Clique encoding in recurrent neural networks

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Introduction

Internal brain activity is only modulated, not driven, by sensory input [1].

Therefore the semantic learning that takes place in the brain is a result of the interaction of external sensory stimuli with an autonomously active network. Clique encoding is a form of sparse coding, that is backed up by experimental findings about real-time memory representation in the hippocampus [2]. For this reason we work with a network with competing cliques, where the activity "flows" from one clique to

another, with a transient state dynamics. We propose a learning rule that correlates such transient states with sensory inputs from the bars and stripes problem, prompted by [3].

Network architecture

The term clique comes from graph theory and it refers to maximal fully connected sub-graphs.

In a network formed by cliques with excitatory synapses within and inhibitory ones across, the dynamics is characterized by competing cliques.

An active clique inhibits every other, so that this is a stable state.

We use networks as the one shown in Fig. 1, in which activity can spread from one clique to any other.

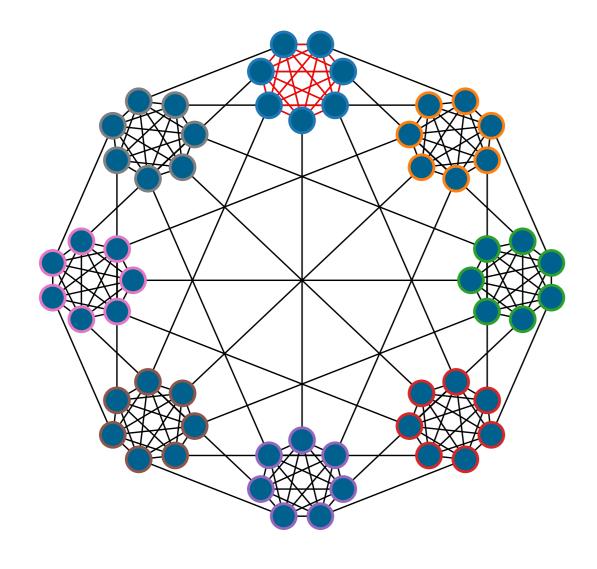


Figure 1: An eight clique network, each with seven nodes. Only excitatory links are shown. Each node within a clique excite a neuron in a different clique. The network is completed with an inhibitory background of connections.

Neuronal dynamics

The j-th neuron has membrane potential x_j , activity y_j and receives excitatory and inhibitory input, respectively $oldsymbol{E}_j$ and $oldsymbol{I}_j$. The rate-encoding model is governed by the following equations:

$$egin{aligned} au_x\dot{x}_j&=-x_j+E_j+I_j\ y_j&=\sigma\left(x_k
ight)&=rac{1}{1+\exp\left(-ax_k
ight)}\ E_j&=\sum_k w_{jk}y_k\ I_j&=\sum_k z_{jk}y_k. \end{aligned}$$

These equations lead to an attractor network, and the system rapidly relaxes to a stable state. Inserting a pre-synaptic reservoir variable the effective signal changes, i.e. $y_k o ilde y_k$, and a transient state dynamics is obtained, compare Fig. 2.

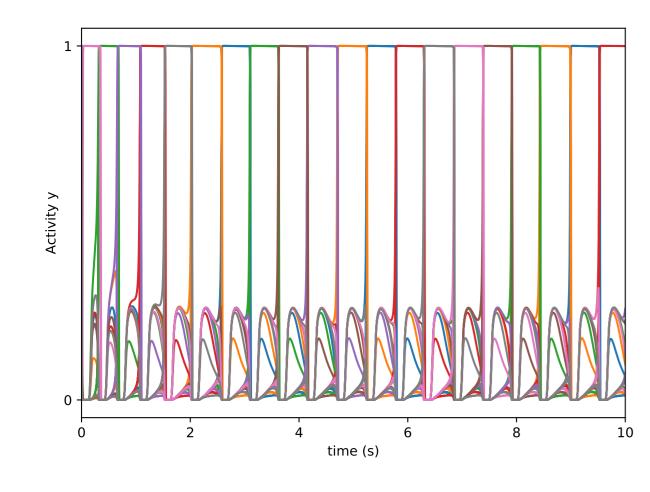


Figure 2: Internal activity in the network shown above. The same colour is used for neurons belonging to the same clique, as shown in Fig. 1.

