# Dynamic market potential, $m(t)$

A generalization of the Bass Model considers a dynamic market potential, $m(t)$

$$z'(t) = m(t)\left\{ \left(p + q\frac{z(t)}{m}\right)\left(1 - \frac{z(t)}{m(t)}\right) \right\} + z(t)\frac{m'(t)}{m(t)}$$

$$\frac{z'(t)m(t) - z(t)m'(t)}{m^2(t)} = \left(\frac{z(t)}{m(t)}\right)' = \left(p + q\frac{z(t)}{m(t)}\right)\left(1 - \frac{z(t)}{m(t)}\right)$$

and, by setting $y(t) = z(t)/m(t)$, we have

$$y'(t) = p + qy(t)(1 - y(t))$$

which is a standard Bass Model.

# Dynamic market potential, $m(t)$

1. Market of new products is unstable and uncertain in the first phase of diffusion: incubation
2. Advertising and promotional efforts play a central role to overcome this phase
3. These efforts influence the structure of the market potential, which depends on information on the product
4. Communication and adoption are two separate phases, needing a distinct modelling

# Dynamic market potential, $m(t)$

We may notice that di $m(t)$ is 'free'

$$z(t) = m(t)F(t) = m(t)\ \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}}$$

# Dynamic market potential, $m(t)$: GGM

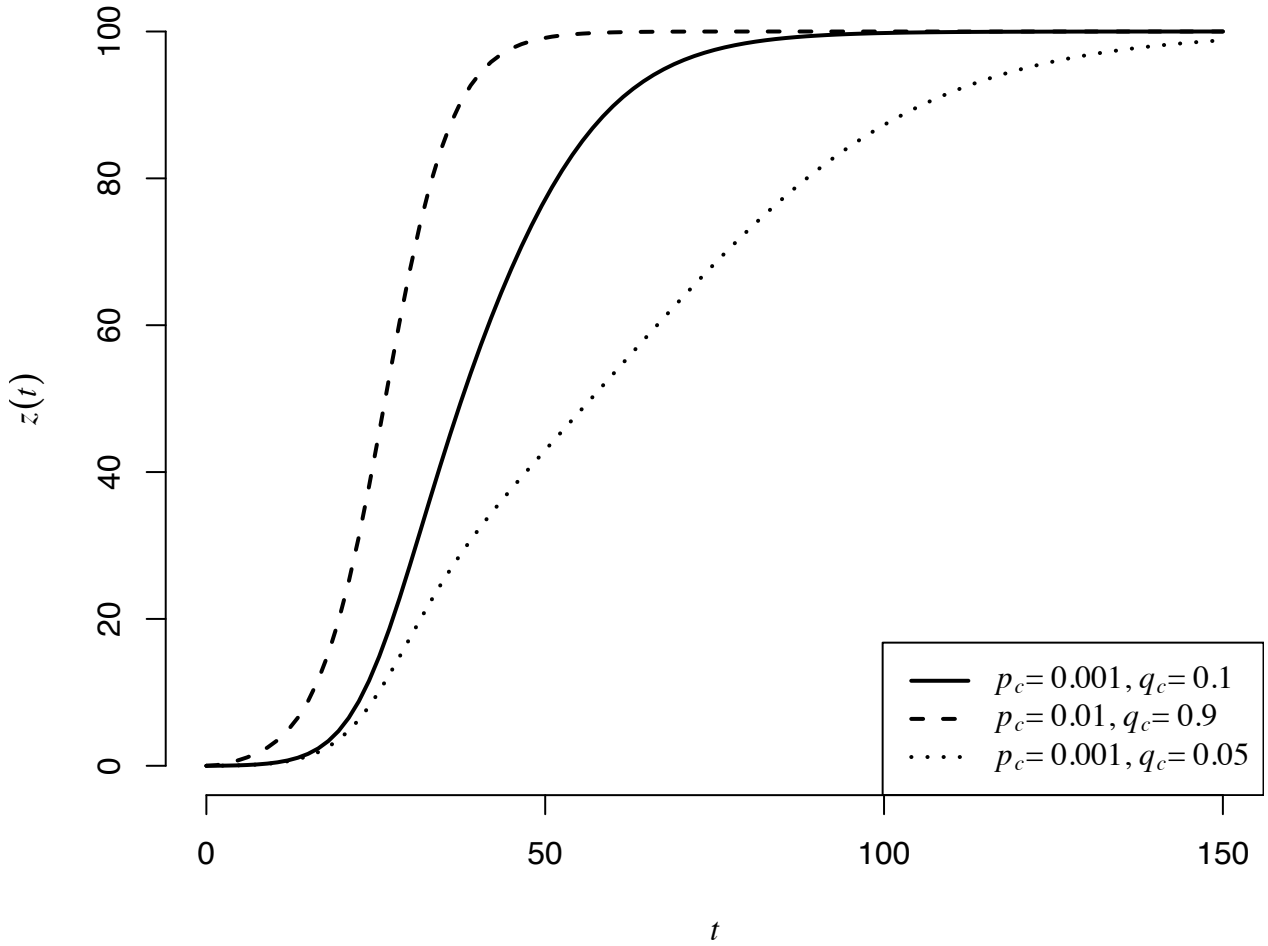
The GGM (Guseo and Guidolin, 2009) is a generalization of the Bass Model, where $m(t)$ is time-dependent

