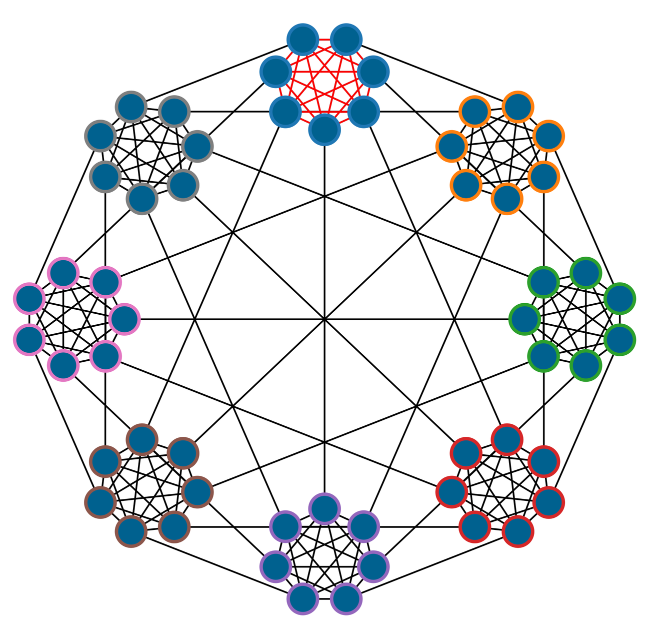


Figure 1: An eight clique network, each with seven nodes. Only excitatory links are shown. Each node within a clique excite a neuron in a different clique. The network is completed with an inhibitory background of connections.

Neuronal dynamics

The j-th neuron has membrane potential x_j, activity y_j and receives excitatory and inhibitory input, respectively E_j and I_j. The rate-encoding model is governed by the following equations:

$$\tau_x \dot{x}_j = -x_j + E_j + I_j$$
$$y_j = \sigma(x_k) = \frac{1}{1+\exp(-a x_k)}$$
$$E_j = \sum_k w_{jk} y_k$$
$$I_j = \sum_k z_{jk} y_k.$$

These equations lead to an attractor network, and the system rapidly relaxes to a stable state.
Inserting a presynaptic reservoir variable the effective output changes, i.e. $y_k \to \tilde{y}_k$, and a transient state dynamics is obtained, as in Fig. 2.

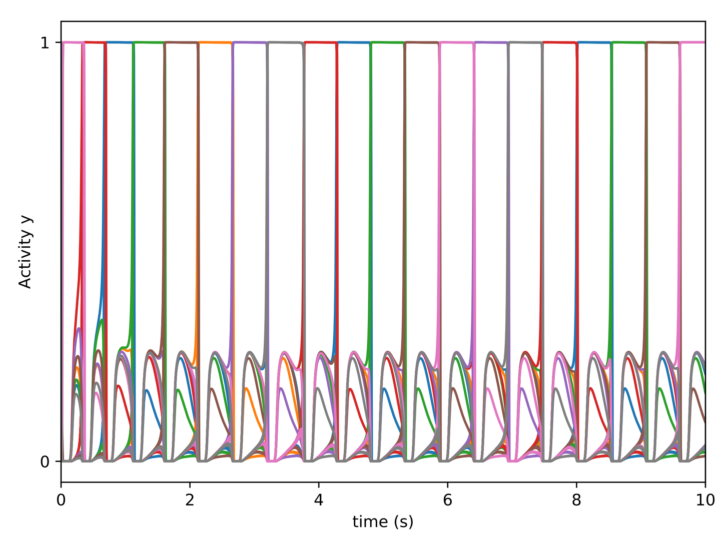


Figure 2: Internal activity in the network shown above. The same colour is used for neurons belonging to the same clique, as shown in Fig. 1.

