Nonlinear models for new product growth

New product life cycle: phases

1. Introduction
2. Growth
3. Maturity
4. Decline

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What are the variables influencing a product's life cycle?
Marketing strategies play an essential role ...
but the success of a new product ultimately depends on consumers accepting them.

Diffusion of innovations

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- Typical distinction: radical vs incremental innovations.
- Radical innovations could be hindered from barriers and social inertia.

New product growth models

General aim:

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- Fourt and Woodlock model (1960)
- Mansfield model (1961)
- Bass model (1969)
- Generalized Bass model (1994)

Bass Model

The Bass Model is defined by a first order differential equation

$$z'(t) = \left(p + q\frac{z(t)}{m}\right)(m - z(t))$$

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word-of-mouth

Bass Model

If we pose $\frac{z(t)}{m} = y(t)$ the model becomes

$$y'(t) = (p + qy(t))(1 - y(t))$$

Bass Model: solution

The Bass Model has a closed-form solution

$$y(t) = F(t; p, q) = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \qquad t > 0.$$

or, by posing $z = ym$

$$z(t) = m\, F(t; p, q) = m\, \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \qquad t > 0.$$

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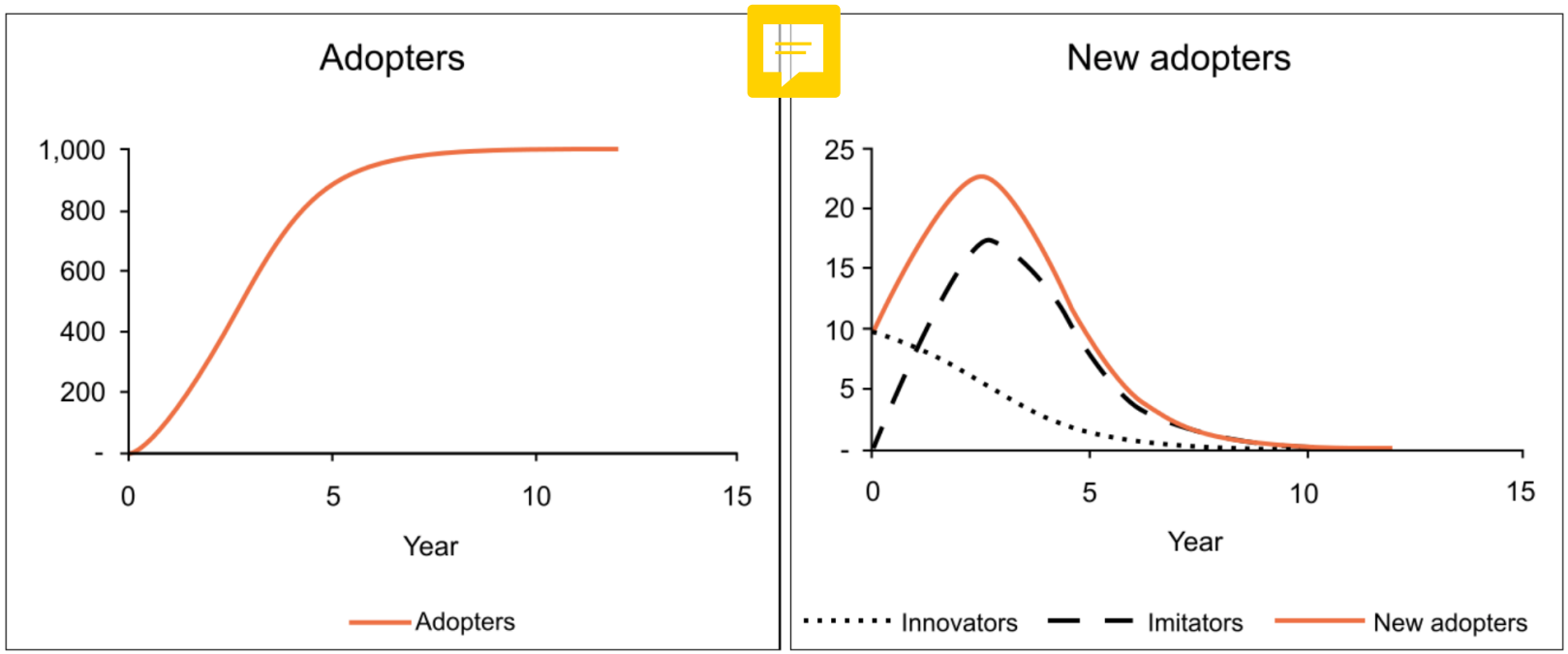
Cumulative sales $z(t)$ 'depend' on parameters p and q.
The market potential m is a scale parameter and is assumed constant.