$$z(t) = m(t)F(t) = m(t)\,\frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}}$$

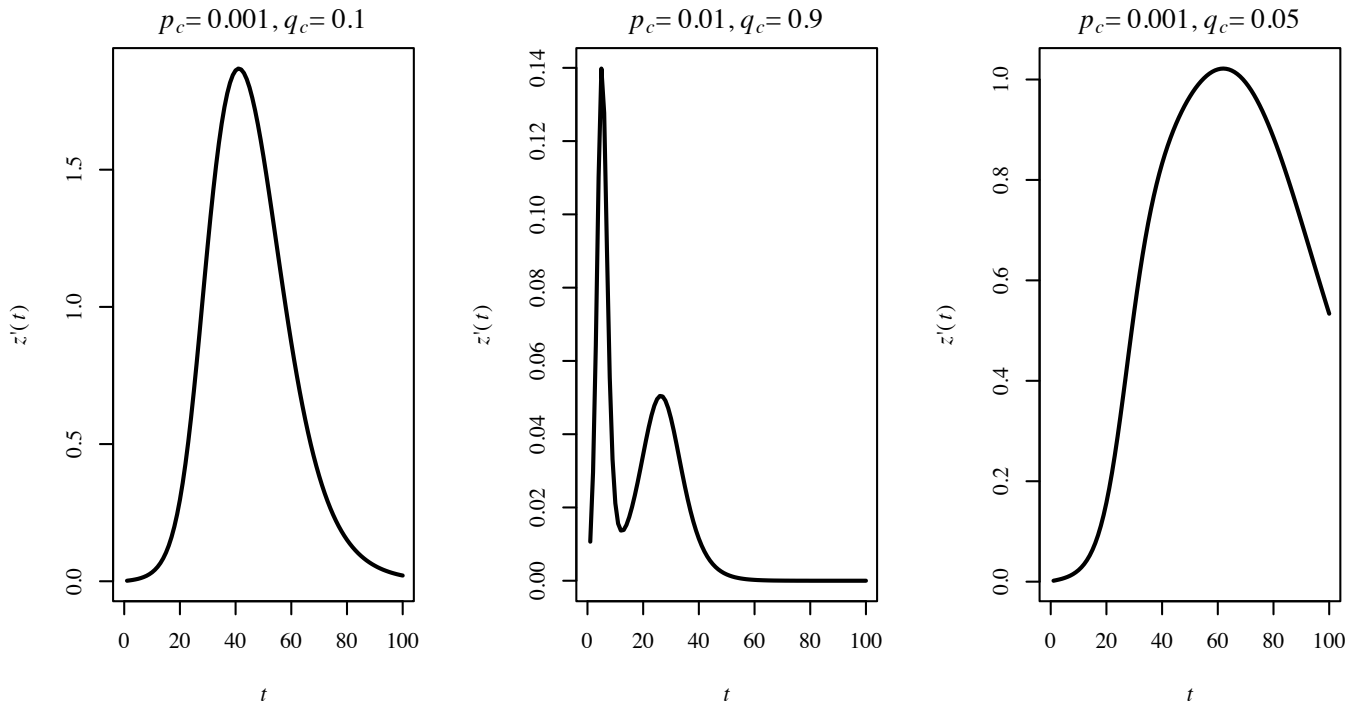
and function of a communication process

$$z(t) = KG(t)F(t) = K\,\sqrt{\frac{1-e^{-(p_c+q_c)t}}{1+\frac{q_c}{p_c}e