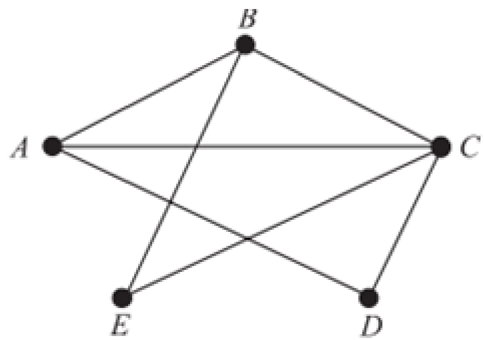
Assignment #8 – Graph Theory

1. Consider the graph G shown below.

(a) Find the set $V(G)$ of vertices of G and the set $E(G)$ of edges of G.

(b) Find the degree of each vertex.



(a) 5 veritces: $V(G) = \{A, B, C, D, E\}$.

7 edges: $E(G) = [\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, E\}, \{C, D\}, \{C, E\}]$

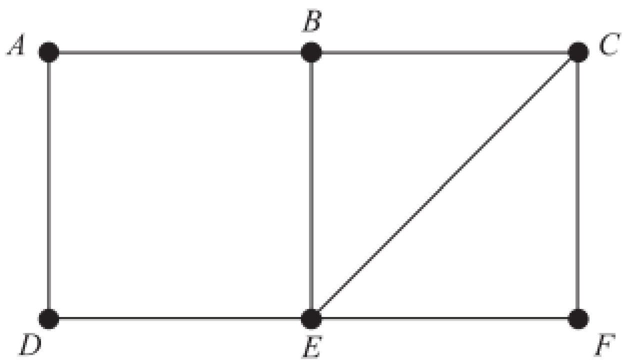
(b) $\deg(A) = 3$, $\deg(B) = 3$, $\deg(C) = 4$, $\deg(D) = 2$, $\deg(E) = 2$

2. Consider the graph G shown below. Find:

(a) all simple paths from A to F;

(b) all cycles which include vertex A;

(c) all cycles in G.



(a) A simple path from A to F is a path such that no vertex, and hence no edge, is repeated.

$$(A, B, C, F), \quad (A, B, C, E, F), \quad (A, B, E, F), \quad (A, B, E, C, F),$$
$$(A, D, E, F), \quad (A, D, E, B, C, F), \quad (A, D, E, C, F).$$

four beginning with the edges $\{A, B\}$ and three beginning with the ege $\{A, D\}$:

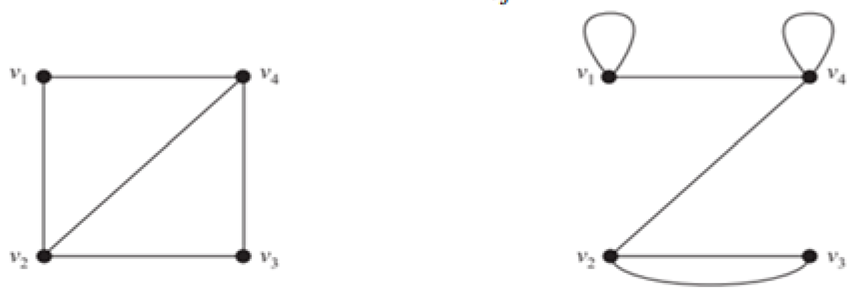
(b) There are three cycles which include vertex A:

$$(A, B, E, D, A), \quad (A, B, C, E, D, A), \quad (A, B, C, F, E, D, A).$$

(c) There are six cycles in G; the three in (b) and

$$(B, C, E, B), \quad (C, F, E, C), \quad (B, C, F, E, B).$$

3. Find the adjacency matrix $A = [a_{ij}]$ of each graph G shown below.

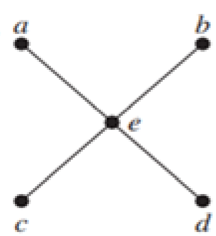


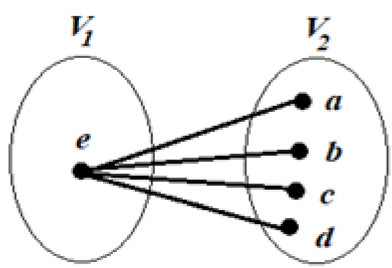
(a) (b)

Set $a_{ij} = n$ if there are n edges $\{v_i, v_j\}$ and $a_{ij} = 0$ otherwise. Hence:

$$(a)\ A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}; \quad (b)\ A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

4. Determine whether the graph is bipartite.

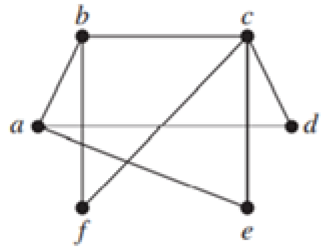




The set of vertices V is split into 2 sets V_1 and V_2.
Every edge of the original graph has one vertex in V_1 and the other in V_2.

