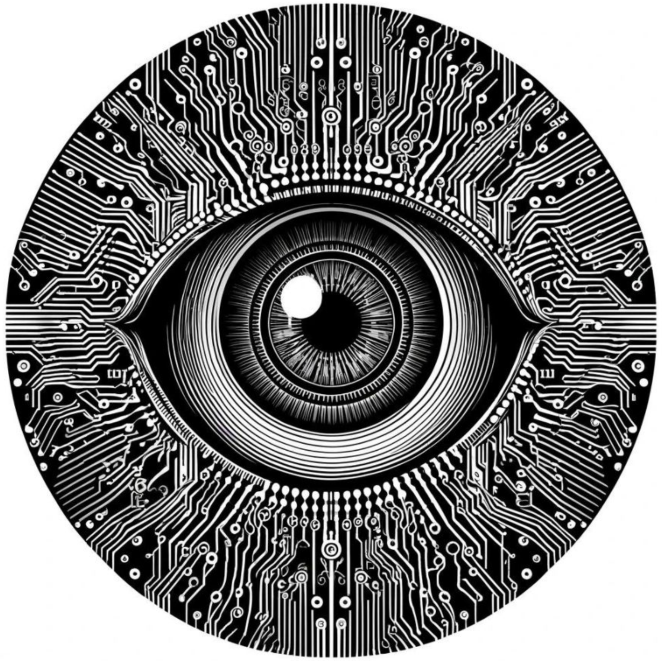
Building a Multilayer Perceptron for Regression in PyTorch



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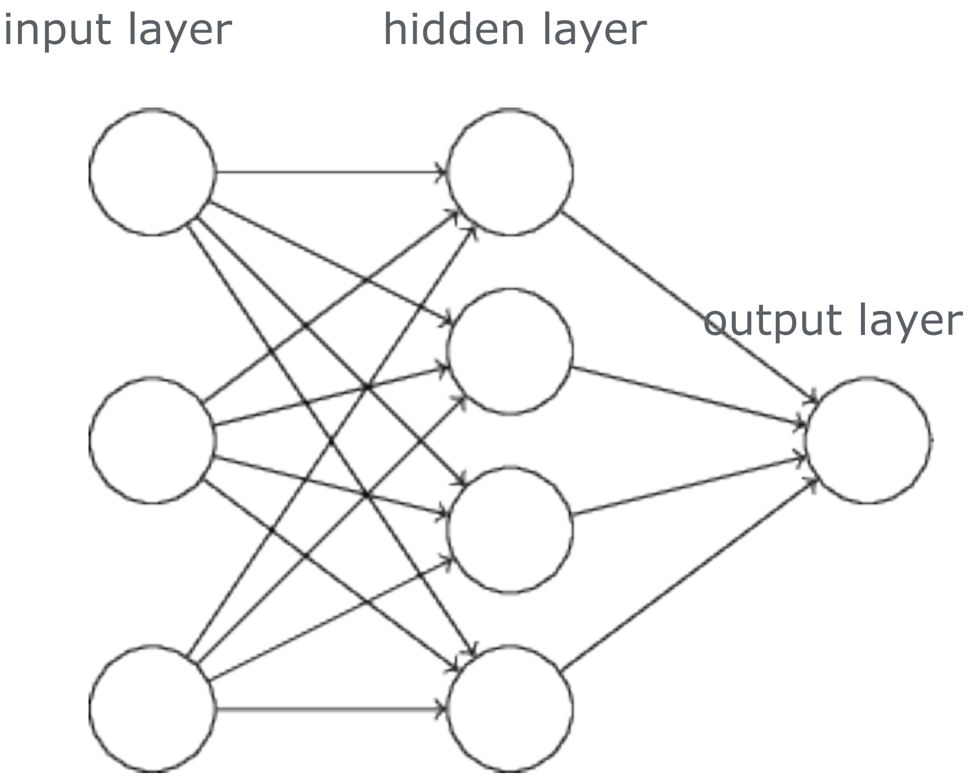
Learning goals