^{-(p_c+q_c)t}}\,\frac{1-e^{-(p_s+q_s)t}}{1+\frac{q_s}{p_s}e^{-(p_s+q_s)t}}}$$

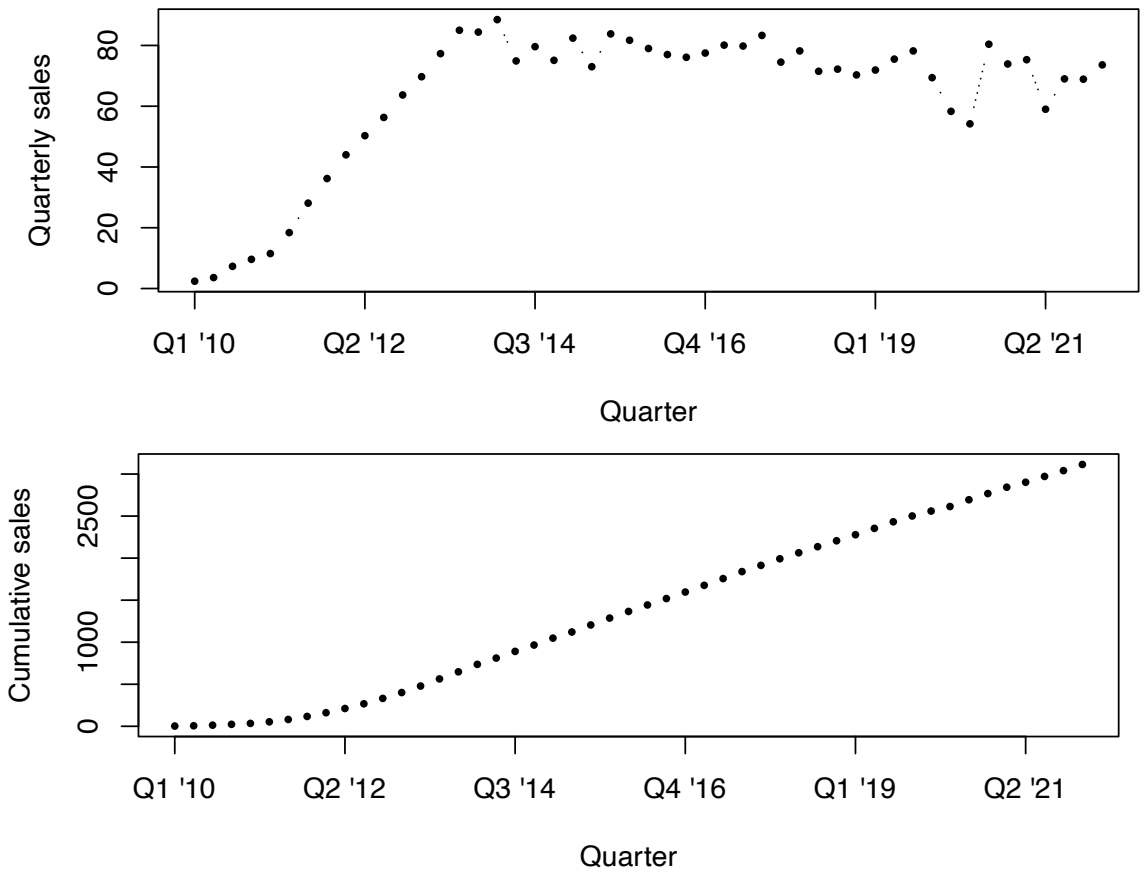
# GGM


z(t)
0
20
40
60
80
100
0
50
100
150
t
$p_c = 0.001, q_c = 0.1$
$p_c = 0.01, q_c = 0.9$
$p_c = 0.001, q_c = 0.05$