Full depletion model

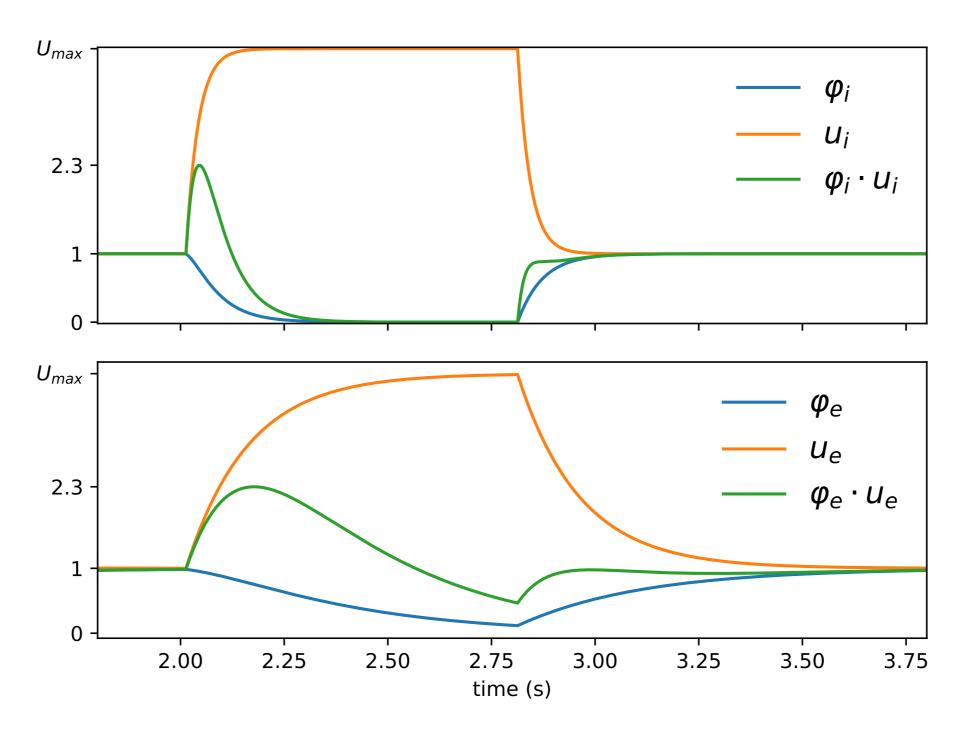


Figure 3: Dynamics of the full depletion model, with high presynaptic activity. Inhibitory short-term plasticity is faster than excitatory.

Transient state dynamics requires winning clique states not to be stable.

This is achieved through the Full-depletion model for short term plasticity, similar to the Tsodyks-Markram model [4]. Each neuron has a pre-synaptic reservoir variable that is completely depleted after sustained firing, so that pre-synaptic activity is modulated by two terms:

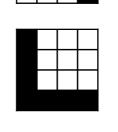
- $lackbox{} u_j$ represents the likelihood of neurotransmitter release, which increases with Ca^{2+} traces,
- $\triangleright \varphi_i$ represents the concentration of available vesicles, that depletes while firing.

The over-all short term plasticity is given by:

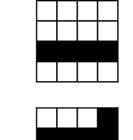
$$egin{align} ilde{y}_j &= y_j u_j arphi_j \ ilde{u}_j &= rac{U_y - u}{T_u}, \quad U_y &= 1 + \left(U_{\mathsf{max}} - 1
ight) y_j \ ilde{arphi}_j &= rac{oldsymbol{\Phi}_u - arphi}{T_arphi}, \quad oldsymbol{\Phi}_u &= 1 - rac{u y_j}{U_{\mathsf{max}}} \ T^{\mathsf{exc}} &= 5 \cdot T^{\mathsf{inh}}. \end{align}$$

Sensory input

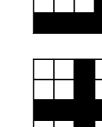
We want our network to extract information from an external environment via signal separation and feature extraction. An example would be to identify recurring patterns and to separate them, even if they occur simultaneously.



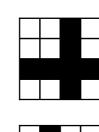
We use the bars problem, in which horizontal and vertical bars are presented on a retina of L x L pixels. Bars are independently drawn so that there is, on average, one bar per input. Active bar pixels have $y_l^{
m ext}=1$, and inactive ones $y_l^{\rm ext}=0$, as shown in the Figure.



The bar intersection pixels also have value 1, so that a non linear independent component analysis has to be performed in order to separate bars.