Bass Model

BM: cumulative and rate data



Bass Model: estimation

The Bass Model is a nonlinear model

$$Z(t) = f(\beta, t) + \varepsilon(t)$$

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where $Z(t)$ is the dependent variable, $f(\beta, t)$ is the deterministic term, function of $\beta \in R^k$ and of time t.
The second term, $\varepsilon(t)$, is the error term, for which usual assumptions hold, namely $M(\varepsilon(t)) = 0$,$Var(\varepsilon(t)) = \sigma^2$, $Cov(\varepsilon(t), \varepsilon(t')) = 0, t \neq t'$.

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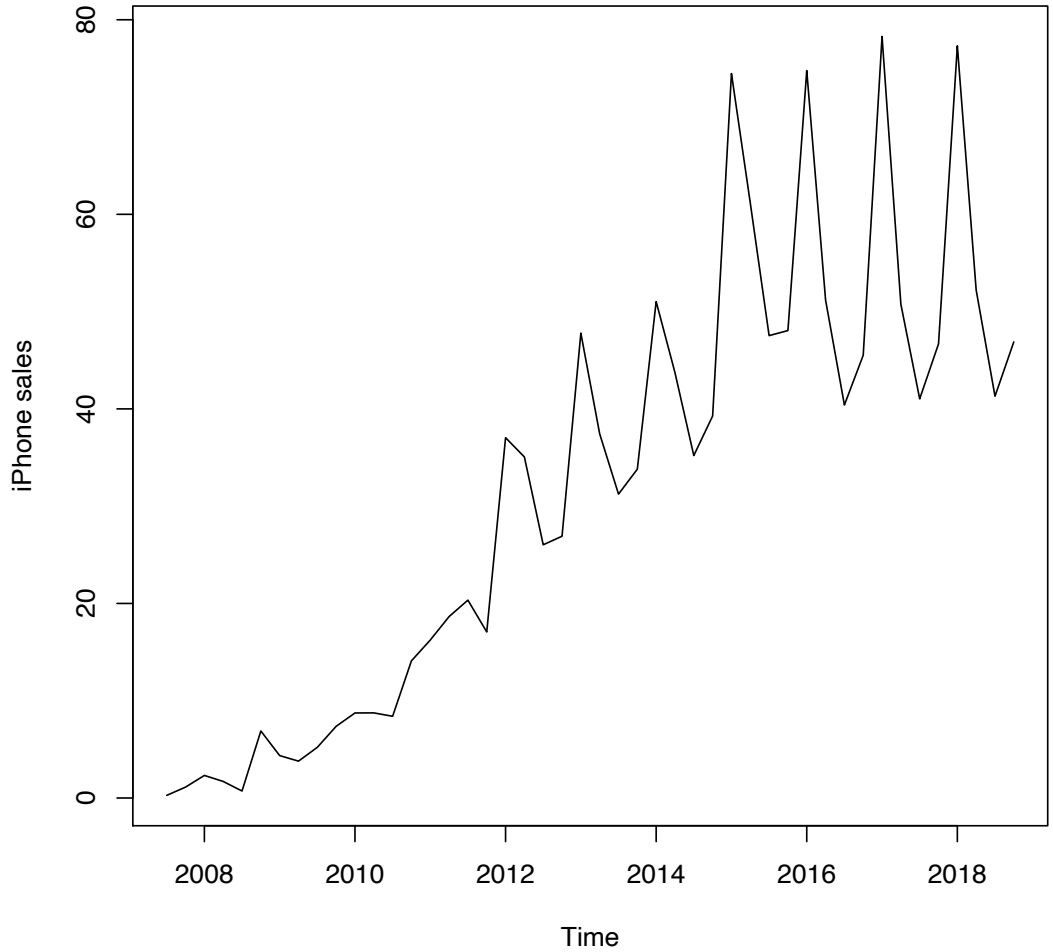
- Typical starting values for p and q are 0.01 and 0.1.
- Estimating m is the most difficult task.
- Parameter estimates are very sensitive to the number of available data.
- Reliable estimates are obtained after the maximum peak, but ... *"By the time sufficient observations have been developed for realiable estimation, it is too late to use the estimates for forecasting purposes"* (Mahajan, Muller, Bass, 1990).