- Implement a fully connected network in PyTorch with `nn.Linear` and `nn.Sequential` syntax
- Explore the necessary agreements between processing units and input size
- Understand the motivations for image input resizing
- Code input units, hidden layers, and output units
- Examine the representation of weights, gradients, and loss in PyTorch

A minimal network




```python
import torch.nn as nn

# Input layer to hidden layer (3 -> 4)
i_h_layer = nn.Linear(in_features=3, out_features=4, bias = False)

# Hidden layer to output layer (4 -> 1)
h_o_layer = nn.Linear(in_features=4, out_features=1, bias = False)


# Connecting the layers, notice that 'hidden layer' is implicit
model = nn.Sequential(
            i_h_layer,
            nn.ReLU(),
            h_o_layer,
        )

# The input has as many entries as we have input units
input_data = torch.tensor([1.0, 2.0, 3.0], dtype=torch.float32)

# Feedforward pass
prediction = model(input_data)

# The value that we want to match
target = torch.tensor([11.], dtype=torch.float32)

# Our measure of error
loss_function = nn.L1Loss()

# Evaluate how good our prediction is
loss_value = loss_function(target, prediction)
```

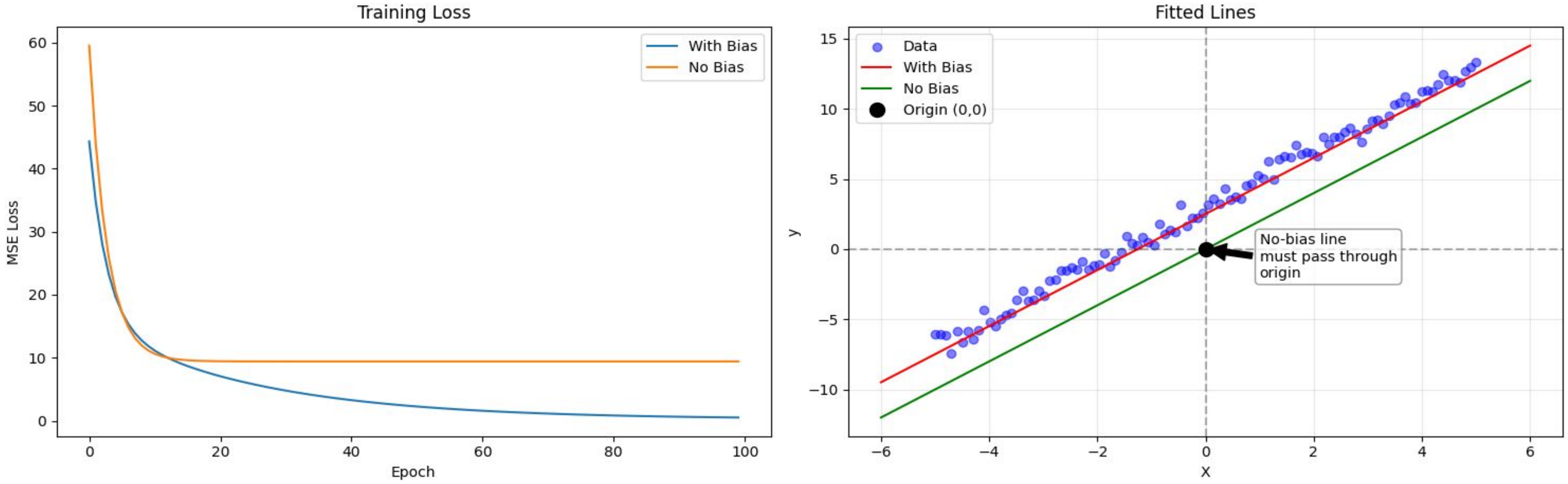
Inspecting nn.Linear

```python
import torch
import torch.nn as nn

# Define a linear layer
layer = nn.Linear(in_features=4, out_features=1, bias=True)


# Examine parameters, weights and bias
for name, param in layer.named_parameters():
    print(f"{name}: {param.shape}")
    print(param)

# Forward pass
x = torch.randn(5, 4)  # Batch of 5 samples, 4 features each
output = layer(x)

# Batch of 5 samples, single output value
print(output.shape)
```

