

MAM4xx Units Conversions (`conversions.hpp`)

Employing the SI definition for molar mass (molecular weight) and the following (slightly tortured) notation:

- Number concentration of species: N_C
- Mass mixing ratio: M_R
- Volume mixing ratio (equivalent to molar mixing ratio--thanks ideal gas law!): V_R
- Molecular weight of species/dry air: $M_z, z \in \{s, \alpha\}$
- Density of species/dry air: $\rho_z, z \in \{s, \alpha\}$
- Avogadro's number: N_A
- Units of meters, kilograms, moles denoted as per convention and number of molecules of species/dry air as: $\#_z, z \in \{s, \alpha\}$

We have, for the following conversion functions:

- `mmr_from_number_conc()`

$$M_R = \frac{N_C \times M_s}{\rho_\alpha \times N_A} \rightarrow \frac{\left[\frac{\#_s}{\cancel{\text{m}^3}} \right] \left[\frac{\text{kg}_s}{\text{mol}_s} \right]}{\left[\frac{\text{kg}_\alpha}{\cancel{\text{m}^3}} \right] \left[\frac{\#}{\text{mol}} \right]} = \frac{\left[\cancel{\#_s} \right] [\text{kg}_s]}{[\text{kg}_\alpha] \left[\frac{\#}{\text{mol}} \right] [\cancel{\text{mol}_s}]} = \frac{[\text{kg}_s]}{[\text{kg}_\alpha]}$$

- `number_conc_from_mmr()`

$$N_C = \frac{M_R \times \rho_\alpha \times N_A}{M_s} \rightarrow \frac{\left[\frac{\cancel{\text{kg}_s}}{\cancel{\text{kg}_\alpha}} \right] \left[\frac{\cancel{\text{kg}_\alpha}}{\text{m}^3} \right] \left[\frac{\#}{\text{mol}} \right]}{\left[\frac{\cancel{\text{kg}_s}}{\text{mol}_s} \right]} = \frac{\left[\frac{\#}{\text{mol}} \right] [\text{mol}_s]}{\text{m}^3} = \frac{\#_s}{\text{m}^3}$$

- `mmr_from_vmr()`

$$M_R = \frac{V_R \times M_s}{M_\alpha} \rightarrow \frac{\left[\frac{\cancel{\text{mol}_s}}{\cancel{\text{mol}_\alpha}} \right] \left[\frac{\text{kg}_s}{\cancel{\text{mol}_s}} \right]}{\left[\frac{\text{kg}_\alpha}{\cancel{\text{mol}_\alpha}} \right]} = \left[\frac{\text{kg}_s}{\text{kg}_\alpha} \right]$$

- `vmr_from_mmr()`

$$V_R = \frac{M_R \times M_\alpha}{M_s} \rightarrow \frac{\left[\frac{\cancel{\text{kg}_s}}{\cancel{\text{kg}_\alpha}} \right] \left[\frac{\cancel{\text{kg}_\alpha}}{\text{mol}_\alpha} \right]}{\left[\frac{\cancel{\text{kg}_s}}{\text{mol}_s} \right]} = \left[\frac{\text{mol}_s}{\text{mol}_\alpha} \right]$$