Technical University of Denmark

Computational Imaging and Spectroscopy

Lecture 1: Introduction to Color imaging Science

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DTU Fotonik
Department of Photonics Engineering

Computational Imaging and Spectroscopy

Course plan

14 Lectures (7 days)

- ❑Introduction to color imaging science
- ❑Digital Imaging processing
- ❑Sparse and redundant representation
- ❑Image restoration and inverse problems
- ❑Segmentation and local keypoints
- ❑Introduction to Deep Learning in imaging
- ❑Computational spectroscopy and image analysis

Group projects (7 days)

Oral exam + written report (10 pages max)

Computational Imaging and Spectroscopy

Suggestion of projects

Groups of 3-4 persons

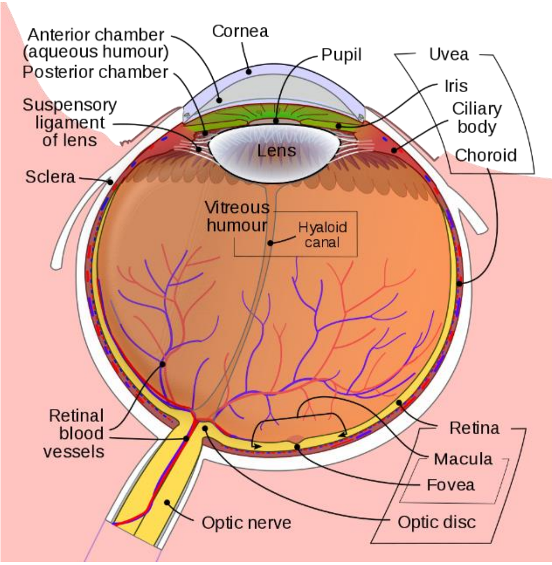
- ❑Colorization of Black and white images
- ❑Low light enhancement
- ❑Image super-resolution
- ❑Classification of hyperspectral images
- ❑Hyperspectral images reconstruction from RGB image
- ❑Classification of medical images

Computational Imaging and Spectroscopy

COLOUR VISION

Physiology and anatomy of the human visual system

The eye's main components for vision are

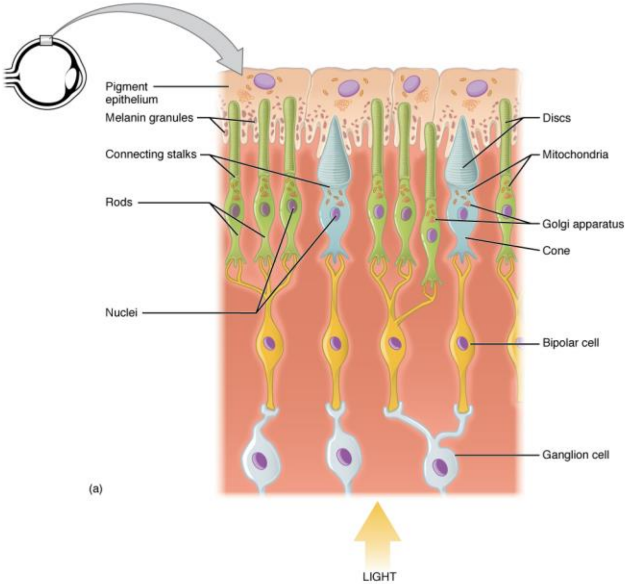


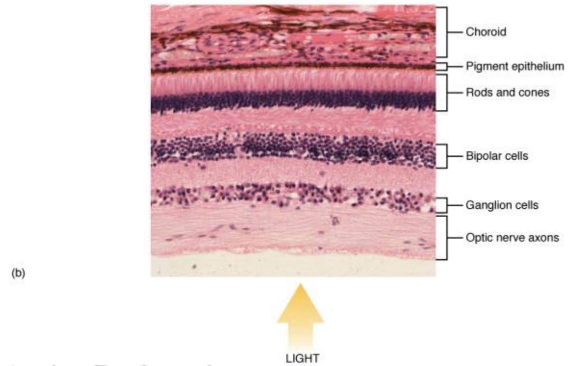
- ❑ The retina, which contains neurons that convert light into:
  - Image forming (IF) signals
  - Non image forming (NIF) signals

