Multiple-type, Two-dimensional Finite Bin Packing Problem

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Course dates
October 2022 - March 2023
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Abstract

The Multiple-type, Two-dimensional Finite Bin Packing Problem (MT2FBPP) involves assigning a set of rectangular items to a finite set of rectangular bins with varying sizes and costs to minimize the overall cost of the bins used while ensuring that no item overlaps. This study proposes three techniques: a Constraint Programming (CP) model, a Mixed Integer Programming (MIP) model, and a Heuristic-based method, with consideration given to item rotation by 90 degrees. Results show that the heuristic-based method outperforms the other two techniques, and a detailed analysis of each approach's benefits and shortcomings is provided, along with potential extensions for future work. Graphical representations aid in visualizing the packing solutions generated by the CP model and heuristic technique. The findings of the investigation contribute to the advancement of MT2FBPP research and offer valuable insights for practitioners seeking to optimize rectangular item packing into finite bins.

1 Introduction and Objectives

K trucks $1, 2, \ldots, K$ are available for transporting N packages $1, 2, \ldots, N$. Each truck k has the container size of $W_k * H_k$. The dimensions of each package i are $w_i * h_i$. Packages that are placed in the same container must not overlap. Assume that the number K can be large, leading to a great number of trucks that are not being used. C_k represents the cost of using the truck k. Find a solution that loads all the packages into those given trucks such that the total cost of trucks used is minimal.

The data format

- First line: N and K
- Next N lines: w_i and h_i
- Last K lines: W_k, H_k , and C_k

Generated data

*The input size is denoted by (number_of_items * number_of_bins).*

- 1 sample test: $(7 * 3)$
- 45 tests: $(10 * 10)$, $(11 * 11)$, ..., $(53 * 53)$, $(54 * 54)$
- 10 tests: $(90 * 90)$, $(120 * 120)$, $(150 * 150)$, ..., $(330 * 330)$, $(360 * 360)$
- 10 tests: $(550 * 550)$, $(600 * 600)$, ..., $(950 * 950)$, $(1000 * 1000)$
- 09 tests mainly for heuristic testing: $(2000 * 2000)$, $(3000 * 3000)$, ..., $(10000 * 10000)$

For each test case, we randomly generate the dimensions of each item and bin as follows:

- Each side of an item is a random integer between 1 and 10. This means that each item can have a width and length that range from 1 to 10 units.
- For each test case, we can find the largest side of any item (i.e., the maximum of all item widths and item lengths). Then generate the dimensions of each bin as follows: each side of a bin is a random integer between the largest side of any item in the test case and 20. This means that each bin will be at least as large as the largest item in the test case, and can be up to 20 units in width or length.

- And the cost for each bin is a random integer between 50 and 100.

2 Modeling the problem

2.1 CP model

In this model. Let:

- N_{bins}: the number of bins given
- N_{items}: the number of items given
- W_j, H_j, C_j: the width, height, and cost of bin j, respectively
- w_i, h_i: the width and height of item i, respectively
- l_i, r_i, b_i, t_i: left, right, bottom and top coordinates of item i

2.1.1 Decision Variables

- $X_{ij} = 1$: item i packed in bin j

$$\Rightarrow \sum_{i=1}^{N_{items}} X_{ij} \geq 1 \Leftrightarrow Z_j = 1 : \text{bin j has been used} \tag{1}$$

- $R_i = 1$: item i rotated 90 degree
- **Item's Coordinate**:
  - The first way to approach:
    - if item i not rotated: $R_i = 0$

$$\Rightarrow \begin{cases} r_i = l_i + w_i \\ t_i = b_i + h_i \end{cases} \tag{2}$$

    - if item i rotated: $R_i = 1$

$$\Rightarrow \begin{cases} r_i = l_i + h_i \\ t_i = b_i + w_i \end{cases} \tag{3}$$

  - Another way to approach:
    - if item i not rotated: $R_i = 0$

$$\Rightarrow \begin{cases} w_i = w_i \\ h_i = h_i \end{cases} \tag{4}$$

    - if item i rotated: $R_i = 1$

$$\Rightarrow \begin{cases} w_i = h_i \\ h_i = w_i \end{cases} \tag{5}$$

2.1.2 Constraints

- Each item has to be packed in exactly 1 bin:

$$\sum_{j=1}^{N_{bins}} X_{ij} = 1 \quad \forall i \in \{1, 2, \ldots, N\_items\} \tag{6}$$

- No two items overlap:
  if $X_{i_1 j} = X_{i_2 j} = 1$

$$r_{i_1} \leq l_{i_2} \text{ or } r_{i_2} \leq l_{i_1} \text{ or } t_{i_1} \leq b_{i_2} \text{ or } t_{i_2} \leq b_{i_1} \tag{7}$$

- Items cannot exceed the bin:
  if $X_{ij} = 1$

$$\Rightarrow \begin{cases} w_i \leq r_i \leq W_j \\ h_i \leq t_i \leq H_j \end{cases} \tag{8}$$

2.1.3 Objective Function

$$\min \sum_{j=1}^{N_{bins}} Z_j * C_j \tag{9}$$

2.2 MIP model

In this model. Let:

- Constant value M
- N_{bins}: the number of bins given
- N_{items}: the number of items given
- W_j, H_j, C_j: the width, height, and cost of bin j, respectively
- w_i, h_i: the width and height of item i, respectively
- l_i, r_i, b_i, t_i: left, right, bottom and top coordinates of item i

2.2.1 Decision Variables

- $X_{ij} = 1$: item i packed in bin j

$$\Rightarrow \begin{cases} Z_j \leq \sum_{i=1}^{N_{items}} X_{ij} * M \\ Z_j * M \geq \sum_{i=1}^{N_{items}} X_{ij} \end{cases} \tag{10}$$

