KI Service Zentrum
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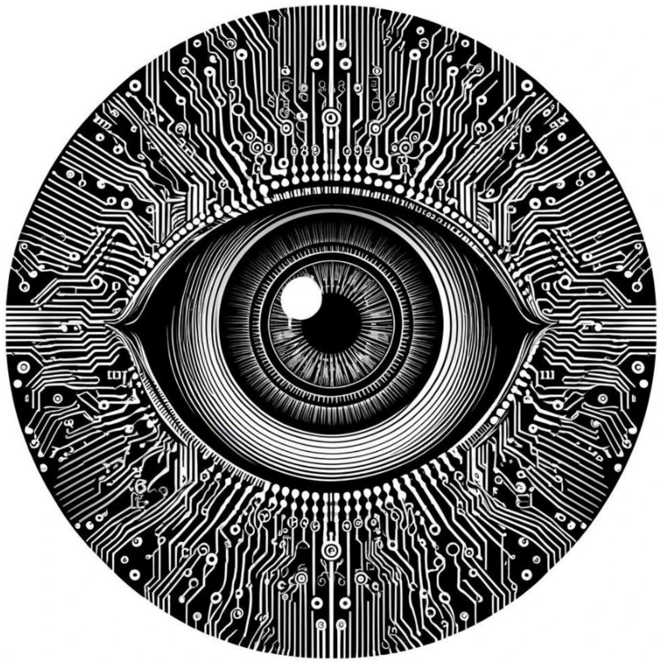
Introduction to Neural Networks



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Learning goals

- Explain how neural networks learn during training
- Identify the components of input units, hidden layers, and output units
- Apply concepts of weights, gradients and loss
- Understand the purpose of validation and test data

What is an artificial neural network?

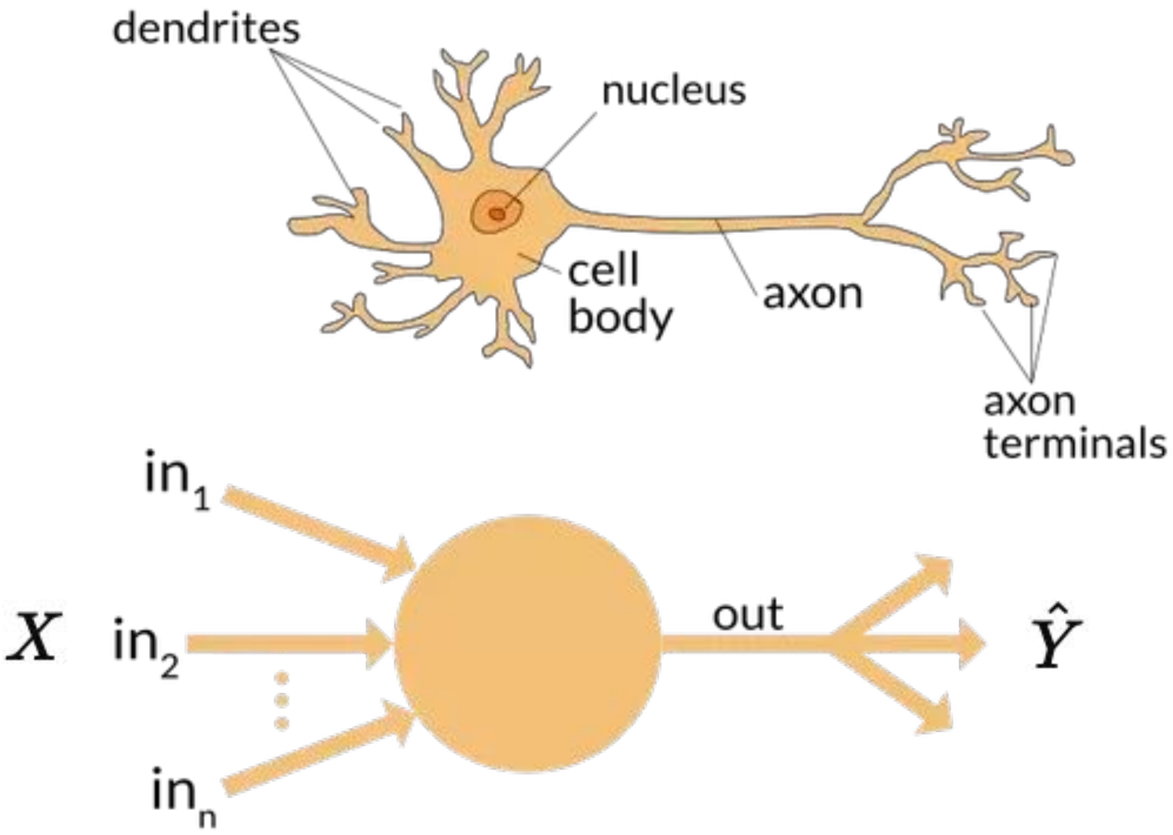
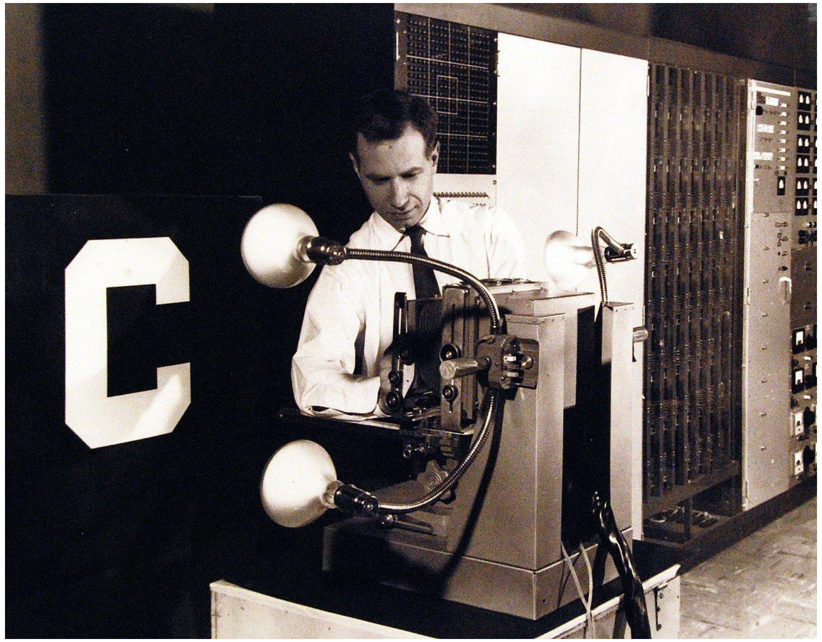
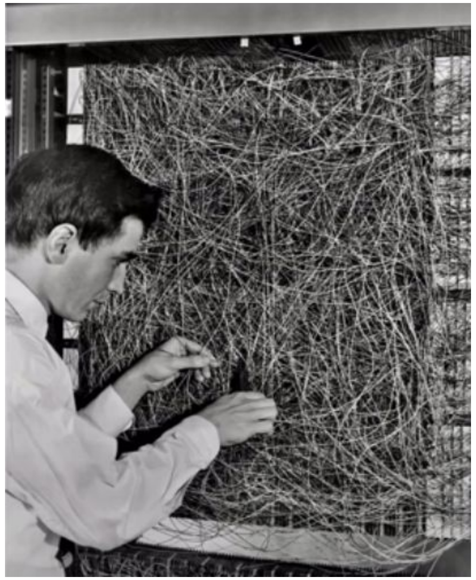


