The ISM and Galaxies: In-Class Problems

Contents

#1 (Continuum Radiation)

The intensity spectrum of free-free (Brehmsstrahlung) radiation as a function of frequency is described by:

$$S(\nu) = T^{-0.5} \exp(-h\nu/kT)\, n_e\, n_i$$

For an ionized nebula (an HII region) with a characteristic temperature of 10,000 K, at what wavelength would you expect the brightness of the HII region to begin to steeply decline?

#2 (Spectral Lines)

Calculate the frequency spread or line width for the Paschen α line of hydrogen (λ_0=1875 nm) emitted from a nebula with a temperature of 10,000 K.

From the textbook, $\Delta\nu^2 = (\nu_0^2\ k\ T) / (\ m\ c^2)$

Assume that the gas in the nebula (being stirred up by winds from a nearby star) has a turbulent velocity width of 30 km/s. How does this compare to the thermal broadening of the line that you calculated above?

#3 (Collisions)

Estimate: How many molecules of air are in our classroom?

#4 (Gas Cooling)

Find the abundance of H_2 in a cloud with density $n(\mathrm{H}) = 10^8\ \mathrm{m}^{-3}$ at a temperature of 100 K that is necessary to give a cooling rate equal to that of C^+ and e^- assuming $n(e^-) = n(C^+) = 10^4\ \mathrm{m}^{-3}$

Recall that at T = 100 K, $\Lambda(H_2) = 10^{-33}\ \mathrm{J\ s^{-1}}$ **per H_2 molecule.**

#5 (Heating Rate)

Show that at roughly T = 15 K that the heating and cooling rates involving C^+ will balance.

#6 (Astrochemistry part 1)

The molecule AB is involved in the following reactions in the interstellar medium, with associated rate coefficients k (with units of reactions s^{-1} m^3 per molecule):

$$A + B \rightarrow AB + h\nu \qquad (k_1)$$

$$A + BC \rightarrow AB + C \qquad (k_2)$$

$$AB + D \rightarrow AD + B \qquad (k_3)$$

$$AB + h\nu \rightarrow A + B \qquad (\beta)$$

(note that the last coefficient is a photodissociation rate coefficient with units of s^{-1} per molecule)

Write an expression for the equilibrium abundance of AB (that is, n_{AB}) in terms of these rates, and the abundances of the various individual reactants.

#7 (Dust)

Supernovae are believed to be responsible for destroying dust in the ISM, as the high shock velocities (> 50 km/s) driven by these explosions are capable of moving free atoms at sufficient speed to knock other atoms out of dust grain lattices. What is the typical lifetime of the dust grains in the Milky Way if they are primarily destroyed in this way?

Assume that the supernova rate in the Milky Way is one per 50 years, and that a single supernova can destroy 300 M_{SUN} of dust. Take the total ISM mass in the Milky Way to be 7×10^9 M_{SUN}.

#8 (Astrochemistry part 2)

Calculate the approximate thermal velocity of air in this classroom!

#9 (Ionization/Recombination)

An O star ($S_* = 10^{49}\ s^{-1}$) illuminates a uniform-density nebula.

How many B stars ($S_* = 2\times10^{47}\ s^{-1}$) are needed to keep the same mass of gas ionized?

(Assume all UV photons are absorbed, and that the densities are the same in both cases.)

How much larger would the Stromgren sphere around a single O star be compared to the Stromgren sphere around a single B star?

#10 (HII Region Cooling)

Two nebulae (A and B) excited by identical stars, are observed to have electron temperatures of $T_{e,A}$ = 9300 K and $T_{e,B}$ = 7000 K. The oxygen : hydrogen abundance ratio in nebula A is $[n_O/n_H]_A = 6 \times 10^{-4}$. What is this ratio for nebula B? (Assume that the only cooling is through O^+ lines, that all of the O present is in the form of O^+, and that the two nebulae have identical n_e).

#11 (Equations of State)

Determine the mean molecular weight (μ) of the following gas mixtures:

(a) A fully-ionized, pure-hydrogen gas

(b) A fully-ionized, pure-helium gas

(c) Neutral gas with solar metallicity ($X=0.7$, $Y=0.28$, $Z=0.02$) where X, Y, and Z are the mass fractions of hydrogen, helium, and metals (you can assume a 'typical' metal is oxygen).

