Item Response Theory for NLP

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https://eacl2024irt.github.io/

In this session

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Improving Model Training

Finding Annotation Error

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Introduction

IRT for NLP

Overview of IRT Applications:

- Dataset Construction
- Model Training
- Evaluation

Assumptions for IRT + NLP

Basic assumptions of the data and parameterization we have:

- A dataset with items indexed by i.
- A set of subjects indexed by j.
- Responses r_{ij} from graded responses of subjects to each item.
- An IRT parameterization, e.g., one with item difficulty β_i, discriminability γ_i, and ability θ_j might assume:

$$p(r_{ij} = 1 | \beta_i, \theta_j) = \frac{1}{1 + e^{-\gamma_i(\theta_j - \beta_i)}}$$

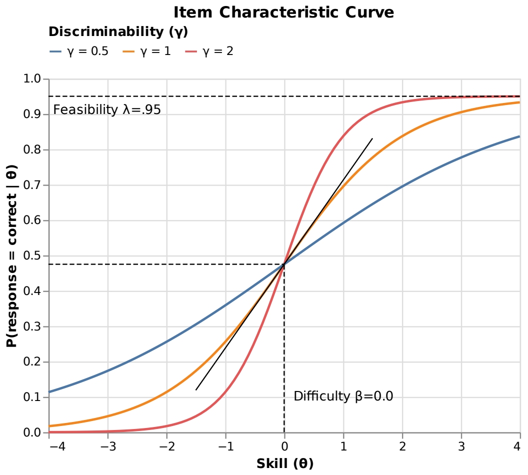
IRT Applications: Example of Model Behavior

$$\overbrace{p(y_{ij} = 1 | \gamma_i, \beta_i, \lambda_i, \theta_j)}^{\text{Likelihood of correct answer for subject } j \text{ on item } i.} = \frac{\lambda_i}{1 + e^{-\gamma_i(\theta_j - \beta_i)}}$$

Discriminability of item i

Ability of subject j

Difficulty of item i



What IRT Yields

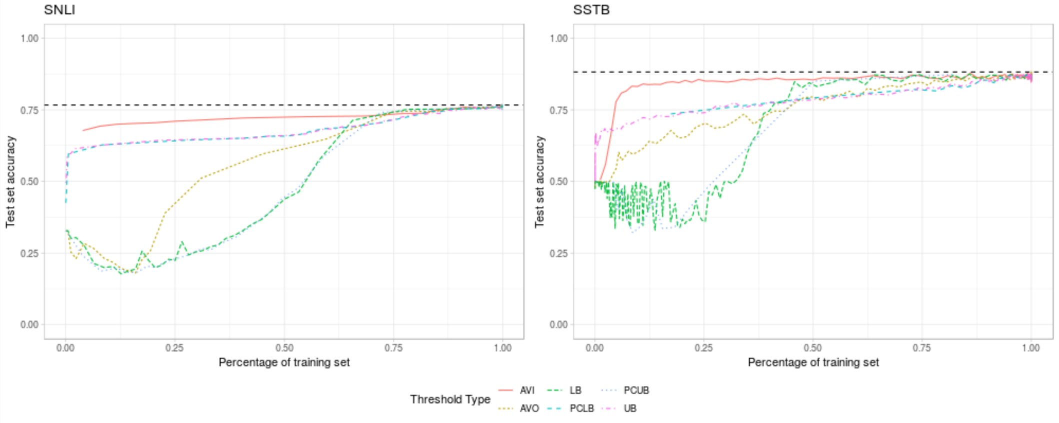
Given the previous information, IRT will yield estimates for chosen parameters, i.e.: item difficulty β_i, discriminability γ_i, and ability θ_j.

Consider two scenarios:

- What if the dataset is the training data?
- What if the dataset is a test set?