Image from source

- Artificial “neurons” are also called “processing units”
- We can think of them as functions that map X inputs to Ŷ outputs

The perceptron





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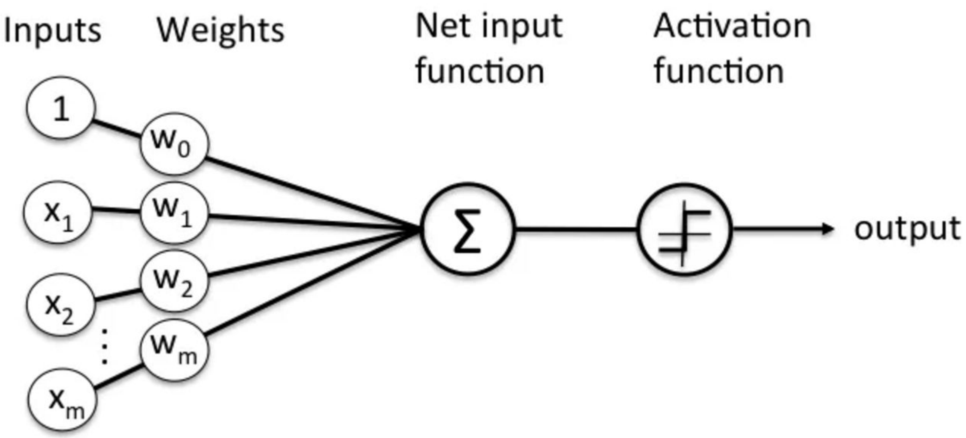
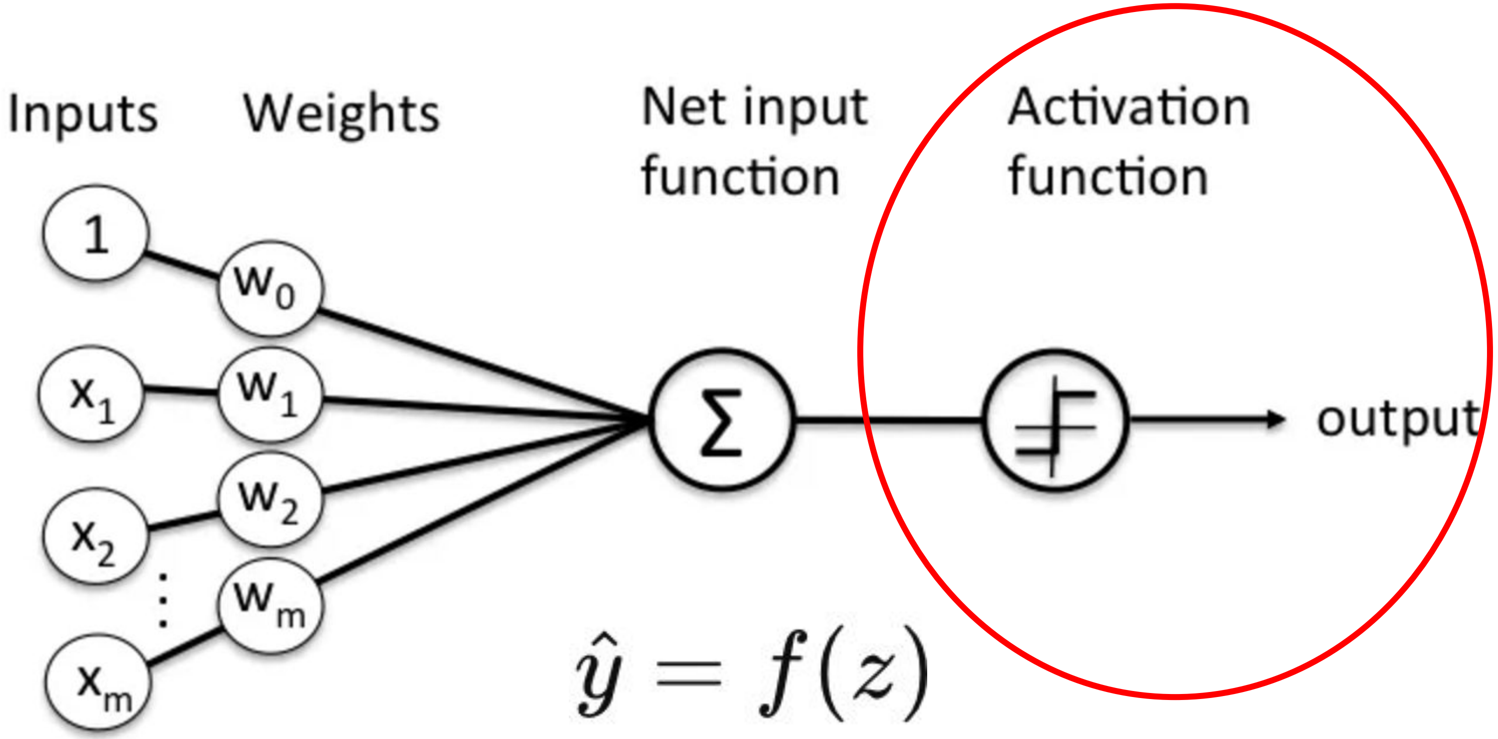