Full depletion model

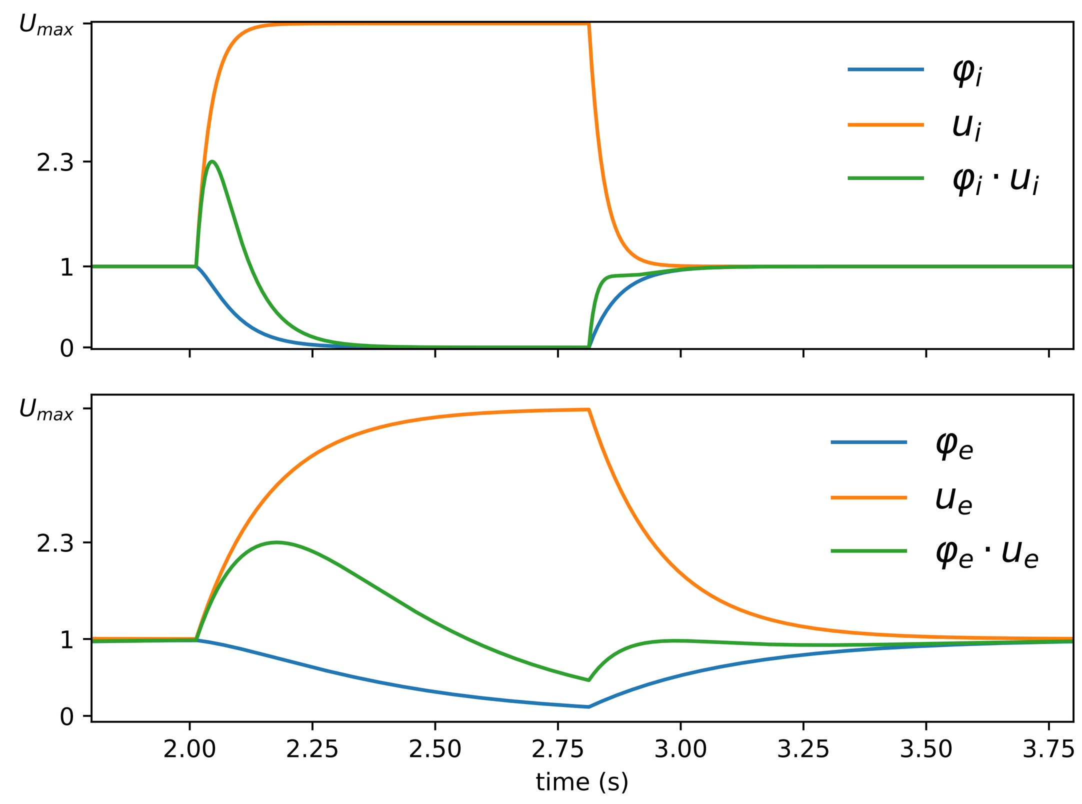


Figure 3: Dynamics of the full depletion model, with high presynaptic activity. Inhibitory short-term plasticity is faster than excitatory.

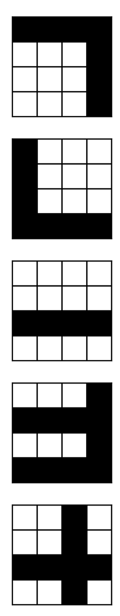
Transient state dynamics requires winning clique states not to be stable.
This is achieved through the Full depletion model for short term plasticity, similar to the Tsodyks-Markram model [4].
Each neuron has a presynaptic reservoir variable that is completely depleted after sustained firing, more specifically presynaptic activity is modulated by two terms:

- u_j represents the likelihood of neurotransmitter release, which increases with Ca^{2+} traces,
- φ_j represents the concentration of available vesicles, that depletes while firing.

The Full depletion model is given by:

$$\tilde{y}_j = y_j u_j \varphi_j$$
$$\dot{u}_j = \frac{U_y - u}{T_u}, \quad U_y = 1 + (U_{\text{max}} - 1)\, y_j$$
$$\dot{\varphi}_j = \frac{\Phi_u - \varphi}{T_\varphi}, \quad \Phi_u = 1 - \frac{u y_j}{U_{\text{max}}}$$
$$T^{\text{exc}} = 5 \cdot T^{\text{inh}}.$$