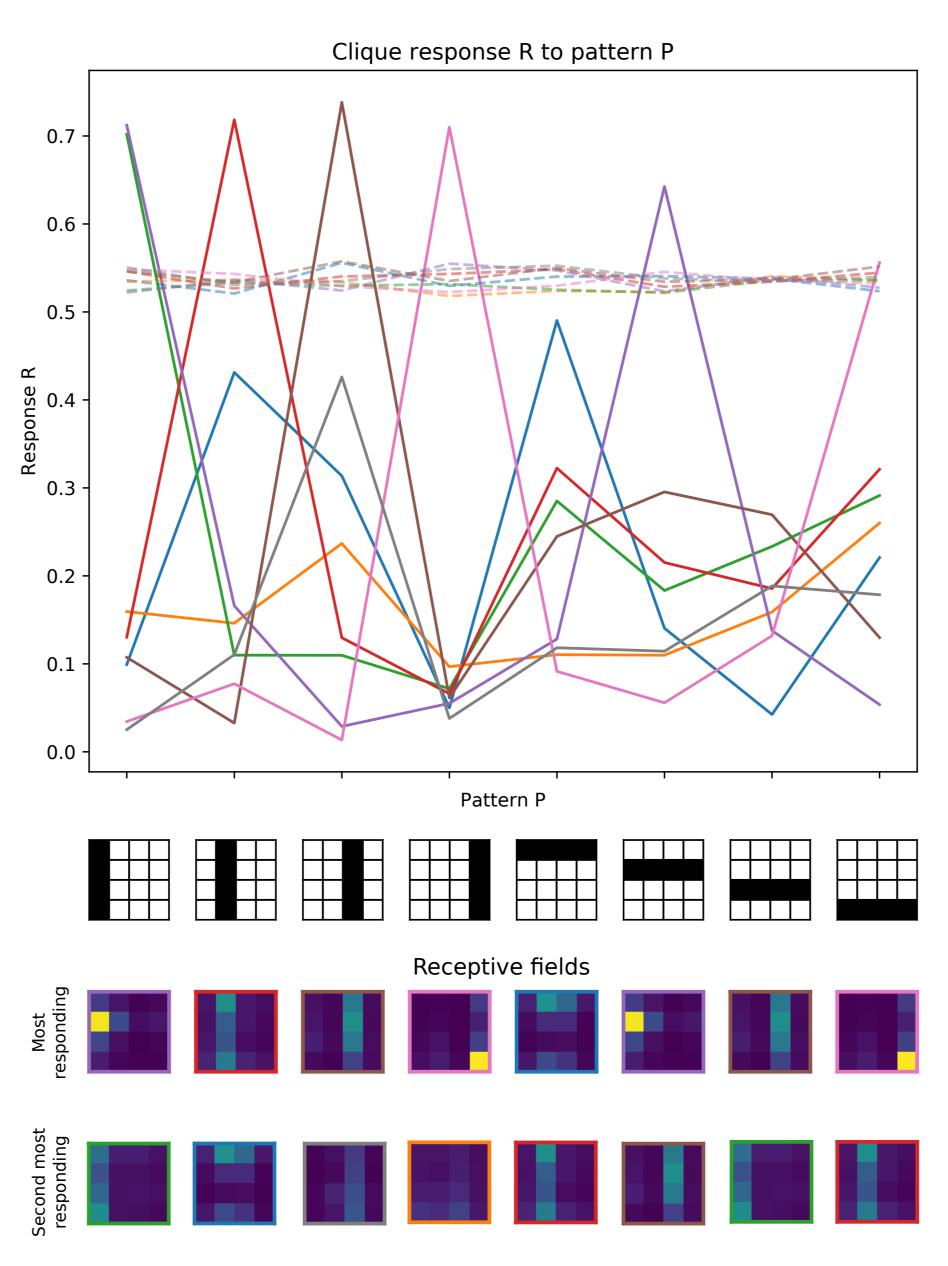
This sensory input layer is connected to the network with excitatory connections v_{il} so that an extra input term ΔE_{j} is added:



 $\Delta E_j = \sum_{l=1}^{L^2} v_{jl} y_l^{\mathsf{ext}}$

The initial weights are chosen so that, on average, $\Delta E_j \approx 0.5$. With these weights external input can, on average, change the internal dynamics only when the short-term plasticity tunes down inhibitory weights.

Learning rule



The learning procedure has to map different bars onto different cliques, by taking advantage of the fact that the sensory input can "steer" the competing dynamics, thus changing the order of winning cliques.

We employ the following learning rule:

$$egin{aligned} rac{d}{dt}v_{jl} &= \left(\dot{y}_{j}c_{j}/ au_{v} - 1/ au_{d}
ight)y_{l}^{ ext{ext}}v_{jl} \ c_{j} &= anh\left[a\left(V_{t} - \Delta E_{j}
ight)
ight] \ V_{t} &= V^{ ext{ina}} + y_{j}\left(V^{ ext{act}} - V^{ ext{ina}}
ight) \ au_{v} &\ll au_{d}, \quad V^{ ext{ina}} &< V^{ ext{act}}. \end{aligned}$$

We note that:

- \blacktriangleright the factor v_{il} ensures that the weights do not change sign,
- lacktriangle pre-synaptic activity $oldsymbol{y}_{l}^{
 m ext}$ is necessary to learning,
- $\dot{m{y}}_j$ ensures that learning only takes place when the activity changes significantly,
- lacktriangle the term c_i prevents runaway growth or shrinking of weights, with the sliding threshold V_t ,
- ightharpoonup the slow decay term $-1/ au_d$ shrinks non-useful synapses.

$$R(lpha,eta) = rac{1}{S(C_lpha)} \sum_{i \in C_lpha,l} v_{il} y_l^{ ext{ext}}$$
 (1)

$$F(\alpha, l) = \frac{1}{S(C_{\alpha})} \sum_{i \in C} v_{il}$$
 (2)

References

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- [2] Lin, L., Osan, R. & Tsien, J. Z. Organizing principles of real-time memory encoding: neural clique assemblies and universal neural codes. Trends in Neurosciences 29, 48-57 (2006).
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- [4] Mongillo, G., Barak, O. & Tsodyks, M. Synaptic Theory of Working Memory. Science 319, 1543-1546 (2008).