- $R_i = 1$: item i rotated 90 degree
- **Item's Coordinate**:

$$\Rightarrow \begin{cases} r_i = l_i + w_i * (1 - R_i) + h_i * R_i \\ t_i = b_i + h_i * (1 - R_i) + w_i * R_i \end{cases} \tag{11}$$

2.2.2 Constraints

- Each item has to be packed in exactly 1 bin:

$$\sum_{j=1}^{N_{bins}} X_{ij} = 1 \quad \forall i \in \{1, 2, \ldots, N\_items\} \tag{12}$$

- No two items overlap:
  New variable e such that:

$$\begin{cases} e \geq X_{i_1 j} + X_{i_2 j} - 1 \\ e \leq X_{i_1 j} \\ e \leq X_{i_2 j} \end{cases} \tag{13}$$

$$\Rightarrow \begin{cases} r_{i_1} \leq l_{i_2} + M * (1 - (r_{i_1} \leq l_{i_2})) \\ r_{i_2} \leq l_{i_1} + M * (1 - (r_{i_2} \leq l_{i_1})) \\ t_{i_1} \leq b_{i_2} + M * (1 - (t_{i_1} \leq b_{i_2})) \\ t_{i_2} \leq b_{i_1} + M * (1 - (t_{i_2} \leq b_{i_1})) \\ (r_{i_1} \leq l_{i_2}) + (r_{i_2} \leq l_{i_1}) + (t_{i_1} \leq b_{i_2}) + (t_{i_2} \leq b_{i_1}) + (1 - e) * M \geq 1 \\ (r_{i_1} \leq l_{i_2}) + (r_{i_2} \leq l_{i_1}) + (t_{i_1} \leq b_{i_2}) + (t_{i_2} \leq b_{i_1}) \leq e * M \end{cases} \tag{14}$$

- Items cannot exceed the bin:

$$\Rightarrow \begin{cases} r_i \leq (1 - X_{ij}) * M + W_j \\ t_i \leq (1 - X_{ij}) * M + H_j \end{cases} \tag{15}$$

2.2.3 Objective Function

$$\min \sum_{j=1}^{N_{bins}} Z_j * C_j \tag{16}$$

2.3 Heuristic

2.3.1 Overview

The proposed approach combines two heuristic algorithms, Guillotine and Maximal Rectangles [5], to solve the MT2FBPP.

Concept of free rectangles: A list of free rectangles represents the free space of the bin. In Guillotine algorithm, these rectangles are pairwise disjoint.

2.3.2 Sorting Input

Bins: Ascending order of density (cost/area), ties broken with the descending order of the longer side, followed by the descending order of the shorter side.

Items: Descending order of the longer side, ties broken with the descending order of the shorter side.

2.3.3 Choosing the Destination for Items

Destination Bin: Bin First Fit rule – pack the item into the bin with the lowest index (after the process of sorting bins from 2.3.2); in other words, pack in the first bin that the item fits.

Destination Free Rectangle of a Bin: Best Short Side Fit rule – choose a free rectangle where the shorter remainder side after insertion is minimized; in other words, minimize the length of the shorter leftover side, ties broken with best longer side (longer leftover side is minimized).

2.3.4 Packing process

Guillotine:

- Pack the item into the first free rectangle of the bin, which means the bin itself, starting with its bottom-left corner.
- For each insertion, split the initial free rectangle into smaller free rectangle(s) by the Guillotine split rule, then tracked in a list.
- Whenever a new item is inserted into the bin, choose a free rectangle (with the rule 2.3.3) and place the item into its bottom-left corner, then split the chosen rectangle using the Guillotine split rule to produce at most two new rectangles.
- Merge some free rectangles into larger ones if possible.
- **Splitting rule**: Best Short Side rule - split by horizontal axis if the free rectangle's width is less than its height; otherwise, split by vertical axis.
- **Rectangle merging**: If exists a pair of neighboring rectangles F_i and F_j such that the union $F_i \cup F_j$ can be exactly represented by a single bigger rectangle, merge these two into one.

Maximal Rectangles

- Rather than choosing one of the two split axes like in the Guillotine algorithm, the Maximal Rectangles algorithm picks both split axes at the same time to ensure that the largest possible rectangular areas are present in the list of free rectangles.

- Because the free rectangles are no longer pairwise disjoint, any free rectangle that intersects the area occupied by the newly inserted item is split such to remove the intersection.
- Delete every free rectangle which is fully overlapped by others in the list.

2.3.5 Combined Approach and Solution Selection Strategy

To further improve the solution quality, the proposed approach uses a solution selection strategy that aims to choose the best solution from either the Guillotine or Maximal Rectangles algorithms. The strategy involves running both algorithms on the same input, saving the solution produced by the first algorithm in temporary variables, and resetting the state of the algorithm to its initial state, including the free rectangles. The second algorithm is then applied to the same input, and the solutions produced by both algorithms are then combined to select the solution with the minimum cost as the final solution. This approach is more versatile and can handle different types of instances, achieving better performance compared to using only one algorithm.

Using multiple algorithms for solution selection involves processing the input twice, resulting in longer running times compared to a single algorithmic approach. However, the use of multiple algorithms can leverage the strengths of different techniques to explore the problem space more thoroughly, leading to a better outcome. By combining their solutions, it is possible to compensate for their individual limitations and achieve a more robust solution. The resulting solutions are typically of lower cost and more thoroughly explored, making this approach a viable option for solving optimization problems.