Improving Model Training

Data set filtering



- AVI: $|b_i| < \tau$
- UB: $b_i < \tau$
- PCUB: $pc_i < \tau$
- AVO: $|b_i| > \tau$
- LB: $b_i > \tau$
- PCLB: $pc_i > \tau$

Source: Lalor et al. (2019)

Biggest Differences

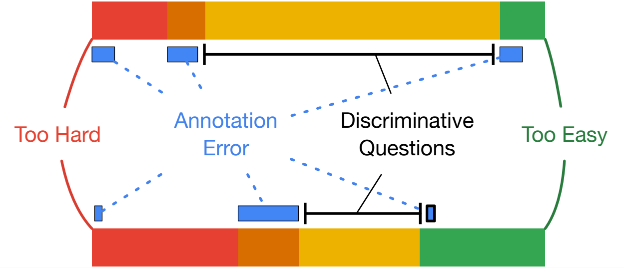
Task	Label	Item Text	Difficulty ranking		
			Humans	LSTM	NSE
SNLI	Con.	*P:* Two dogs playing in snow. *H:* A cat sleeps on floor	168	1	5
	Ent.	*P:* A girl in a newspaper hat with a bow is unwrapping an item. *H:* The girl is going to find out what is under the wrapping paper.	55	172	176
SSTB	Pos.	Only two words will tell you what you know when deciding to see it: Anthony. Hopkins.	9	103	110
	Neg.	...are of course stultifyingly contrived and too stylized by half. Still, it gets the job done–a sleepy afternoon rental.	128	46	41

Source: Lalor et al. (2019)

Finding Annotation Error

IRT Applications: Finding Annotation Error

Test examples can be: too hard, discriminative, too easy, or erroneous [1]



How can we use IRT to identify each example type?

[1] Boyd-Graber and Börschinger (2020)

IRT Applications: Finding Annotation Error

What makes examples bad?

IRT Applications: Finding Annotation Error

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- Examples that do not discriminate between good and bad subjects

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- Non-Example: Difficult example few models get correct

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- What parameter could identify this?

IRT Applications: Finding Annotation Error

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- Examples that do not discriminate between good and bad subjects
- Example: Bad label $\rightarrow$ all models get wrong
- Example: Correctness is a coinflip
- Non-Example: Difficult example few models get correct
- What parameter could identify this?
- We can use IRT discriminability γ_i to find bad examples!

IRT Applications: Setup for Finding Annotation Error

Can follow along in notebook! Setup/Assumptions:

- Run a simulation where:

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Then, train a 3PL IRT model with py-irt

IRT Applications: 3PL Model

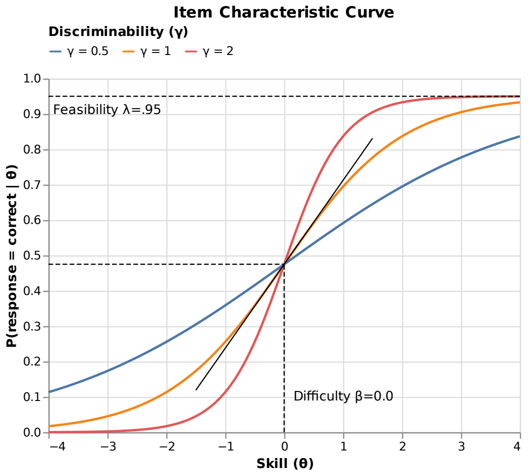
Likelihood of correct answer for subject j on item i.

$$\underbrace{p(y_{ij} = 1 \mid \gamma_i, \beta_i, \lambda_i, \theta_j)}_{\text{Likelihood of correct answer for subject } j \text{ on item } i} = \frac{\lambda_i}{1 + e^{-\gamma_i(\theta_j - \beta_i)}}$$

Discriminability of item i

Ability of subject j

Difficulty of item i



IRT Applications: Setup for Finding Annotation Error

IRT Parameters

- Item Difficulty: $\beta_i \sim$ Normal
- Item Discriminability: $\gamma_i \sim$ LogNormal
- Subject Ability $\theta_j \sim$ Normal

IRT Model

$$p(r_{ij} = 1 | \beta_i, \gamma_i, \theta_j) = \frac{1}{1 + e^{-\gamma_i(\theta_j - \beta_i)}}$$

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Note:

- Why $\gamma_i \sim$ LogNormal? Following Vania et al. (2021), forces γ_i to be non-negative.
- Other variables are zero centered.

