

# Property Graphs

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# Graph Operations

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# Basic graph operations

- Adjacency
- Path queries
  - Path existence (reachability)
  - Regular path queries (RPQs)
- Graph patterns
  - Basic Graph Patterns (BGPs)
  - Navigational Graph Patterns (NGPs)
- *Graph metrics*

# Adjacency

Formal definition:

$$\begin{aligned} \text{Adjacency}(n) &= \bar{N} \\ n_i \in \bar{N} &\iff \exists e_1 \mid \rho(e_1) = (n_i, n) \vee \rho(e_1) = (n, n_i) \end{aligned}$$

Computational cost: linear cost on the number of edges to visit

Examples:

- Find all friends of a person
- Airports with a direct connection
- Movies watched by a person
- Products bought by a customer
- ...

# Path Queries

Based on the general concept of reachability in graph theory

- A node  $n1$  can reach a node  $n2$  ( $n2$  is reachable from  $n1$ ) if there exists a sequence of adjacent nodes which starts with  $n1$  and ends with  $n2$

Path query:

$$x \xrightarrow{\alpha} y$$

- $x, y$  are nodes
- $\alpha$  is a regular expression over  $Lab$  that specifies conditions on the path
  - $\alpha = *$  denotes path existence without any further constraints (reachability)

# Regular Path Queries (RPQs)

Path query:

$$P = x \xrightarrow{\alpha} y$$

- $x, y$  are nodes
- $\alpha$  is a regular expression over  $Lab$  that specifies conditions on the path

Regular expression operators

- \* Kleene star
- + Kleene plus
- Concatenation
- | Union
- Inverse
- ... and combinations of them

Examples:

$$P := x \xrightarrow{\text{knows}^+} y$$

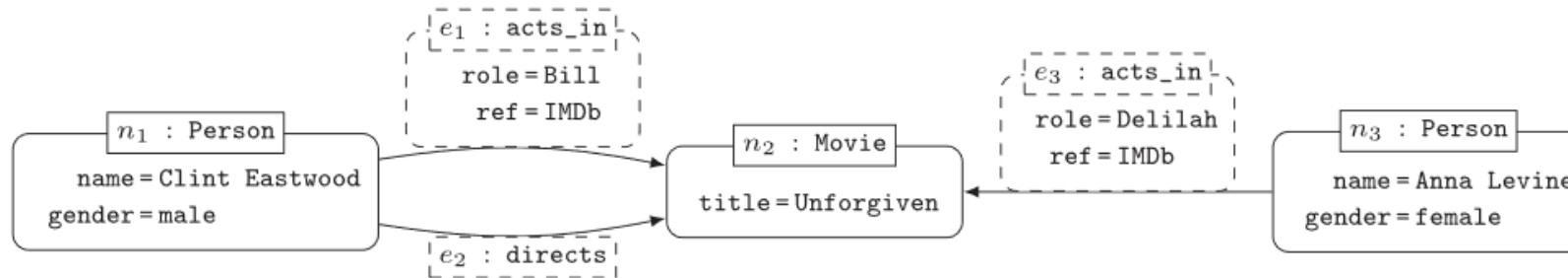
$$P' := x \xrightarrow{\text{knows}^+ \cdot \text{likes}} y$$

$$P'' := x \xrightarrow{\text{knows}^+ \cdot (\text{likes} \mid \text{dislikes})} y$$

# Activity

Assume a graph containing relationships and nodes like the ones shown below

- Define a RPQ including expressions from the previous slide to find *all co-actors of all actors*
  - Which are the solutions you will get?
- Define a RPQ to retrieve *all actors you can reach by following the co-acting path at least once*
- Define a RPQ to find *all persons that participate in the same movie*



# Pattern Matching

Based on the subgraph isomorphism problem in graph theory

- Input: property graph **G**, and a graph pattern **P**
- Output: all sub-graphs of **G** that are isomorphic to **P**

Computational cost: hard to compute, in general, NP-complete

Examples:

- Group of cities all of them directly connected by flights
- People who have ordered the same item
- ...



