1 Exercise *(Probabilities (2p))*

1.

$$
\begin{aligned}
P(red|b_1) = \frac{h(red, b_1)}{h(b1)} &= \frac{5}{5+3+2} = \frac{5}{10} &= \frac{1}{2} \\
P(green|b_1) = \frac{h(green, b_1)}{h(b1)} &= \frac{3}{5+3+2} &= \frac{3}{10} \\
P(yellow|b_1) = \frac{h(yellow, b_1)}{h(b1)} &= \frac{2}{5+3+2} = \frac{2}{10} &= \frac{1}{5}
\end{aligned}
$$

2. $p(b_1) = 0.2, p(b_2) = 0.3, p(b_3) = 0.5$

$$
\begin{aligned}
P(red) &= P(red|b_1)P(b_1) + P(red|b_2)P(b_2) + P(red|b_3)P(b_3) \\
&= 0.2\frac{5}{5+3+2} + 0.3\frac{1}{1+2+3} + 0.5\frac{4}{4+2+5} \\
&= \frac{73}{220} \\
&= 0.3319 \\
P(yellow) &= P(yellow|b_1)P(b_1) + P(yellow|b_2)P(b_2) + P(yellow|b_3)P(b_3) \\
&= 0.2\frac{3}{5+3+2} + 0.3\frac{2}{1+2+3} + 0.5\frac{2}{4+2+5} \\
&= \frac{69}{275} \\
&= 0.251 \\
P(yellow) &= P(yellow|b_1)P(b_1) + P(yellow|b_2)P(b_2) + P(green) \\
&= P(green|b_1)P(b_1) + P(green|b_2)P(b_2) + P(green|b_3)P(b_3) \\
&= 0.2\frac{2}{5+3+2} + 0.3\frac{3}{1+2+3} + 0.5\frac{5}{4+2+5} \\
&= \frac{459}{1100} \\
&= 0.417
\end{aligned}
$$

2 Exercise *(Bayes Classifier (8p))*

1. Bayes' rule for illness given som symptom s: $P(i|s) = \frac{P(s|i)P(i)}{P(s)}$

- $P(s|i)$:
  $P(n|i) = \frac{2}{3}, P(c|i) = \frac{2}{3}, P(r|i) = \frac{2}{3}, P(f|i) = \frac{1}{3}$
  $P(\neg n|i) = \frac{1}{3}, P(\neg c|i) = \frac{1}{3}, P(\neg r|i) = \frac{1}{3}, P(\neg f|i) = \frac{2}{3}$
- $P(i) = \frac{3}{6} = 0.5$

- $P(s)$:
  $P(n) = \frac{3}{6} = 0.5, P(c) = \frac{3}{6} = 0.5, P(r) = \frac{3}{6} = 0.5, P(f) = \frac{1}{6}$
  $P(\neg n) = \frac{3}{6} = 0.5, P(\neg c) = \frac{3}{6} = 0.5, P(\neg r) = \frac{3}{6} = 0.5, P(\neg f) = \frac{5}{6}$

2.

$$d1 : P(i|n,c,r,\neg f) = \frac{P(n|i)P(c|i)P(r|i)P(\neg f|i)P(i)}{P(n)P(c)P(r)P(\neg f)} = \frac{\frac{2}{3}^4 0.5}{0.5^3 \frac{5}{6}} = 0.94$$

$$d2 : P(i|n,c,\neg r,\neg f) = \frac{P(n|i)P(c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(n)P(c)P(\neg r)P(\neg f)} = \frac{\frac{2}{3}^4 \frac{1}{3} 0.5}{0.5^3 \frac{5}{6}} = 0.316$$

$$d3 : P(i|\neg n,\neg c,r,f) = \frac{P(\neg n|i)P(\neg c|i)P(r|i)P(f|i)P(i)}{P(\neg n)P(\neg c)P(r)P(f)} = \frac{\frac{1}{3}^3 \frac{2}{3}^2 0.5}{0.5^3 \frac{1}{3}} = 0.198$$

$$d4 : P(i|n,\neg c,\neg r,\neg f) = \frac{P(n|i)P(\neg c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(n)P(\neg c)P(\neg r)P(\neg f)} = \frac{\frac{1}{3}^2 \frac{2}{3}^2 0.5}{0.5^3 \frac{5}{6}} = 0.237$$

$$d5 : P(i|\neg n,\neg c,\neg r,\neg f) = \frac{P(\neg n|i)P(\neg c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(\neg n)P(\neg c)P(\neg r)P(\neg f)} = \frac{\frac{1}{3}^3 \frac{2}{3}^2 0.5}{0.5^3 \frac{5}{6}} = 0.079$$

$$d6 : P(i|\neg n,c,r,\neg f) = \frac{P(\neg n|i)P(c|i)P(r|i)P(\neg f|i)P(i)}{P(\neg n)P(c)P(r)P(\neg f)} = \frac{\frac{1}{3} \frac{2}{3}^3 0.5}{0.5^3 \frac{5}{6}} = 0.474$$

3.

$$P(i|c,f) = \frac{P(c|i)P(f|i)P(i)}{P(c)P(f)} = \frac{\frac{2}{3} \frac{1}{3} 0.5}{0.5 \frac{1}{6}} = 1.333$$

$$P(i|n,f) = \frac{P(n|i)P(f|i)P(i)}{P(n)P(f)} = \frac{\frac{2}{3} \frac{1}{3} 0.5}{0.5 \frac{1}{6}} = 1.333$$

$$P(i|n,r) = \frac{P(n|i)P(r|i)P(i)}{P(n)P(r)} = \frac{\frac{2}{3}^2 0.5}{0.5^2} = 0.889$$

3 Exercise *(Reinforcement Learning (10p))*

1. $V(s_t) = 0*0.9 + 0*0.9^2 + 0*0.9^3 + 0*0.9^4 + 0*0.9^5 + 100*0.9^6 = 53.1441$
2. Three episodes of Q-learning:
   $q(s,a) = r + \gamma max_a q(s,a)$

States are described by their coordinates in $[1,3] \times [1,3]$.
Normally, the initial state is chosen at random. Today, our totally legitimate nine-sided dice always lets us take (1,1).
Furthermore, we choose a probabilistic approach of choosing the next action. We use a dice from the same company as the previously used nine-sided one and always end up with the same path from $(1,1)$ to $(3,3)$.

(a) $q((1,1), up) = 0 + 0.9 * 0 = 0$
$q((2,1), up) = 0 + 0.9 * 0 = 0$
$q((3,1), right) = 0 + 0.9 * 0 = 0$
$q((3,2), right) = 100 + 0.9 * 0 = 100$

(b) $q((1,1), up) = 0 + 0.9 * 0 = 0$
$q((2,1), up) = 0 + 0.9 * 0 = 0$
$q((3,1), right) = 0 + 0.9 * 100 = 90$
$q((3,2), right) = 100 + 0.9 * 0 = 100$

(c) $q((1,1), up) = 0 + 0.9 * 0 = 0$
$q((2,1), up) = 0 + 0.9 * 90 = 81$
$q((3,1), right) = 0 + 0.9 * 100 = 90$
$q((3,2), right) = 100 + 0.9 * 0 = 100$

4 Exercise *(LDA (6p))*

1.
2.
3.