Ontology Languages
Description Logics

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Examples in this section are based on:

- D. Calvanese and D. Lembo (tutorial on DL @ISCW’07)
- F. Baader et al. The Description Logic Handbook

DESCRIPTION LOGICS

HOW TO MODEL KNOWLEDGE AND ASSERT INSTANCES

Logics-Based Knowledge Representation

First-Order Logic (FOL)

- Suitable for knowledge representation
  - Classes as unary predicates
  - Properties / relationships as binary predicates
  - Constraints as logical formulas using those predicates
- Undecidability
  - In the general case, there is no algorithm that determines if a FOL formula implies another

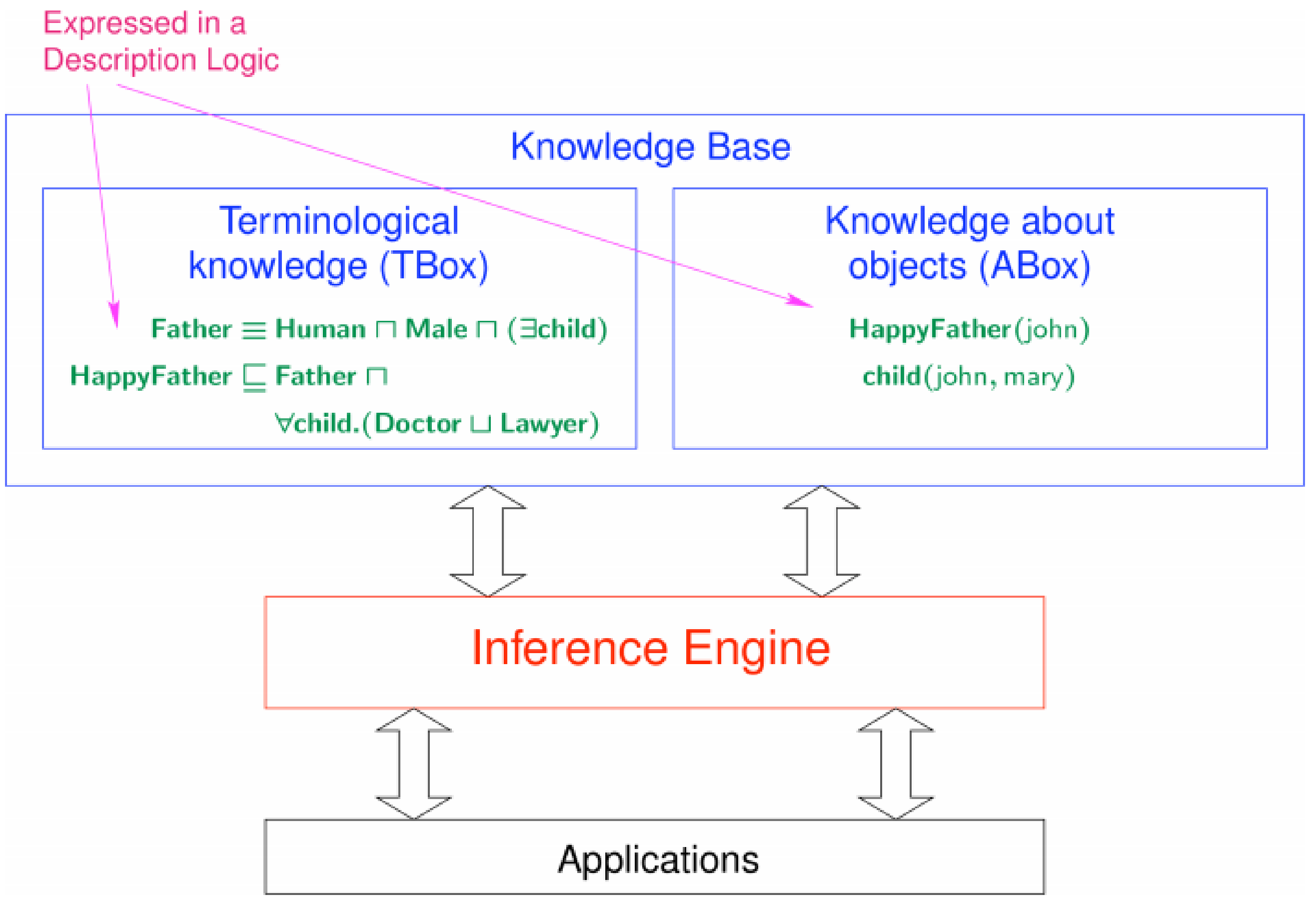
Decidable Fragments of FOL

- Description Logics (binary predicates bounded number of variables)
- Datalog (Horn-clauses)