$$z = \mathbf{WX} = \begin{bmatrix} w_1 & w_2 & \cdots & w_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \sum_{i=1}^{m} w_i x_i$$

```
weight: torch.Size([1, 4])
Parameter containing:
tensor([[-0.0576,  0.3385, -0.3604, -0.1795]], requires_grad=True)
bias: torch.Size([1])
Parameter containing:
tensor([0.3461], requires_grad=True)
```

Bias vs no bias

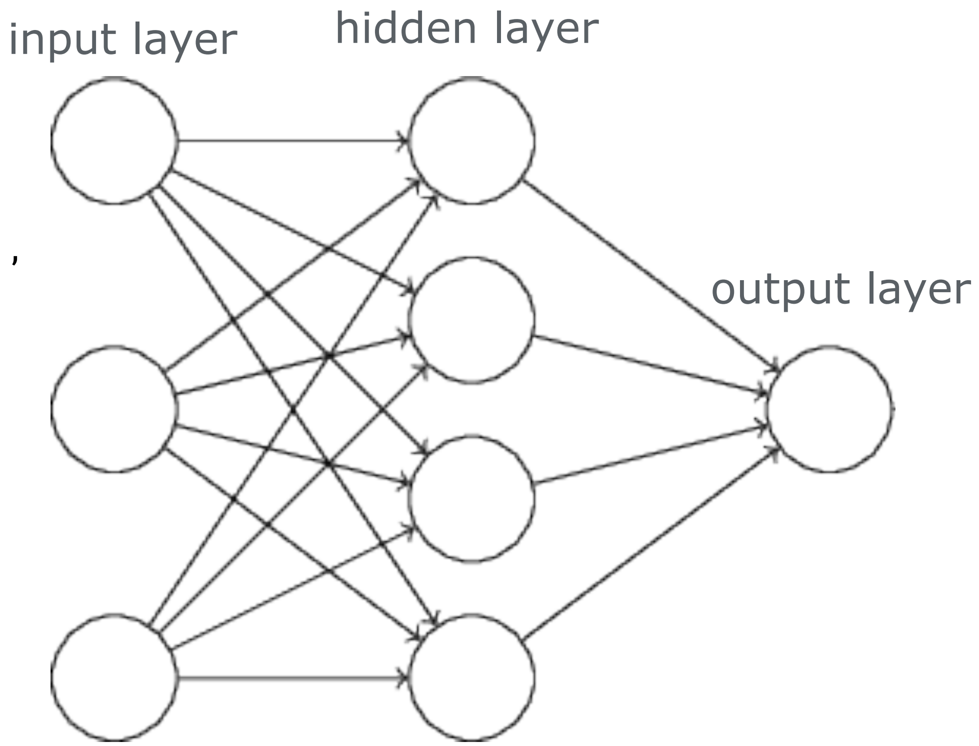


$$\text{Mean Squared Error (MSE) Loss} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

