Clique encoding in recurrent neural networks

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Introduction

- Internal brain activity is only modulated, not driven, by sensory input [1].
- Semantic learning has to result from the interaction of sensory stimuli with an autonomously active network.
- Clique encoding [2] can be achieved with a network of competing cliques, that has transient state dynamics.
- We propose a learning rule that correlates such transient states with sensory inputs from the bars and stripes problem, prompted by [3].

Network architecture

- A clique is a maximal fully connected sub-graphs.
- Cliques compete with one another if they have predominantly inhibitory connections across and excitatory ones within.
- A single active clique is a stable state, as long as other cliques are inhibited.

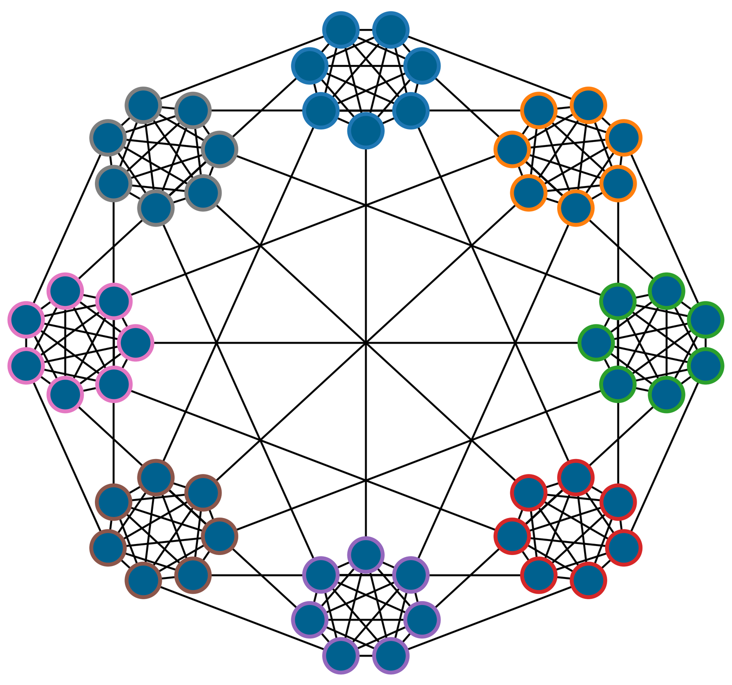


Figure 1: An eight clique network, each with seven nodes. Only excitatory links are shown. Each neuron within a clique excites an extra-clique neuron. The network is completed with inhibitory connections.

Neuronal dynamics

- A rate-encoding model is used.
- Neuron have membrane potential x_j, activity y_j and excitatory and inhibitory input, E_j and I_j.

$$\tau_x \dot{x}_j = -x_j + E_j + I_j$$
$$y_j = \sigma(x_j) = \frac{1}{1+\exp(-a x_j)}$$
$$E_j = \sum_k w_{jk} y_k$$
$$I_j = \sum_k z_{jk} y_k .$$

- With these equation the system rapidly relaxes to a stable active clique state.
- The inclusion of presynaptic short-term plasticity, i.e. $y_k \to \tilde{y}_k$, gives transient state dynamics, as in Fig. 2.