$$\hat{y} = f\left(\sum_{j=0}^{m} w_j x_{ij}\right)$$

Images: Mark I Perceptron at Cornell University (1961) source

A modern fully connected layer

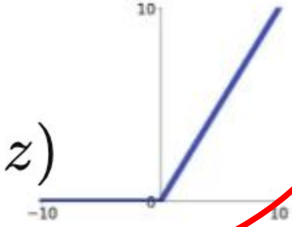


$$\hat{y} = f(z)$$

$$z = \sum_{i=0}^{m} w_i x_i$$

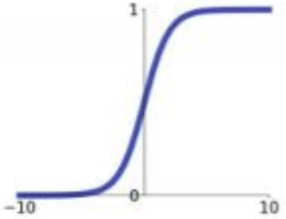
ReLU

$$f(z) = \max(0, z)$$



Sigmoid

$$f(z) = \frac{1}{1 + e^{-z}}$$



Making a perceptron learn the price of a vintage car

training

1964 Chevrolet Corvette Stingray

Image source

y = 83999 EUR (real price, target value)

ŷ = 84000 EUR (a possible prediction)

Absolute error = |ŷ - y | = 1 EUR

validation

Image source

testing

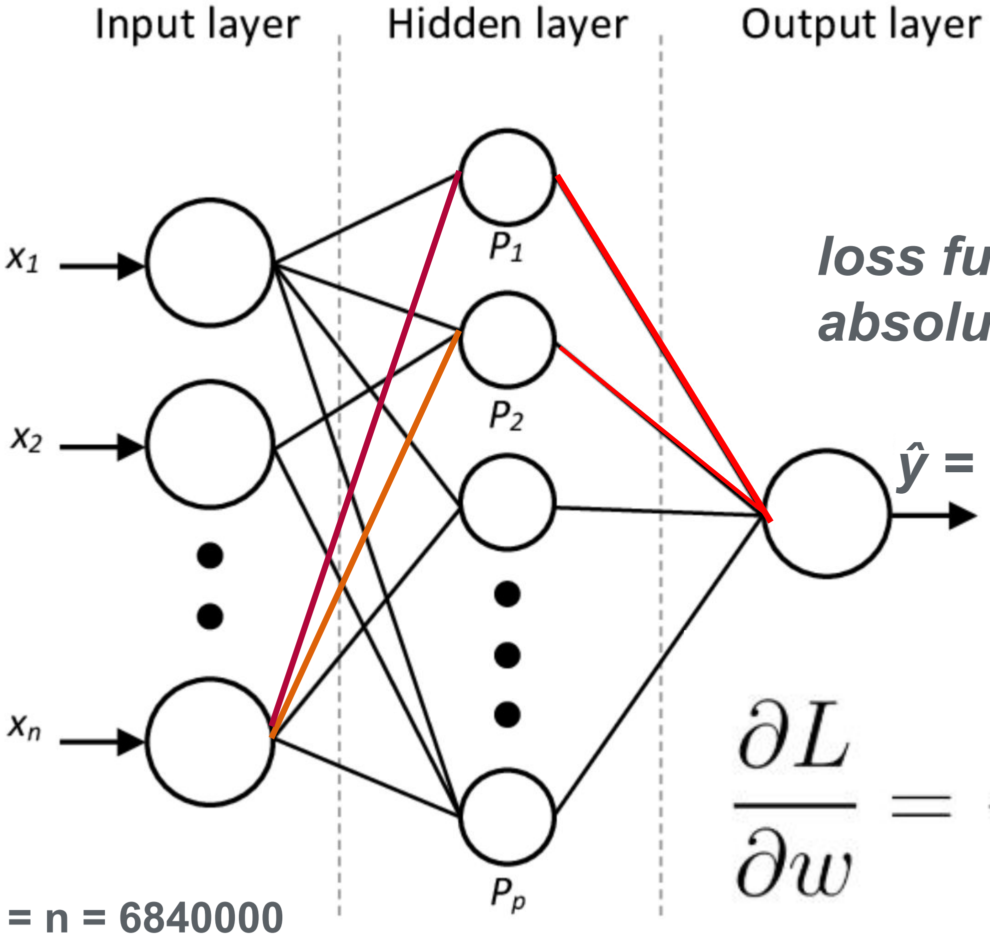
Image source

Training a fully connected network (multilayer perceptron)

y = 83999 EUR

H = 1900
W = 1200
C = 3 (RGB)