A Colab demo notebook

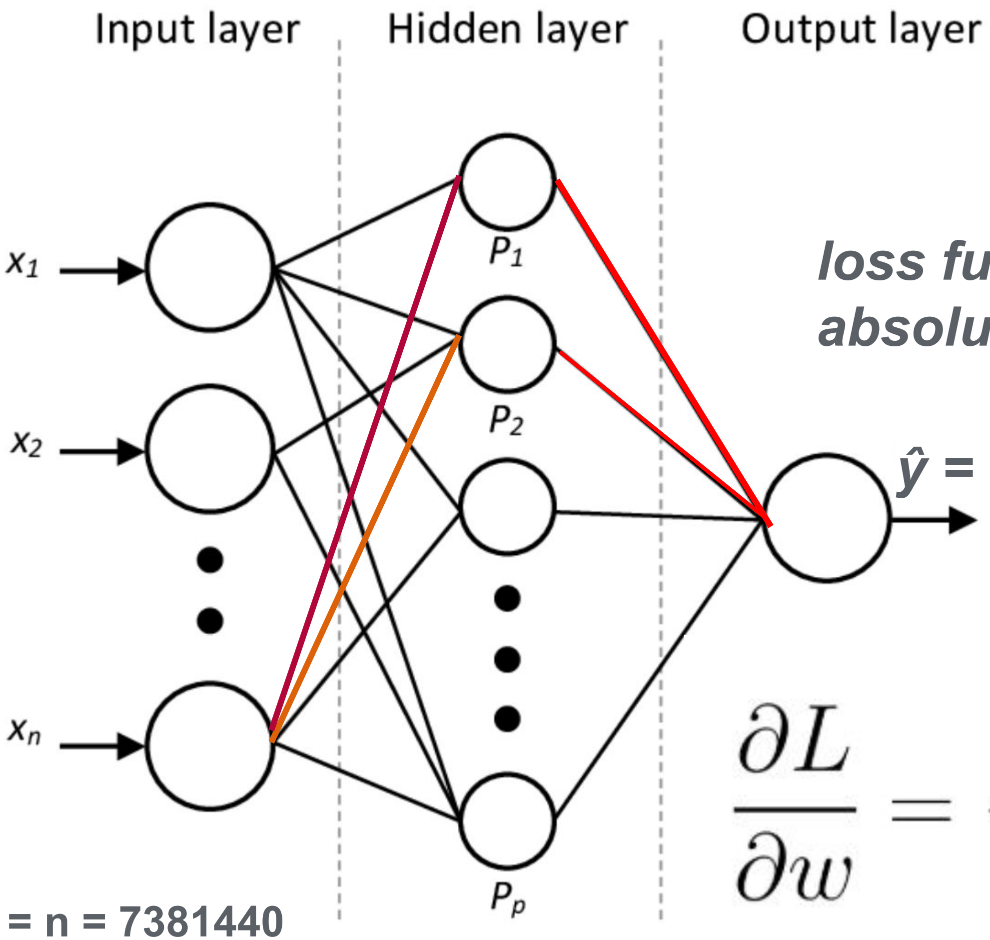
Inspecting nn.Sequential

```python
# Connecting the layers,
# notice that 'hidden layer' is implicit

model = nn.Sequential(
          nn.Linear(in_features=3, out_features=4, bias = False),
          nn.ReLU(),
          nn.Linear(in_features=4, out_features=1, bias = False)
        )

# Forward pass
x = torch.randn(5, 3) # Batch of 5 samples, 3 features each
output = model(x)


# Batch of 5 outputs, each one having one regression target value
print(output.shape) # prints torch.Size([5, 1])
```



Fully connected networks (aka multilayer perceptrons)

y = 83999 EUR



loss function =
absolute error = |ŷ - y| = 12188

$\hat{y} = f(x_1, \ldots, x_n) = 71811$

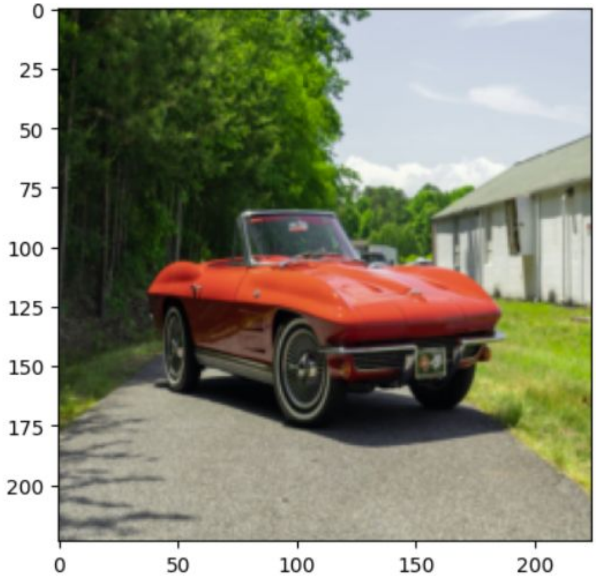
$$\frac{\partial L}{\partial w} = -\,\mathrm{sign}(y - f(x))\frac{\partial f(x)}{\partial w}$$

```
H = 1920
W = 1282
C = 3 (RGB)
```