Decidable Subsets of FOL

	Datalog	Description Logics
Focus	Instances	Knowledge
Approach	Centralized	Decentralized
Reasoning	Closed-world assumption	Open-world assumption
Unique name	Unique name assumption	Non-unique name assumption

Description Logic (DL) Knowledge Base



Description Logics and Ontologies

Description Logics are used to assert **knowledge** and **instances**

- The knowledge is asserted in the TBOX (DL terminology)
- The instances are asserted in the ABOX (DL assertions)

A DL TBOX and ABOX is a decidable subset of FOL. DL defines accordingly reasoning services for DL KBs

We say a *knowledge base* is an ontology if:

- It defines the ontology terminology (TBOX)
- The asserted instances (ABOX) are complaint with the terminology (i.e., TBOX)
- It provides **sound** reasoning services

Thus:

- Any Description Logic KB is always an ontology
- A RDFS KB is an ontology if:
  - You define a TBOX
  - The RDFS ABOX is compliant with the TBOX
  - You use sound inference rules (e.g., those defined by the SPARQL community)

Description Logic: TBOX

A DL TBOX is characterized by a set of constructs for building **complex concepts and roles** from **atomic concepts and roles**:

- Concepts correspond to classes
- Roles correspond to relationships

Atomic concepts / roles:

- Must be explicitly defined by the user (e.g., the *person* concept or the *lectures* role)

Complex concepts / roles:

- They are derived from atomic concepts or roles (e.g., a *lecturer* is a *person* who *lectures*)
- They must be derived using the pre-defined **concept and role constructs** provided by the description logic

It is called TBOX because it defines the **terminology** (of the domain)

- It is equivalent to the metadata / schema layer we have used for RDFS

Description Logic: TBOX

A DL TBOX is characterized by a set of constructs for building **complex concepts and roles** from **atomic concepts and roles**:

- Concepts correspond to classes
- Roles correspond to relationships

A DL TBOX formal semantics are given in terms of interpretations:

An interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of:

- a nonempty set $\Delta^{\mathcal{I}}$, the domain of $\mathcal{I}$
- an interpretation function $\cdot^{\mathcal{I}}$, which maps
  - each individual a to an element $a^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$
  - each atomic concept A to a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$
  - each atomic role P to a subset $P^{\mathcal{I}}$ of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

Concept Constructs

Atomic concepts and roles are defined explicitly by the user!

Construct	Syntax	Example	Semantics
atomic concept	A	Doctor	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
atomic role	P	hasChild	$P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
atomic negation	$\neg A$	¬Doctor	$\Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$
conjunction	$C \sqcap D$	Hum ⊓ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
(unqual.) exist. res.	$\exists R$	∃hasChild	$\{ a \mid \exists b. (a,b) \in R^{\mathcal{I}} \}$
value restriction	$\forall R.C$	∀hasChild.Male	$\{a \mid \forall b. (a,b) \in R^{\mathcal{I}} \to b \in C^{\mathcal{I}}\}$
bottom	$\bot$		$\emptyset$

(C, D denote arbitrary concepts and R an arbitrary role)

The above constructs form the basic language $\mathcal{AL}$ of the family of $\mathcal{AL}$ languages.