#12 (Sound Speed)

Compare the (isothermal) sound-crossing time and the cooling time (due to C^+) for a molecular cloud at 15 K, with $n(e) = 10^4$ m^{-3}, $n(C^+) = 10^4$ m^{-3}, and $n(H_2) = 10^8$ m^{-3}. Assume the molecular cloud has a length of 10 pc. Is it reasonable to assume that a molecular cloud is an isothermal system?

#13 (Adiabatic Shocks)

A strong shock with a velocity of $v_s = 100$km/s moves into atomic gas with density $n_0 = 10^8$m^{-3}. Assume that we can describe the volumetric cooling rate of the post-shock gas as $L_s = 4 \times 10^{-37} n_f^2$ where L has the typical units of J m^{-3} s^{-1}.

Estimate the cooling time of the post-shock gas.

#14 (ISM Energy Input)

The average mass loss rate of the sun is $10^{-14} M_{\odot}$ per year. Assuming a wind speed of $v \sim 400$ km/s calculate the total kinetic energy released by the solar wind in a year

How does this compare to the total energy radiated by the sun in a year?

#15 (Virial Theorem)

A cloud of molecular hydrogen with a uniform density, a radius of 0.1 pc, and a temperature of 15 K is in virial equilibrium. What is its mass?

Staying in virial equilibrium, the cloud loses energy through radiation, and contracts to half of its original size (no mass is lost in this process). What is its new temperature?

#16 (Star Formation Scales)

If a molecular cloud with a number density of $10^{10}\ m^{-3}$ and a radius of 0.5 pc has just started to collapse, what is its temperature?

#17 (Physics of Star Formation)

Compare the gravitational potential energy of Barnard 68 (a protostellar core in the process of forming a star more massive than the sun) and that of the star Sirius. For simplicity, assume that each of these has a uniform density.

	Radius	Mass
Barnard 68	2×10^{15} m	2 solar masses
Sirius	1×10^{9} m	2 solar masses

#18 (Physics of Stars)

How much longer will a star twice as massive as the sun live compared to a star with twenty times its mass? You can assume each star will fuse the same proportion of its mass from hydrogen into helium.

#19 (Initial Mass Function)

Using the Salpeter initial mass function, for a cluster of 10,000 stars, how many stars have a mass less than the sun?

For a different cluster with a mass of $10^4\ M_{\odot}$, how much of the cluster mass is in stars more massive than the sun?

#20 (Star Clusters)

What is the interaction timescale (time between strong gravitational interactions, for a single “typical” 0.5 solar-mass star) in:

(A) A globular cluster, with a number density of 1000 stars per cubic parsec and typical random speeds of 10 km/s.

(B) The Milky Way, with a number density of 0.1 stars per cubic parsec and typical random speeds of 30 km/s

#21 (Collisional Dynamics)

