Wedge Product Expansion (3 vectors)

$$
\begin{aligned}
a \wedge b \wedge c = &\left(a_1 e_1 + a_2 e_2 + a_3 e_3 \right) \\
&\wedge \left(b_1 e_1 + b_2 e_2 + b_3 e_3 \right) \\
&\wedge \left(c_1 e_1 + c_2 e_2 + c_3 e_3 \right)
\end{aligned}
\qquad
\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}
$$

Wedge Product Expansion (3 vectors)

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a \wedge b \wedge c = (\; & a_1 e_1 + a_2 e_2 + a_3 e_3 \;) \\
\wedge \; (\; & b_1 e_1 + b_2 e_2 + b_3 e_3 \;) \\
\wedge \; (\; & c_1 e_1 + c_2 e_2 + c_3 e_3 \;)
\end{aligned}
\qquad
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Wedge Product Expansion (3 vectors)

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\end{aligned}
\qquad
\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}
$$

$$
a \wedge b \wedge c = a_1 (b_2 c_3 - b_3 c_2)\, e_1 \wedge e_{23}
$$

Wedge Product Expansion (3 vectors)

$$
\begin{aligned}
a \wedge b \wedge c = & \left(a_1 e_1 + a_2 e_2 + a_3 e_3 \right) \\
& \wedge \left(b_1 e_1 + b_2 e_2 + b_3 e_3 \right) \\
& \wedge \left(c_1 e_1 + c_2 e_2 + c_3 e_3 \right)
\end{aligned}
\qquad
\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}
$$

$$
a \wedge b \wedge c = a_1 (b_2 c_3 - b_3 c_2)\, e_1 \wedge e_{23}
\qquad
\Delta = a_1 (b_2 c_3 - b_3 c_2)
$$

Wedge Product Expansion (3 vectors)

$$a \wedge b \wedge c = (a_1 e_1 + a_2 e_2 + a_3 e_3) \wedge (b_1 e_1 + b_2 e_2 + b_3 e_3) \wedge (c_1 e_1 + c_2 e_2 + c_3 e_3)$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$a \wedge b \wedge c = a_1(b_2 c_3 - b_3 c_2)\ e_1 \wedge e_{23} \qquad \Delta = a_1(b_2 c_3 - b_3 c_2)$$

Wedge Product Expansion (3 vectors)

$$a \wedge b \wedge c = (a_1 e_1 + a_2 e_2 + a_3 e_3) \wedge (b_1 e_1 + b_2 e_2 + b_3 e_3) \wedge (c_1 e_1 + c_2 e_2 + c_3 e_3)$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$a \wedge b \wedge c = a_1(b_2 c_3 - b_3 c_2)\, e_1 \wedge e_{23} + a_2(b_1 c_3 - b_3 c_1)\, e_2 \wedge e_{13}$$

$$\Delta = a_1(b_2 c_3 - b_3 c_2)$$

Wedge Product Expansion (3 vectors)

$$a \wedge b \wedge c = (a_1 e_1 + a_2 e_2 + a_3 e_3) \wedge (b_1 e_1 + b_2 e_2 + b_3 e_3) \wedge (c_1 e_1 + c_2 e_2 + c_3 e_3)$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$a \wedge b \wedge c = a_1(b_2 c_3 - b_3 c_2)\ e_1 \wedge e_{23} + a_2(b_1 c_3 - b_3 c_1)\ e_2 \wedge e_{13}$$

$$\Delta = a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1)$$

Wedge Product Expansion (3 vectors)

$$a \wedge b \wedge c = (a_1 e_1 + a_2 e_2 + a_3 e_3) \wedge (b_1 e_1 + b_2 e_2 + b_3 e_3) \wedge (c_1 e_1 + c_2 e_2 + c_3 e_3)$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$a \wedge b \wedge c = a_1(b_2 c_3 - b_3 c_2)\ e_1 \wedge e_{23} + a_2(b_1 c_3 - b_3 c_1)\ e_2 \wedge e_{13}$$

$$\Delta = a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1)$$

Wedge Product Expansion (3 vectors)

$$a \wedge b \wedge c = (a_1 e_1 + a_2 e_2 + a_3 e_3) \wedge (b_1 e_1 + b_2 e_2 + b_3 e_3) \wedge (c_1 e_1 + c_2 e_2 + c_3 e_3)$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$a \wedge b \wedge c = a_1(b_2 c_3 - b_3 c_2)\, e_1 \wedge e_{23} + a_2(b_1 c_3 - b_3 c_1)\, e_2 \wedge e_{13} + a_3(b_1 c_2 - b_2 c_1)\, e_3 \wedge e_{12}$$

$$\Delta = a_1(b_2 c_3 - b_3 c_2) - a_2(b_1 c_3 - b_3 c_1)$$

Wedge Product Expansion (3 vectors)

$$a \wedge b \wedge c = (\ a_1 e_1 + a_2 e_2 + a_3 e_3\ )$$
$$\wedge\ (\ b_1 e_1 + b_2 e_2 + b_3 e_3\ )$$
$$\wedge\ (\ c_1 e_1 + c_2 e_2 + c_3 e_3\ )$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$a \wedge b \wedge c = a_1(b_2 c_3 - b_3 c_2)\ e_1 \wedge e_{23}$$
$$+ a_2(b_1 c_3 - b_3 c_1)\ e_2 \wedge e_{13}$$
$$+ a_3(b_1 c_2 - b_2 c_1)\ e_3 \wedge e_{12}$$

$$\Delta = a_1(b_2 c_3 - b_3 c_2)$$
$$- a_2(b_1 c_3 - b_3 c_1)$$
$$+ a_3(b_1 c_2 - b_2 c_1)$$

Wedge Product Expansion (3 vectors)

$$
\begin{aligned}
a \wedge b \wedge c = &(\; a_1 e_1 + a_2 e_2 + a_3 e_3 \;) \\
&\wedge (\; b_1 e_1 + b_2 e_2 + b_3 e_3 \;) \\
&\wedge (\; c_1 e_1 + c_2 e_2 + c_3 e_3 \;)
\end{aligned}
\qquad
\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}
$$

$$
\begin{aligned}
a \wedge b \wedge c = &\; a_1(b_2 c_3 - b_3 c_2) \; e_1 \wedge e_{23} \\
&+ a_2(b_1 c_3 - b_3 c_1) \; e_2 \wedge e_{13} \\
&+ a_3(b_1 c_2 - b_2 c_1) \; e_3 \wedge e_{12} \\
&= \Delta e_{123}
\end{aligned}
\qquad
\begin{aligned}
\Delta = &\; a_1(b_2 c_3 - b_3 c_2) \\
&- a_2(b_1 c_3 - b_3 c_1) \\
&+ a_3(b_1 c_2 - b_2 c_1)
\end{aligned}
$$

Wedge Product Expansion (3 vectors)

$$a \wedge b \wedge c = (\ a_1 e_1 + a_2 e_2 + a_3 e_3\ ) \wedge (\ b_1 e_1 + b_2 e_2 + b_3 e_3\ ) \wedge (\ c_1 e_1 + c_2 e_2 + c_3 e_3\ )$$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$a \wedge b \wedge c = a_1(b_2c_3 - b_3c_2)\ e_1 \wedge e_{23} + a_2(b_1c_3 - b_3c_1)\ e_2 \wedge e_{13} + a_3(b_1c_2 - b_2c_1)\ e_3 \wedge e_{12} = \Delta e_{123}$$

$$\Delta = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

Summary

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = +a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$