Additional Concept and Role Constructs

Construct	$\mathcal{AL}\cdot$	Syntax	Semantics
disjunction	$\mathcal{U}$	$C \sqcup D$	$C^\mathcal{I} \cup D^\mathcal{I}$
top		$\top$	$\Delta^\mathcal{I}$
qual. exist. res.	$\mathcal{E}$	$\exists R.C$	$\{\ a \mid \exists b.\ (a,b) \in R^\mathcal{I} \wedge b \in C^\mathcal{I}\ \}$
(full) negation	$\mathcal{C}$	$\neg C$	$\Delta^\mathcal{I} \setminus C^\mathcal{I}$
number	$\mathcal{N}$	$(\geq k\,R)$	$\{\ a \mid \#\{b \mid (a,b) \in R^\mathcal{I}\} \geq k\ \}$
restrictions		$(\leq k\,R)$	$\{\ a \mid \#\{b \mid (a,b) \in R^\mathcal{I}\} \leq k\ \}$
qual. number	$\mathcal{Q}$	$(\geq k\,R.C)$	$\{\ a \mid \#\{b \mid (a,b) \in R^\mathcal{I} \wedge b \in C^\mathcal{I}\} \geq k\ \}$
restrictions		$(\leq k\,R.C)$	$\{\ a \mid \#\{b \mid (a,b) \in R^\mathcal{I} \wedge b \in C^\mathcal{I}\} \leq k\ \}$
inverse role	$\mathcal{I}$	R^-	$\{\ (a,b) \mid (b,a) \in R^\mathcal{I}\ \}$
role closure	reg	$\mathcal{R}^*$	$(R^\mathcal{I})^*$

Understanding DL Axioms

What is the meaning of these axioms? Write the **interpretation** corresponding to each axiom

$$\forall \mathsf{hasChild}.(\mathsf{Doctor} \sqcup \mathsf{Lawyer})$$

$$\exists \mathsf{hasChild}.\mathsf{Doctor}$$

$$\neg(\mathsf{Doctor} \sqcup \mathsf{Lawyer})$$

$$(\geq 2\, \mathsf{hasChild}) \sqcap (\leq 1\, \mathsf{sibling})$$

$$(\geq 2\, \mathsf{hasChild}.\, \mathsf{Doctor})$$

$$\forall \mathsf{hasChild}^{-}.\mathsf{Doctor}$$

$$\exists \mathsf{hasChild}^{*}.\mathsf{Doctor}$$

TBOX Definition

A DL TBOX only includes terminological axioms of the following form

- Inclusion (*subsumption*)

  $C_1 \sqsubseteq C_2$ is satisfied by $\mathcal{I}$ if $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$

  $R_1 \sqsubseteq R_2$ is satisfied by $\mathcal{I}$ if $R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$

  Example: $\mathrm{PhDStudent} \sqsubseteq \mathrm{Student} \sqcap \mathrm{Researcher}$

- Equivalence $C_1 \sqsubseteq C_2,\ C_2 \sqsubseteq C_1$

  Example: $\mathrm{PhDStudent} \equiv \mathrm{Student} \sqcap \mathrm{Researcher}$

Description Logics: ABOX

Defines instances in terms of the terminological axioms defined in the TBOX

- Concept assertions
  - Student(Pere)
- Role assertions
  - Teaches(Oscar, Pere)

We **cannot** assert instances for a concept not defined previously in the TBOX

We can assert instances of both atomic and complex concepts / roles

It is called ABOX because it defines **assertions** on the TBOX concepts and roles

- It is equivalent to the instance layer we have used for RDFS

Example of DL Knowledge Base

TBox assertions:

- Inclusion assertions on concepts:

$$
\begin{aligned}
\mathsf{Father} &\equiv \mathsf{Human} \sqcap \mathsf{Male} \sqcap \exists \mathsf{hasChild} \\
\mathsf{HappyFather} &\sqsubseteq \mathsf{Father} \sqcap \forall \mathsf{hasChild}.(\mathsf{Doctor} \sqcup \mathsf{Lawyer} \sqcup \mathsf{HappyPerson}) \\
\mathsf{HappyAnc} &\sqsubseteq \forall \mathsf{descendant}.\mathsf{HappyFather} \\
\mathsf{Teacher} &\sqsubseteq \neg \mathsf{Doctor} \sqcap \neg \mathsf{Lawyer}
\end{aligned}
$$

- Inclusion assertions on roles:

