cw2

December 13, 2023

1 Experimental Evaluation

MIFE scheme, we used ElGamal as the public-key encryption scheme

This notebook was runned with a HP Laptop 14s-dq1xxx with a 1GHz Intel Core i5 and 8GB RAM running Windows 10, 64 bit and Python 3.10 using `PyCryptoDome` and `numpy`

We first begin by generating some 1024 bit G, P parameters that will be recycled for the whole experiment, since it is the most time consuming operation

[1]:
```
from utils import generate_gp

LOAD_GP = True

if LOAD_GP:
    # loading the generated values
    with open('gp.txt', 'r') as f:
        G = int(f.readline().split('=')[1])
        P = int(f.readline().split('=')[1])
else:
    G, P = generate_gp(nbits=1024, num_processes=8)
    print("G =", G)
    print("P =", P)
    # saving the generated values
    with open('gp.txt', 'w') as f:
        f.write("G = " + str(G) + "\n")
        f.write("P = " + str(P) + "\n")
```

After this we proceed to show an initial representation of the tree, and the following operations

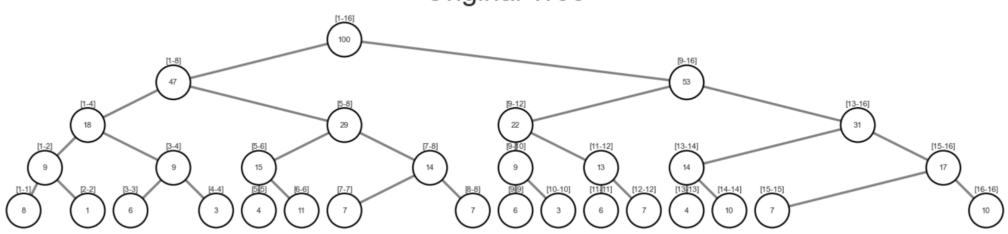
1. **Tree generation and population**: The tree nodes are initialized depending on the N value and a dataset is added
2. **Noise addition**: Noise is added to each node
3. **Encryption**: Each node is encypted. Note that the result of *ElGamal* encryption gives two large values of which only the first two digits are represented in the tree

[13]:
```
import random
import math
import numpy as np
from mife import MIFE, ElGamal
```

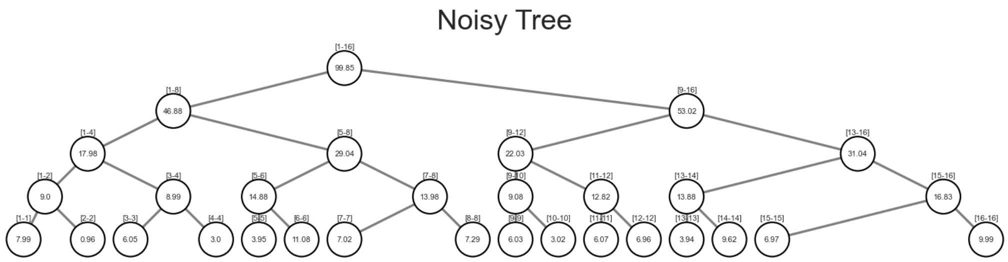
```python
from utils import *
from plm import PLM_H
from entities import BinaryRangeTree, Curator
import matplotlib as mpl

dpi_0 = mpl.rcParams['figure.dpi']
mpl.rcParams['figure.dpi'] = 200

import matplotlib.pyplot as plt


N = 4
DATASET_SIZE = 100

print(f"Num_leaves: {2**N}, Num_nodes: {2**(N+1)-1}")

x = np.random.randint(1, 2**N+1, DATASET_SIZE)
C = Curator(N, x, G=G, P=P)
query = random.randint(1, 2**N-1)
query = [query, random.randint(query+1, 2**N)]
plot_tree(C.T, "Original Tree")
print(f"Query {query}: {C.read(query)}")
C.add_noise(10)
plot_tree(C.T, "Noisy Tree")
print(f"Query {query}: {C.read(query)}")
C.encrypt()
plot_tree(C.T, "Encrypted Tree")
print(f"Query {query}: {C.read(query)}")
print(f"Query {query}: {C.read(query, f_key=True)}")
print(C.times)
# print(C.mife.dec_l1(C.T.get_values()))
```

```
Num_leaves: 16, Num_nodes: 31
```

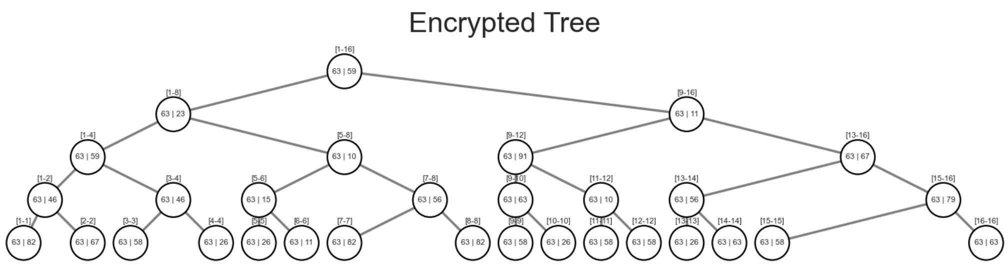
Original Tree



```
Query [14, 16]: 27
```




```
Query [14, 16]: 26.45577396671319
```




```
Query [14, 16]: 25
Query [14, 16]: 25
{'generateAndPopulate': 0.0007702000439167023, 'generateKeys':
0.5788941383361816, 'addNoise': 6.560003384947777e-05, 'encrypt':
0.35041284561157227}
```