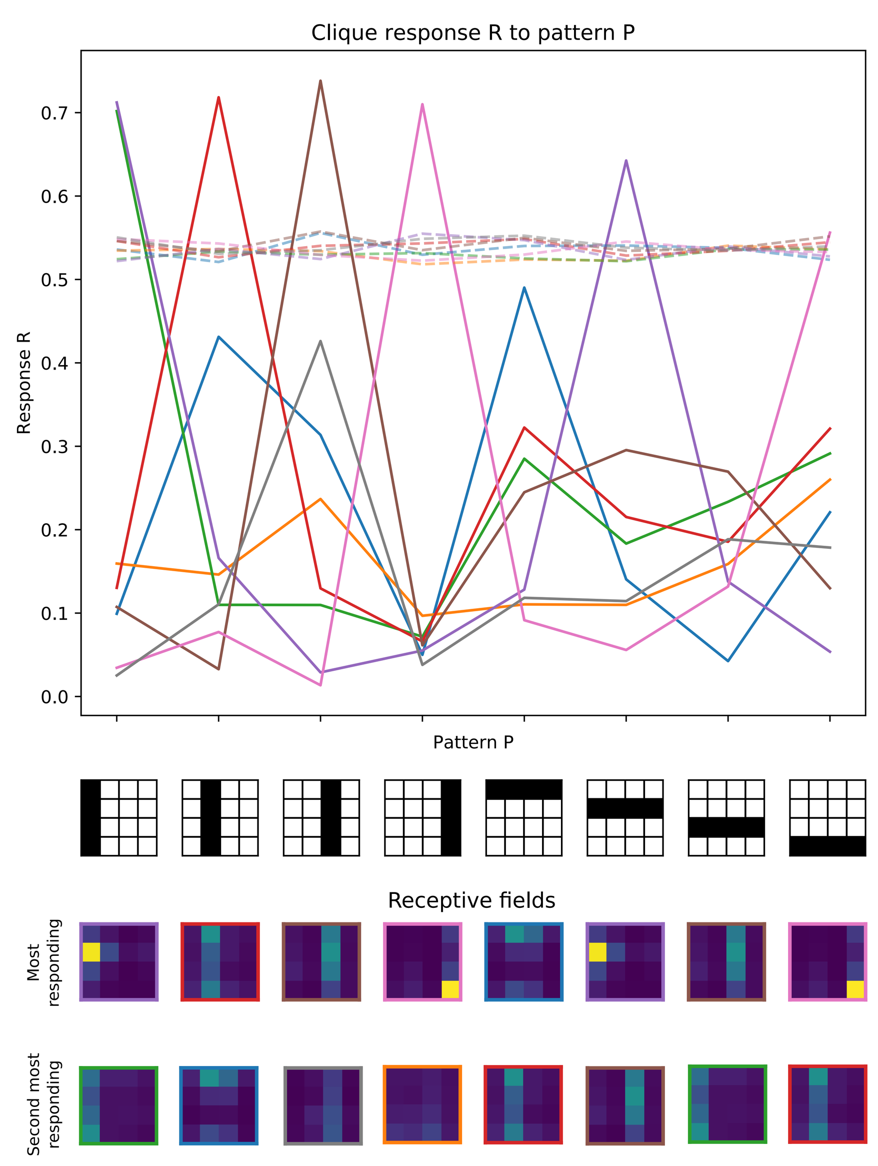
Sensory input



A critical task for a cognitive system is to identify recurring patterns, i.e. objects, in a noisy environment, a task that falls under the domain of Independent Component Analysis.
The autonomous activity of the network has no semantic content, being independent from the external world.
Meaning can be acquired by correlating sensory signals to specific cliques.
We use the bars problem, in which horizontal and vertical bars are presented on a retina of $L \times L$ pixels. Each of the $2L$ bars is independently drawn with probability $p = 1/2L$. Inactive pixels have value $y_l^{\text{ext}} = 0$, whereas active pixels always $y_l^{\text{ext}} = 1$, even if they are at the intersection of two bars.
Because of this, a *non linear* independent component analysis has to be performed in order to separate bars.
This sensory input layer is connected to every node in the network with excitatory connections v_{jl}. At regular intervals the network receives an extra input term:

$$\Delta E_j = \sum_{l=1}^{L^2} v_{jl} y_l^{\text{ext}}$$

Learning rule



The learning procedure has to map different bars onto different cliques, by taking advantage of the fact that the sensory input can "steer" the competitive dynamics by changing the order of winning cliques.
We employ the following learning rule:

$$\frac{d}{dt} v_{jl} = (\dot{y}_j c_j / \tau_v - 1/\tau_d)\, y_l^{\text{ext}} v_{jl}$$
$$c_j = \tanh\left[a\,(V_t - \Delta E_j)\right]$$
$$V_t = V^{\text{ina}} + y_j\,(V^{\text{act}} - V^{\text{ina}})$$
$$\tau_v \ll \tau_d, \quad V^{\text{ina}} < V^{\text{act}}.$$

We note that:

- the factor v_{jl} ensures that the weights do not change sign,
- presynaptic activity y_l^{ext} is necessary to learning,
- $\dot{y}_j$ ensures that learning only takes place when the activity changes significantly,
- the term c_j prevents runaway growth or shrinking of weights, with the sliding threshold V_t,
- the slow decay term $-1/\tau_d$ shrinks non-useful synapses.

To assess the performance of the learning rule we compute the response $R(\alpha, \beta)$ of clique α to the pattern β and the receptive field $F(\alpha, l)$:

$$R(\alpha, \beta) = \frac{1}{S(C_\alpha)} \sum_{\substack{i \in C_\alpha \\ l \in P_\beta}} v_{il} y_l^{\text{ext}}$$
$$F(\alpha, l) = \frac{1}{S(C_\alpha)} \sum_{i \in C_\alpha} v_{il}$$

References

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varriale@itp.uni-frankfurt.de