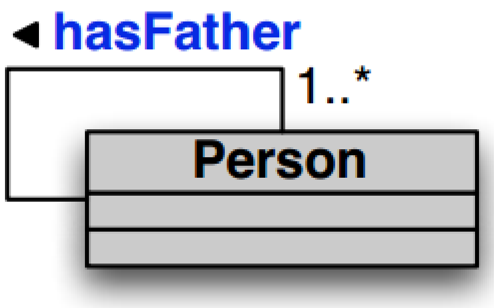
$$
\mathsf{hasChild} \sqsubseteq \mathsf{descendant} \qquad\qquad \mathsf{hasFather} \sqsubseteq \mathsf{hasChild}^{-}
$$

ABox membership assertions:

- $\mathsf{Teacher}(\mathtt{mary}), \quad \mathsf{hasFather}(\mathtt{mary}, \mathtt{john}), \quad \mathsf{HappyAnc}(\mathtt{john})$

Exercise

Represent as concept expressions the following UML diagram

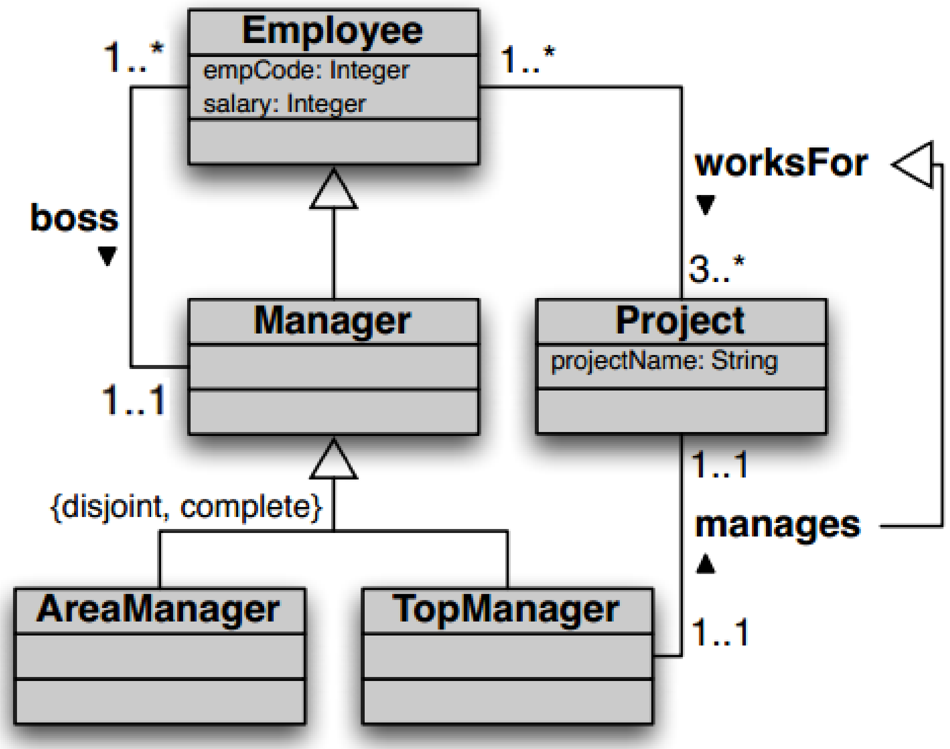


TBox $\mathcal{T}$:

$$\begin{aligned} \exists \mathsf{hasFather} &\sqsubseteq \mathsf{Person} \\ \exists \mathsf{hasFather}^{-} &\sqsubseteq \mathsf{Person} \\ \mathsf{Person} &\sqsubseteq \exists \mathsf{hasFather} \end{aligned}$$