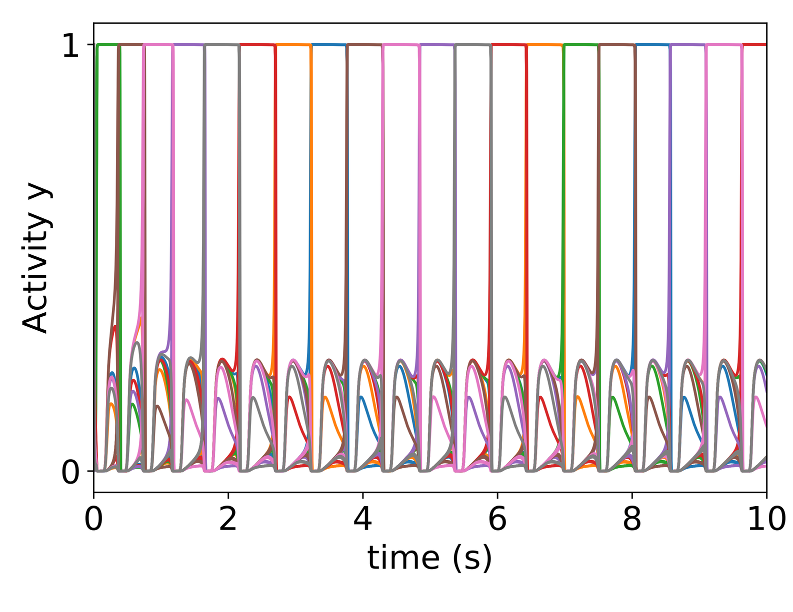


Figure 2: Internal activity in the network shown above with the full depletion model. Lines are plotted according to the clique a neuron belongs to, matching the colours in Fig. 1.

Full depletion model

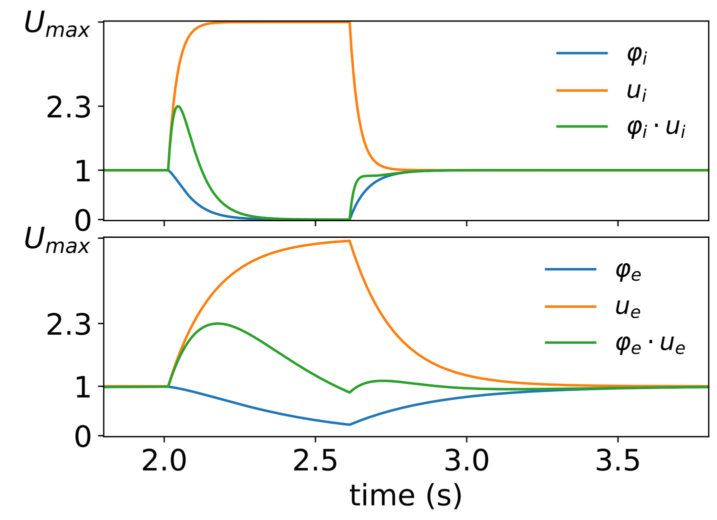


Figure 3: Dynamics of the full depletion model, with high presynaptic activity, for inhibitory (top) and excitatory (bottom) synapses.

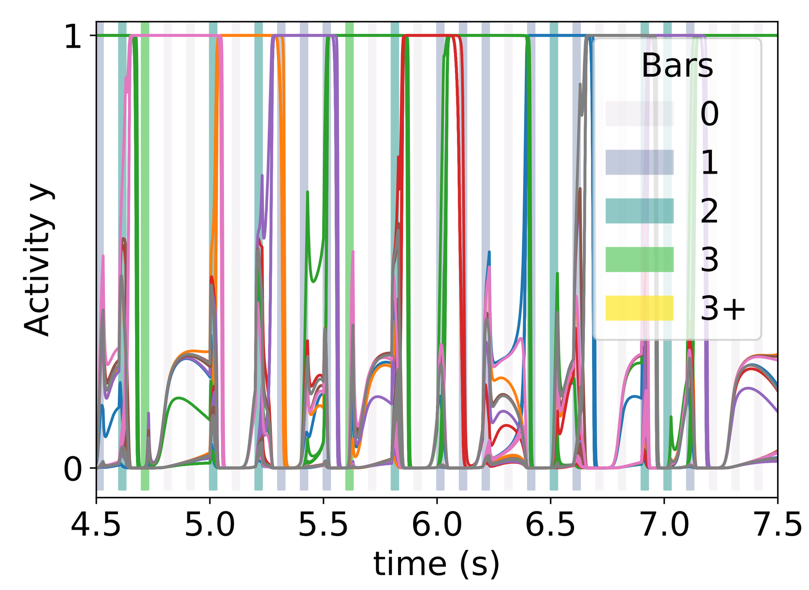
- Winning clique states become unstable with the STP *full depletion model*, inspired by the Tsodyks-Markram model [4].
- Transient states are obtained by modelling neurotransmitter vesicles with:
  - u_k: likelihood of neurotransmitter release, increases with input
  - φ_k: concentration of vesicles, that deplete while firing.
- Inhibitory synapses deplete faster than excitatory ones.

