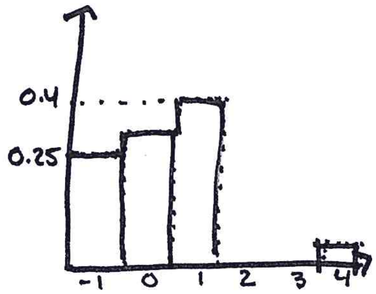
PROBLEM SET 1

1a)



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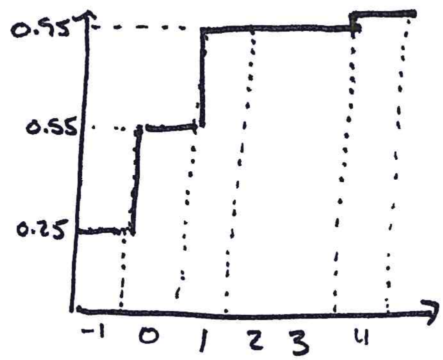


$$E[x] = \sum x P(x)$$

1b) $E[X] = -1(0.25) + 0(0.3) + 1(0.4) + 4(0.05)$

$$= -0.25 + 0.4 + 0.2$$

$$\boxed{= 0.35}$$

1c) $Var[x] = \sum (x-\mu)^2 P(x)$

$$= (-1-0.35)^2(0.25) + (0-0.35)^2(0.3)$$

$$+ (1-0.35)^2(0.4) + (4-0.35)^2(0.25)$$

$$\boxed{= 1.3275}$$

Problem Set 1

2.6

a) $E[Y] = \sum_y yP(y) = 0(0.068) + 1(0.932)$

$= \boxed{0.932}$

b) Unemployment rate $= \dfrac{\#\text{ Unemployed}}{\#\text{ Labor force}} = 0.068$

$0.068 = 1 - E[Y] = 1 - 0.932$

d) ~~unemp rate = 1 - E E~~ Note from b that unemp rate $= 1 - E[Y]$

(i) unemp rate for college grads $= 1 - E[Y|X=1]$

$= 1 - \sum_y y P(y|X=1)$ $\Rightarrow$ Calculate conditional probabilities

c)

$E[Y|X=1] = \sum y P(y|X=1)$

$= 0(0.0416) + 1(0.958)$

$\boxed{= 0.958}$

$P(0|X=1) = \dfrac{P(Y=0, X=1)}{P(X=1)} = \dfrac{0.015}{0.361} = 0.0416$

$E[Y|X=0] = \sum y P(y|X=0)$

$= 0(0.083) + 1(0.917)$

$\boxed{= 0.917}$

(see next page for prob.)

$P(1|X=1) = \dfrac{P(Y=1, X=1)}{P(X=1)} = \dfrac{0.346}{0.361} = 0.958$

plug in $\Leftarrow$

$= 1 - \left[[0(0.0416)] + [1(0.958)] \right]$

$= 1 - 0.958$

$\boxed{= 0.0416}$

PROBLEM SET 1

2.6

d)

(ii) unemp rate non-college graduates

$= 1 - E[Y|X=0]$

first, calculate conditional probabilities

$$P(0|X=0) = \frac{P(Y=0, X=0)}{P(X=0)} = \frac{0.053}{0.639} = 0.083$$

$$P(1|X=0) = \frac{P(Y=1, X=0)}{P(X=0)} = \frac{0.586}{0.639} = 0.917$$

(Note $P(1/X=0) = 1 - P(0/X=0)$!)

ie: $1 - E[Y|X=0] = 1 - [0(0.083) + 1(0.917)]$

$\boxed{= 0.083}$

e) $P(X=1|Y=0) = \dfrac{P(X=1, Y=0)}{P(Y=0)} = \dfrac{0.015}{0.068} = 0.221$

[There is a 22% chance this person is a college graduate, and a $(1-0.221=0.779)$ 78% chance he/she is not]

f. They are independent iff $P(x,y) = P(x)P(y)$

Consider $P(0,0) = 0.053$

Note $P(X=0)P(Y=0) = 0.068 \cdot 0.639 = 0.043$

$\Rightarrow$ they are not independent!

PS1

2.10

a) $P(Y \le 3) = P\left(\frac{Y-\mu}{\sigma} < \frac{3-\mu}{\sigma}\right)$

$Y \sim N(1,4)$

$= P\left(Z < \frac{3-1}{2}\right)$

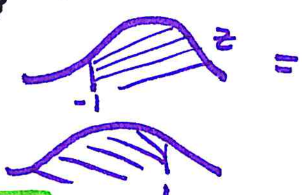
$= P(Z < 1)$

$= \Phi(1) = \boxed{0.8413}$

b) $P(Y > 0) = 1 - P(Y < 0)$

$Y \sim N(3,9)$ $= 1 - P\left(Z < \frac{0-3}{3}\right)$

$= 1 - P(Z < -1)$



$= 1 - \Phi(-1)$

$= \Phi(1) = \boxed{0.8413}$

c) $P(40 \le Y \le 52) = P\left(\frac{40-50}{5} < Y < \frac{52-50}{5}\right)$

$= P(-2 < Z < 0.4)$

$= P(Y < 0.4) - P(Y < -2)$

$= \Phi(0.4) - \Phi(-2)$

$= \Phi(0.4) - [1 - \Phi(-2)$

$= 0.6554 - 1 + 0.9772$

$= \boxed{0.6326}$

d) $P(6 \le Y \le 8) = P\left(\frac{6-5}{\sqrt{2}} < Z < \frac{8-5}{\sqrt{2}}\right)$

$= (0.707 < Z < 2.121)$

$= P(Z < 2.121) - P(Z < 0.707)$

$= \Phi(2.121) - \Phi(0.707)$

$= 0.9381 - 0.7602$

$\boxed{= 0.2229}$

3)

a) $P(X > 0.5) = 1 - P(X < 0.5)$

$= 1 - F(0.5)$

$= 1 - [3 \cdot 0.5^2 - 2 \cdot 0.5^3]$

$= 1 - [0.75 - 0.25]$

$= 0.5$

b) $P(0.4 < X < 0.6)$

$= P(X < 0.6) - P(X < 0.4)$

$= F(0.6) - F(0.4)$

$= [3(0.6)^2 - 2(0.6)^3] - [3(0.4)^2 - 2(0.4)^3]$

$= [1.08 - 0.432] - [0.48 - 0.128$

$= 0.648 - 0.352$

$= 0.296$