1 Exercise *(Probabilities (2p))*

1.

$$
\begin{aligned}
P(red|b_1) &= \frac{h(red, b_1)}{h(b1)} &&= \frac{5}{5+3+2} = \frac{5}{10} &&= \frac{1}{2} \\
P(green|b_1) &= \frac{h(green, b_1)}{h(b1)} &&= \frac{3}{5+3+2} &&= \frac{3}{10} \\
P(yellow|b_1) &= \frac{h(yellow, b_1)}{h(b1)} &&= \frac{2}{5+3+2} = \frac{2}{10} &&= \frac{1}{5}
\end{aligned}
$$

2. $p(b_1) = 0.2, p(b_2) = 0.3, p(b_3) = 0.5$

$$
\begin{aligned}
P(red) &= P(red|b_1)P(b_1) + P(red|b_2)P(b_2) + P(red|b_3)P(b_3) \\
&= 0.2\frac{5}{5+3+2} + 0.3\frac{1}{1+2+3} + 0.5\frac{4}{4+2+5} \\
&= \frac{73}{220} \\
&= 0.3319 \\
P(yellow) &= P(yellow|b_1)P(b_1) + P(yellow|b_2)P(b_2) + P(yellow|b_3)P(b_3) \\
&= 0.2\frac{3}{5+3+2} + 0.3\frac{2}{1+2+3} + 0.5\frac{2}{4+2+5} \\
&= \frac{69}{275} \\
&= 0.251 \\
P(yellow) &= P(yellow|b_1)P(b_1) + P(yellow|b_2)P(b_2) + P(green) \\
&= P(green|b_1)P(b_1) + P(green|b_2)P(b_2) + P(green|b_3)P(b_3) \\
&= 0.2\frac{2}{5+3+2} + 0.3\frac{3}{1+2+3} + 0.5\frac{5}{4+2+5} \\
&= \frac{459}{1100} \\
&= 0.417
\end{aligned}
$$

2 Exercise *(Bayes Classifier (8p))*

1. Bayes' rule for illness given som symptom s: $P(i|s) = \frac{P(s|i)P(i)}{P(s)}$

- $P(s|i)$:
  $P(n|i) = \frac{2}{3}, P(c|i) = \frac{2}{3}, P(r|i) = \frac{2}{3}, P(f|i) = \frac{1}{3}$
  $P(\neg n|i) = \frac{1}{3}, P(\neg c|i) = \frac{1}{3}, P(\neg r|i) = \frac{1}{3}, P(\neg f|i) = \frac{2}{3}$
- $P(i) = \frac{3}{6} = 0.5$

- $P(s)$:
  $P(n) = \frac{3}{6} = 0.5, P(c) = \frac{3}{6} = 0.5, P(r) = \frac{3}{6} = 0.5, P(f) = \frac{1}{6}$
  $P(\neg n) = \frac{3}{6} = 0.5, P(\neg c) = \frac{3}{6} = 0.5, P(\neg r) = \frac{3}{6} = 0.5, P(\neg f) = \frac{5}{6}$

2.

$$
\begin{aligned}
d1 : P(i|n, c, r, \neg f) &= \frac{P(n|i)P(c|i)P(r|i)P(\neg f|i)P(i)}{P(n)P(c)P(r)P(\neg f)} &&= \frac{\frac{2}{3}^4 0.5}{0.5^3 \frac{5}{6}} = 0.94 \\
d2 : P(i|n, c, \neg r, \neg f) &= \frac{P(n|i)P(c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(n)P(c)P(\neg r)P(\neg f)} &&= \frac{\frac{2}{3}^4 \frac{1}{3} 0.5}{0.5^3 \frac{5}{6}} = 0.316 \\
d3 : P(i|\neg n, \neg c, r, f) &= \frac{P(\neg n|i)P(\neg c|i)P(r|i)P(f|i)P(i)}{P(\neg n)P(\neg c)P(r)P(f)} &&= \frac{\frac{1}{3}^3 \frac{2}{3}^2 0.5}{0.5^3 \frac{1}{3}} = 0.198 \\
d4 : P(i|n, \neg c, \neg r, \neg f) &= \frac{P(n|i)P(\neg c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(n)P(\neg c)P(\neg r)P(\neg f)} &&= \frac{\frac{1}{3}^2 \frac{2}{3}^2 0.5}{0.5^3 \frac{5}{6}} = 0.237 \\
d5 : P(i|\neg n, \neg c, \neg r, \neg f) &= \frac{P(\neg n|i)P(\neg c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(\neg n)P(\neg c)P(\neg r)P(\neg f)} &&= \frac{\frac{1}{3}^3 \frac{2}{3}^2 0.5}{0.5^3 \frac{5}{6}} = 0.079 \\
d6 : P(i|\neg n, c, r, \neg f) &= \frac{P(\neg n|i)P(c|i)P(r|i)P(\neg f|i)P(i)}{P(\neg n)P(c)P(r)P(\neg f)} &&= \frac{\frac{1}{3} \frac{2}{3}^3 0.5}{0.5^3 \frac{5}{6}} = 0.474
\end{aligned}
$$

3.

3 Exercise *(Reinforcement Learning (10p))*

1. $V(s_t) = 0 * 0.9 + 0 * 0.9^2 + 0 * 0.9^3 + 0 * 0.9^4 + 0 * 0.9^5 + 100 * 0.9^6 = 53.1441$

2. Three episodes of Q-learning:
   $q(s, a) = r + \gamma max_a q(s, a)$
   States are described by their coordinates in $[1, 3] \times [1, 3]$.
   Normally, the initial state is chosen at random. Today, our totally legitimate nine-sided dice always lets us take (1,1).
   Furthermore, we choose a probabilistic approach of choosing the next action. We use a dice from the same company as the previously used nine-sided one and always end up with the same path from $(1, 1)$ to $(3, 3)$.

   (a) $q((1, 1), up) = 0 + 0.9 * 0 = 0$
   $q((2, 1), up) = 0 + 0.9 * 0 = 0$

$q((3,1), right) = 0 + 0.9 * 0 = 0$
$q((3,2), right) = 100 + 0.9 * 0 = 100$

(b) $q((1,1), up) = 0 + 0.9 * 0 = 0$
$q((2,1), up) = 0 + 0.9 * 0 = 0$
$q((3,1), right) = 0 + 0.9 * 100 = 90$
$q((3,2), right) = 100 + 0.9 * 0 = 100$

(c) $q((1,1), up) = 0 + 0.9 * 0 = 0$
$q((2,1), up) = 0 + 0.9 * 90 = 81$
$q((3,1), right) = 0 + 0.9 * 100 = 90$
$q((3,2), right) = 100 + 0.9 * 0 = 100$

4 Exercise *(Classification (6p))*

1. (a) If we want to use SVM to solve multi-class classification problem such as MNIST, shortly explain how can we solve this problem?

   We could use a SVM for each possible pair of classes. Then, during each iteration, we assign a class label to each example. The class such an example is assigned to most often is taken as its predicted output class.

5 Exercise *(LDA (6p))*

1. Discuss the purpose of LDA in comparison with PCA.

   Both LDA and PCA search for linear combinations of variables that explain the data. They are therefore unsuited for nonlinear distributions in the case of PCA or distributions that are not linearly seperable in the case of LDA. While LDA is specifically used to assign class labels to examples, PCA does not take into account different classes and only searches for the directions of the largest variance of the data.