C * H * W = n = 6840000

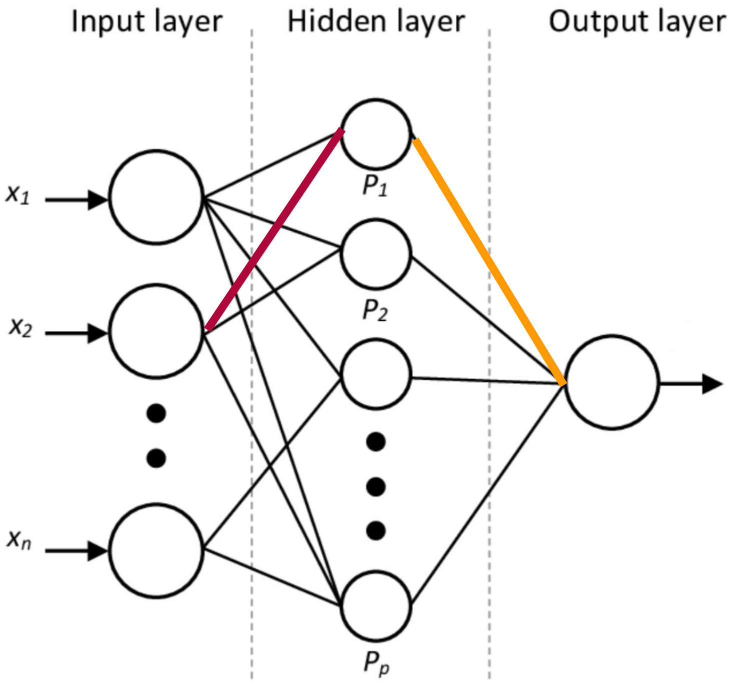


loss function =
absolute error = $|\hat{y} - y|$ = 12188

$\hat{y} = f(z)$ = 71811

$$\frac{\partial L}{\partial w} = -\operatorname{sign}(y - f(x))\frac{\partial f(x)}{\partial w}$$

Understanding network weights and their updates



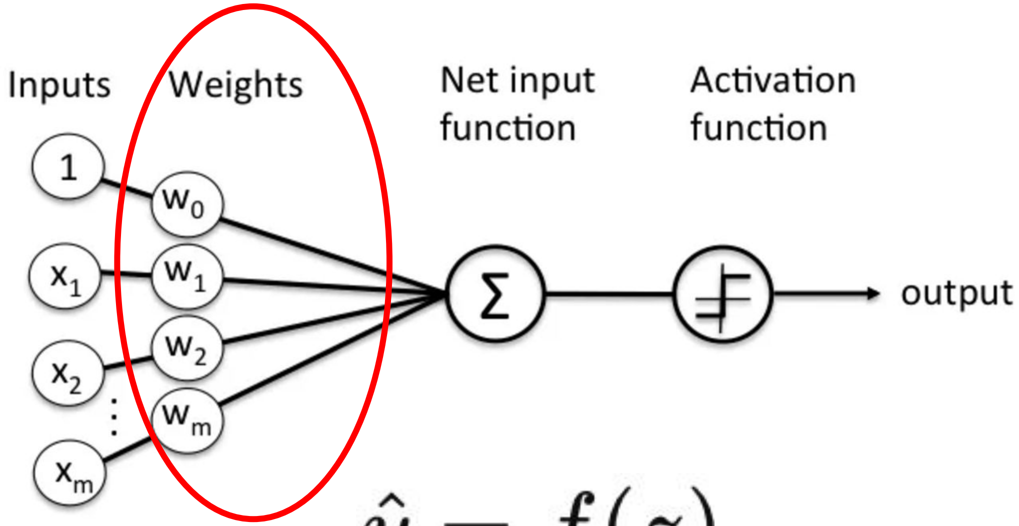
w_{ij} = weight from neuron i to neuron j