C * H * W = n = 7381440

Resizing the input to reduce the number of parameters

```
transforms.Compose([
 transforms.ToImage(),
 transforms.Resize((224, 224)),
transforms.ToDtype(torch.float32,scale=True)
])
```



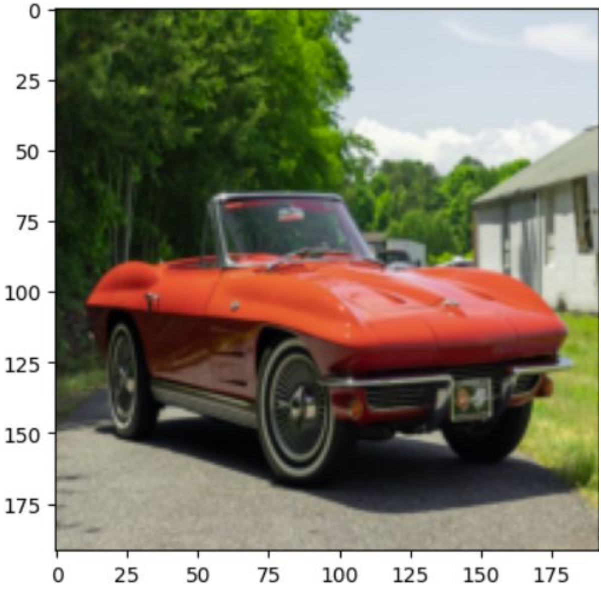
H = 1920
W = 1282
C = 3 (RGB)

H = 224
W = 224
C = 3 (RGB)

CenterCrop to capture more of the object of interest

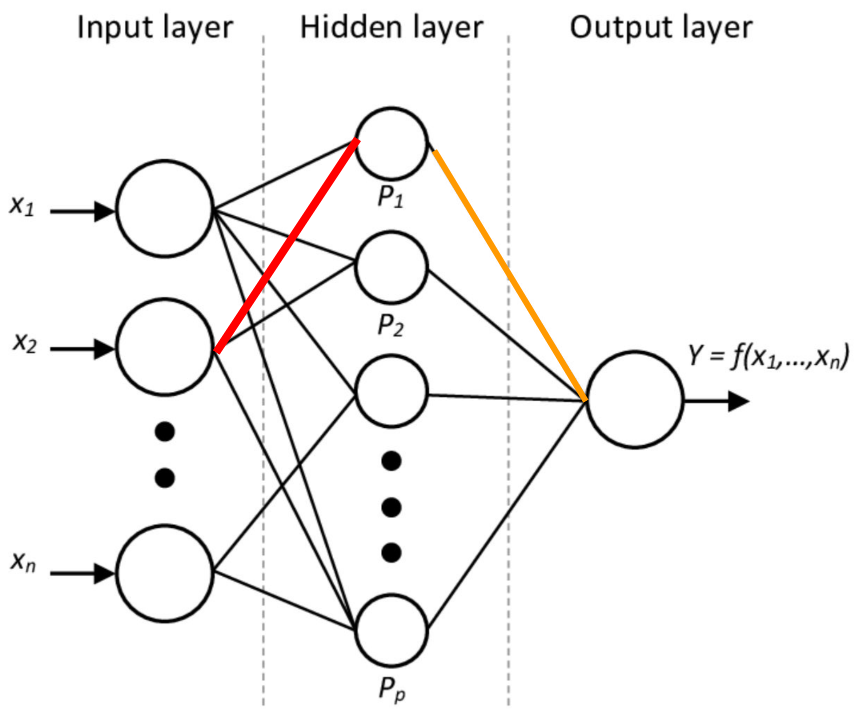
```
H = 1920
W = 1282
C = 3 (RGB)
```

```
transforms.Compose([
 transforms.ToImage(),
 transforms.Resize((256, 256)),
 transforms.CenterCrop(192),
transforms.ToDtype(torch.float32,scale=True)
])
```




```
H = 192
W = 192
C = 3 (RGB)
```

