

1 Exercise (*Probabilities (2p)*)

1.

$$\begin{aligned}
 P(\text{red}|b_1) &= \frac{h(\text{red}, b_1)}{h(b_1)} = \frac{5}{5+3+2} = \frac{5}{10} = \frac{1}{2} \\
 P(\text{green}|b_1) &= \frac{h(\text{green}, b_1)}{h(b_1)} = \frac{3}{5+3+2} = \frac{3}{10} \\
 P(\text{yellow}|b_1) &= \frac{h(\text{yellow}, b_1)}{h(b_1)} = \frac{2}{5+3+2} = \frac{2}{10} = \frac{1}{5}
 \end{aligned}$$

2. $p(b_1) = 0.2, p(b_2) = 0.3, p(b_3) = 0.5$

$$\begin{aligned}
 P(\text{red}) &= P(\text{red}|b_1)P(b_1) + P(\text{red}|b_2)P(b_2) + P(\text{red}|b_3)P(b_3) \\
 &= 0.2 \frac{5}{5+3+2} + 0.3 \frac{1}{1+2+3} + 0.5 \frac{4}{4+2+5} \\
 &= \frac{73}{220} \\
 &= 0.3319 \\
 P(\text{yellow}) &= P(\text{yellow}|b_1)P(b_1) + P(\text{yellow}|b_2)P(b_2) + P(\text{yellow}|b_3)P(b_3) \\
 &= 0.2 \frac{3}{5+3+2} + 0.3 \frac{2}{1+2+3} + 0.5 \frac{2}{4+2+5} \\
 &= \frac{69}{275} \\
 &= 0.251 \\
 P(\text{green}) &= P(\text{green}|b_1)P(b_1) + P(\text{green}|b_2)P(b_2) + P(\text{green}|b_3)P(b_3) \\
 &= P(\text{green}|b_1)P(b_1) + P(\text{green}|b_2)P(b_2) + P(\text{green}|b_3)P(b_3) \\
 &= 0.2 \frac{2}{5+3+2} + 0.3 \frac{3}{1+2+3} + 0.5 \frac{5}{4+2+5} \\
 &= \frac{459}{1100} \\
 &= 0.417
 \end{aligned}$$

2 Exercise (*Bayes Classifier (8p)*)

1. Bayes' rule for illness given some symptom s : $P(i|s) = \frac{P(s|i)P(i)}{P(s)}$

- $P(s|i) :$
 $P(n|i) = \frac{2}{3}, P(c|i) = \frac{2}{3}, P(r|i) = \frac{2}{3}, P(f|i) = \frac{1}{3}$
 $P(\neg n|i) = \frac{1}{3}, P(\neg c|i) = \frac{1}{3}, P(\neg r|i) = \frac{1}{3}, P(\neg f|i) = \frac{2}{3}$
- $P(i) = \frac{3}{6} = 0.5$
- $P(s) :$
 $P(n) = \frac{3}{6} = 0.5, P(c) = \frac{3}{6} = 0.5, P(r) = \frac{3}{6} = 0.5, P(f) = \frac{1}{6}$
 $P(\neg n) = \frac{3}{6} = 0.5, P(\neg c) = \frac{3}{6} = 0.5, P(\neg r) = \frac{3}{6} = 0.5, P(\neg f) = \frac{5}{6}$

2.

$$\begin{aligned}
d1 : P(i|n, c, r, \neg f) &= \frac{P(n|i)P(c|i)P(r|i)P(\neg f|i)P(i)}{P(n)P(c)P(r)P(\neg f)} = \frac{\frac{2}{3}^4 0.5}{0.5^3 \frac{5}{6}} = 0.94 \\
d2 : P(i|n, c, \neg r, \neg f) &= \frac{P(n|i)P(c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(n)P(c)P(\neg r)P(\neg f)} = \frac{\frac{2}{3}^4 \frac{1}{3} 0.5}{0.5^3 \frac{5}{6}} = 0.316 \\
d3 : P(i|\neg n, \neg c, r, f) &= \frac{P(\neg n|i)P(\neg c|i)P(r|i)P(f|i)P(i)}{P(\neg n)P(\neg c)P(r)P(f)} = \frac{\frac{1}{3}^3 \frac{2}{3}^2 0.5}{0.5^3 \frac{1}{3}} = 0.198 \\
d4 : P(i|n, \neg c, \neg r, \neg f) &= \frac{P(n|i)P(\neg c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(n)P(\neg c)P(\neg r)P(\neg f)} = \frac{\frac{1}{3}^2 \frac{2}{3}^2 0.5}{0.5^3 \frac{5}{6}} = 0.237 \\
d5 : P(i|\neg n, \neg c, \neg r, \neg f) &= \frac{P(\neg n|i)P(\neg c|i)P(\neg r|i)P(\neg f|i)P(i)}{P(\neg n)P(\neg c)P(\neg r)P(\neg f)} = \frac{\frac{1}{3}^3 \frac{2}{3}^2 0.5}{0.5^3 \frac{5}{6}} = 0.079 \\
d6 : P(i|\neg n, c, r, \neg f) &= \frac{P(\neg n|i)P(c|i)P(r|i)P(\neg f|i)P(i)}{P(\neg n)P(c)P(r)P(\neg f)} = \frac{\frac{1}{3} \frac{2}{3}^3 0.5}{0.5^3 \frac{5}{6}} = 0.474
\end{aligned}$$

3.

$$\begin{aligned}
P(i|c, f) &= \frac{P(c|i)P(f|i)P(i)}{P(c)P(f)} = \frac{\frac{2}{3} \frac{1}{3} 0.5}{0.5 \frac{1}{6}} = 1.333 \\
P(i|n, f) &= \frac{P(n|i)P(f|i)P(i)}{P(n)P(f)} = \frac{\frac{2}{3} \frac{1}{3} 0.5}{0.5 \frac{1}{6}} = 1.333 \\
P(i|n, r) &= \frac{P(n|i)P(r|i)P(i)}{P(n)P(r)} = \frac{\frac{2}{3}^2 0.5}{0.5^2} = 0.889
\end{aligned}$$

3 Exercise (*Reinforcement Learning (10p)*)

$$1. V(s_t) = 0 * 0.9 + 0 * 0.9^2 + 0 * 0.9^3 + 0 * 0.9^4 + 0 * 0.9^5 + 100 * 0.9^6 = 53.1441$$

2. Three episodes of Q-learning:
 $Q(s, a) = r(s, a) + \gamma V^*(\delta(s, a))$
 $a^*(s) = \operatorname{argmax}_a Q(s, a)$
(a)

4 Exercise (*LDA (6p)*)

- 1.
- 2.
- 3.