[3]:

```
import pandas as pd

N_values = range(5, 11)
Dataset_Sizes = [100, 500, 1000, 10000]

df1 = pd.DataFrame(columns=['Dataset Size', 'Number of Leaves', 'Time (ms)'])
df2 = pd.DataFrame(columns=['Total Number of Tree Nodes', 'Laplacian Noise',␣
↪'Key Generation', 'Encryption', 'Tree Generation(Dataset size = 10000)',␣
↪'Total Time'])

for dataset_size in Dataset_Sizes:
    for N in N_values:
        x = np.random.randint(1, 2**N, dataset_size)
        C = Curator(N, x, G=G, P=P)
        C.add_noise(10)
        C.encrypt()
```

```
        df1 = pd.concat([df1, pd.DataFrame({'Dataset Size': [dataset_size],␣
↪'Number of Leaves': [2**N], 'Time (ms)': [C.
↪times["generateAndPopulate"]*1000]})], ignore_index=True)
        if dataset_size == 10000:
            df2 = pd.concat([df2, pd.DataFrame({'Total Number of Tree Nodes':␣
↪[2**(N+1)-1], 'Laplacian Noise': [C.times["addNoise"]], 'Key Generation': [C.
↪times["generateKeys"]], 'Encryption': [C.times["encrypt"]],'Tree␣
↪Generation(Dataset size = 10000)': [C.times["generateAndPopulate"]], 'Total␣
↪Time': [sum(C.times.values())]})], ignore_index=True)
        # print(N, dataset_size, C.times)

# save to excel
df1.to_excel("cw2_1.xlsx")
df2.to_excel("cw2_2.xlsx")
```

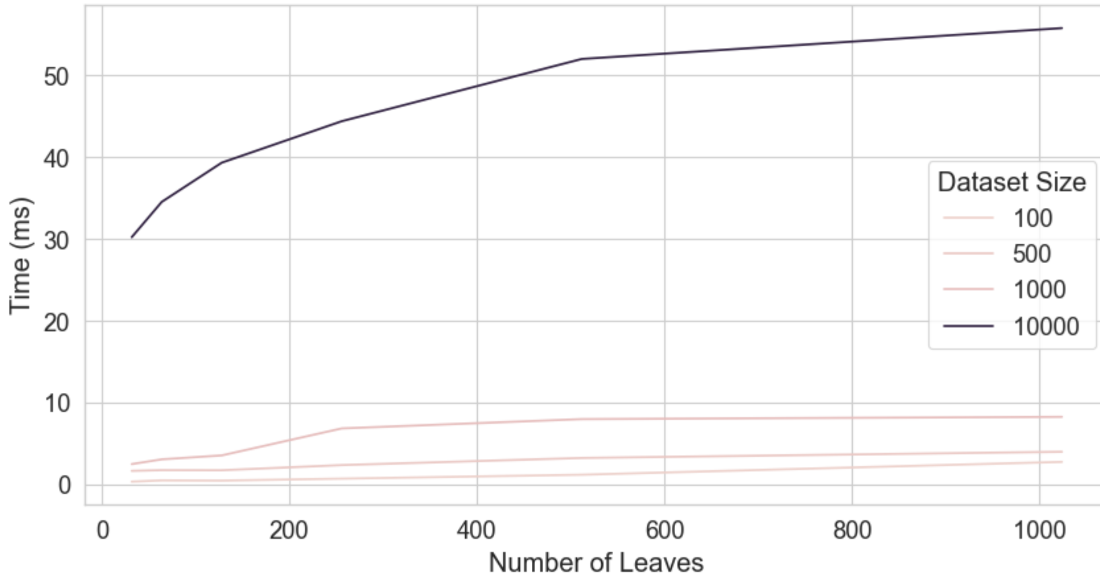
[4]:
```
# show the table
df1
```

[4]:

	Dataset Size	Number of Leaves	Time (ms)
0	100	32	0.3275
1	100	64	0.4815
2	100	128	0.4539
3	100	256	0.6926
4	100	512	1.1724
5	100	1024	2.7412
6	500	32	1.6426
7	500	64	1.7322
8	500	128	1.7188
9	500	256	2.3515
10	500	512	3.2219
11	500	1024	3.9804
12	1000	32	2.4731
13	1000	64	3.0524
14	1000	128	3.5333
15	1000	256	6.8361
16	1000	512	7.9736
17	1000	1024	8.2514
18	10000	32	30.2225
19	10000	64	34.5394
20	10000	128	39.3356
21	10000	256	44.4070
22	10000	512	52.0214
23	10000	1024	55.7928