IRT Applications: Sample Code for Finding Errors

Sample Code

```
dataset = Dataset.from_jsonlines("/tmp/irt_dataset.jsonlines")
config = IrtConfig(
  model_type='tutorial', log_every=500, dropout=.2
)
trainer = IrtModelTrainer(
  config=config, data_path=None, dataset=dataset
)
trainer.train(epochs=5000, device='cuda')
```

IRT Applications: Simulation Results

Can we distinguish valid from invalid items based on discriminability γ_i?

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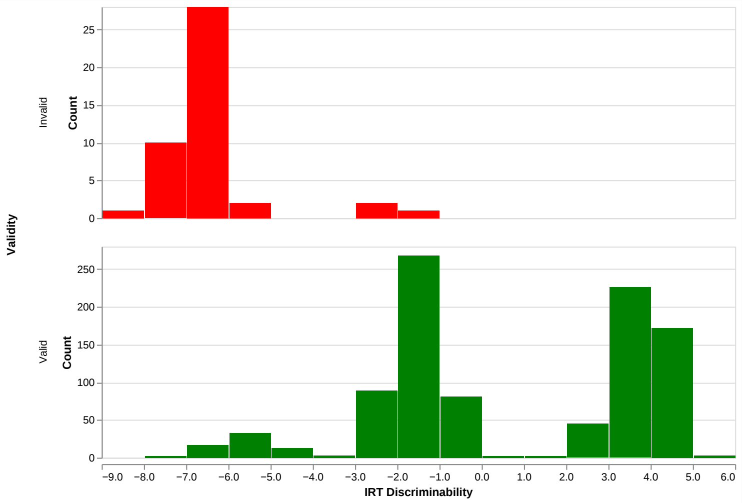
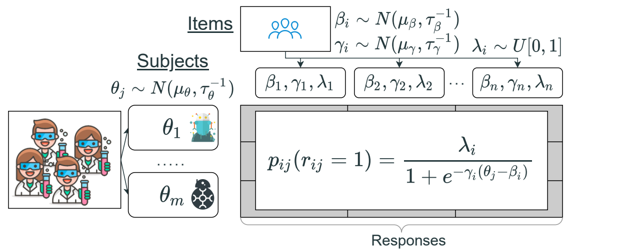


IRT Applications: Finding Annotation Error

In Rodriguez et al. (2021), we used a slightly different model to do this for SQuAD:

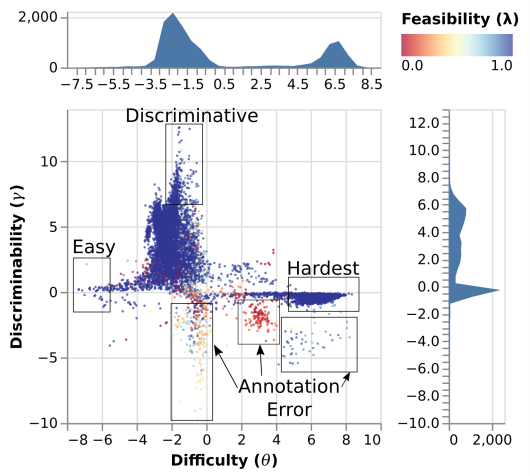


Differences

- Discriminability γ_i could be negative, which is inconvenient.
- Feasibility λ_i.

