ASTR 592
M/W/F 3:00 - 3:50 PM
Due February 21

Problem Set #2

1. Making the Simplest Molecule: H_2

(a) Let's try to form the molecule H_2 from a gas of pure H. The first way we might try to do this is by having two H atoms collide. Any time particles collide, there will be excess energy that must be gotten rid of, or the newly-formed molecule will bust right back apart. If you only have two colliding particles, then this energy must be radiated away as a photon. The reaction rate can then be expressed as $R_2 = P\,Z$. Here, P is the the probability of giving off a photon during the collision, expressed as

$$P = A_{ul}\,t$$

and Z is the collision rate (collisions per unit volume and time), which for two identical particles can be expressed as:

$$Z_2 = 2\,n_H^2\,d^2\,\left(\frac{\pi\,k\,T}{m}\right)^{1/2}$$

Assume the following values apply to the collision:

- A_{ul} (the Einstein A or rate of emitting a photon) is $\sim \left(\frac{1}{127}\right)^4 \times 10^8\ \mathrm{s}^{-1}$.
  This extra factor of α^4 is because this transition is forbidden: it can't happen the 'usual' way because H_2 doesn't have a dipole moment, thus the emission of a photon depends instead on the quadrupole moment, and so is much less likely.
- d (the size scale for a collision interaction) is $\sim$ 1 Å.
- t (the time over which the collision happens) is $\sim 10^{-13}$ s
- T is 500 K
- n_H is $10^8\ \mathrm{m}^{-3}$

Calculate R_2 for the collision of two identical H atoms. For a 1 m^3 volume of gas (containing 100 million H atoms), how long would it be before you formed a single H_2 molecule?

(b) An alternative to emitting a photon in order to get rid of the collision energy is to involve another particle in the interaction, which will carry away the collision energy and stabilize the union of the other two particles. The reaction rate R_3 should just be proportional to the collision rate, which for three identical particles can be expressed as:

$$Z_3 = \frac{32\pi}{3!}\, n^3\, d^5\, \left(\frac{\pi\, k\, T}{m}\right)^{1/2}$$

Compare R_3 for the same set of conditions you used to calculate R_2. Is this any more likely? What gas density is required so that $R_3 > R_2$?

(c) At the same time as reactions are forming H_2, there will also be reactions that destroy it. One common way H_2 is destroyed is through photodissociation by UV photons. The photodissociation cross section for a single H_2 molecule is $\sigma \sim 6 \times 10^{-18}$ m^2. Assuming that every sufficiently energetic photon that hits H_2 results in dissociation, and that a typical flux of sufficiently-energetic UV photons in the ISM is $S = 10^{10}$ m^{-2} s^{-1}, at what rate will H_2 be destroyed? How does this compare to the rates calculated in parts (a) and (b)?

(d) We know that actually there is a lot of H_2 around– it is by far (99.99999%) the most common molecule! This means we have to find some way to make it efficiently. This brings us to forming H_2 on grain surfaces. Assume that if an H atom collides with a grain it sticks, and that if it sticks it inevitably meets up with another H atom and forms H_2. Then the formation rate on dust grains can be described as

$$R_{dust} = 0.5\, v_H\, \sigma_d\, n_H\, n_d$$

We can evaluate this by assuming v_H is just the thermal velocity, and taking $n_d = 10^{-12}\, n_H$ and a typical grain radius to be $a_{dust} = 3 \times 10^{-7}$ m. What then is the reaction rate R_{dust} for the formation of H_2 on grain surfaces for the same conditions as parts (a) and (b)?

