ASTR 792
T/R 9:30 - 10:45 AM
Due September 26

# Week #6

**Draine 1.3**

The "very local" interstellar medium has $n_H \approx 0.22$ cm$^{-3}$ (Lallement et al. 2004: A&A 426, 875; Slavin & Frisch 2007: Sp. Sci. Revs. 130, 409). The Sun is moving at $v_W = 261$ km s$^{-1}$ relative to this local gas (Möbius et al. 2004: A&A 426, 897).

Suppose that this gas has He/H=0.1, and contains dust particles with total mass equal to 0.5% of the mass of the gas. Suppose these particles are of radius $a = 0.1\mu$m and density $\rho = 2$ g cm$^{-3}$, and we wish to design a spacecraft to collect them for study.

How large a collecting area $A$ should this spacecraft have in order to have an expected collection rate of 1 interstellar grain per hour? Neglect the motion of the spacecraft relative to the Sun, and assume that the interstellar grains are unaffected by solar gravity, radiation pressure, and the solar wind (and interplanetary magnetic field).

**SOLUTION:**

This is equivalent to asking what is the collisional cross section $\sigma$ that would lead to the required collisional rate (collisions per time)– note that this is *not* the same as a volumetric collision rate! Looking at the units, we can express this rate as the velocity divided by the mean free path of the spacecraft through the dust:

$$\text{Rate} = \frac{v}{\tau} = n_{dust}\sigma v$$

We want to solve this for $\sigma$:

$$\sigma = \frac{\text{Rate}}{n_{dust}v}$$

We are interested in the collision rates specifically with dust, so we need to know the dust grain number density $n_{dust}$:

$$n_{dust} = n_{\text{H}}x_{dust}.$$

This is a function of $x_{dust}$, which is the (numerical) fraction of dust grains, compared to hydrogen atoms. Unfortunately, this problem does not directly give us the ratio of hydrogen atoms to dust grains, instead it gives us a mass fraction. We can define the mass fraction of dust as:

$$w_{dust} = x_{dust}\frac{M_{dust}}{\bar{M}_{gas}}$$

Finally, the average mass of the gas ($\bar{M}_{gas}$) is given by

$$\bar{M}_{gas} = [\mathrm{H/He}]m_{He} + m_H$$

and the mass of a dust grain is given by:

$$M_{dust} = V\rho = \frac{4}{3}\pi(a)^3\rho.$$

Using this information to put the collisional cross section in terms of the quantities we are given :

$$\sigma = \frac{\mathrm{Rate}\,(4\pi a^3)\rho}{3v\,n_H\,w_{dust}([\mathrm{H/He}]m_{He} + m_H)}$$

$$= \frac{(1/3600\,\mathrm{s}^{-1})(4\pi)(0.00001\,\mathrm{cm})^3(2\,\mathrm{g\,cm}^{-3})}{3(0.22\,\mathrm{cm}^{-3})(0.005)(0.1(6.65\times10^{-24})\,\mathrm{g}\,+1.67\times10^{-24}\,\mathrm{g})(2.61\times10^5\,\mathrm{cm\,s}^{-1})}$$

$$= 3.46\times10^3\,\mathrm{cm}^2$$

or, 0.3 square meters!