IRT Applications: Finding Annotation Error

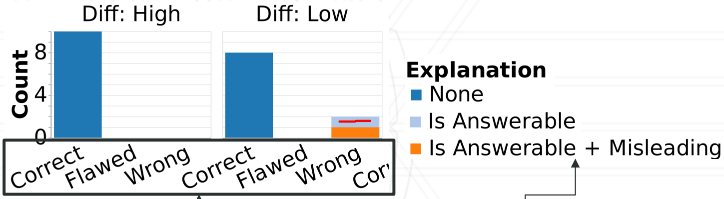
Plotting IRT parameters:



IRT Applications: Finding Annotation Error

Use IRT parameters to find partitions of data with annotation errors

Was the example correct?
- Question makes sense
- Answer is correct
- No ambiguity
- …

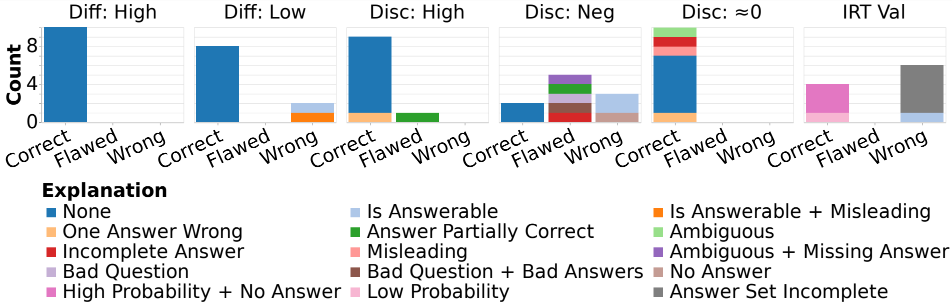


If the example is wrong, then why?
- It is *"Wrong/Flawed"* because it *"Explanation"*

Example:
One low difficulty questionwas wrong, because although the label says it is not answerable, it is answerable

IRT Applications: Finding Annotation Error

Use IRT parameters to find partitions of data with annotation errors



Things to note:

- Negative discriminability identifies errors

IRT Applications: Finding Annotation Error

Example of bad example identified by IRT

discriminability: -9.63 **Difficulty**: -0.479 **Feasibility**: 0.614 **Mean Exact Match**: 0.472
Wikipedia Page: Economic inequality **Question ID:** 572a1c943f37b319004786e3
Question: Why did the demand for rentals decrease?
Official Answer: demand for higher quality housing
Context: A number of researchers (David Rodda, Jacob Vigdor, and Janna Matlack), argue that a shortage of affordable housing – at least in the US – is caused in part by income inequality. David Rodda noted that from 1984 and 1991, the number of quality rental units decreased as the demand for higher quality housing increased (Rhoda 1994:148). Through gentrification of older neighbourhoods, for example, in East New York, rental prices increased rapidly as landlords found new residents willing to pay higher market rate for housing and left lower income families without rental units. The ad valorem property tax policy combined with rising prices made it difficult or impossible for low income residents to keep pace.

Evaluation Metrics

IRT Applications: Evaluation Metrics

Simple Idea: Instead of accuracy, use subject ability θ_j to rank.

IRT Applications: Evaluation Metrics

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IRT Applications: Evaluation Metrics Example

Suppose the following:

- 10 Subjects, similar setup as before
- As before, 1,000 Test Examples

IRT Applications: Evaluation Metrics Example

Suppose the following:

- 10 Subjects, similar setup as before
- As before, 1,000 Test Examples
- A set of 800 easy examples
- A set of 150 moderate examples
- A set of 50 hard examples

