DS 3000: Foundations of Data Science **(Practice)**

Midterm Exam (In-Class Math Practice)

Professor: Dr. Gerber *Name:* _____

Instructions: Please read the below carefully before continuing:

1. Write your name **legibly** in the box above.
2. You have **100 minutes** to complete the exam. There are **??? parts**. You should move on if a part is taking you more than ??? minutes.
3. Write **only on the front side of the exam**; the exams will be scanned into Gradescope one-sided. Anything written on the back will not be considered when grading.
4. You may use a **calculator** (**NOT** a mobile phone) and **one note sheet** as resources on this exam. You may not use any other resources (aside from your brain). You will sign an academic integrity pledge before beginning the exam.
5. Show as much work as possible to obtain partial credit. Correct answers **without supporting work will NOT receive any credit**. Also **clearly indicate your answers by circling/putting boxes around them**.
6. Round all numeric answers with decimals to **3 decimal places**.
7. When you finish the exam, please bring the exam to the front of the room. You can keep your note sheet. You may leave as soon as you hand in your exam.

Failure to sign the below academic integrity pledge will result in a score of 0 on this exam!

Academic Integrity Pledge

I pledge on my personal honor and integrity that the work on this exam is entirely my own, and that no outside sources were used in helping me answer the questions.

Signature:_____

Problem 1: Vectors (12 points)

Find the resultant vector and its magnitude (Euclidean length/L2-norm) given the vectors in each part:

1. Find $p \odot q + r \cdot s$ and it's length.

$$p = \begin{bmatrix} -7 \\ -6 \end{bmatrix} \quad q = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad r = \begin{bmatrix} -3 \\ -9 \end{bmatrix} \quad s = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

2. Find $-3a - 7(b \odot c)$ and it's length.

$$a = \begin{bmatrix} -3 \\ -1 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ \frac{1}{7} \end{bmatrix} \quad c = \begin{bmatrix} \frac{1}{4} \\ 2 \end{bmatrix}$$

Problem 2: Span (12 points)

Is the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the span of the following set in R^3?

$$T = \left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Problem 3: Matrix Operations I (16 points)

Given:

$$w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, Y = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}, Z = \begin{bmatrix} 0 & 5 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

Perform the following operations. If the operation is not possible, indicate the reason. You must show all work/derivations to receive full credit.

1. $Y^T w$
2. $Y \odot Z$
3. wZ
4. YZ^T

Problem 4: Matrix Operations II (12 points)

Mr. Hwan writes the following matrix on the board and asks his students to write down a matrix that they could multiply to that matrix (i.e., that would work as a right matrix in a matrix multiplication):

$$\begin{bmatrix} -3 & 1 \\ 2 & 0 \\ 4 & -3 \end{bmatrix}$$

Three students, Wanda (W), Xavier (X) and Zach (Z) write:

$$W = \begin{bmatrix} -1 & 2 \\ 5 & 3 \\ 0 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & -3 \\ 7 & 1 \end{bmatrix}$$

$$Z = \begin{bmatrix} -4 & 1 & 0 \\ 8 & 5 & -3 \end{bmatrix}$$

"One of you has made a mistake," Mr. Hwan says. Is Mr. Hwan correct? Who made the mistake? Calculate the final matrices for those who did not make a mistake.

Problem 5: Projections I (12 points)

For each of the below, find the point in the span of the $\vec{a}$ vectors closest to the $\vec{b}$ vector.

1. $\vec{a} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

2. $\vec{a_0} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\vec{a_1} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

3. $\vec{a_0} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $\vec{a_1} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$

Problem 6: Projections II (12 points)

Let $W = \text{span}\{v_1, v_2\}$ where

$$v_1 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Find the closest point $\vec{w}$ in W to:

$$\vec{x} = \begin{bmatrix} 0 \\ 14 \\ -4 \end{bmatrix}$$

Problem 7: Dependence/Independence (12 points)

Decide whether each set of vectors is linearly dependent or linearly independent.

1. $\left\{\vec{a}_1 = \begin{bmatrix}-1\\4\end{bmatrix}, \vec{a}_2 = \begin{bmatrix}2\\-8\end{bmatrix}\right\}$

2. $\left\{\vec{a}_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}, \vec{a}_2 = \begin{bmatrix}4\\5\\6\end{bmatrix}, \vec{a}_2 = \begin{bmatrix}7\\8\\9\end{bmatrix}\right\}$

3. $\left\{\vec{a}_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}, \vec{a}_2 = \begin{bmatrix}1\\2\\3\end{bmatrix}, \vec{a}_2 = \begin{bmatrix}1\\3\\4\end{bmatrix}\right\}$

Problem 8: Eigenvectors/Eigenvalues (12 points)

Find the eigenvalues and eigenvectors for the following matrices

1. $$A = \begin{bmatrix} -1 & -2 \\ -3 & 1 \end{bmatrix}$$
2. $$B = \begin{bmatrix} 2 & 0 \\ 4 & -4 \end{bmatrix}$$
3. $$C = \begin{bmatrix} 1 & 1 \\ 0 & 5 \end{bmatrix}$$