# GGM


$p_c = 0.001, q_c = 0.1$
$z'(t)$
0.0
0.5
1.0
1.5
0
20
40
60
80
100
$t$
$p_c = 0.01, q_c = 0.9$
$z'(t)$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0
20
40
60
80
100
$t$
$p_c = 0.001, q_c = 0.05$
$z'(t)$
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100
$t$

# Samsung smartphones



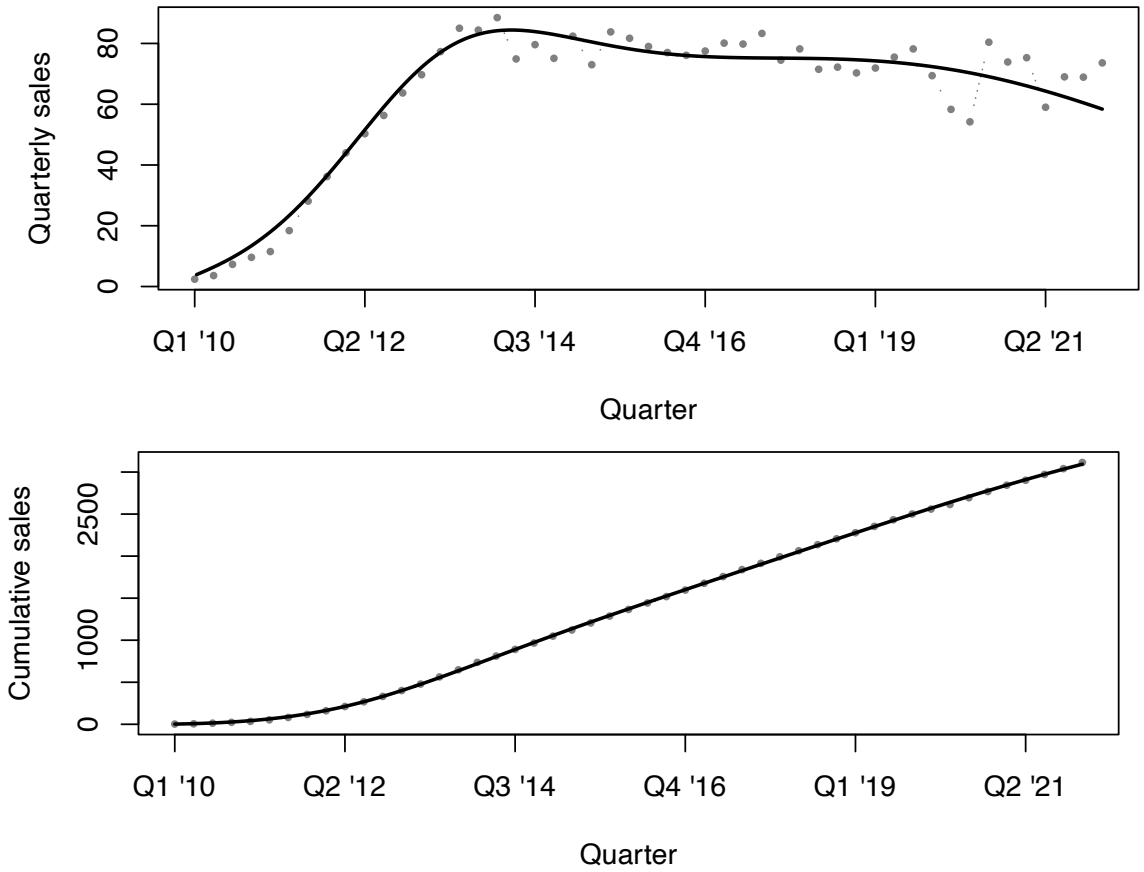
# Samsung smartphones

## GGM for Samsung: estimates and 95% CIs

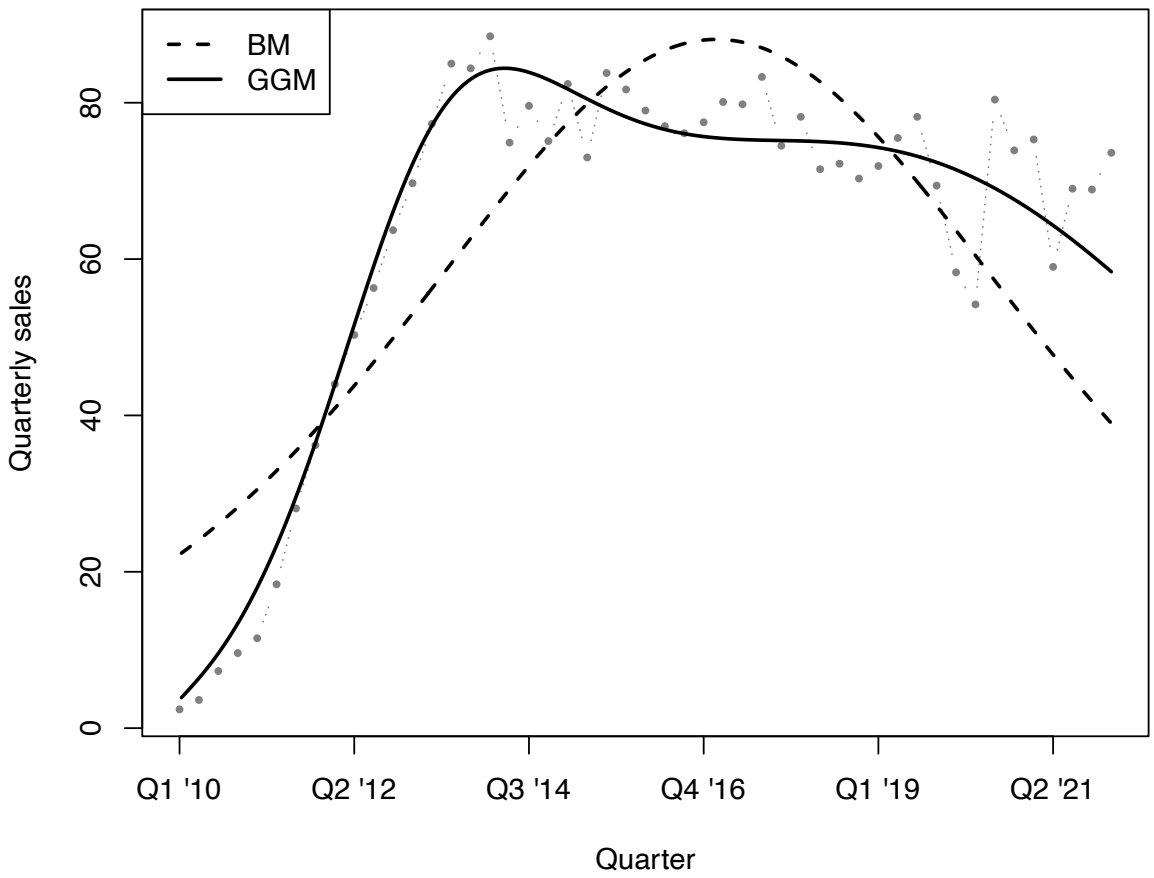
| | Estimate | Std.Error | Lower | Upper | P-value |
|---|---|---|---|---|---|
| $K$ | 4030.7 | 75.47 | 3882.8 | 4178.6 | $< 0.0001$ |
| $p_c$ | 0.0015 | 0.00001 | 0.0014 | 0.0016 | $< 0.0001$ |
| $q_c$ | 0.08 | 0.0026 | 0.08 | 0.09 | $< 0.0001$ |
| $p_s$ | 0.012 | 0.0006 | 0.011 | 0.014 | $< 0.0001$ |
| $q_s$ | 0.21 | 0.008 | 0.20 | 0.23 | $< 0.0001$ |