[5]:
```
mpl.rcParams['figure.dpi'] = dpi_0

# draw the graph number of leaves vs time as a graph and points
```

```python
import matplotlib.pyplot as plt
import seaborn as sns
sns.set_theme(style="whitegrid")
sns.set_context("paper", font_scale=1.5)
plt.figure(figsize=(10, 5))
ax = sns.lineplot(x="Number of Leaves", y="Time (ms)", hue="Dataset Size",␣
↪data=df1)
```



[6]:
```python
df2
```

[6]:

	Total Number of Tree Nodes	Laplacian Noise	Key Generation	Encryption
0	63	0.000146	0.958605	0.653254
1	127	0.000219	1.892498	1.289893
2	255	0.000436	4.106747	2.655997
3	511	0.000779	7.964437	5.252489
4	1023	0.001660	16.386053	10.780267
5	2047	0.004975	34.205822	21.332881

	Tree Generation(Dataset size = 10000)	Total Time
0	0.030223	1.642227
1	0.034539	3.217149
2	0.039336	6.802516
3	0.044407	13.262111
4	0.052021	27.220001
5	0.055793	55.599471

Finally we capture a, fully unrealistic/worst-case scenario, retrieve the values from all the leaves of a 1024 leaves tree

[7]:
```python
import time
import numpy as np

N = 10

x = np.random.randint(1, 2**N, 10000)
C = Curator(N, x, G=G, P=P)
interval = (10, 16)
interval = (interval, C.read(interval))
C.encrypt()
```

[8]:
```python
t0 = time.perf_counter()
checksum = sum([C.T.query_interval([i, i]) for i in range(1, 2**N+1)])
if checksum == 10000:
    print("Time to retrieve all leaves: ", round(time.perf_counter() - t0,␣
 ↪5)*1000, "ms", sep="")
else:
    print("Error", time.time() - t0)
t0 = time.perf_counter()

print(f"Query {interval[0]}: {interval[1]}")
t0 = time.perf_counter()
checksum = C.read(interval[0])
print(f"Time to retrieve interval: {round(1000*(time.perf_counter() - t0),␣
 ↪2)}ms, checksum: {checksum}")
t0 = time.perf_counter()
checksum = C.read((interval[0]), f_key=True)
print(f"Time to retrieve interval with functional key: {round(1000*(time.
 ↪perf_counter() - t0), 2)}ms, checksum: {checksum}")

from entities import generate_f_key

f_key = generate_f_key(C.mife.msk, P)
print(f"Time to generate functional key for 1024 secret keys: {round(1000*(time.
 ↪perf_counter() - t0), 2)}ms")
```

```
Time to retrieve all leaves: 347.21000000000004ms
Query (10, 16): 76
Time to retrieve interval: 2.05ms, checksum: 76
Time to retrieve interval with functional key: 8.78ms, checksum: 76
Time to generate functional key for 1024 secret keys: 11.77ms
```

1.0.1 LCH Demo

[9]:
```python
from mife import ElGamal
from utils import enc_gamal_additive, dec_gamal_additive
import random
```

```python
# generating random r
r = random.randint(1, P-1)

# generating random messages
m1, m2 = random.randint(1, 100), random.randint(1, 100)
print(f"{m1} + {m2} = {m1+m2}")

# generating two ElGamal instances
eg1 = ElGamal(nbits=1024, G=G, P=P)
eg2 = ElGamal(nbits=1024, G=G, P=P)


enc = [eg1.enc(m1, r=r), eg2.enc(m2, r=r)]
c_1 = enc[0][0]*enc[1][0] % P, enc[0][1]*enc[1][1] % P
c_2 = enc_gamal_additive((m1 + m2)%P, (eg1.pk*eg2.pk)%P, G, P, r=r)
```

```
14 + 27 = 41
```

$$c_1 = (\text{Enc}\,(m_1, pk_1) \cdot \text{Enc}\,(m_2, pk_2))\,\%P$$

[10]:
```python
print(f"c_1:(...{str(c_1[0])[-5:]}, ...{str(c_1[1])[-5:]})")
```

```
c_1:(…58223, …01765)
```

$$c_2 = \text{Enc}\,((m_1 + m_2)\%P, (pk_1 \cdot pk_2)\%P)$$

[11]:
```python
print(F"c_2:(...{str(c_2[0])[-5:]}, ...{str(c_2[1])[-5:]})")
```

```
c_2:(…30192, …01765)
```

[12]:
```python
if c_1[1] == c_2[1]:
    print("The two ciphertexts are equal")
d1 = dec_gamal_additive(c_1, eg2.sk, G, P)
d2 = dec_gamal_additive(c_2, eg1.sk + eg2.sk % P, G, P)
d3 = dec_gamal_additive((pow(G, r, P), c_1[1]), eg1.sk + eg2.sk, G, P)
if d1 == d2 == d3:
    print(f"Decryption: {d1}")
```

```
The two ciphertexts are equal
Decryption: 41
```