Ecem Yildirim – March 1st, 2025

Fair Price of Coffee Futures Contract Analysis

To determine the fair price of a coffee futures contract and its associated option, we applied three key quantitative models: the **Cost-of-Carry model**, the **Black-Scholes model**, and **Monte Carlo simulations**.

1. Cost-of-Carry Model

The Cost-of-Carry model is used to price futures contracts by incorporating the spot price, risk-free interest rate, and storage costs. The formula is:

$F = S_0 * e^{((r + s)T)}$

where:

• F = Futures price

• S_0 = Current spot price ($1.20 per pound)

• r = Risk-free interest rate (2% per annum)

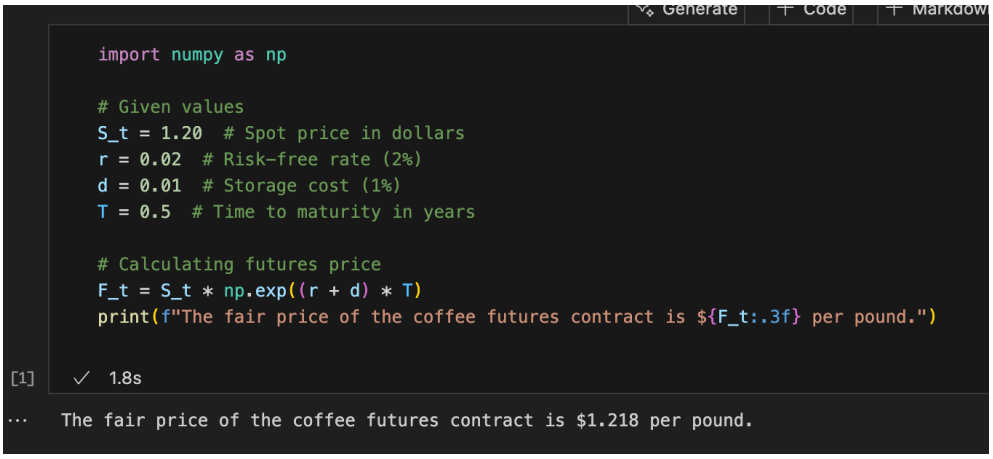
• s = Storage cost (1% per annum)

• T = Time to maturity (6 months or 0.5 years)

Using this model, the calculated **fair price of the coffee futures contract** is:

$1.218 per pound

Python Implementation:



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2. Black-Scholes Model for Coffee Option Pricing

The Black-Scholes model is applied to price a European call option on the coffee futures contract. The formula used is:

Call Price = S_0 * N(d_1) - X * e^(-rT) * N(d_2)

where:

• X = Strike price ($1.25)

• σ = Volatility (25% per annum)

• N(d_1) and N(d_2) = Cumulative normal distribution functions

Using the Black-Scholes model, the calculated **coffee option price** is:

$0.068 per pound

Python Implementation:

```
from scipy.stats import norm
import numpy as np

# Given values
S_0 = 1.20  # Spot price in dollars
X = 1.25  # Strike price in dollars
r = 0.02  # Risk-free rate (2%)
T = 0.5  # Time to maturity in years
sigma = 0.25  # Volatility (25%)

# Calculating d1 and d2
d1 = (np.log(S_0 / X) + (r + 0.5 * sigma ** 2) * T) / (sigma * np.sqrt(T))
d2 = d1 - sigma * np.sqrt(T)

# Calculating call option price using Black-Scholes formula
C = S_0 * norm.cdf(d1) - X * np.exp(-r * T) * norm.cdf(d2)
print(f"The price of the call option is ${C:.3f}.")
```

[2] ✓ 1.6s

```
The price of the call option is $0.068.
```

3. Monte Carlo Simulation for Option Pricing

Monte Carlo simulations were performed to estimate the **fair price of the option** by simulating thousands of potential future price paths based on **geometric Brownian motion**.

By running **10,000 simulations**, the **Monte Carlo simulated price** of the coffee futures contract at maturity is:

$1.210 per pound

Python Implementation:

```python
import numpy as np

# Simulation parameters
S_0 = 1.20  # Spot price in dollars
r = 0.02  # Risk-free rate (2%)
sigma = 0.25  # Volatility (25%)
T = 0.5  # Time to maturity in years
num_simulations = 10000  # Number of simulations
num_steps = 252  # Number of steps (daily)

# Time increment
dt = T / num_steps

# Simulating price paths
np.random.seed(42)  # For reproducibility
price_paths = np.zeros((num_steps, num_simulations))
price_paths[0] = S_0

for t in range(1, num_steps):
    z = np.random.standard_normal(num_simulations)
    price_paths[t] = price_paths[t-1] * np.exp((r - 0.5 * sigma ** 2) * dt + sigma * np.sqrt(dt) * z)

# Calculating the average simulated price at maturity
average_simulated_price = np.mean(price_paths[-1])
print(f"The average simulated price of the coffee futures contract at maturity is ${average_simulated_price:.3f}.")
```

✓ 0.1s

```
The average simulated price of the coffee futures contract at maturity is $1.210.
```

Impact of Market Factors on Pricing

The fair price of coffee futures and options is significantly influenced by **supply shocks, demand shifts, weather conditions, and geopolitical factors**:

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1. Supply Shocks:

• A drought, frost, or pest outbreak in coffee-producing regions (e.g., Brazil, Colombia) reduces supply, increasing futures and option prices.

• A bumper crop due to favorable weather would increase supply and lower prices.

2. Demand Shifts:

• Higher global coffee consumption, especially in emerging markets, drives prices higher.

• Shifts in consumer preferences or health trends reducing coffee demand would lower prices.

3. Weather Conditions:

• Extreme weather events increase price volatility, making options more valuable due to greater uncertainty.

• Stable weather conditions result in lower volatility and lower option premiums.

4. Geopolitical Factors:

• Trade restrictions, tariffs, and currency fluctuations can increase costs and disrupt supply chains, leading to higher prices.

• Political stability and smooth trade relations help keep prices stable with lower volatility.

Summary:

- Fair price of the coffee futures contract (Cost-of-Carry model): **$1.218 per pound**
- Call option price (Black-Scholes model): **$0.068 per pound**
- The average simulated price of the coffee futures contract at maturity (Monte Carlo simulation): **$1.210 per pound**