Exercise II

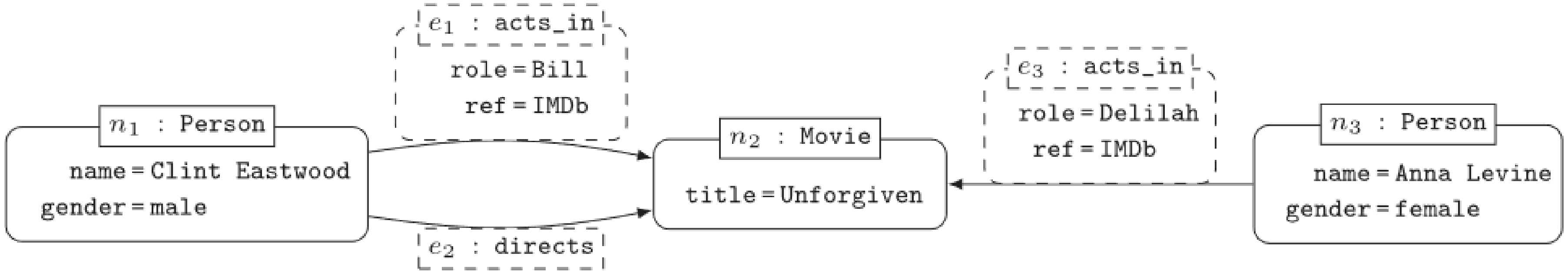
Represent as concept expressions the following UML diagram



$$
\begin{aligned}
\mathsf{Manager} &\sqsubseteq \mathsf{Employee}\\
\mathsf{AreaManager} &\sqsubseteq \mathsf{Manager}\\
\mathsf{TopManager} &\sqsubseteq \mathsf{Manager}\\
\mathsf{Manager} &\sqsubseteq \mathsf{AreaManager} \sqcup\\
&\ \mathsf{TopManager}\\
\mathsf{AreaManager} &\sqsubseteq \neg\mathsf{TopManager}\\
\mathsf{Employee} &\sqsubseteq \exists\mathsf{salary}\\
\exists\mathsf{salary}^{-} &\sqsubseteq \mathsf{Integer}\\
\exists\mathsf{worksFor} &\sqsubseteq \mathsf{Employee}\\
\exists\mathsf{worksFor}^{-} &\sqsubseteq \mathsf{Project}\\
\mathsf{Project} &\sqsubseteq \exists\mathsf{worksFor}^{-}\\
\mathsf{Employee} &\sqsubseteq\ \geq 3\ \mathsf{worksFor}\\
&\ldots
\end{aligned}
$$

Exercise III

Create a DL KB capturing as much constraints as possible from the following graph:



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DESCRIPTION LOGICS

REASONING

Model of a DL Ontology

Model of a DL knowledge base

An interpretation $\mathcal{I}$ is a model of $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ if it satisfies all assertions in $\mathcal{T}$ and all assertions in $\mathcal{A}$.

$\mathcal{O}$ is said to be satisfiable if it admits a model.

The fundamental reasoning service from which all other ones can be easily derived is ...

Logical implication

$\mathcal{O}$ logically implies and assertion α, written $\mathcal{O} \models \alpha$, if α is satisfied by all models of $\mathcal{O}$.

TBOX Reasoning

- Concept Satisfiability: C is satisfiable wrt $\mathcal{T}$, if there is a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}}$ is not empty, i.e., $\mathcal{T} \not\models C \equiv \bot$.
- Subsumption: C_1 is subsumed by C_2 wrt $\mathcal{T}$, if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \sqsubseteq C_2$.
- Equivalence: C_1 and C_2 are equivalent wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} = C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \equiv C_2$.
- Disjointness: C_1 and C_2 are disjoint wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}} = \emptyset$, i.e., $\mathcal{T} \models C_1 \sqcap C_2 \equiv \bot$.
- Functionality implication: A functionality assertion $(\textbf{funct}\ R)$ is logically implied by $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$, we have that $(o, o_1) \in R^{\mathcal{I}}$ and $(o, o_2) \in R^{\mathcal{I}}$ implies $o_1 = o_2$, i.e., $\mathcal{T} \models (\textbf{funct}\ R)$

Reasoning Complexity

Complexity of concept satisfiability: [DLNN97]

$\mathcal{AL}$, $\mathcal{ALN}$	PTIME
$\mathcal{ALU}$, $\mathcal{ALUN}$	NP-complete
$\mathcal{ALE}$	coNP-complete
$\mathcal{ALC}$, $\mathcal{ALCN}$, $\mathcal{ALCI}$, $\mathcal{ALCQI}$	PSPACE-complete

Observations:

- Two sources of complexity:
  - union ($\mathcal{U}$) of type NP,
  - existential quantification ($\mathcal{E}$) of type coNP.

