

PROBLEM - SET 1

Problem 1. Consider the random experiment of rolling twice a balanced die with six faces.

- (a) Describe the probability space for this random experiment.
- (b) Compute the probability the number 6 appears *exactly* once in the outcomes.
- (c) Compute the probability the number 6 appears *at least* once in the outcomes *knowing* that the total score is 9.

Problem 2. A deck of 52 cards is accurately shuffled.

- (a) Describe the probability space for this random experiment.
- (b) What is the probability that the ace of spades is found in its original (before shuffling) position?
- (c) What is the probability that the four aces occupy their four original positions, but not necessarily in the same order?
- (d) What is the probability that the ace of spades and the ace of clubs are found in neighboring positions?

Problem 3. An urn contains 10 red balls and 20 black balls. We make five successive draws *without replacement* (drawn balls are *not* re-inserted in the urn).

- (a) Describe the probability space for this random experiment.
- (b) Compute the probability of drawing *at least* a black ball.
- (c) Knowing that at least one red ball was drawn, compute the probability of drawing *at least* a black ball.

Problem 4. Consider a set of n individuals, identified with the numbers $\{1, 2, \dots, n\}$. To each individual i we assign a random binary label $\sigma_i \in \{0, 1\}$, in such a way that all assignments are equally likely.

- (a) Describe the probability space for this random experiment.
- (b) What is the probability that exactly k individuals have label 1 ($0 \leq k \leq n$)? For which values of k this probability is maximized?

Problem 5. Consider again a set of n individuals. To each *unordered pair* $\{i, j\}$, with $i \neq j$, we assign a random binary label $\sigma_{ij} \in \{0, 1\}$, in such a way that all assignments are equally likely. If $\sigma_{ij} = 1$ we say i and j are *friends*.

- (a) Describe the probability space for this random experiment.
- (b) What is the probability that individual 1 has exactly k friends?
- (c) What is the probability that 1 is friend of 2, 2 is friend of 3 but 1 is *not* friend of 3?

Problem 6. A set A of n elements is randomly partitioned into two *nonempty* subsets B and B^c . All such partitions are equally likely.

- (a) Describe the probability space for this random experiment.
- (b) Two distinct elements x and y of A are said to be connected if they belong to the same element of the partition (i.e. to either B or B^c). Given x and y , what is the probability that they are connected?
- (c) An element $x \in A$ is said to be isolated if it is not connected to any other element of A . Find the probability that there exists an isolated element.