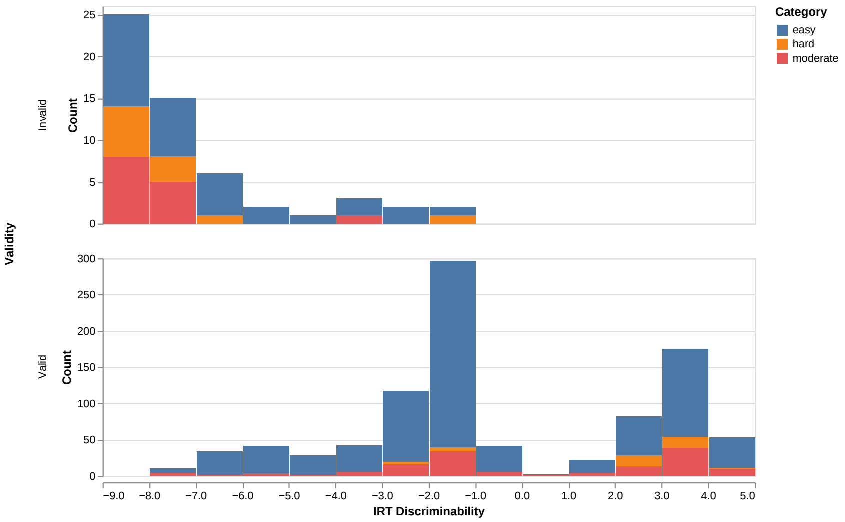
IRT Applications: Evaluation Metrics Example

- Subjects sorted by True Ability
- Accuracy gaps vary
- IRT can account for some of this variability

Ability		Accuracy			
True	IRT	Total	Easy	Mod	Hard
-3.506	-12.1	0.194	0.218	0.093	0.100
-3.000	-7.61	0.256	0.301	0.066	0.100
-2.645	-4.88	0.325	0.380	0.093	0.140
-1.214	0.348	0.543	0.650	0.113	0.120
-1.156	1.40	0.560	0.667	0.120	0.160
-0.748	2.68	0.602	0.712	0.146	0.200
-0.455	3.36	0.631	0.746	0.193	0.100
0.232	5.76	0.729	0.848	0.293	0.120
2.16	11.1	0.865	0.956	0.586	0.240
2.50	14.2	0.897	0.971	0.686	0.340

IRT Applications: Discounting Bad Examples

- Invalid examples sorted down
- Harder examples tend to be more dis- criminating



IRT Applications: Rank Reliability in Evaluation Metrics

In Rodriguez et al. (2021), we examined a case where:

- The cost of annotation model responses is high.

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- We want to:

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- We want to:
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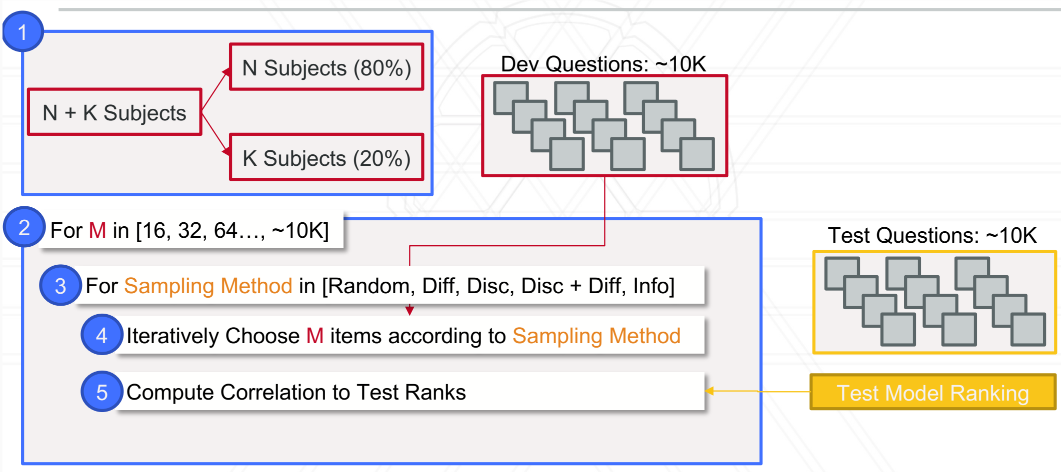
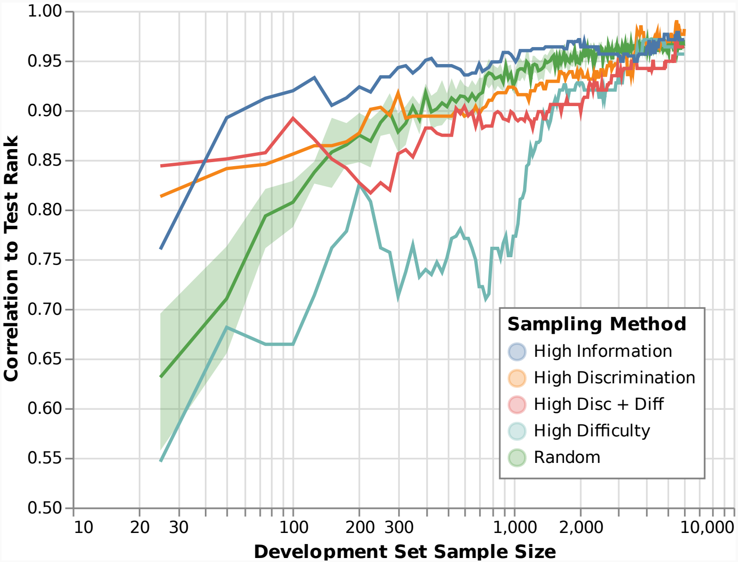
  - Maximize correlation to ranking if fully annotate
- Experiment: What method for selecting subset to annotate is best?