Although the use of multiple algorithms does not guarantee finding the optimal solution, it can provide a more diverse search that increases the chances of finding a better solution. Therefore, the proposed solution selection strategy that employs multiple algorithms can be beneficial in situations where the goal is to generate high-quality solutions, even if the computational cost is slightly higher than that of a single algorithmic approach. For the MT2FBPP, where the problem can be complex and the optimal solution may be difficult to find using a single algorithm, the solution selection strategy ensures that the best solution is chosen from the two algorithms based on the cost of using the bin.

2.3.6 Pseudo-code

The following pseudo-codes describe the implementation of the Guillotine and Maximal Rectangles algorithms used in the proposed approach to solve the MT2FBPP. These algorithms are commonly used to pack items into bins optimally, and the pseudo-codes outline the steps involved in the packing process using these algorithms.

The pseudo-codes can serve as a guideline for implementing the algorithms in the proposed approach and can be used as a starting point for further optimizations.

Algorithm 1: The Guillotine algorithm

Set $F = \{(W, H)\}$;
foreach *item* $i = (w, h)$ *in the list of inserted items of the bin* **do**
    Decide the free rectangle $F_j \in F$ to pack the item into;
    Decide the orientation for the item and place it at the bottom-left of F_j;
    Use the guillotine split scheme to subdivide F_j into two new free rectangles F_{j1} and F_{j2};
    Set $F \leftarrow (F \cup \{F_{j1}, F_{j2}\}) \setminus \{F_j\}$;
    foreach *ordered pair of free rectangles* F_{j1} *and* F_{j2} *in* F **do**
        if F_{j1} *and* F_{j2} *can be merged together* **then**
            $F_{merge} \leftarrow$ Merge F_{j1} and F_{j2};
            Set $F \leftarrow (F \cup \{F_{merge}\}) \setminus \{F_{j1}, F_{j2}\}$;
        end
    end
end

Algorithm 2: The Maximal Rectangles algorithm

Set $F = \{(W, H)\}$;
foreach *item* $i = (w, h)$ *in the list of inserted items of the bin* **do**
  Decide the free rectangle $F_j \in F$ to pack the item into;
  Decide the orientation for the item and place it at the bottom-left of F_j;
  Use the max_rec split scheme to subdivide F_j into two new free rectangles F_{j1} and F_{j2};
  Set $F \leftarrow (F \cup \{F_{j1}, F_{j2}\}) \setminus \{F_j\}$;
  foreach *free rectangle* F_j *in* F **do**
    Compute $F_j \setminus i$ and subdivide the result into at most four new free rectangles $F_{j1}, \ldots, F_{j4}$;
    Set $F \leftarrow (F \cup \{F_{j1}, \ldots, F_{j4}\}) \setminus \{F_j\}$;
  end
  foreach *ordered pair of free rectangles* F_{j1} *and* F_{j2} *in* F **do**
    if F_{j1} *contains* F_{j2} **then**
      Set $F \leftarrow F \setminus \{F_{j2}\}$;
    end
  end
end

3 Result and analysis

3.1 Result

3.1.1 CP model and MIP model solver

Exact solution

Input sizes		CP 1		CP 2		MIP	
n_packs	n_bins	f	t(s)	f	t(s)	f	t(s)
7	3	250	0.027932056	250	0.029741828	250	3.176666667
10	10	51	0.049313393	51	0.082288480	51	2.642000000
11	11	79	0.053943779	79	0.194204638	79	14.00300000
12	12	54	0.057868931	54	0.088900393	54	7.898000000
13	13	103	0.109191075	103	0.248680102	F	F
14	14	50	0.218952388	50	0.156991295	50	27.73466667
15	15	106	0.513134012	106	0.859766784	F	F
16	16	113	0.905138111	113	0.434518921	F	F
17	17	105	117.0229775	105	48.88790709	F	F
18	18	F	F	F	F	F	F
19	19	106	5.214479066	106	4.515408114	F	F
20	20	171	2.519827466	171	3.259184459	F	F
21	21	108	8.500796449	108	13.22533175	F	F

Table 1: Results only exact solution of 2 CP solver and MIP solver

All solution

As a result of the large size and intricate nature of the optimization problems under consideration, we were only able to obtain feasible solutions for test instances with dimensions greater than or equal to $(21*21)$ for the CP solver, and above $(15*15)$ for the MIP solver, within a time limit of 300 seconds. This indicates that the optimization problems are highly complex and computationally demanding, requiring significant computational resources to solve.

To ensure the reliability of our findings, we conducted three runs of each solver and computed the average results, thereby mitigating the potential impact of random variations and ensuring the robustness of our numerical experiments.