$\Rightarrow$ Bipartite

5. Determine whether the graph is bipartite.

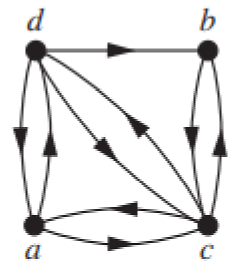


The set of vertices V can not be split into 2 sets V_1 and V_2 such that every edge in the original graph has onr vertex in V_1 and the other in V_2.

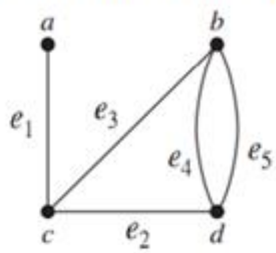
Not bipartite

6. Draw a graph with the given adjacency matrix.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



7. Use an incidence matrix to represent the graph



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

8. Let G be the directed graph shown below.

(a) Describe G formally.

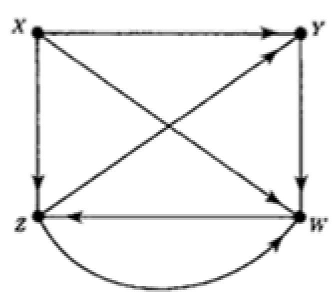
(b) Find all simple paths from X to Z.

(c) Find all simple paths from Y to Z.

(d) Find all cycles in G.

(e) Is G unilaterally connected?

(f) Is G strongly connected?



(*a*) 4 vertices: $V = \{X, Y, Z, W\}$
7 edges: $E = \{(X, Y), (X, Z), (X, W), (Y, W), (Z, Y), (Z, W), (W, Z)\}$

(*b*) 3 simple paths from X to Z: (X, Z), (X, W, Z), and (X, Y, W, Z).

(*c*) 1 simple path from Y to Z: (Y, W, Z).

(*d*) 1 cycle in G: (Y, W, Z, Y).

(*e*) G is unilaterally connected since (X, Y, W, Z) is a spanning path.

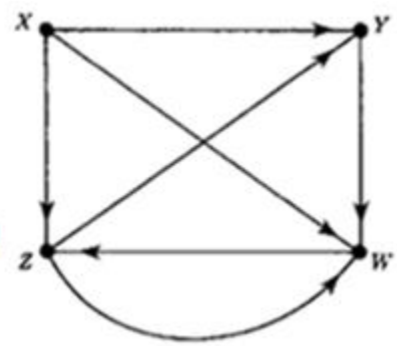
(*f*) G is not strongly connected since there is no closed spanning path.

9. Let G be the directed graph shown below.

(*a*) Find the indegree and outdegree of each vertex of G.

(*b*) Are there any sources or sinks?

(*c*) Find the subgraph H of G determined by the vertex set $V' = X, Y, Z$.



(*a*)

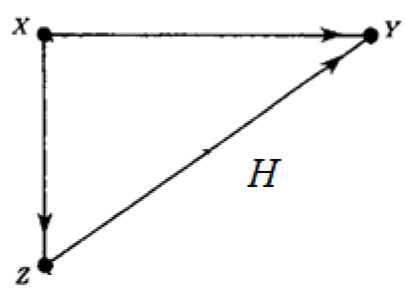
	X	Y	Z	W	Total
indeg	0	2	2	3	7
outdeg	3	1	2	1	7

(*b*) Source: X
Sink: none

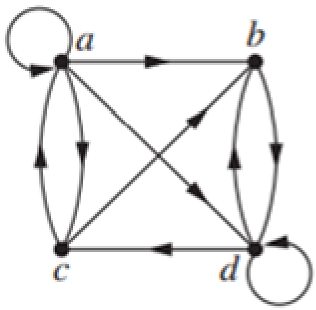
(*c*) Let E' consists of all edges of G whose endpoints lie in V'.