Example: Apple iPhone life cycle



Cumulative quarterly sales data from 2007 to 2019 (source: Apple Inc.)

Example: Apple iPhone life cycle

Bass Model for iPhone life cycle: parameter estimates and 95% CIs

```
Estimate
        Estimate    Std.Error        Lower        Upper  p-value
m : 1.823747e+03 3.412507e+01 1.756863e+03 1.890631e+03 5.84e-41
p : 1.412817e-03 5.410927e-05 1.306765e-03 1.518869e-03 5.26e-28
q : 1.258732e-01 2.675751e-03 1.206289e-01 1.311176e-01 1.29e-38

R-squared
0.9995498
```

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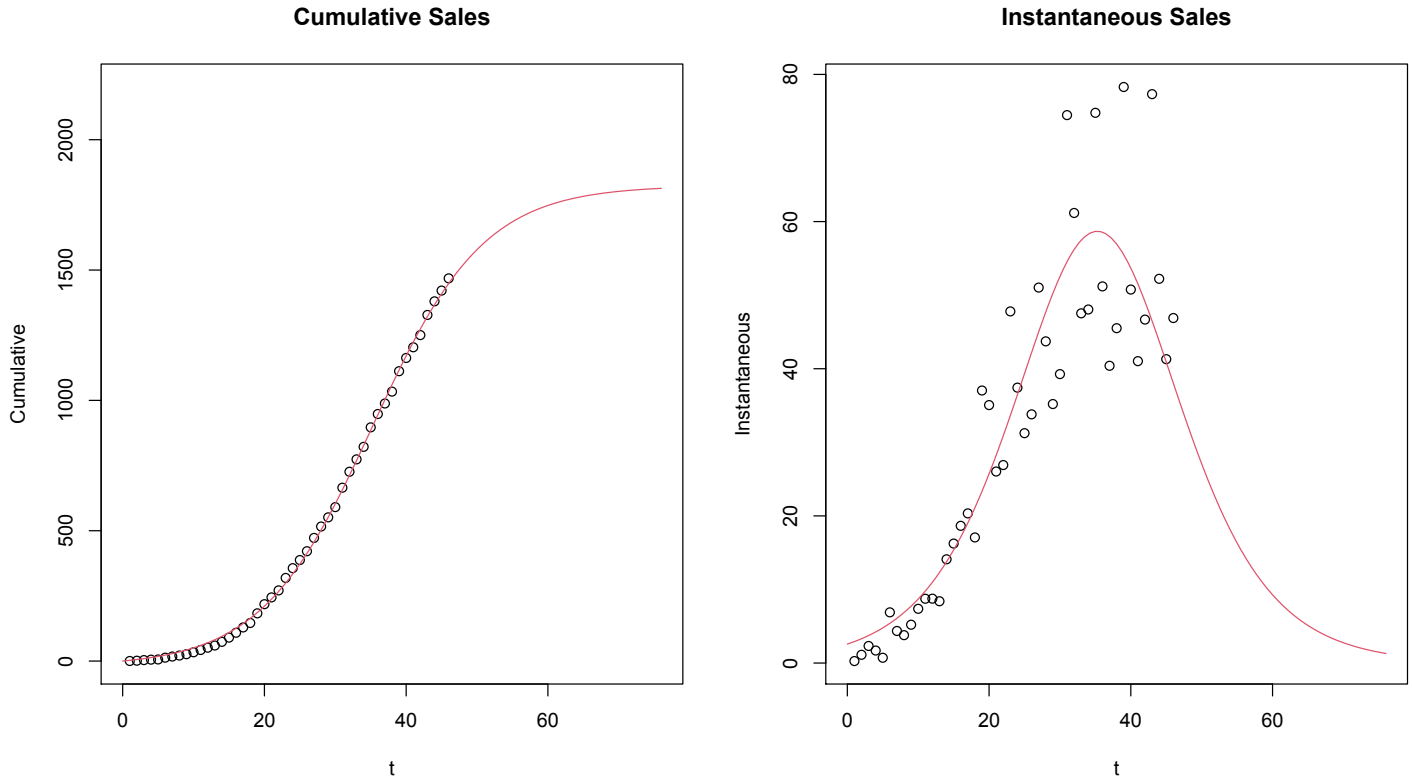

Cumulative and instantaneous sales data and forecasts with BM

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- Only needs aggregate sales data.
- Easy to interpret.

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- The Bass Model does not account for marketing mix strategies.
- It is a model for products with a limited life cycle: needs a hypothesis.