$R^2 = 0.9999$

# Samsung smartphones

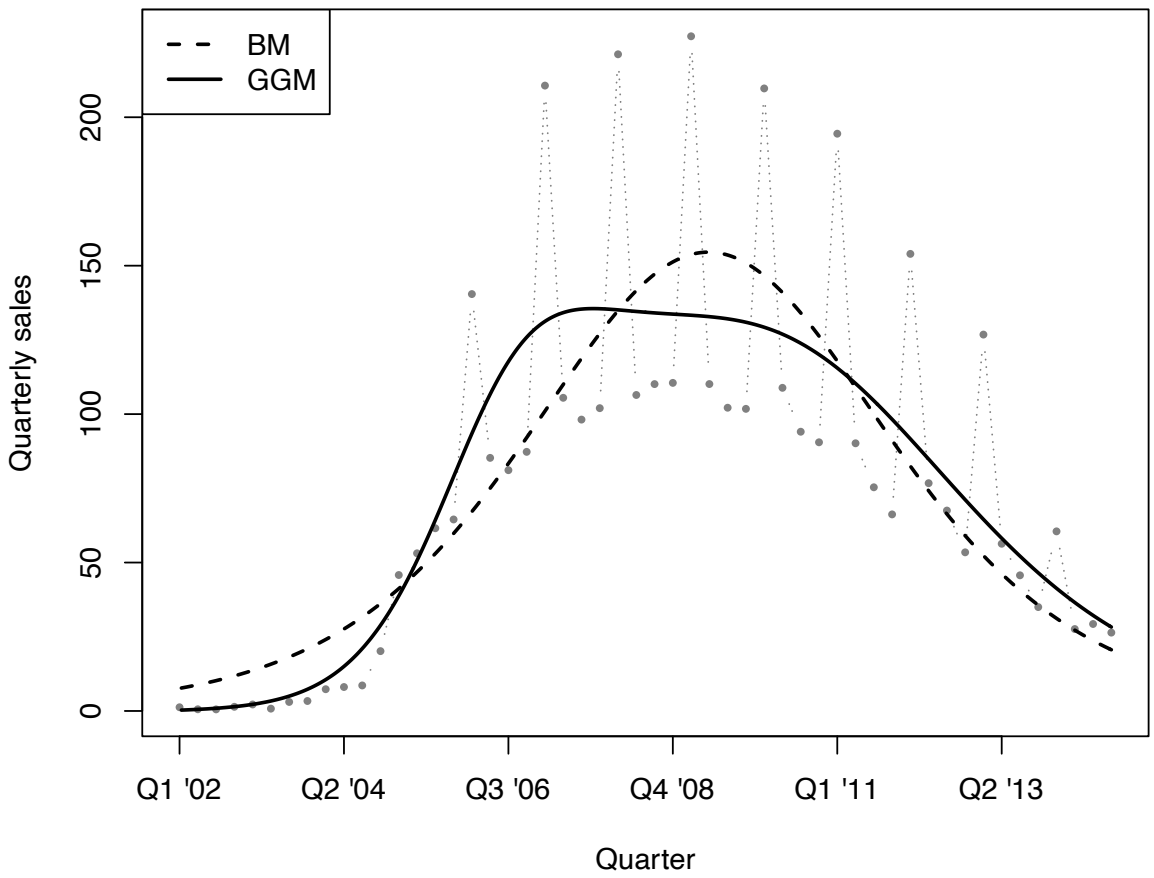


# Samsung smartphones




Model comparison . . .

# Apple iPod




Model comparison . . .

# Competition between two products

## Unbalanced competition and regime change diachronic model

$$
\begin{aligned}
z_1'(t) &= m\Bigg\{ \left[p_{1a} + q_{1a}\frac{z(t)}{m}\right](1 - I_{t>c_2}) \\
&\quad + \left[p_{1c} + (q_{1c} + \delta)\frac{z_1(t)}{m} + q_{1c}\frac{z_2(t)}{m}\right] I_{t>c_2} \Bigg\} \left[1 - \frac{z(t)}{m}\right], \\
z_2'(t) &= m\left[p_2 + (q_2 - \gamma)\frac{z_1(t)}{m} + q_2\frac{z_2(t)}{m}\right]\left[1 - \frac{z(t)}{m}\right] I_{t>c_2},
\end{aligned}
$$