Stochastic gradient descent - the weight update rule



w_{ij} = weight from neuron i to neuron j

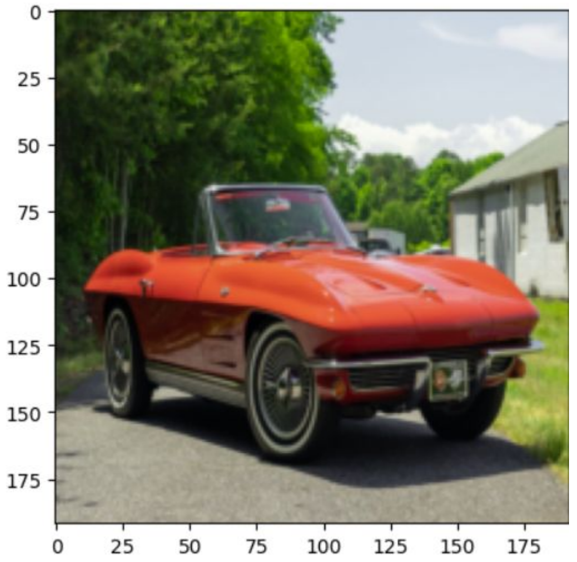
learning_rate = step size for updates, a constant like 0.01

L = loss function

$$w_{ij} = w_{ij} - (\text{learning rate} * \frac{dL}{dw_{ij}})$$

Implementing the network

```python
# Number of units on the hidden layer, this number is arbitrary

p = 100

# We use nn.Sequential to concatenate PyTorch modules in order

multilayer_perceptron=nn.Sequential(

                      nn.Flatten(), # Flattens the images within a batch

                      nn.Linear(in_features= C * H * W, # This matches the image shape

                                out_features= p,

                                bias=True),

                      nn.ReLU(),

                      nn.Linear(in_features=p,

                                out_features=1, # We are predicting a single value

                                bias=True),

)
```



H = 192
W = 192
C = 3 (RGB)