2. Measuring Dust Properties

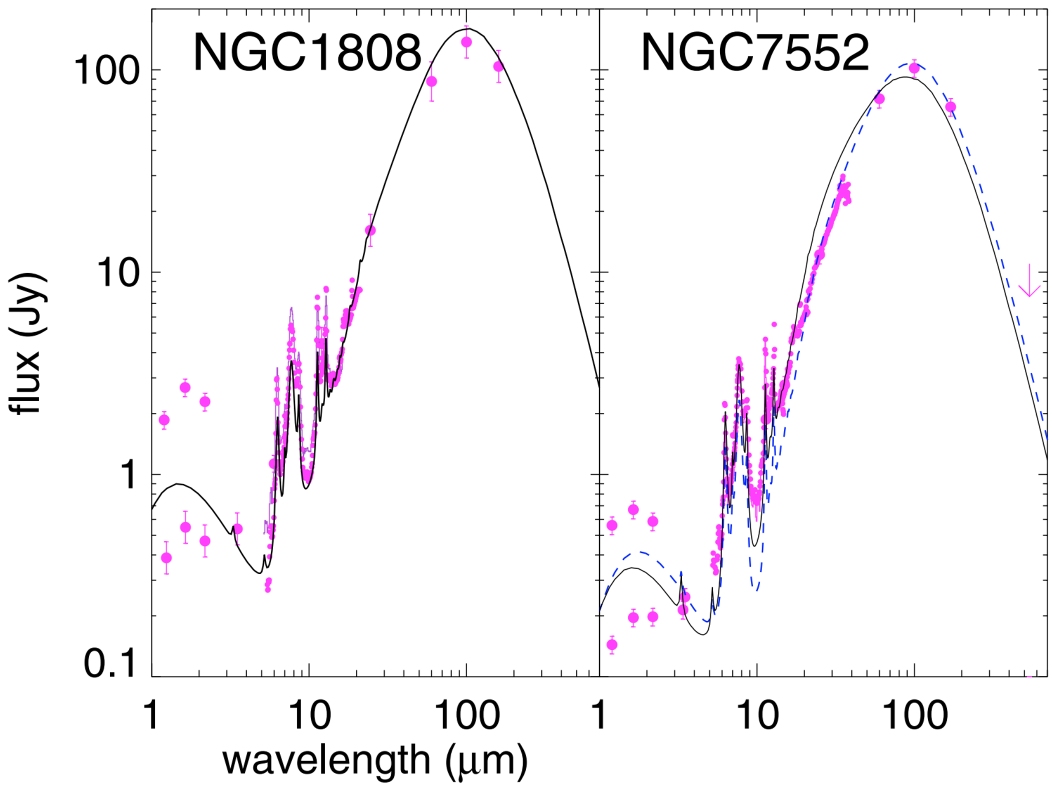

Figure 1: Spectral energy distribution (dominated by dust continuum at long wavelengths) observed toward the spiral galaxy NGC 7552 (right). Figure from Siebenmorgen & Krügel (2007).

(a) Let's revisit the galaxy NGC 7552, which we first saw in Problem Set #1. Figure 1 shows the dust continuum emission from this galaxy. Using just this information, what is a rough estimate of the dust temperature of this galaxy?

(b) We have talked about how gas can be heated and cooled, but how do we know what temperature it is in the first place? One way is to use Boltzmann statistics to measure the excitation temperature T_{ex} between two different energy states of a molecule:

$$\frac{\#\text{ molecules in state E1}}{\#\text{ molecules in state E2}} = exp\left[\frac{\Delta E}{kT}\right]$$

If the different energy states of a molecule are populated with a thermal distribution, then T_{ex} will be a good estimate of the kinetic temperature.

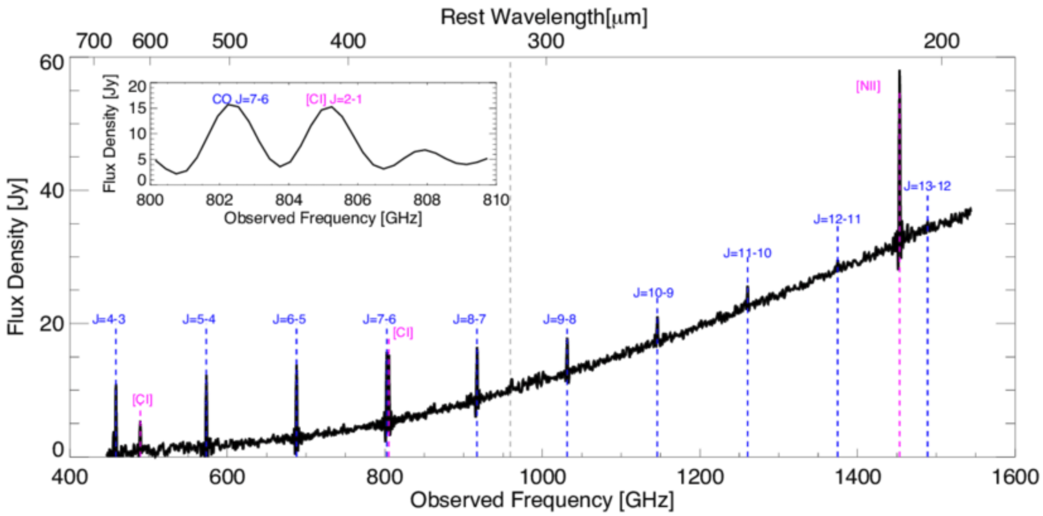

Figure 2: CO lines (on top of dust continuum) observed with Herschel toward the spiral galaxy NGC 7552. Inset shows an enlargement of the spectrum of the CO 7-6 line and a nearby [CI] line. Figure from Rosenberg et al. (2015). The flux of the CO 7-6 line is 1.21×10^{-16} W m^{-2}, and the flux of the CO 13-12 line is 1.83×10^{-17} W m^{-2}.

Assume that the flux of a molecular line is proportional to the number of molecules in the upper energy state (e.g., the flux of the $J = 1-0$ line is proportional to the number of molecules in the J=1 state). What gas temperature would you estimate using the $J = 7-6$ and $J = 13-12$ lines? How does this compare to the dust temperature?

3. HII regions

(a) Estimate the flux of ionizing photons (S_*, with units of photons per time) for an O-star ($T_* = 35{,}000$ K, $R = 10$ R$_\odot$). HINT: Check out DW problem 5.5, and think carefully about the units!!!

(b) What size would a Strömgren sphere around this star be? Assume that the density of the surrounding gas is n = 10^8 m^{-3}.