$$y_k \to \tilde{y}_k = y_k u_k \varphi_k$$
$$\dot{u}_k = \frac{U_y - u_k}{T_u}, \quad U_y = 1 + (U_{\text{max}} - 1)\, y_k$$
$$\dot{\varphi}_k = \frac{\Phi_u - \varphi_k}{T_\varphi}, \quad \Phi_u = 1 - \frac{u_k y_k}{U_{\text{max}}}$$
$$T^{\text{exc}} = 5 \cdot T^{\text{inh}}.$$

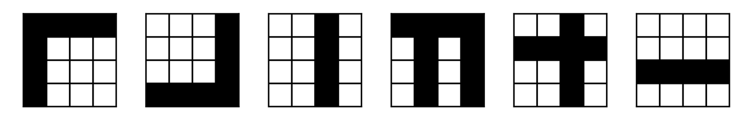
Sensory input

- Cognitive systems must identify objects in a noisy environment, performing independent component analysis.
- Only correlation of sensory signals to cliques can confer semantic meaning to the network.
- Sensory inputs are vertical and horizontal bars presented on a retina of $L \times L$ pixels.
- Each bar is independently drawn with probability $p = 1/2L$.
- Inactive pixels have output $y_l^{\text{ext}} = 0$, and active pixels $y_l^{\text{ext}} = 1$, even at the intersection of two bars.
- Therefore, *non linear* independent component analysis has to be performed to separate bars.
- At regular intervals, the all-to-all sensory connections transmit the extra input

$$\Delta E_j = \sum_l v_{jl} y_l^{\text{ext}}$$



(a) Activity with sensory input shown by vertical stripes.



(b) Examples of input patterns.

Learning rule

- The sensory weights v_{jl} must change so that different clique responds mostly to different bars.
- We hypothesize that learning occurs during transition from one winning clique to another.

$$\dot{v}_{jl} = (\dot{y}_j c_j/\tau_v - 1/\tau_d)\, y_l^{\text{ext}} v_{jl}$$
$$c_j = \tanh\left[a\,(V_t - \Delta E_j)\right]$$
$$V_t = V^{\text{inact}} + y_j \left(V^{\text{act}} - V^{\text{inact}}\right)$$
$$\tau_v \ll \tau_d, \quad V^{\text{inact}} < V^{\text{act}}.$$

- v_{jl} ensures that the weights do not change sign,
- sensory input y_l^{ext} is necessary,
- $\dot{y}_j = 0$ during winning clique states,
- c_j prevents runaway synapses dynamics, through the sliding activity level target V_t,
- the slow decay term $-1/\tau_d$ shrinks orthogonal synapses.

Results

We use two performance measure:

$$R(\alpha,\beta) = \frac{1}{S_\alpha} \sum_{\substack{i \in C_\alpha \\ l \in P_\beta}} v_{il} y_l^{\text{ext}}$$
$$F(\alpha,l) = \frac{1}{S_\alpha} \sum_{i \in C_\alpha} v_{il}$$

- The response $R(\alpha,\beta)$ is the clique averaged input received by clique α of size S_α, when bar β is active.
- Ideally, $R(\alpha,\beta)$ would have a single maximum, that would be different across cliques.
- The receptive field $F(\alpha,l)$ is the clique averaged sensory synaptic weight from the j-th pixel.
- Ideally, $F(\alpha,l)$ would mirror the input from the most important bar.