Input sizes		**CP 1**					**CP 2**					**MIP**				
n_packs	**n_bins**	**f_min**	**f_max**	**f_avg**	**std_dev**	**t_avg(s)**	**f_min**	**f_max**	**f_avg**	**std_dev**	**t_avg(s)**	**f_min**	**f_max**	**f_avg**	**std_dev**	**t_avg(s)**
7	3	250	250	250	0	0.027932056	250	250	250	0	0.02974183	250	250	250	0	3.17666667
10	10	51	51	51	0	0.049313393	51	51	51	0	0.08228848	51	51	51	0	2.64200000
11	11	79	79	79	0	0.053943779	79	79	79	0	0.19420464	79	79	79	0	14.0030000
12	12	54	54	54	0	0.057868931	54	54	54	0	0.08890039	54	54	54	0	7.89800000
13	13	103	103	103	0	0.109191075	103	103	103	0	0.24868010	103	103	103	0	300.245000
14	14	50	50	50	0	0.218952388	50	50	50	0	0.15699130	50	50	50	0	27.7346667
15	15	106	106	106	0	0.513134012	106	106	106	0	0.85976678	106	106	106	0	300.406667
16	16	113	113	113	0	0.905138111	113	113	113	0	0.43451892	113	113	113	0	300.472667
17	17	105	105	105	0	117.0229775	105	105	105	0	48.8879071	105	105	105	0	300.741667
18	18	118	118	118	0	300.0134580	118	118	118	0	300.009749	121	147	133	13.115	300.650000
19	19	106	106	106	0	5.214479066	106	106	106	0	4.51540811	106	106	106	0	300.789000
20	20	171	171	171	0	2.519827466	171	171	171	0	3.25918446	466	466	466	0	301.060667
21	21	108	108	108	0	8.500796449	108	108	108	0	13.2253317	210	210	210	0	301.076333
22	22	156	156	156	0	300.0168832	156	156	156	0	300.015485	461	623	569	93.531	301.211333
23	23	116	116	116	0	300.0116365	116	116	116	0	300.011188	1108	1108	1108	0	301.363333
24	24	158	158	158	0	300.0148793	158	160	158.67	1.1547	300.015304	356	356	356	0	301.663333
25	25	204	204	204	0	300.0179474	217	217	217	0	300.016161	1601	1601	1601	0	301.887000
26	26	158	158	158	0	300.0207270	158	158	158	0	300.019220	622	622	622	0	301.989667
27	27	158	158	158	0	300.0189365	158	158	158	0	300.015856	N/A	N/A	N/A	N/A	N/A
28	28	158	158	158	0	300.0196483	158	158	158	0	300.016423	1190	1190	1190	0	304.034333
29	29	167	178	174.33	6.3509	300.0187450	178	178	178	0	300.016640	1415	1415	1415	0	303.806667
30	30	181	182	181.67	0.5774	300.0173433	181	181	181	0	300.016748	2144	2144	2144	0	303.520000
31	31	215	215	215	0	300.0253246	215	215	215	0	300.022030	2249	2249	2249	0	303.887000
32	32	171	171	171	0	300.0176523	171	171	171	0	300.017343	1224	1224	1224	0	306.495333
33	33	165	165	165	0	300.0201768	165	165	165	0	300.019763	1429	1429	1429	0	306.537000
34	34	165	165	165	0	300.0260553	165	165	165	0	300.020847	803	803	803	0	306.058333
35	35	213	220	215.33	4.0415	300.0196759	220	220	220	0	300.023850	1723	1723	1723	0	309.124333
36	36	299	299	299	0	300.0296591	299	299	299	0	300.023764	2795	2795	2795	0	309.352000
37	37	163	163	163	0	300.0222829	163	163	163	0	300.021046	1750	1750	1750	0	308.550000
38	38	252	252	252	0	300.0218226	252	252	252	0	300.024474	1193	1193	1193	0	309.728667
39	39	229	229	229	0	300.0224621	229	229	229	0	300.022671	2895	2895	2895	0	308.145000
40	40	263	263	263	0	300.0266393	263	263	263	0	300.027277	2964	2964	2964	0	317.509667
41	41	225	228	226	1.7321	300.0271970	225	225	225	0	300.023711	1304	1304	1304	0	315.537000
42	42	367	367	367	0	300.0270539	375	375	375	0	300.030499	3115	3115	3115	0	317.383667
43	43	237	237	237	0	300.0364837	279	279	279	0	300.024842	3178	3178	3178	0	314.327667
44	44	283	304	296.33	11.590	300.0313213	283	292	286	5.1962	300.027890	2030	2030	2030	0	318.902667
45	45	220	220	220	0	300.0270809	220	220	220	0	300.029565	N/A	N/A	N/A	N/A	N/A
46	46	272	272	272	0	300.0306748	274	291	285.33	9.8150	300.030624	N/A	N/A	N/A	N/A	N/A
47	47	313	322	319	5.1962	300.0306518	322	322	322	0	300.030069	2182	N/A	N/A	N/A	N/A
48	48	269	269	269	0	300.0316954	269	269	269	0	300.029771	N/A	N/A	N/A	N/A	N/A
49	49	273	273	273	0	300.0297330	273	273	273	0	300.032365	N/A	N/A	N/A	N/A	N/A
50	50	291	293	291.67	1.1547	300.0346591	290	290	290	0	300.040313	N/A	N/A	N/A	N/A	N/A
51	51	328	328	328	0	300.0348325	328	391	370	36.373	300.035202	N/A	N/A	N/A	N/A	N/A

Input sizes		**CP 1**					**CP 2**					**MIP**				
n_packs	**n_bins**	**f_min**	**f_max**	**f_avg**	**std_dev**	**t_avg(s)**	**f_min**	**f_max**	**f_avg**	**std_dev**	**t_avg(s)**	**f_min**	**f_max**	**f_avg**	**std_dev**	**t_avg(s)**
52	52	330	345	335	8.6603	300.0370899	330	334	332.67	2.3094	300.038526	N/A	N/A	N/A	N/A	N/A
53	53	382	382	382	0	300.0353604	359	384	373	12.767	300.037128	N/A	N/A	N/A	N/A	N/A
54	54	260	260	260	0	300.0374483	260	260	260	0	300.037056	N/A	N/A	N/A	N/A	N/A
90	90	567	585	573	10.392	300.0979588	565	570	567.33	2.5166	300.117140	N/A	N/A	N/A	N/A	N/A
120	120	760	800	776.33	20.984	300.1965074	744	784	770	22.539	300.192483	N/A	N/A	N/A	N/A	N/A
150	150	1169	1454	1301.3	143.58	300.3545031	856	994	911.67	72.762	300.442578	N/A	N/A	N/A	N/A	N/A
180	180	1810	1847	1825	19.468	300.7002796	1314	1787	1490	258.68	300.542184	N/A	N/A	N/A	N/A	N/A
210	210	2595	2828	2731	121.30	300.7788572	2270	2600	2481.7	183.73	300.749900	N/A	N/A	N/A	N/A	N/A
240	240	1861	1992	1941.7	70.571	301.0924356	2074	2434	2308.3	203.12	301.062326	N/A	N/A	N/A	N/A	N/A