learning_rate = step size for updates, a constant like 0.01

L = loss function

$$w_{ij} = w_{ij} - (\text{learning rate} * \frac{dL}{dw_{ij}})$$

Modern perceptrons use loss functions as feedback



$$\hat{y} = f(z)$$

$$z = \sum_{i=0}^{m} w_i x_i$$

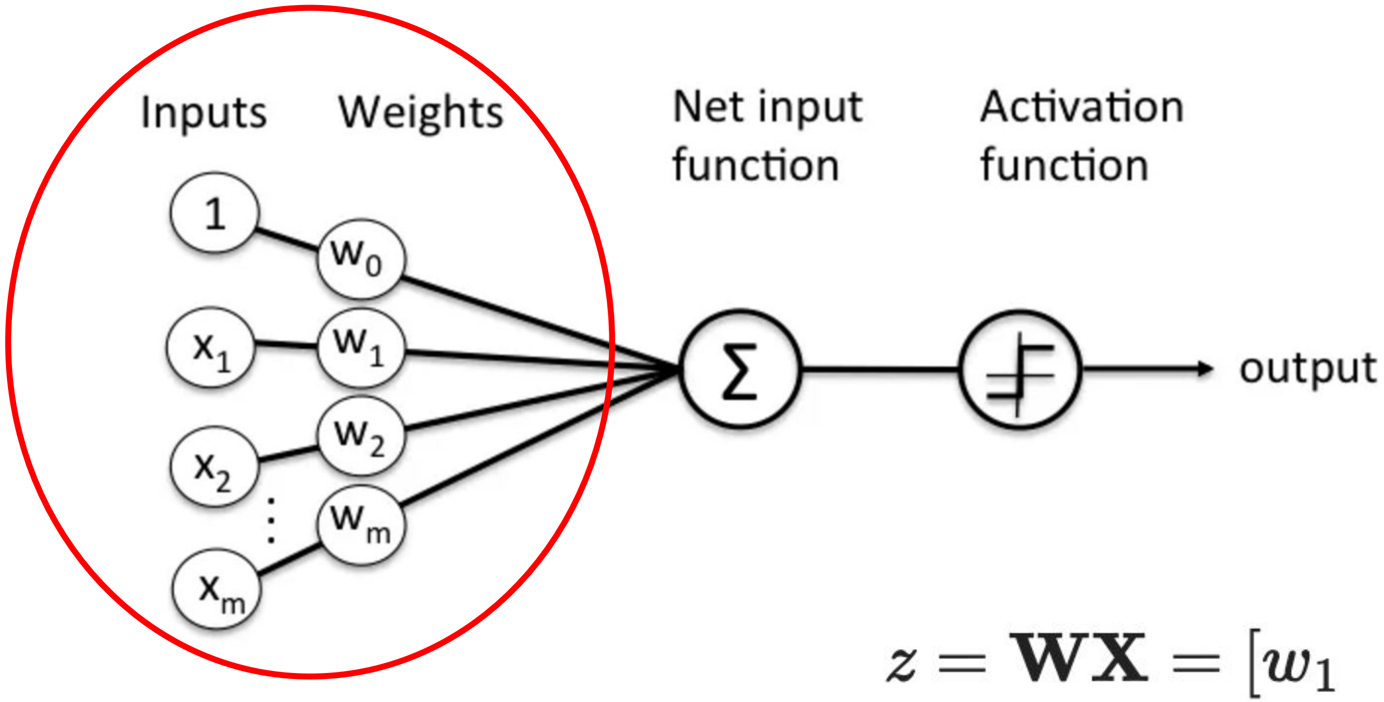
Regression

$$\text{Mean Absolute Error} = \frac{1}{N}\sum_{i=1}^{N} |y_i - \hat{y}_i|$$

Classification

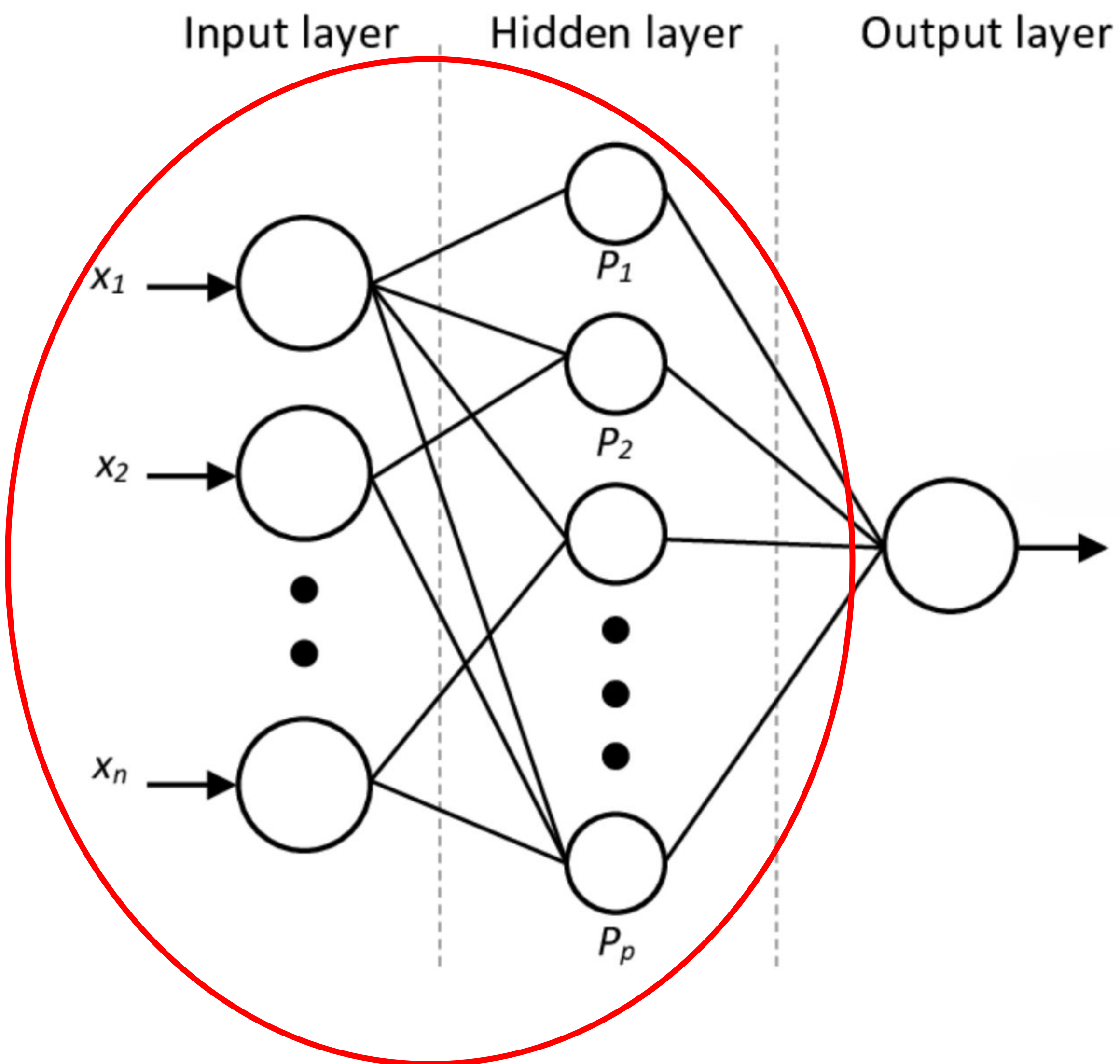
$$\text{Cross Entropy} = -\frac{1}{N}\sum_{i=1}^{N}\sum_{c=1}^{C} y_{i,c}\log(\hat{y}_{i,c})$$