# Competition between two products

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\begin{aligned}
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\end{aligned}
$$

within imitation

# Competition between two products

## Unbalanced competition and regime change diachronic model

$$
\begin{aligned}
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&\quad + \left[p_{1c} + (q_{1c} + \delta)\frac{z_1(t)}{m} + q_{1c}\frac{z_2(t)}{m}\right] I_{t>c_2} \Bigg\} \left[1 - \frac{z(t)}{m}\right], \\
z_2'(t) &= m\left[p_2 + (q_2 - \gamma)\frac{z_1(t)}{m} + q_2\frac{z_2(t)}{m}\right]\left[1 - \frac{z(t)}{m}\right] I_{t>c_2},
\end{aligned}
$$

within imitation

cross imitation

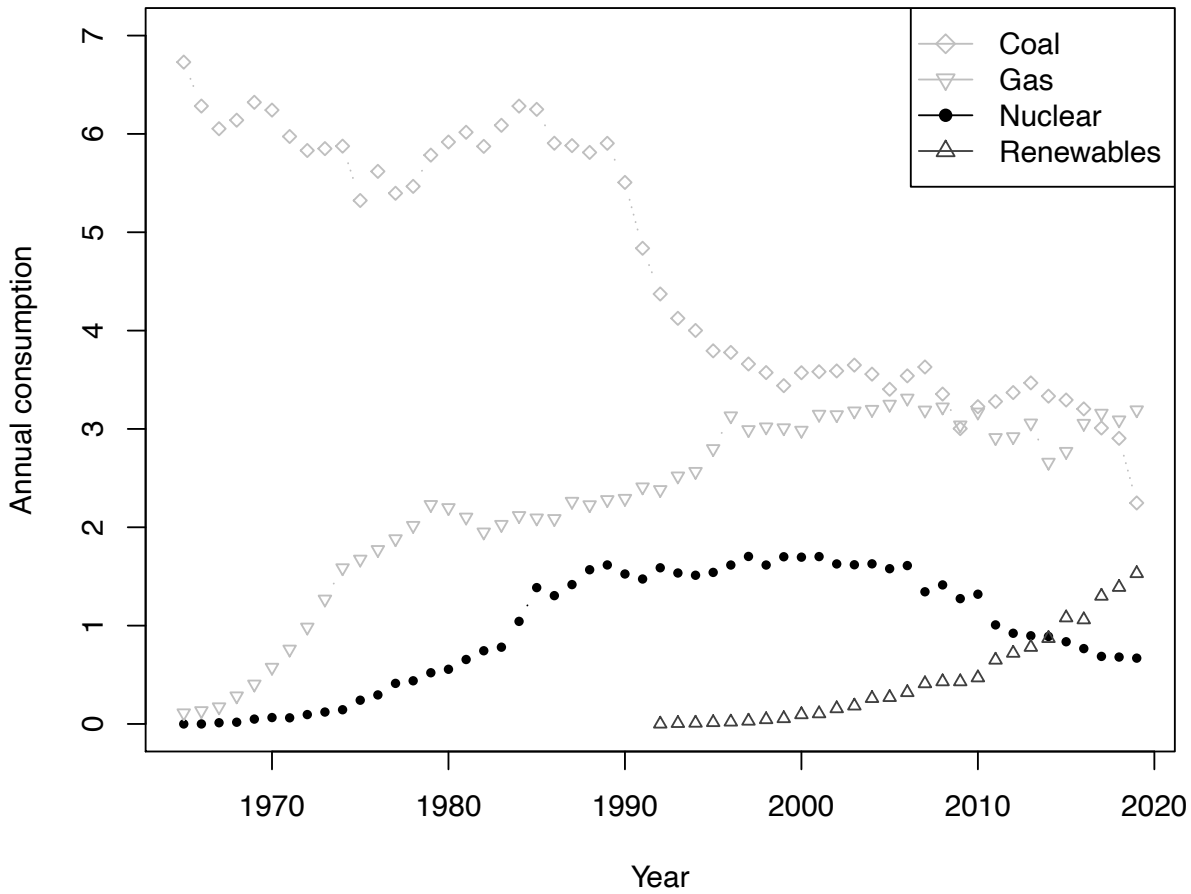
# Model

Sign of cross-imitation coefficients: competition-collaboration

| $q_{1c}$ | $q_2 - \gamma$ | interpretation |
|---|---|---|
| negative | negative | full competition |
| negative | positive | 2 competes with 1, 1 collaborates with 2 |
| positive | negative | 2 collaborates with 1, 1 competes with 2 |
| positive | positive | full collaboration |