# Pattern Matching

Based on *basic graph patterns* (BGPs)

- Equivalent to conjunctive queries

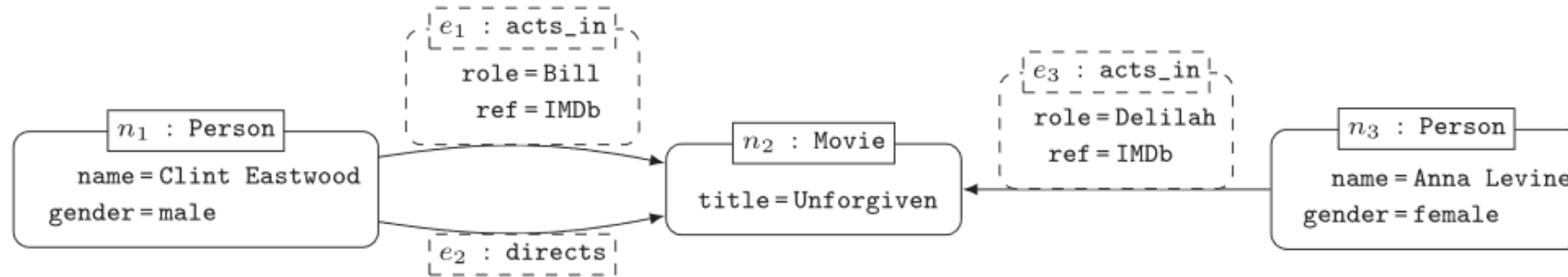
A *BGP* for querying property graphs is a property graph where variables can appear in place of any constant (ids/labels/properties)

A **match** for a *BGP* is a mapping from variables to constants such that when the mapping is applied to the *BGP*, the result is *contained* within the original graph

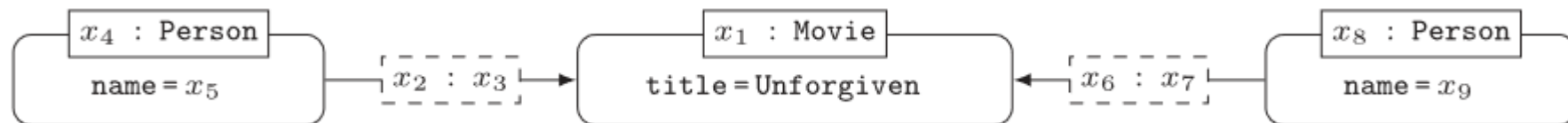
The **results** for a *BGP* are then **all mappings** from variables in the query to constants that comprise a match

# Example of Graph Pattern

Graph:



BGP:



# Evaluating Graph Patterns

This is completely different from SQL and Relational Algebra!

Evaluating a *bgp*  $Q$  against a graph database  $G$  corresponds to listing **all** possible matches of  $Q$  with respect to  $G$

Formally:

*Definition 3.5 (Match).* Given an edge-labelled graph  $G = (V, E)$  and a *bgp*  $Q = (V', E')$ , a *match*  $h$  of  $Q$  in  $G$  is a mapping from  $Const \cup Var$  to  $Const$  such that:

- (1) for each constant  $a \in Const$ , it is the case that  $h(a) = a$ ; that is, the mapping maps constants to themselves; and
- (2) for each edge  $(b, l, c) \in E'$ , it holds that  $(h(b), h(l), h(c)) \in E$ ; this condition imposes that (a) each edge of  $Q$  is mapped to an edge of  $G$ , and (b) the structure of  $Q$  is preserved in its image under  $h$  in  $G$  (that is, when  $h$  is applied to all the terms in  $Q$ , the result is a sub-graph of  $G$ ).

*Extracted from: R. Angles et al. Foundations of Modern Query Languages for Graph Databases*

# Semantics of a Match

## Homomorphism-based semantics:

- The previous definition corresponds to a homomorphism from  $\mathbf{Q}$  to  $\mathbf{G}$ 
  - Multiple variables in  $\mathbf{Q}$  can map to the same term in  $\mathbf{G}$  (within a match)