IRT Applications: Rank Reliability in Evaluation Metrics

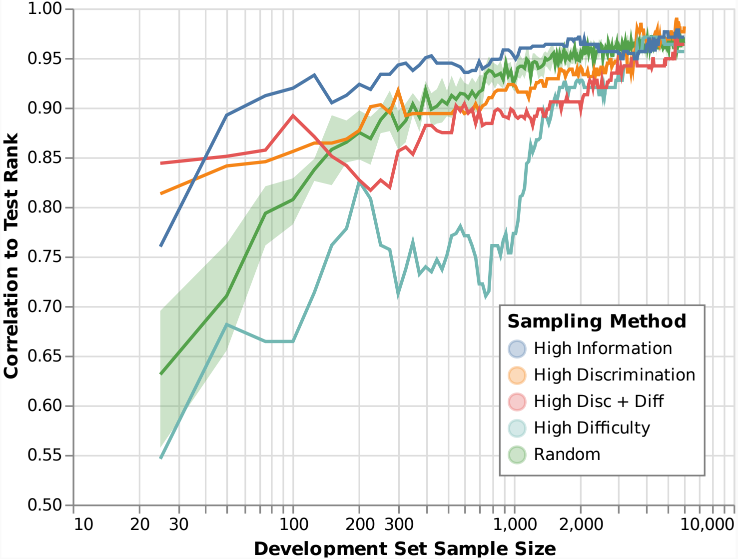
We test this setup with SQuAD leaderboard data:



IRT Applications: Rank Reliability in Evaluation Metrics



IRT Applications: Rank Reliability in Evaluation Metrics



Overall best method: pick item that maximizes Fisher information content, i.e.,

$$I_i(\theta_j) = \gamma_i^2 p_{ij}(1 - p_{ij})$$

$$Info(i) = \sum_j I_i(\theta_j)$$

Additional Work

- Adaptive Language-based Mental Health Assessment with Item-Response Theory (Varadarajan et al., 2023)
- Alternate Evaluation Metrics, e.g., Subject ability θ_j (Lalor et al., 2018)
- Anchor Points: Benchmarking Models with Much Fewer Examples (Vivek et al., 2024)
- tinyBenchmarks: evaluating LLMs with fewer examples (Polo et al., 2024)
- Comparing Test Sets with Item Response Theory (Vania et al., 2021)

Break!

- Back in 15 minutes
- Next section: Advanced Topics

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