This yield $E' = \{(X, Y), (X, Z), (Z, Y)\}$.

Then $H = H(V', E')$

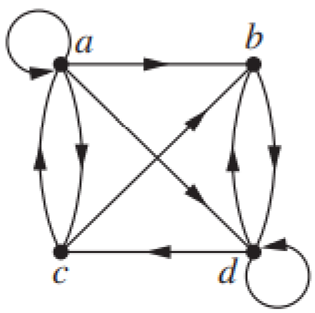


10. Use an adjacency list to represent the given graph.



Vertex	*Terminal Vertices*
a	a, b, c, d
b	d
c	a, b
d	b, c, d

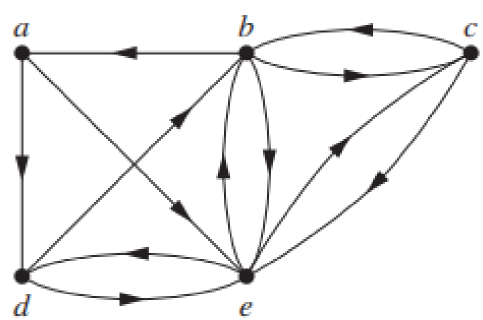
11. Use an adjacency matrix to represent the given graph.



$$\begin{array}{c c} & \begin{array}{cccc} \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{c} & \boldsymbol{d} \end{array} \\ \begin{array}{c} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \\ \boldsymbol{d} \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

12. Determine whether the directed graph shown has an Euler circuit. Construct an Euler circuit if one exists.

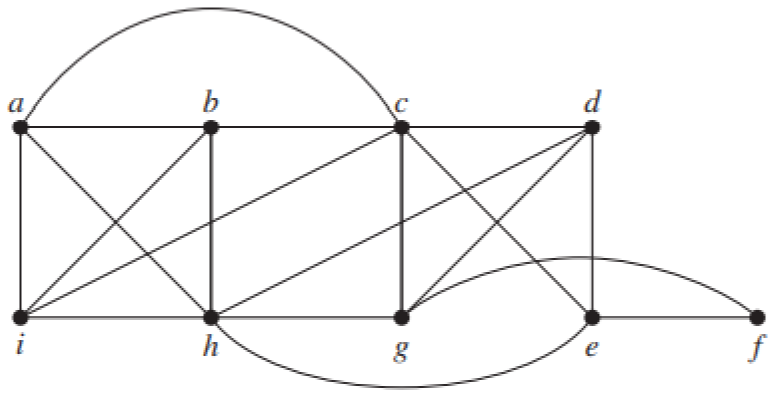
If no Euler circuit exists, determine whether the directed graph has an Euler path. Construct an Euler path if one exists.



No Euler circuit.

Euler path: *a, d, e, d, b, a, e, c, e, b, c, b, e*

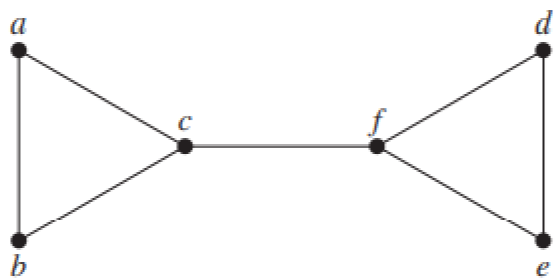
13. Determine whether the given graph has an Euler circuit.
Construct such a circuit when one exists.
If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



Euler circuit: *a, i, h, g, d, e, f, g, c, e, h, d, c, a, b, i, c, b, h, a*

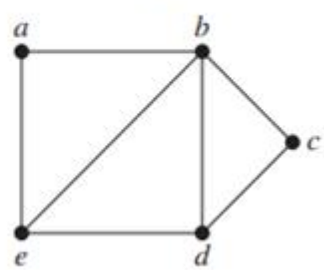
No Euler path.

14. Does the given graph have a Hamilton path? If so, find such a path.
If it does not, give an argument to show why no such path exists.



Yes, *a, b, c, f, d, e* is a Hamilton path.

15. Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit.
If it does not, give an argument to show why no such circuit exists.



a, b, c, d, e, a is a Hamilton circuit.