Table 2: Average results of 2 CP solver and MIP solver

3.1.2 Heuristic solver

Input sizes		Heuristics				
n_packs	n_bins	f_min	f_max	f_avg	std_dev	t_avg(s)
7	3	300	300	300	0	0.000029500
10	10	51	51	51	0	0.000037000
11	11	79	79	79	0	0.000035500
12	12	54	54	54	0	0.000040500
13	13	145	145	145	0	0.000039000
14	14	55	55	55	0	0.000051000
15	15	106	106	106	0	0.000057500
16	16	130	130	130	0	0.000049000
17	17	105	105	105	0	0.000052500
18	18	121	121	121	0	0.000059000
19	19	129	129	129	0	0.000058500
20	20	188	188	188	0	0.000053500
21	21	120	120	120	0	0.000060000
22	22	171	171	171	0	0.000060500
23	23	147	147	147	0	0.000075500
24	24	181	181	181	0	0.000084500
25	25	237	237	237	0	0.000071500
26	26	161	161	161	0	0.000070500
27	27	180	180	180	0	0.000069000
28	28	158	158	158	0	0.000079000
29	29	186	186	186	0	0.000077000
30	30	182	182	182	0	0.000080000
31	31	215	215	215	0	0.000090000
32	32	187	187	187	0	0.000096500
33	33	178	178	178	0	0.000101500
34	34	182	182	182	0	0.000091500
35	35	237	237	237	0	0.000098000
36	36	345	345	345	0	0.000098500
37	37	163	163	163	0	0.000095000
38	38	260	260	260	0	0.000102000
39	39	229	229	229	0	0.000116500
40	40	236	236	236	0	0.000121500
41	41	251	251	251	0	0.000117500
42	42	412	412	412	0	0.000109000
43	43	243	243	243	0	0.000109000
44	44	356	356	356	0	0.000149500
45	45	239	239	239	0	0.000117000
46	46	301	301	301	0	0.000118000
47	47	284	284	284	0	0.000160000
48	48	287	287	287	0	0.000124500
49	49	275	275	275	0	0.000134500
50	50	314	314	314	0	0.000125000
51	51	291	291	291	0	0.000162000
52	52	358	358	358	0	0.000135500
53	53	404	404	404	0	0.000130500
54	54	278	278	278	0	0.000134000
90	90	591	591	591	0	0.000233000
120	120	701	701	701	0	0.000335500
150	150	770	770	770	0	0.000483500
180	180	965	965	965	0	0.000588500

Input sizes		Heuristics				
n_packs	n_bins	f_min	f_max	f_avg	std_dev	t_avg(s)
210	210	1098	1098	1098	0	0.000721500
240	240	1161	1161	1161	0	0.000794500
270	270	1243	1243	1243	0	0.001095500
300	300	1817	1817	1817	0	0.001208500
330	330	1942	1942	1942	0	0.001404500
360	360	2052	2052	2052	0	0.001529000
550	550	2841	2841	2841	0	0.003081500
600	600	3633	3633	3633	0	0.004040500
650	650	3775	3775	3775	0	0.004719500
700	700	3835	3835	3835	0	0.005014500
750	750	4017	4017	4017	0	0.005300000
800	800	4489	4489	4489	0	0.006496500
850	850	4503	4503	4503	0	0.007256500
900	900	4625	4625	4625	0	0.007605000
950	950	5308	5308	5308	0	0.008947000
10000	10000	53952	53952	53952	0	0.774925500
1000	1000	5579	5579	5579	0	0.010288000
2000	2000	11217	11217	11217	0	0.032937001
3000	3000	16019	16019	16019	0	0.057311000
4000	4000	21701	21701	21701	0	0.101622500
5000	5000	27568	27568	27568	0	0.127237998
6000	6000	32746	32746	32746	0	0.178141996
7000	7000	38368	38368	38368	0	0.224725999
8000	8000	43364	43364	43364	0	0.288765997
9000	9000	49632	49632	49632	0	0.362545997

Table 3: Results of Heuristics solver

3.2 Analysis

3.2.1 Exact solution

Our evaluation of the accuracy of each algorithm revealed that CP and MIP were capable of generating exact solutions for a subset of test cases:

- CP was able to produce exact solutions for tests with sizes: $(7 * 3)$, $(10 * 10)$, $(11 * 11)$, $(12 * 12)$, $(13 * 13)$, $(14 * 14)$, $(15 * 15)$, $(16 * 16)$, $(17 * 17)$, $(19 * 19)$, $(20 * 20)$ and $(21 * 21)$.
- MIP was only able to produce exact solutions for tests with sizes: $(7*3)$, $(10*10)$, $(11*11)$, $(12*12)$ and $(14 * 14)$.