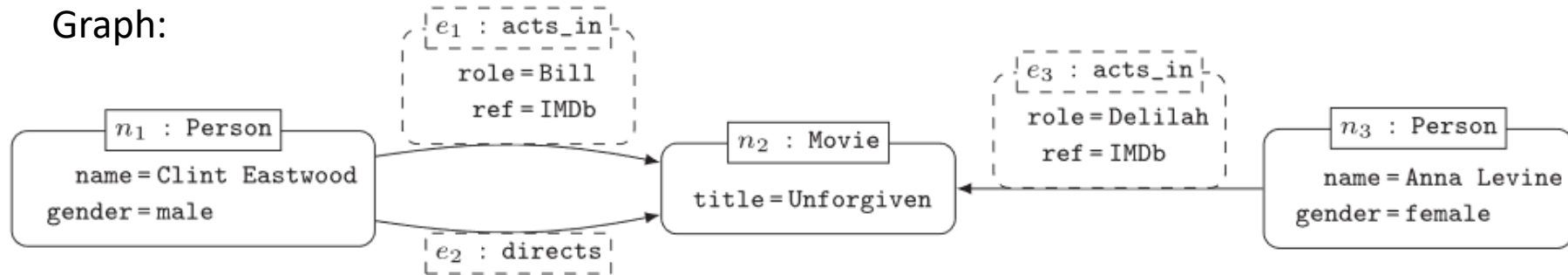
## Isomorphism-based semantics: adds constraints to the mapping function

- Strict isomorphism semantics (no-repeated-anything)
  - Each variable in  $\mathbf{Q}$  maps to a different term in  $\mathbf{G}$  (within a match)
- No repeated-node semantics
  - Each variable representing a node in  $\mathbf{Q}$  maps to a different node in  $\mathbf{G}$  (within a match)
- No repeated-edge semantics:
  - Each variable representing an edge in  $\mathbf{Q}$  maps to a different edge in  $\mathbf{G}$  (within a match)

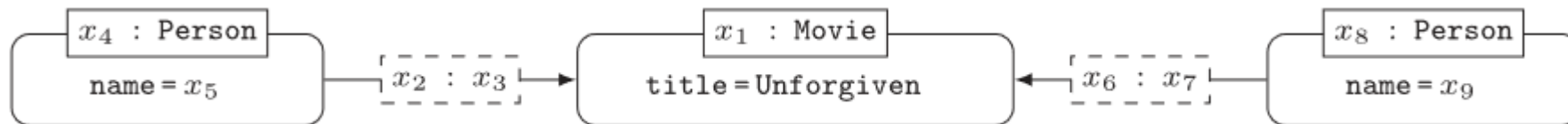
# Activity

*Objective: Understand the differences between isomorphism-based and homomorphism-based semantics in pattern matching*

- Given the following graph, bgp and potential results...



BGP:



# Activity

*Objective: Understand the differences between isomorphism-based and homomorphism-based semantics in pattern matching*

- *Given the following graph, bgp and potential results...*

Results:

	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$
1	$n_2$	$e_2$	directs	$n_1$	Clint Eastwood	$e_3$	acts_in	$n_3$	Anna Levine
2	$n_2$	$e_3$	acts_in	$n_3$	Anna Levine	$e_2$	directs	$n_1$	Clint Eastwood
3	$n_2$	$e_1$	acts_in	$n_1$	Clint Eastwood	$e_3$	acts_in	$n_3$	Anna Levine
4	$n_2$	$e_3$	acts_in	$n_3$	Anna Levine	$e_1$	acts_in	$n_1$	Clint Eastwood
5	$n_2$	$e_2$	directs	$n_1$	Clint Eastwood	$e_1$	acts_in	$n_1$	Clint Eastwood
6	$n_2$	$e_1$	acts_in	$n_1$	Clint Eastwood	$e_2$	directs	$n_1$	Clint Eastwood
7	$n_2$	$e_1$	acts_in	$n_1$	Clint Eastwood	$e_1$	acts_in	$n_1$	Clint Eastwood
8	$n_2$	$e_2$	directs	$n_1$	Clint Eastwood	$e_2$	directs	$n_1$	Clint Eastwood
9	$n_2$	$e_3$	acts_in	$n_1$	Anna Levine	$e_3$	acts_in	$n_1$	Anna Levine

Which results would be obtained when applying **isomorphism-based** semantics? And **homomorphism-based** semantics?

