DATA SCIENCE **Stochastic Methods**

June 26, 2025

Problem 1. [12]

Let X be a Binomial random variable of parameters $(2,0,4)$ and be Y be a Poisson random variable of parameter $X+1$, i.e. $Y|X=n \sim Poisson(n+1)$.

(i) Compute $\mathrm{P}[Y=k|X=n]$ for any $k \in \mathbb{N}, n=0,1,2$;

(ii) Compute $h(n) = \mathrm{E}[Y|X=n]$ for any n;

(iii) Compute $\mathrm{E}[\mathrm{E}[Y|X]]$;

(iv) Compute $\mathrm{E}[Y]$.

Problem 2. [12] Let $(X_n)_{n \geq 0}$ be a Markov chain on $\{1,2,3,4\}$ with transition probabilities given by

$$p_{i,j} = \frac{\max\{i,j\}}{12} \quad , \quad i \neq j$$

(i) Is the Markov chain irreducible?

(ii) Is the Markov chain aperiodic?

(iii) Compute $\mathrm{P}[X_2 = k|X_0 = 1]$ for any $k = 1,2,3,4$;

(iv) Determine the invariant distribution.

Problem 3. [12] Let X_1 and X_2 be two independent uniform $U(0,1)$ random variables. Define $Y = \min\{X_1, X_2\}$ and $Z = \max\{X_1, X_2\}$.

(i) Compute the density, the expectation and the variance of Y;

(ii) Compute the density, the expectation and the variance of Z;

(iii) Compute the joint distribution of (Y,Z), i.e. $\mathrm{P}[Y \leq y, Z \leq z]$ for any $y,z \in \mathbb{R}^2$.