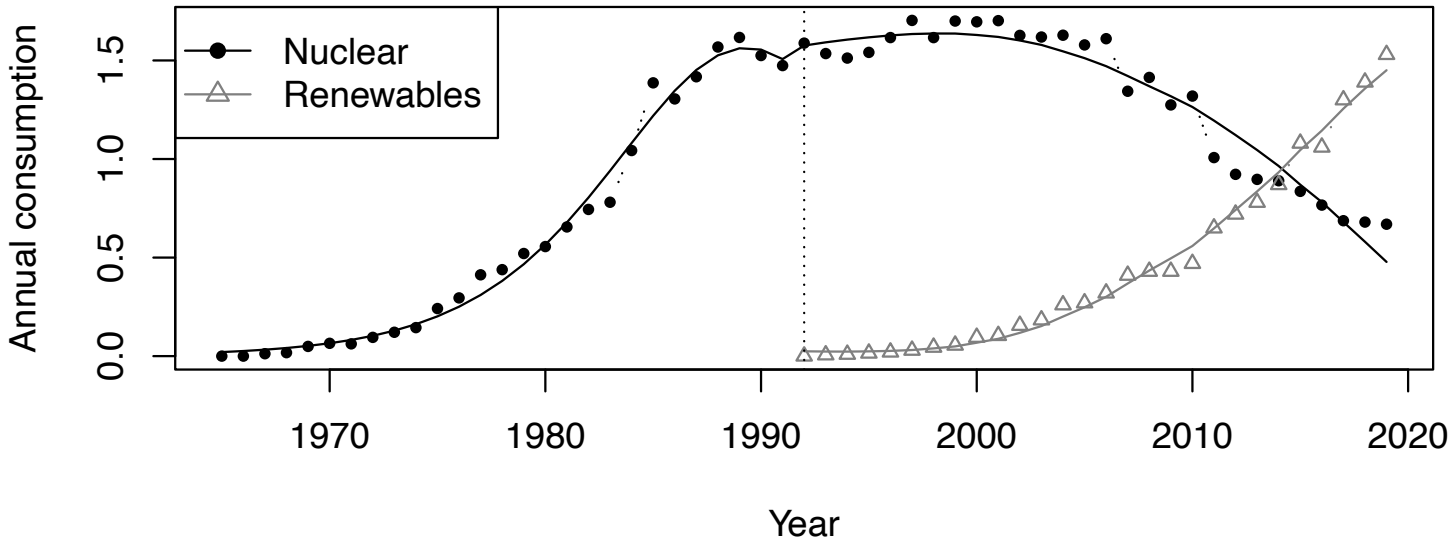
# Germany energy transition

## Energy consumption in Germany



# Germany energy transition

## Competition between renewables and nuclear in Germany



# Germany energy transition

## UCRCD for renewables vs nuclear: estimates and 95% CIs

| | Estimate | Std.Error | Lower | Upper |
|---|---|---|---|---|
| $m_a$ | 26.6 | 0.73 | 25.1 | 28.0 |
| $p_{1a}$ | 0.0007 | 0.0000 | 0.0006 | 0.0007 |
| $q_{1a}$ | 0.23 | 0.004 | 0.22 | 0.24 |
| $m_c$ | 99.9 | 9.87 | 80.5 | 119.2 |
| $p_{1c}$ | 0.012 | 0.0012 | 0.010 | 0.014 |
| $p_2$ | 0.001 | 0.0015 | −0.002 | 0.003 |
| $q_{1c}$ | −0.145 | 0.015 | −0.176 | −0.114 |
| $q_2$ | 0.342 | 0.0683 | 0.208 | 0.475 |
| $\delta$ | 0.183 | 0.0186 | 0.146 | 0.219 |
| $\gamma$ | 0.343 | 0.0730 | 0.200 | 0.487 |

$R^2 = 0.9915$

- nuclear $q_{1c} = -0.145$,
- renewables $q_2 - \gamma = -0.002$