  When they are combined, the complexity jumps to PSPACE.
- Number restrictions ($\mathcal{N}$) do not add to the complexity.

Ontology Reasoning

- Ontology Satisfiability: Verify whether an ontology $\mathcal{O}$ is satisfiable, i.e., whether $\mathcal{O}$ admits at least one model.
- Concept Instance Checking: Verify whether an individual c is an instance of a concept C in $\mathcal{O}$, i.e., whether $\mathcal{O} \models C(c)$.
- Role Instance Checking: Verify whether a pair (c_1, c_2) of individuals is an instance of a role R in $\mathcal{O}$, i.e., whether $\mathcal{O} \models R(c_1, c_2)$.
- Query Answering:

The certain answers to $q(\vec{x})$ over $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, denoted $cert(q, \mathcal{O})$, . . .

are the tuples $\vec{c}$ of constants of $\mathcal{A}$ such that $\vec{c} \in q^{\mathcal{I}}$, for every model $\mathcal{I}$ of $\mathcal{O}$.

Example

TBOX:

$$Researcher \sqsubseteq \neg Professor$$
$$Researcher \sqsubseteq \neg Lecturer$$
$$\exists TeachesTo^{-} \sqsubseteq Student$$

$$Student \sqcap \neg Undergrad \sqsubseteq GraduateStudent$$
$$\exists TeachesTo.Undergrad \sqsubseteq Professor \sqcup Lecturer$$

TBOX Inferences: $Researcher \sqsubseteq \forall TeachesTo.GraduateStudent$ ⇐ concept subsumption

ABOX:

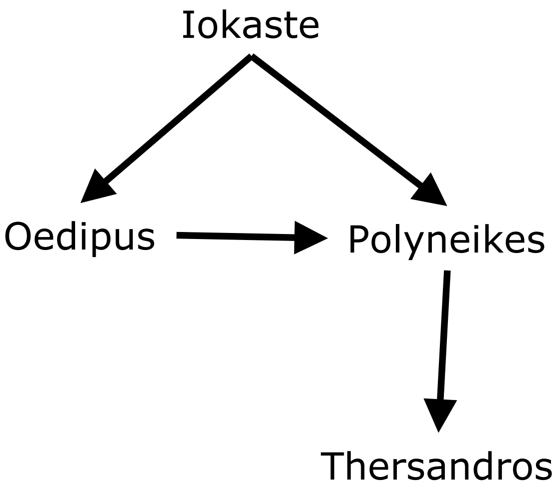
$$TeachesTo(dupond, pierre)$$
$$\neg GraduateStudent(pierre)$$
$$\neg Professor(dupond)$$

Ontology Inferences: $Lecturer(dupond)$ ⇐ concept instance checking

Open-World Assumption

Something evaluates false **only if** it contradicts other information in the ontology

hasSon(Iokaste,Oedipus)
hasSon(Iokaste,Polyneikes)
hasSon(Oedipus,Polyneikes)
hasSon(Polyneikes,Thersandros)
patricide(Oedipus)
¬patricide(Thersandros)



Query≡∃hasSon.(patricide ⊓ ∃hasSon.¬patricide)
ABox ⊨ Query(Iokaste)?

Modeling with Description Logics

It is hard to build good ontologies with DL

- The names of the classes are irrelevant
- Classes are overlapping by default
- Domain and range definitions are axioms, not constraints
- Open world assumption
  - Anything might be true unless explicit asserted knowledge contradicts it (negation)
- Non-unique name assumption
  - Although families such as the DL-Lite family assume the unique name assumption

In this course, we aim at modeling usual data models and we will solely focus on modeling UML-like TBOXes (like the examples we have seen during this lecture)

Summary

Description Logics

- TBOX
  - Constructs
  - Formal Semantics
- ABOX
- Reasoning
  - Open-World Assumption