ASTR 792
T/R 9:30 - 10:45 AM
Due September 12

Week #4

Draine 1.1 a b d

1.1 The total mass of neutral gas in the Galaxy is $\sim 4 \times 10^9$ $M_\odot$. Assume that it is uniformly distributed in a disk of radius $R_{\rm disk} = 15$ kpc and thickness $H = 200$ pc, and that it is a mixture of H and He with He/H=0.1 (by number). Assume ionized hydrogen to be negligible in this problem. [Note: even though the assumptions in this problem are very approximate, please carry out calculations to two significant digits.]

(a) What is the average number density of hydrogen nuclei within the disk?

SOLUTION:

There are two steps to this question. First, we need to determine the total number of just the hydrogen atoms. Then we need to determine the volume over which they are distributed.

To find the total number of hydrogen atoms, we can write:

$$M_{neutral} = N_{\rm H}\, m_{\rm H} + 0.1 N_{\rm H}\, m_{\rm He}$$

Solving this (with all masses in units of kg):

$$4 \times 10^9\,(1.99 \times 10^{30}) = N_{\rm H}\,(1.67 \times 10^{-27}) + 0.1\, N_{\rm H}\,(6.64 \times 10^{-27})$$

$$N_{\rm H} = 3.43 \times 10^{66}$$

Now for the volume. Even though it is huge, we want this in units of $\rm cm^{-3}$:

$$V = \pi r^2 h = 3.14\,(15 \times 10^3)^2\,(200)\,(3.086 \times 10^{18})^3$$

$$V = 4.15 \times 10^{66}\ \rm cm^{-3}$$

Putting this together:

$$n = \frac{N_H}{V} = 0.83\ \rm cm^{-3}$$

(b) If 0.7% of the interstellar mass is in the form of dust in spherical particles of radius $a = 1000$ Å$= 0.1\ \mu$m and density $\rho = 2$ g cm^{-3}, what is the mean number density of dust grains in interstellar space?

SOLUTION:

This is very similar to the previous problem, except that to find the total number of dust grains, we must first find the typical mass of a dust grain:

$$m_{dust} = \rho\, V_{grain} = \rho\, \frac{4}{3}\pi a^3$$

$$m_{dust} = 2 \text{ g cm}^{-3} \left(\frac{4}{3}\right) \pi\ (10^{-5}\text{ cm})^3 = 8.37 \times 10^{-15} \text{ g}$$

$$N_{dust} = \frac{M_{dust}}{m_{dust}} = \frac{0.007\, M_{neutral}}{m_{dust}}$$

$$N_{dust} = \frac{0.007\, (4 \times 10^9)(1.99 \times 10^{33}) \text{ g}}{8.37 \times 10^{-15} \text{ g}} = 6.66 \times 10^{54}$$

$$n_{dust} = \frac{N_{dust}}{V} = \frac{6.66 \times 10^{54}}{4.15 \times 10^{66}} = 1.6 \times 10^{-12} \text{ cm}^{-3}$$

(d) Now assume that 30% of the gas and dust mass is in spherical molecular clouds of radius 15 pc and mean density $n(\text{H2}) = 100$ cm^{-3}. What would be the mass of one such cloud? How many such molecular clouds would there be in the Galaxy?

SOLUTION:

This is basically the same as the previous problem, except that our ‘particles’ are now entire clouds and we only do the first two steps:

$$m_{cloud} = \rho\, V_{cloud} = n_{H2}\, m_{H2}\, \frac{4}{3}\pi r^3$$

$$m_{cloud} = (100 \text{ cm}^{-3})(2)(1.67 \times 10^{-24} \text{ g}) \left(\frac{4}{3}\right) \pi\ \left((15)\ (3.086 \times 10^{18}\text{ cm})\right)^3$$

$$m_{cloud} = 1.39 \times 10^{38} \text{ g} = 7 \times 10^4\, M_\odot$$

$$N_{clouds} = \frac{M_{clouds}}{m_{cloud}} = \frac{0.3\, M_{neutral}}{m_{cloud}}$$

$$N_{clouds} = \frac{0.3\, (4 \times 10^9) M_\odot}{7 \times 10^4\, M_\odot} = 1.7 \times 10^4$$