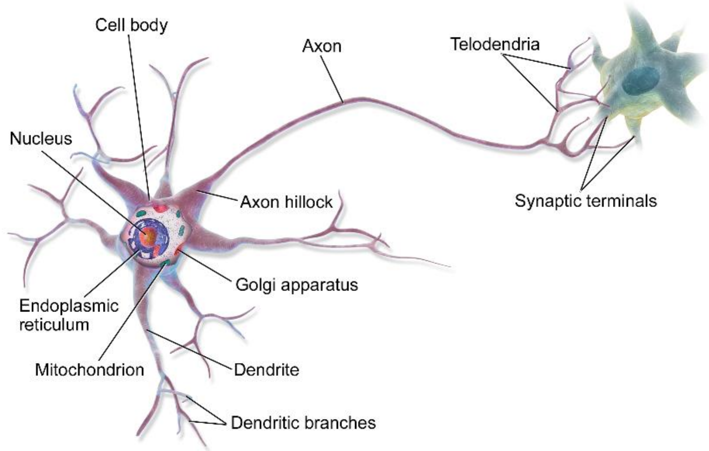
- ❑ Optics (lens, pupil) that focus light on the retina

Physiology and anatomy of the human visual system

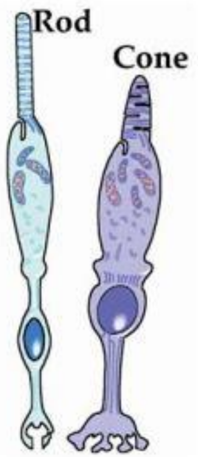
The retina comprises two photoreceptor cells for sensing light, rods and cones, that project into retinal neurons