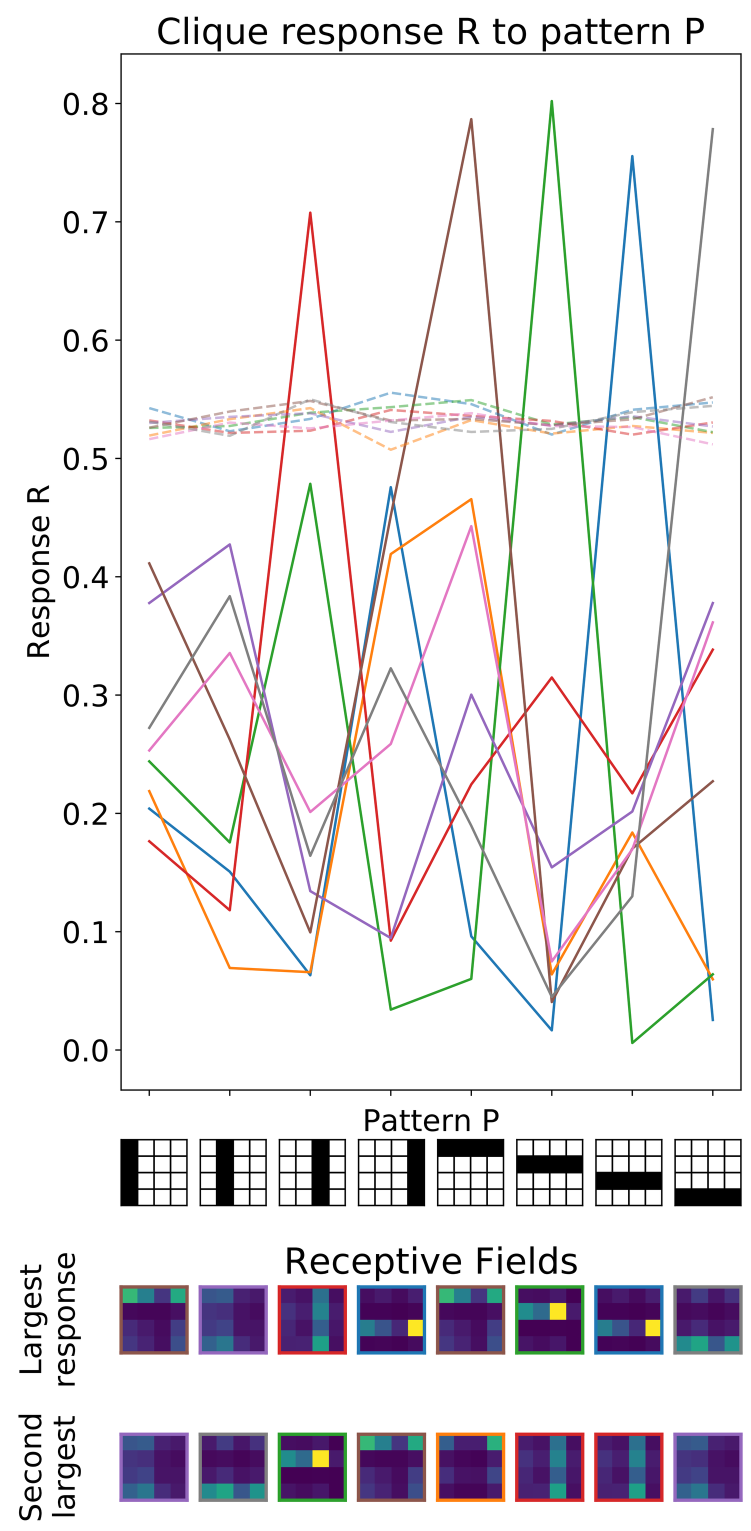


Figure 4: Performance of the learning rule. Top: clique response for every bar, dashed lines show initial response. Bottom: receptive fields of two most responding cliques. The colour coding spans from violet (< 0.01) to yellow (≈ 0.25).

Conclusions

- In neural networks designed for feature extraction activity usually dies out without input. This is not, however, how actual brains operate.
- This work is a proof of concept showing how feature extraction with an autonomously active network can be achieved.
- Internal activity is characterized by transient states and competitive dynamics, and learning occurs during transitions between active cliques.
- More work is needed to improve performance and robustness of the algorithm.

References

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[2] Lin, L., Osan, R. & Tsien, J. Z. Organizing principles of real-time memory encoding: neural clique assemblies and universal neural codes. Trends in Neurosciences 29, 48–57 (2006).

[3] Gros, C. & Kaczor, G. Semantic learning in autonomously active recurrent neural networks. Log J IGPL 18, 686–704 (2010).

[4] Mongillo, G., Barak, O. & Tsodyks, M. Synaptic Theory of Working Memory. Science 319, 1543–1546 (2008).

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