Matrix multiplication: features times weights



$$z = \mathbf{WX} = \begin{bmatrix} w_1 & w_2 & \cdots & w_m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \sum_{i=1}^{m} w_i x_i$$

Dimensions of the weight matrix define the output's

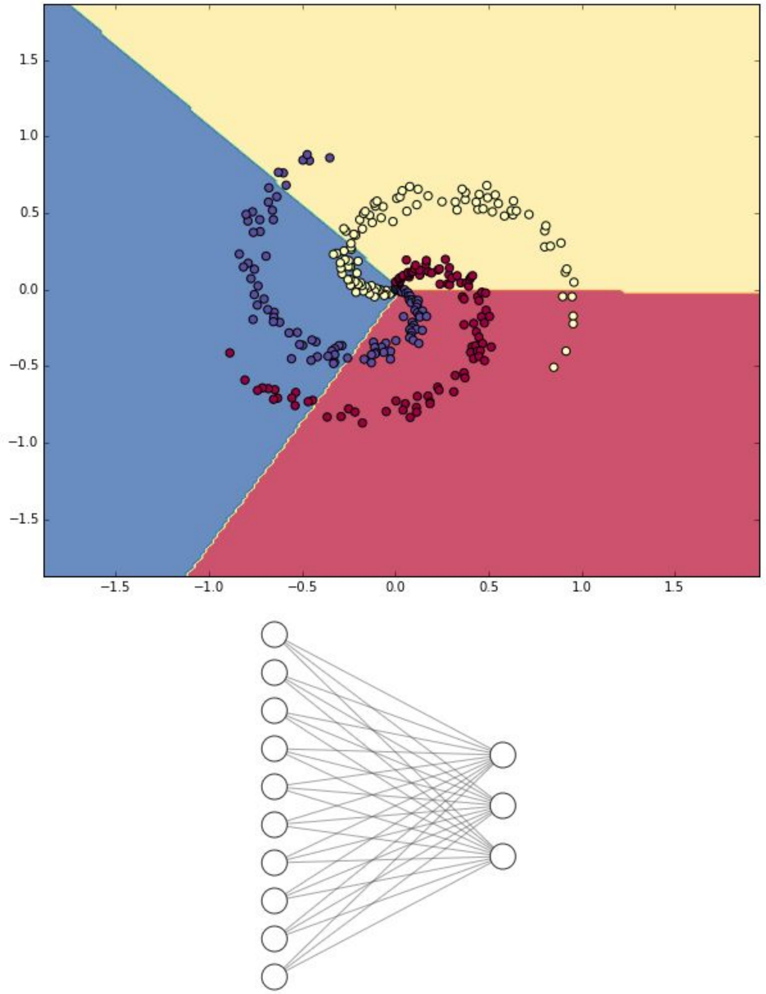


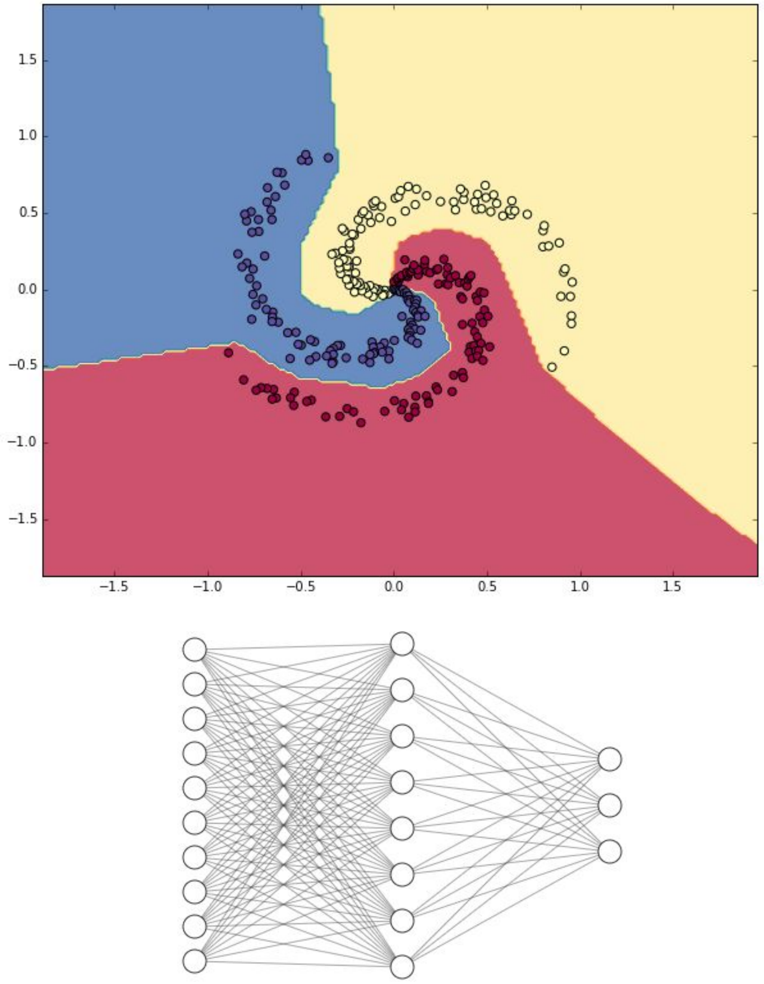
$$P = WX$$

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_p \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{p1} & w_{p2} & \cdots & w_{pn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

For example: $p = 4 \quad n = 3$

Why do we add hidden layers?