y = 83999 EUR

Implementing the weight update rule

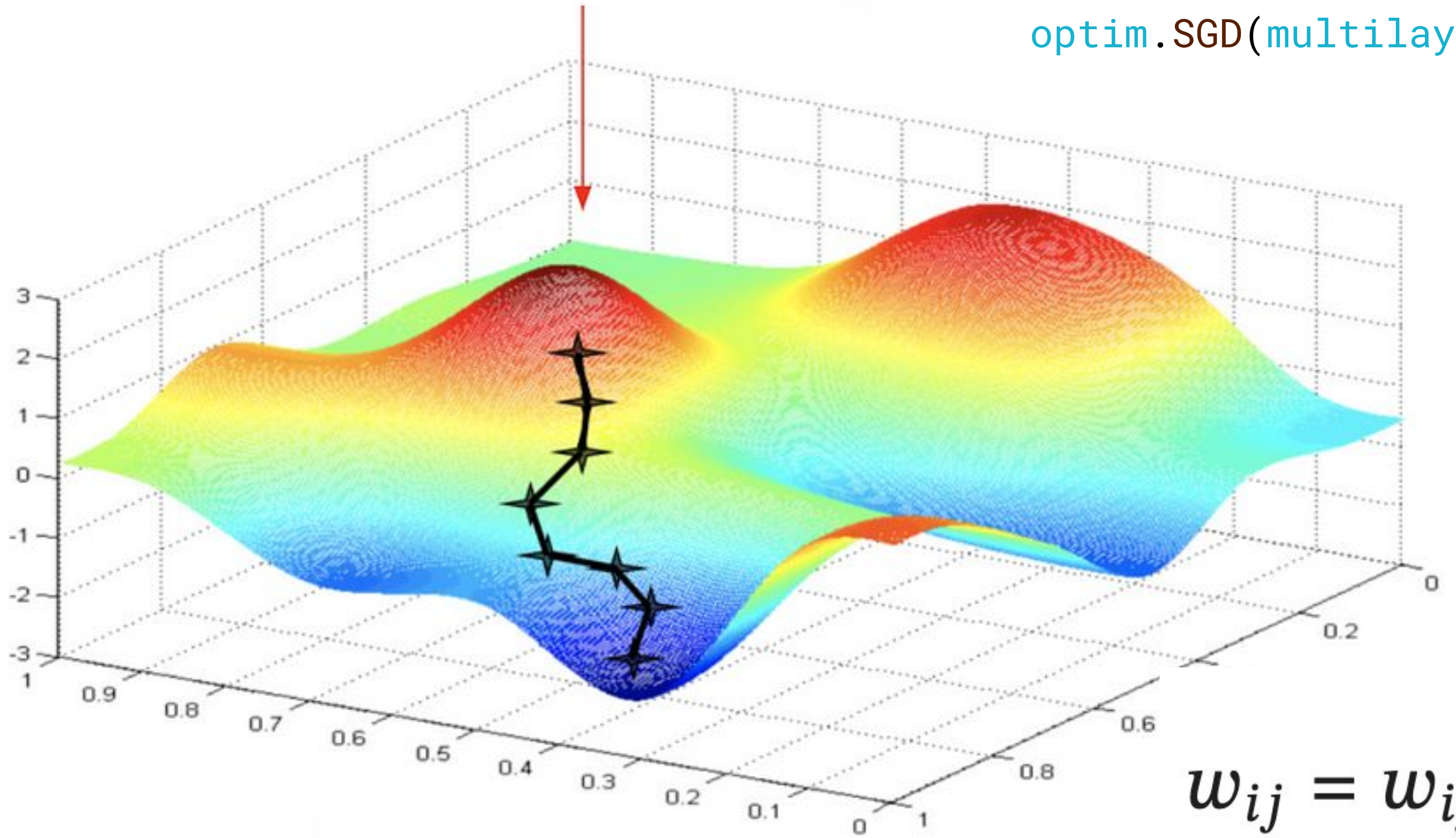
```python
def update_rule(weight, grad, learning_rate):
    return nn.Parameter((weight - learning_rate * grad))

learning_rate = 0.1
new_weights_0 = update_rule(multilayer_perceptron[0].weight,
                            multilayer_perceptron[0].weight.grad,
                            learning_rate)

new_weights_2 = update_rule(multilayer_perceptron[2].weight,
                            multilayer_perceptron[2].weight.grad,
                            learning_rate)

# We reassign the weights to our model
multilayer_perceptron[0].weight = new_weights_0
multilayer_perceptron[2].weight = new_weights_2
```

Image from ideogram.ai

$$w_{ij} = w_{ij} - (\text{learning rate} * \frac{dL}{dw_{ij}})$$

Training with an optimizer

```python
import torch.optim as optim

optim.SGD(multilayer_perceptron.parameters(), lr=1e-2)
```



$$w_{ij} = w_{ij} - (\text{learning rate} * \frac{dL}{dw_{ij}})$$

Training with an optimizer

```python
import torch
import torch.nn as nn
import torch.optim as optim

# Define an Stochastic Gradient Descent optimizer
optimizer = optim.SGD(multilayer_perceptron.parameters(), lr=1e-2)

# Define a loss function
criterion = nn.L1Loss()

# Forward pass
pred = multilayer_perceptron(x)

# Compute loss
loss = criterion(pred, y)

# Backprop
optimizer.zero_grad()
loss.backward()

# Update weights
optimizer.step()
```

