DATA SCIENCE **Stochastic Methods**

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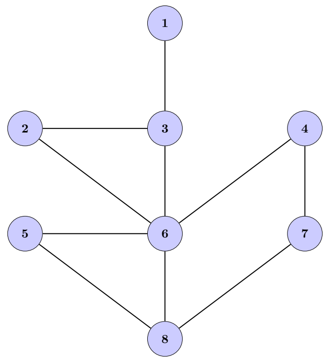
Problem 1. [9] We choose two points, X and Y, randomly, uniformly and independently on the segment [0,1].

(i) Compute $\mathrm{P}[X < Y]$;

(ii) Compute $\mathrm{P}[\max\{X,Y\} < 1/2]$;

(iii) Compute the expected length of the segment with endpoints X and Y.

Problem 2. [9] Define a simple Random Walk $\{X_n, n \geq 0\}$ on the undirected graph:



(i) Compute the probability of going from state 2 to state 8 in three steps.

(ii) Is the chain irreducible? aperiodic?

(iii) Find the invariant distribution.

(iv) Starting from state 1, what is the probability of visiting every state before visiting any state more than once?

Problem 3. [9] Let X be a Binomial random variable with parameters $(2,p)$, where $0 < p < 1$ and define $Y = (X+1)/3$. Assume that Z is a Geometric random variable with parameter Y, i.e. $Z|Y = k \sim Geo(k)$.

(i) Compute the support and the discrete density of Y;

(ii) Compute $h(k) = \mathrm{E}[Z|Y = k]$ for any k in the support of Y;

(iii) Compute $\mathrm{E}[Z]$.

Problem 4. [9] Let $(X_i)_{1 \leq i \leq n}$ be a family of i.i.d. $N(\mu, \sigma^2)$ r.v.'s.

(i) Compute the moment generating function of X_1;

(ii) Defined $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, determine an exponential decay for the "lower tail" of $\overline{Y}_n - \mu$.