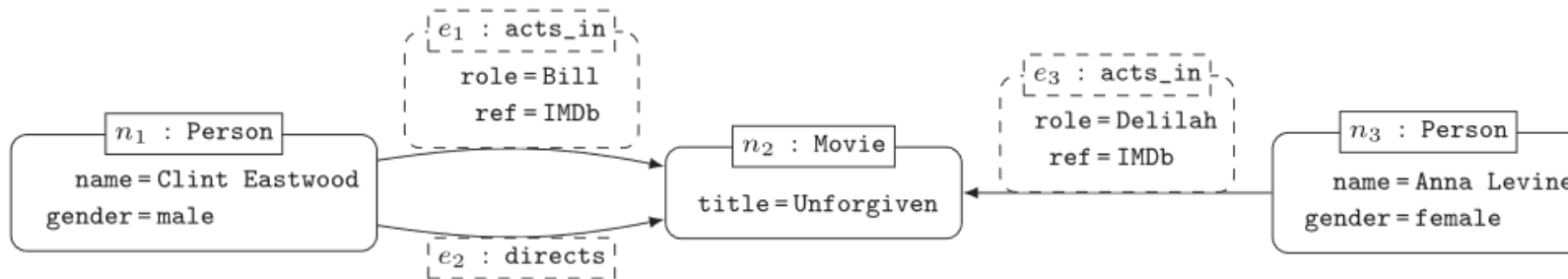
- For isomorphism, **distinguish the three isomorphism semantics** presented

# Activity

*Objective: Understand the relationship between RPQs and BGPs*

Assume a graph containing relationships and nodes like the ones shown below

- Define a BGP to find *all co-actors of all actors*
- Define a BGP to retrieve *all actors you can reach by following the co-acting path at least once*
- Define a BGP to find *all persons that participate in the same movie*

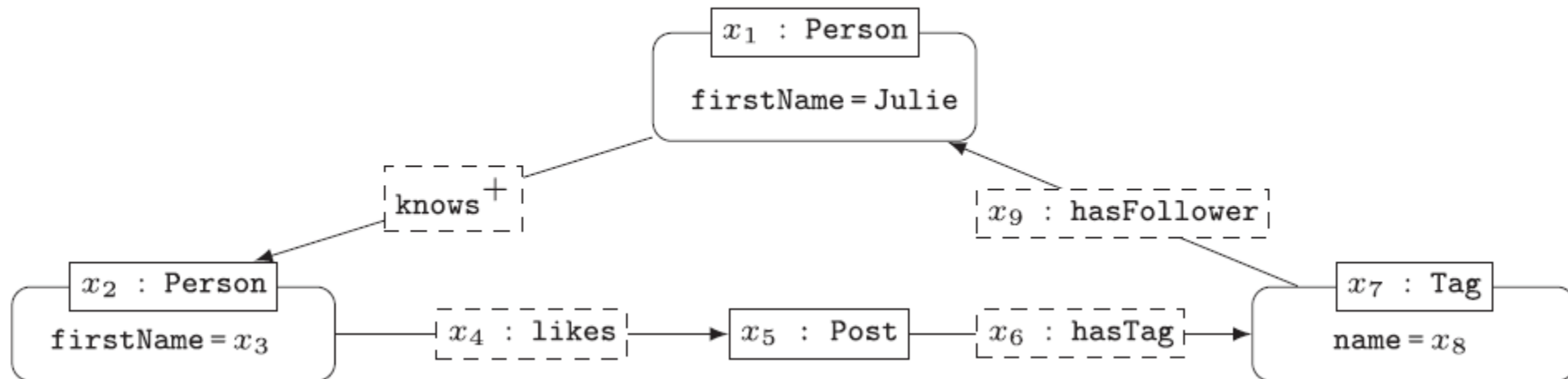


# Navigational Graph Patterns (NGPs)

NGPs are a combination of Pattern Matching and Path Queries

- BGPs where edge labels can be RPQs

Example:





# Graph Metrics

Take into account the graph topology only

They can be defined as combinations of adjacency, reachability, pattern matching

- Given their relevance, they are typically provided as built-in functions

## Examples:

- the min / max degree in the graph
- the graph diameter
- the graph density / sparsity
- betweenness of a node
- the pageRank of a node
- ...

# Summary

The basic operations on graphs are:

- Adjacency
- Path queries
- Graph patterns

The result of a Pattern Matching query depends on the semantics assumed:

- Homomorphism
- Strict Isomorphism
- No-repeated-node Isomorphism
- No-repeated-edge Isomorphism

Thanks! *Any* Question?

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