Validating and testing the model

sample to decide when to stop training: validation

***y* = 82999**
Image source

sample to use as a final benchmark: test

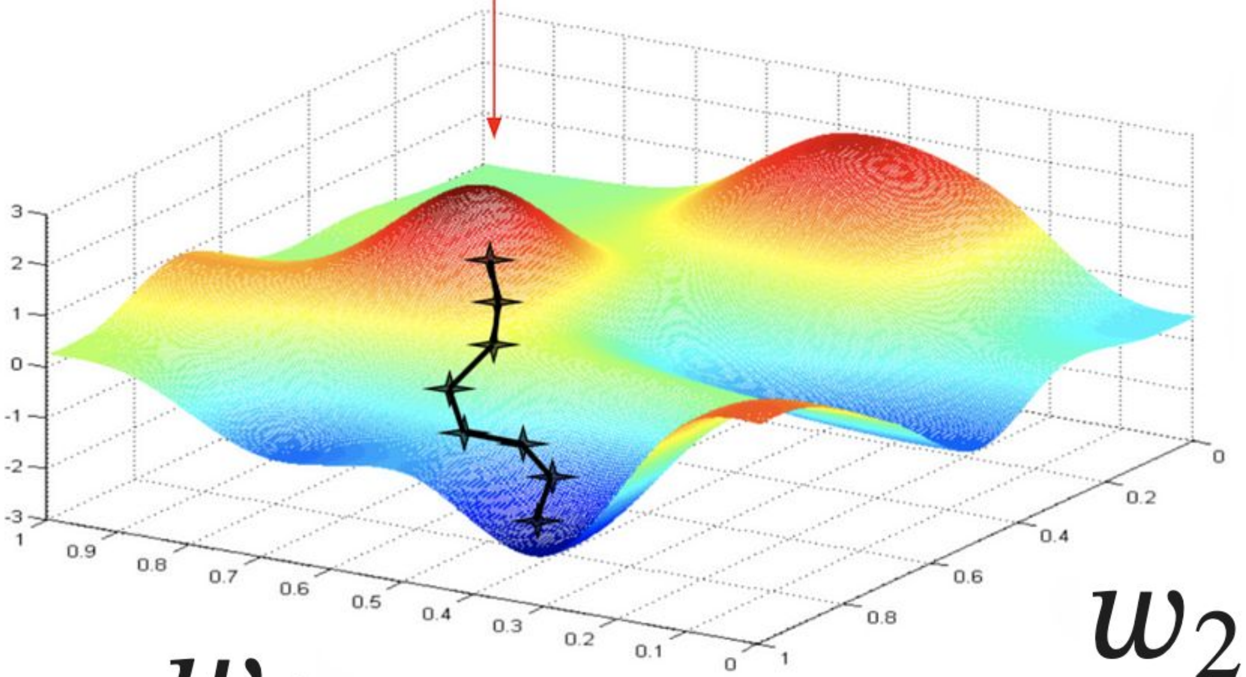
***y* = 82699**

Image source

Gradient descent

we start at a random position in the loss landscape

$L(w_1, w_2)$



w_1

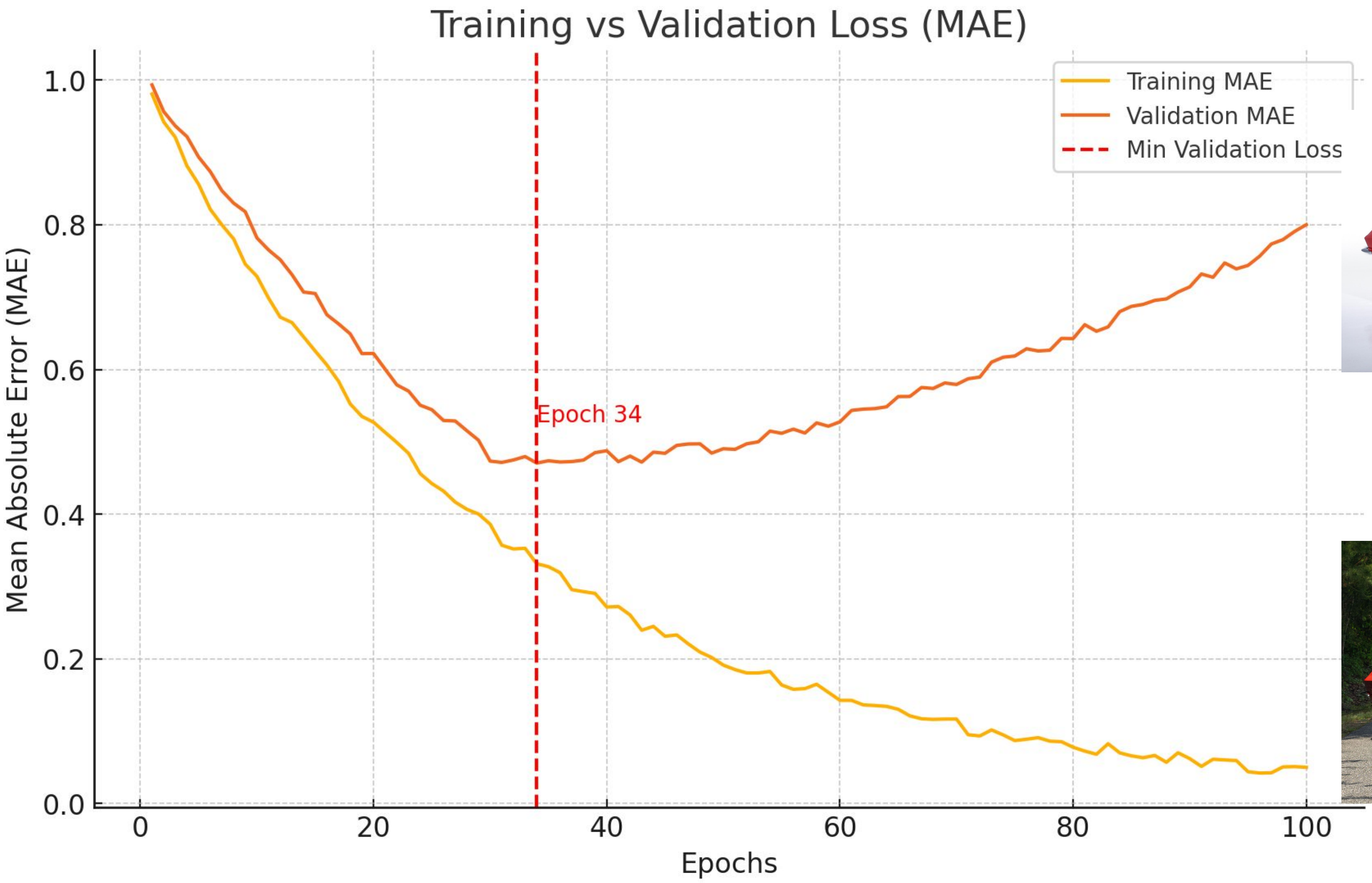
w_2

training image
role: update weights

$$w_{ij} = w_{ij} - (\text{learning rate} * \frac{dL}{dw_{ij}})$$

validation image
role: evaluate best weights

Training vs validation loss



The test set gives the benchmark of generalization

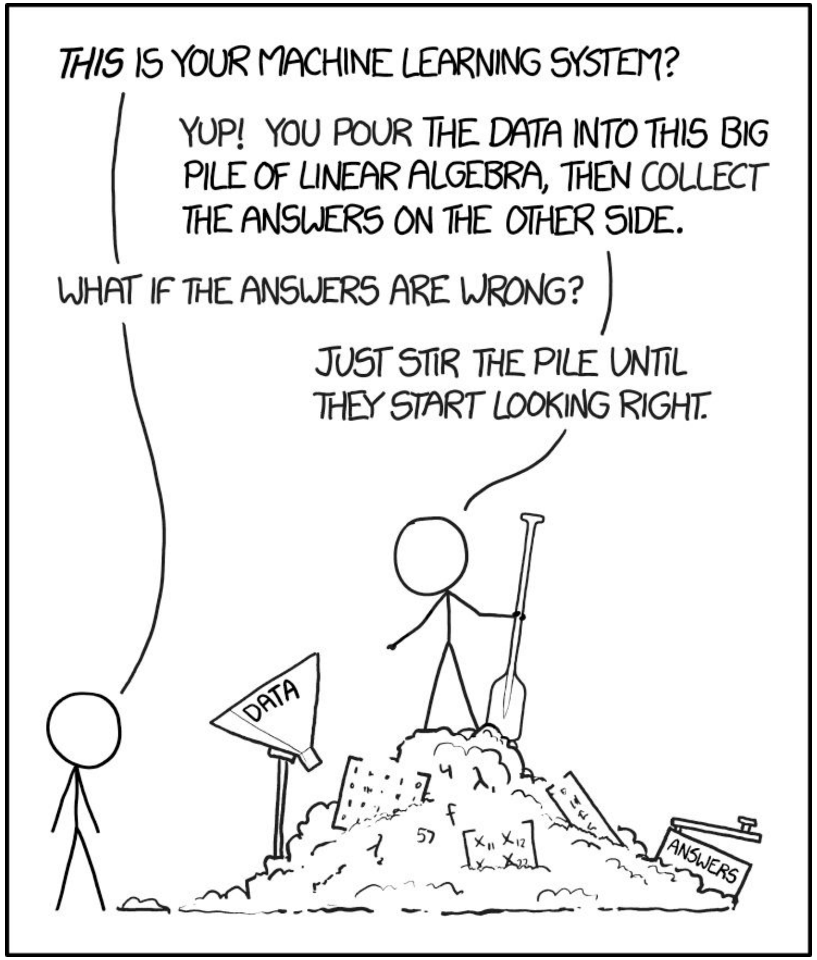


Image from https://xkcd.com/1838/

y = 82699
ŷ = 84015
test set error = |y - ŷ| = 1316

Summary

Neural networks are <u>simplified</u> mathematical models of biological neurons

- They are functions with trainable parameters (aka ‘weights’) that we stack in layers with variable number of processing units (aka ‘neurons’)
- The size of our weight matrices define the dimensions of our hidden and output layers

We use loss functions and gradients to update the weights of a network

- Loss functions provide feedback to the model
- The weight update is an iterative process (gradient descent)

Deciding on training settings (aka “hyperparameters”) is an explorative process

- We use validation sets to decide how and when to stop training. We use tests to provide final benchmarks

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Further reading

On the origin of neural networks

- https://nautil.us/the-man-who-tried-to-redeem-the-world-with-logic-2885/

Annotated history of modern AI and deep learning by Jurgen Schmidhüber

- https://people.idsia.ch/~juergen/deep-learning-history.html

Frank Rosenblatt's Perceptron

- https://news.cornell.edu/stories/2019/09/professors-perceptron-paved-way-ai-60-years-too-soon

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