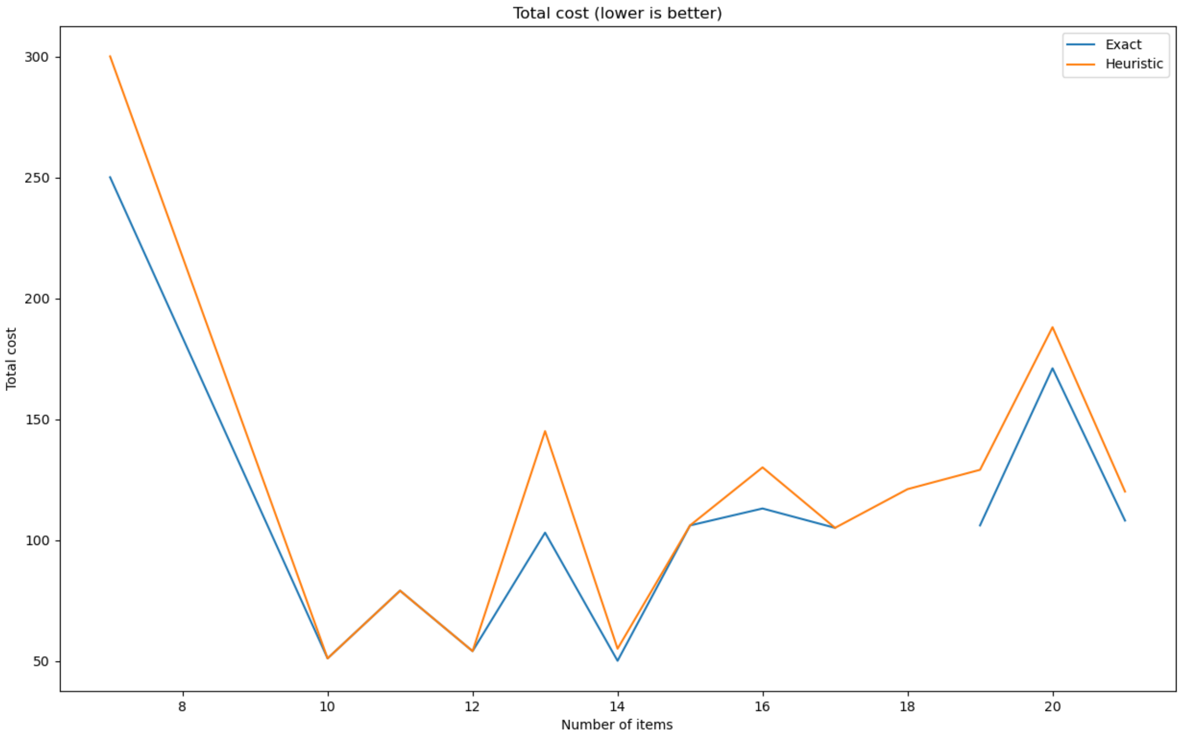


Figure 1: Results compare only exact solution

3.2.2 All solution

Our analysis of the solutions generated by three distinct algorithms revealed the following:

- CP is unable to handle data sets larger than $(240 * 240)$.
- MIP is unable to handle data sets larger than $(44 * 44)$.
- The Heuristic algorithm is capable of handling all test cases, including the largest test size of $(10,000 * 10,000)$.

In terms of total cost, our findings indicate:

- MIP performs the worst.
- CP1 and CP2 generate nearly equivalent results.
- With larger data sets, CP2 performs better than CP1.
- The Heuristic algorithm generates better results and significantly outperforms all other algorithms for all test cases larger than $(100 * 100)$, although it is slightly inferior to CP for test cases smaller than $(100 * 100)$.