$$w_{ij} = w_{ij} - (\text{learning rate} * \frac{dL}{dw_{ij}})$$

The importance of the learning rate

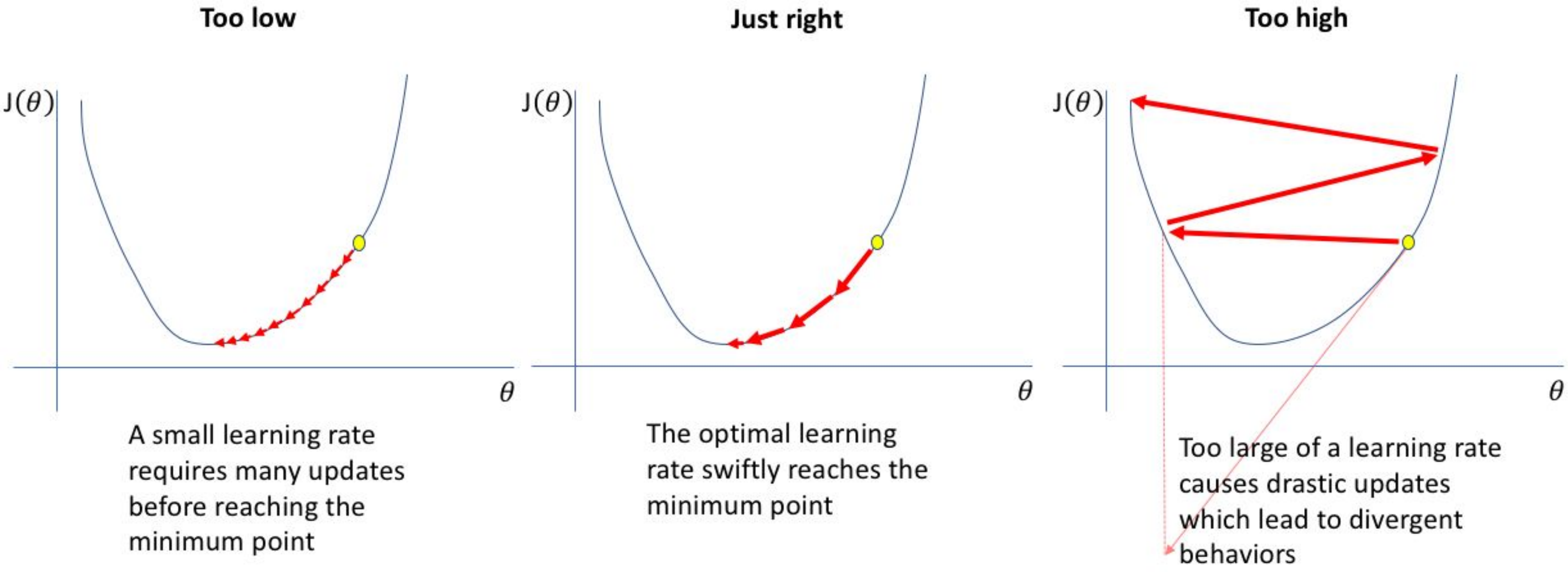


Image from Deep Learning for Coders

Summary

PyTorch building blocks

- Multilayer perceptrons in PyTorch combine `Linear` layers, activation functions, and loss functions. The framework handles gradient computation through autograd.
- We can use the `Sequential` syntax to wrap up components, this is an alternative to subclassing `nn.Module`

Input processing

- Image input requires preprocessing through resizing and normalization to create manageable feature tensors. The input size determines the network architecture.

Model training

- Training combines forward passes, loss computation, backpropagation, and weight updates. The learning rate controls optimization stability and convergence speed.

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Further reading

What is torch.nn really?

- https://pytorch.org/tutorials/beginner/nn_tutorial.html

Tensorflow playground

- https://playground.tensorflow.org/

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