

## PROBLEM - SET 1

**Problem 1.** Consider the random experiment of rolling twice a balanced die with six faces.

- (a) Describe the probability space for this random experiment.
- (b) Compute the probability the number 6 appears *exactly* once in the outcomes.
- (c) Compute the probability the number 6 appears *at least* once in the outcomes *knowing* that the total score is 9.

**Problem 2.** A deck of 52 cards is accurately shuffled.

- (a) Describe the probability space for this random experiment.
- (b) What is the probability that the ace of spades is found in its original (before shuffling) position?
- (c) What is the probability that the four aces occupy their four original positions, but not necessarily in the same order?
- (d) What is the probability that the ace of spades and the ace of clubs are found in neighboring positions?

**Problem 3.** An urn contains 10 red balls and 20 black balls. We make five successive draws *without replacement* (drawn balls are *not* re-inserted in the urn).

- (a) Describe the probability space for this random experiment.
- (b) Compute the probability of drawing *at least* a black ball.
- (c) Knowing that at least one red ball was drawn, compute the probability of drawing *at least* a black ball.

**Problem 4.** Consider a set of  $n$  individuals, identified with the numbers  $\{1, 2, \dots, n\}$ . To each individual  $i$  we assign a random binary label  $\sigma_i \in \{0, 1\}$ , in such a way that all assignments are equally likely.

- (a) Describe the probability space for this random experiment.
- (b) What is the probability that exactly  $k$  individuals have label 1 ( $0 \leq k \leq n$ )? For which values of  $k$  this probability is maximized?

**Problem 5.** Consider again a set of  $n$  individuals. To each *unordered pair*  $\{i, j\}$ , with  $i \neq j$ , we assign a random binary label  $\sigma_{ij} \in \{0, 1\}$ , in such a way that all assignments are equally likely. If  $\sigma_{ij} = 1$  we say  $i$  and  $j$  are *friends*.

- (a) Describe the probability space for this random experiment.
- (b) What is the probability that individual 1 has exactly  $k$  friends?
- (c) What is the probability that 1 is friend of 2, 2 is friend of 3 but 1 is *not* friend of 3?

**Problem 6.** A set  $A$  of  $n$  elements is randomly partitioned into two *nonempty* subsets  $B$  and  $B^c$ . All such partitions are equally likely.

- (a) Describe the probability space for this random experiment.
- (b) Two distinct elements  $x$  and  $y$  of  $A$  are said to be connected if they belong to the same element of the partition (i.e. to either  $B$  or  $B^c$ ). Given  $x$  and  $y$ , what is the probability that they are connected?
- (c) An element  $x \in A$  is said to be isolated if it is not connected to any other element of  $A$ . Find the probability that there exists an isolated element.