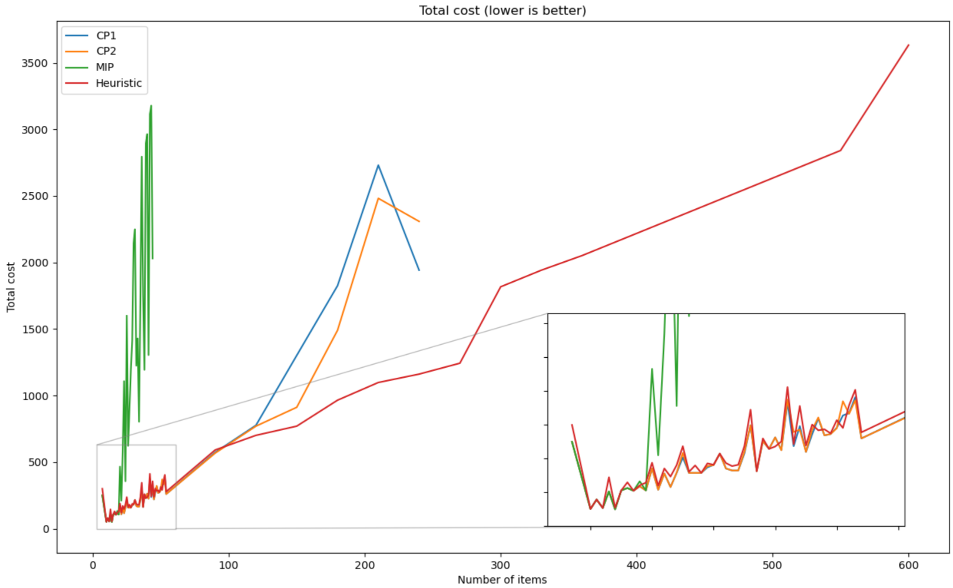


Figure 2: Results compare all solution

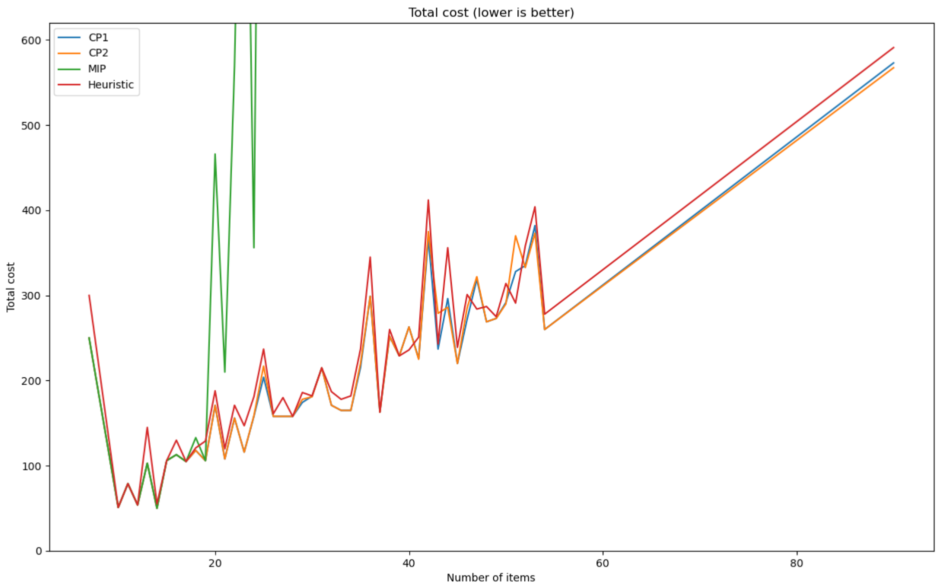


Figure 3: Zoomed results compare all solution

3.2.3 Running time

The running time of each algorithm was also evaluated, and our findings indicate that:

- MIP reaches the time limit of 300 seconds for all tests with size greater than or equal $(15 * 15)$.
- CP reaches the time limit of 300 seconds for all tests with size greater than or equal $(22 * 22)$.
- The Heuristic algorithm has a remarkably short run time, with every test completing in under 1 second, even for the largest test size of $(10,000 * 10,000)$.

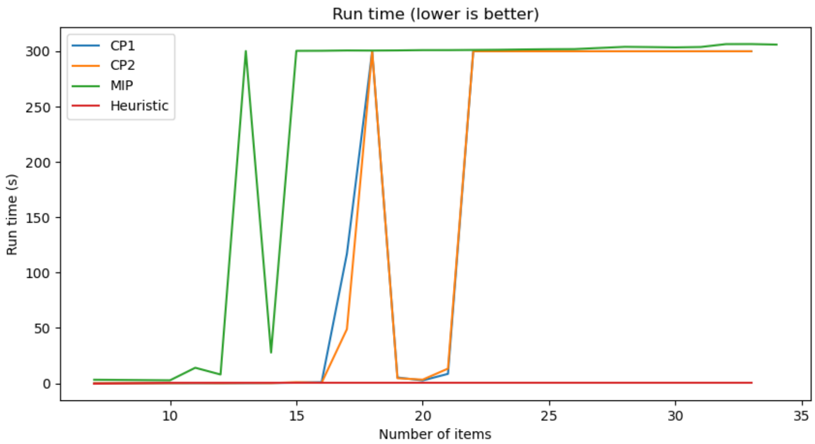


Figure 4: Run time of the first 25 tests

Our comprehensive analysis indicates that:

- MIP consistently performs the worst, as it reaches the time limit of 300 seconds and generates the poorest results.
- CP is a more favorable option than MIP in terms of run time and cost, with CP1 generating nearly equivalent results to CP2 and exhibiting a faster run time for smaller data sets.
- However, it is also revealed that the Heuristic algorithm outperforms both MIP and CP in terms of efficiency, consistently generating accurate results and exhibiting a significantly shorter run time for all test cases.

4 Conclusion

In conclusion, based on our evaluation, the Heuristic algorithm is highly accurate and efficient, making it a valuable tool for practical applications that require both accuracy and speed. While the CP solver is also a viable alternative, particularly for smaller data sets, the MIP solver consistently performs poorly and is not recommended.

5 Limitations and Future Research

Our analysis also revealed some limitations and potential avenues for future research. Firstly, our study only considered a specific set of test cases, and the impact of varying problem sizes and parameter settings was not investigated. Therefore, future research could focus on evaluating the performance of the algorithms on a wider rage of test cases and exploring the impact of different parameter settings. Additionally, future research could focus on optimizing the performance of the Heuristic algorithm while maintaining its accuracy and efficiency, or explore the potential for hybrid algorithms that combine elements of the CP, MIP, and Heuristic models to achieve even better performance.

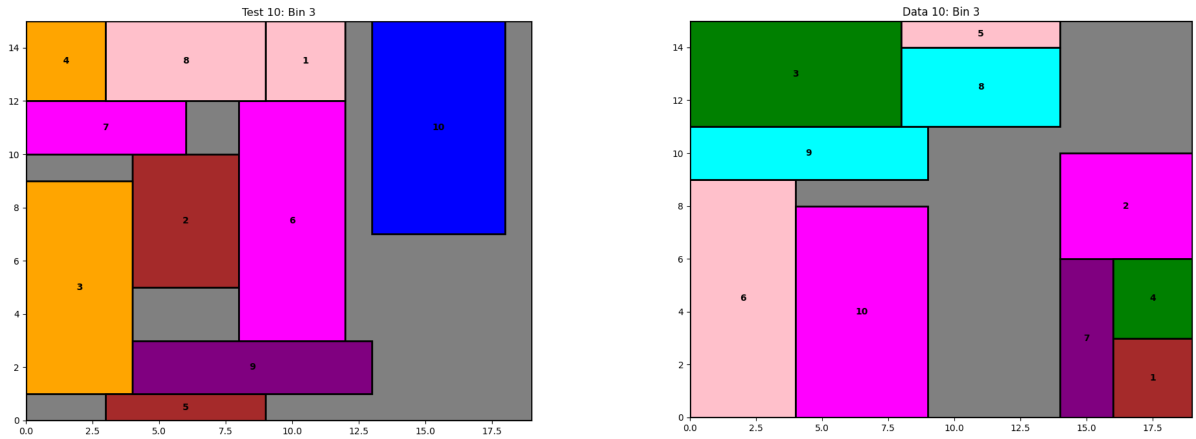
Furthermore, our study only focused on the theoretical aspects of the optimization problem and did not consider the practical implications of implementing the algorithms in real-world scenarios. Thus, future research could investigate the potential for implementing these algorithms in various domains and evaluating their performance in practical settings.