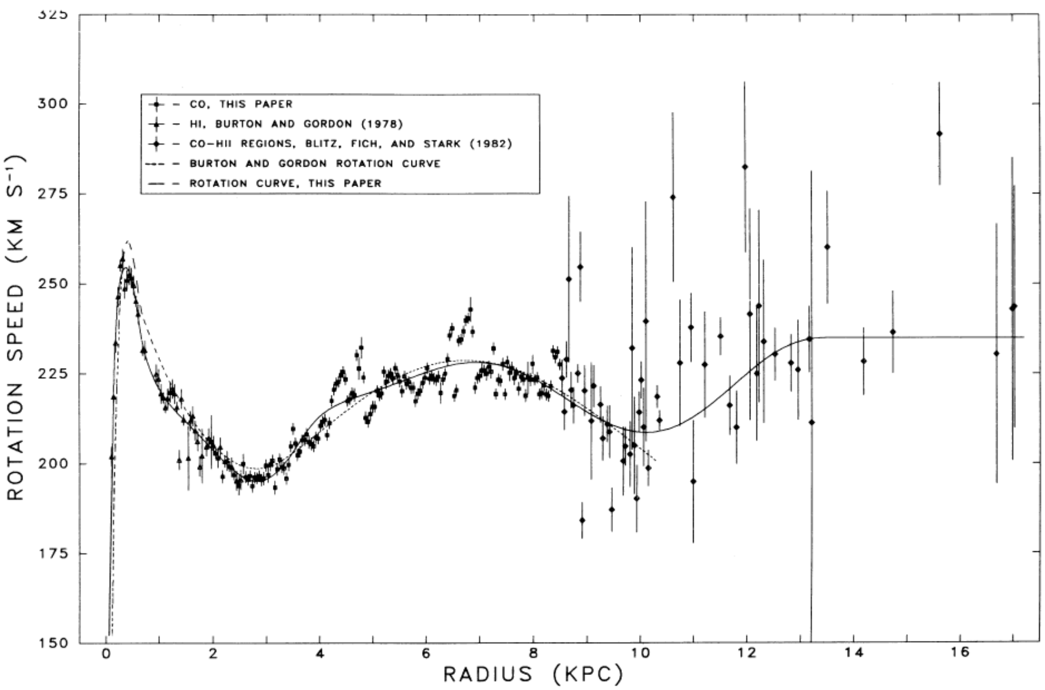
Calculate the relaxation time for a single “typical” 0.5 solar-mass star in the disk of the Milky Way, with a number density of 0.1 stars per cubic parsec, typical random speeds of 30 km/s and $ln(b_{max}/b_{min}) = 18$
(this corresponds to assuming a disk scale height of ~300 pc).

#22 (Dwarf Galaxies)

The galaxy Willman 1 has a stellar mass of about $500\ M_{\odot}$, with a radius of 33 pc.
The velocity dispersion of the stars in this galaxy is $\sigma = 4\ km\ s^{-1}$.
If this galaxy is in virial equilibrium, estimate its total mass, and the ratio of dark matter to luminous matter.

#23 (Galaxy Rotation Curves)

Calculate the mass of the Milky Way interior to the Sun, using the following data



#24 (Black Holes)

The star S-02 has a highly elliptical (e=0.88) orbit that takes it within 17 light-hours of the Milky Way's central supermassive black hole, every 16 years. At its closest approach, it is traveling at ~2.5 percent of the speed of light. Estimate the mass of our Galaxy's central supermassive black hole (Sgr A*). How does this compare to the most recent precision measurements of its mass ($4.15 \times 10^6 M_\odot$)?

Based on the M-σ relation, what would you then estimate is the velocity dispersion (σ or $< v_{RMS} >$) of the Milky Way's bulge? How does this compare with the measured values of 70-100 km/s?

#25 (Galaxies, Groups and Mergers)

Calculate a collision time scale for galaxies in the Local Group. Assume the following group parameters:

- $\sigma = \; < v_{RMS} > \; = 60$ km/s
- $N_{galaxies} = 50$
- $R_{group} = 3$ Mpc
- $R_{galaxy} = 2$ kpc (e.g., the LMC radius)

#26 (Galaxy Scaling Relations)

Using the observed Fundamental Plane relation for elliptical galaxies:

$$R \propto \sigma^{1.4}\, I^{-0.9}$$

And the theoretically-derived relationship between physical properties of a galaxy:

$$L = \frac{\sigma^{4}}{4\pi G^{2}\, I\left(\frac{M}{L}\right)^{2}}$$

Are the observations consistent with all elliptical galaxies having a constant Mass-to-Light ratio $\left(\frac{M}{L}\right)$?

#27 (Galaxy Clusters)

The Coma galaxy cluster (Abell 1656) is a rich galaxy cluster containing more than 10,000 individual galaxies. In the core of this cluster (R= 0.27 Mpc), the number density of galaxies is n~200 Mpc^3, and the typical velocity dispersion is $\sigma = \ < v_{RMS} > \ = 1000$ km/s.

Assuming a typical galaxy radius of 10 kpc, what is the relaxation time for galaxies in the core of this cluster?

How does it compare to the crossing time?

#28 (Supermassive Black Holes and AGN)

Compare the Schwarzschild radius for the black hole at the center of M87 and the black hole at the center of the Milky way