ASTR 792
T/R 9:30 - 10:45 AM
Due October 19

# Week #9

**Draine 5.3abc**

Most interstellar CO is $^{12}C^{16}O$. The $J = 1 \rightarrow 0$ transition is at $\nu = 115.27$ GHz, or $\lambda = 0.261$ cm, and the $v = 1 \rightarrow 0$ transition is at $\lambda = 4.61$ $\mu$m (ignoring rotational effects).

(a) Estimate the frequencies of the $J = 1 - 0$ transitions in $^{13}C^{16}O$ and $^{12}C^{17}O$.

**SOLUTION:**

$$h\nu = \Delta E = ((J+1)(J+2) - (J)(J+1))B_0 \; = \; 2(J+1)\left(\frac{\hbar^2}{2m_r r_0^2}\right)$$

$$\nu \propto \frac{1}{m_r} \qquad \text{where} \qquad m_r = \frac{m_1 m_2}{m_1 + m_2}$$

$$m_r(12\mathrm{CO}) = \frac{12(16)}{12+16} = 6.86$$

$$m_r(13\mathrm{CO}) = \frac{13(16)}{13+16} = 7.17$$

$$m_r(13\mathrm{CO}) = \frac{12(17)}{12+17} = 7.03$$

$$\nu(13\mathrm{CO}) = \nu(12\mathrm{CO})\frac{m_r(12\mathrm{CO})}{m_r(13\mathrm{CO})} = 115.27\text{ GHz}\left(\frac{6.86}{7.17}\right) = 110.29\text{ GHz}$$

$$\nu(\mathrm{C17O}) = \nu(12\mathrm{CO})\frac{m_r(12\mathrm{CO})}{m_r(\mathrm{C17O})} = 115.27\text{ GHz}\left(\frac{6.86}{7.03}\right) = 112.48\text{ GHz}$$

(b) Estimate the wavelengths of the $v = 1 - 0$ transitions in $^{13}C^{16}O$ and $^{12}C^{17}O$. Ignore rotational effects.

**SOLUTION:**

$$\frac{hc}{\lambda} = \Delta E = \frac{h\omega_0}{2\pi} = 2(J+1)\left(\frac{hk^{1/2}}{2\pi m_r^{1/2}}\right)$$

$$\lambda \propto m_r^{1/2}$$

$$\lambda(13\text{CO}) = \lambda(12\text{CO})\left(\frac{m_r(13\text{CO})}{m_r(12\text{CO})}\right)^{1/2} = 0.261\text{ cm}\left(\frac{7.17}{6.86}\right)^{1/2} = 0.267\text{ cm}$$

$$\lambda(\text{C}17\text{O}) = \lambda(12\text{CO})\left(\frac{m_r(\text{C}17\text{O})}{m_r(12\text{CO})}\right)^{1/2} = 0.261\text{ cm}\left(\frac{7.03}{6.86}\right)^{1/2} = 0.264\text{ cm}$$

(c) Suppose that the $^{13}$C$^{16}$O $J = 1 - 0$ line were mistaken for the $^{12}$C$^{16}$O $J = 1 - 0$ line. What would be the error in the inferred radial velocity of the emitting gas?

**SOLUTION:**

Here, we use the relation

$$\frac{\Delta v}{c} = \frac{\Delta \nu}{\nu_0}$$

$$\Delta v = \frac{(115.27 - 110.29)\text{ GHz}}{115.27\text{ GHz}}(2.99 \times 10^5\text{ km s}^{-1}) = 1.3 \times 10^4\text{ km s}^{-1}$$