In summary, our study provides insights into the performance of different algorithms for the optimization problem, highlights their limitations, and identifies potential areas for further research.

6 Visualization

In terms of visualization, we generated figures to display the results of the CP solver and the Heuristic solver. However, due to the MIP solver's long-running time and poor performance, we did not generate any figures for it. Therefore, it is not necessary to incorporate MIP solver results in the figures.

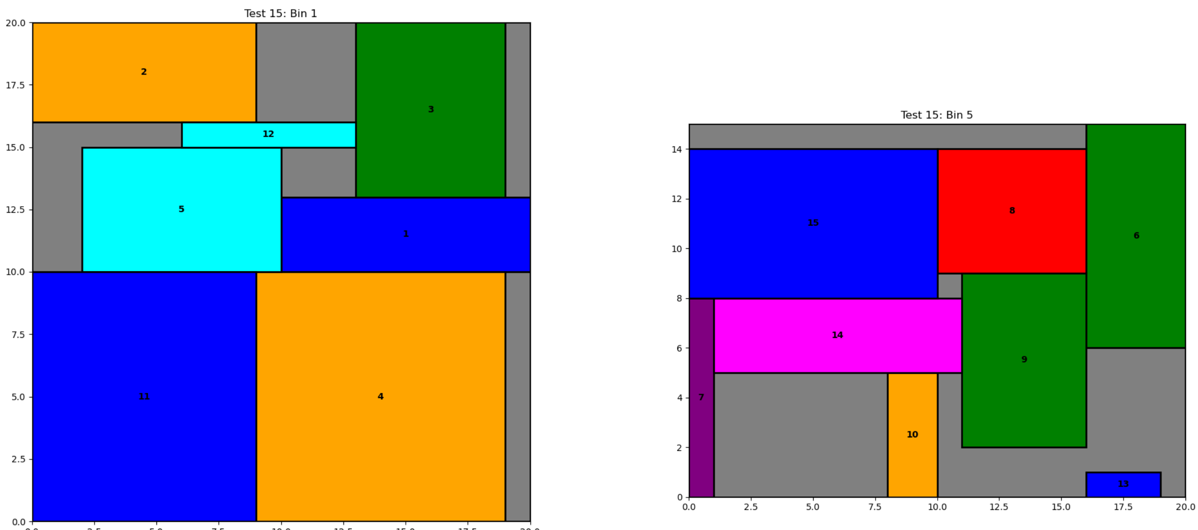
The figures comparing the results of the CP and Heuristic solvers are presented below.



(a) CP solver.

(b) Heuristic solver.

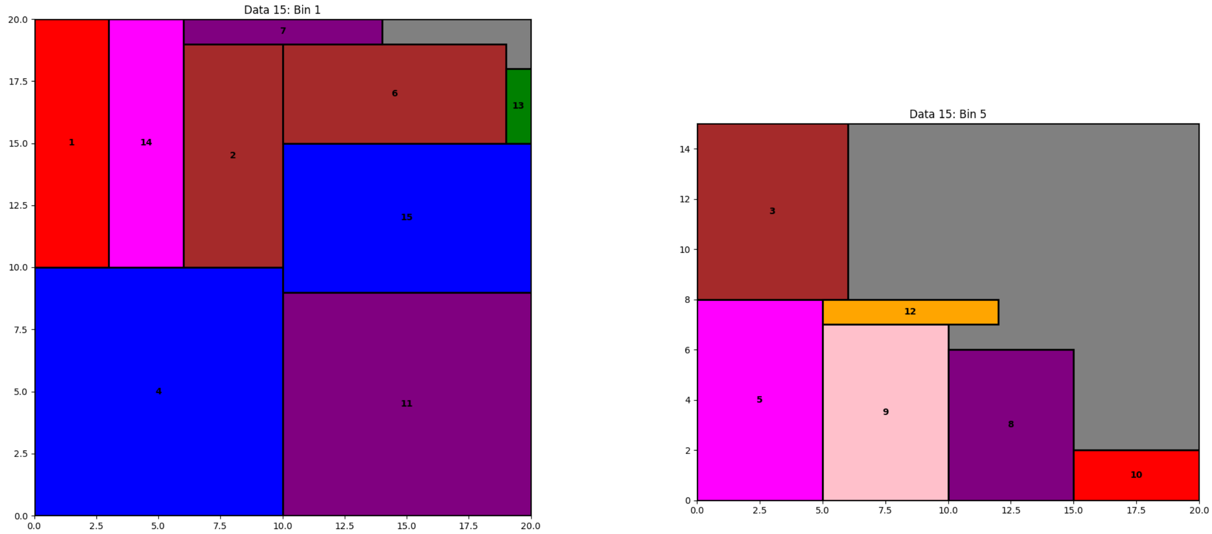
Figure 5: Result for test size $(10 * 10)$.



(a) First bin.

(b) Second bin.

Figure 6: Result for test size $(15 * 15)$ by CP solver.



(a) First bin.

(b) Second bin.

Figure 7: Result for test size $(15 * 15)$ by Heuristic solver.

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