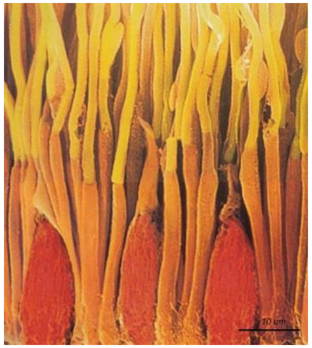


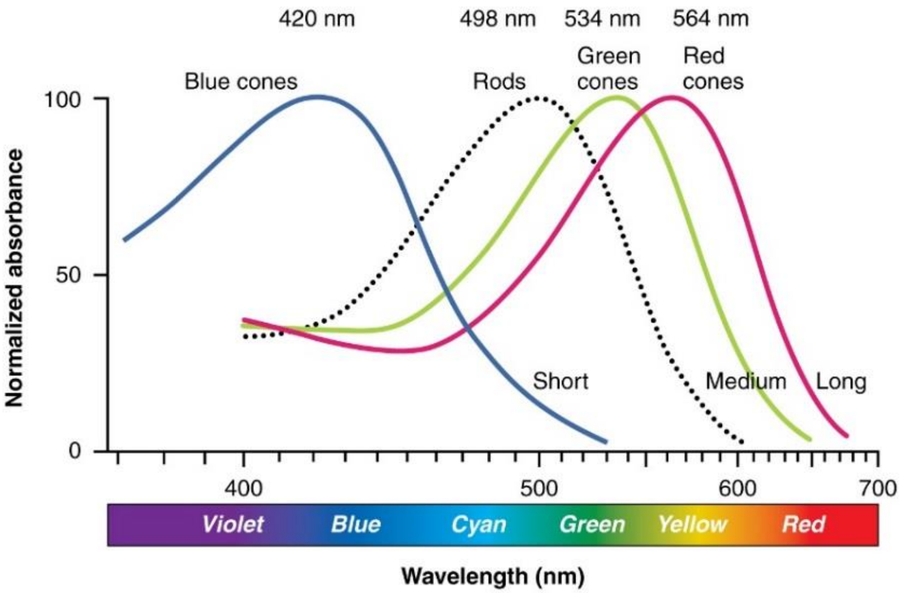




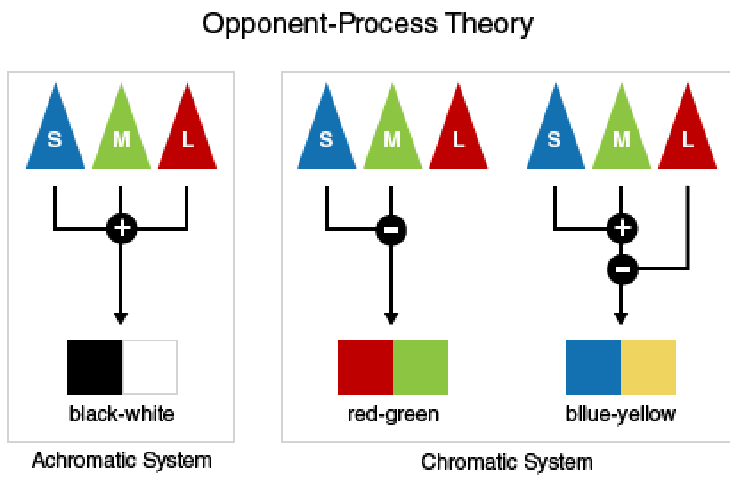
Physiology and anatomy of the human visual system

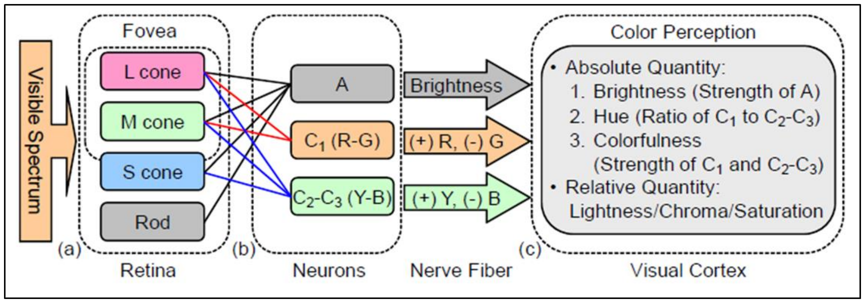






Physiology and anatomy of the human visual system





- ❑ The three types of cones (L, M, S) have different spectral sensitivities
- ❑ Colours and contrast are encoded through an ON/OFF pathway
- ❑ Colours are ultimately processed in the primary visual cortex (V1)
- ❑ Both cones and rods share the same pathway → rods also contribute to colour vision

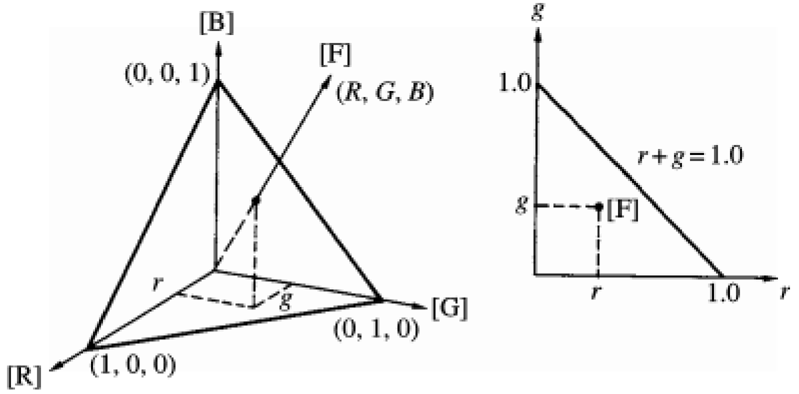
COLORIMETRY

International Commission on Illumination
Commission Internationale de l'Eclairage
Internationale Beleuchtungskommission

Colorimetry

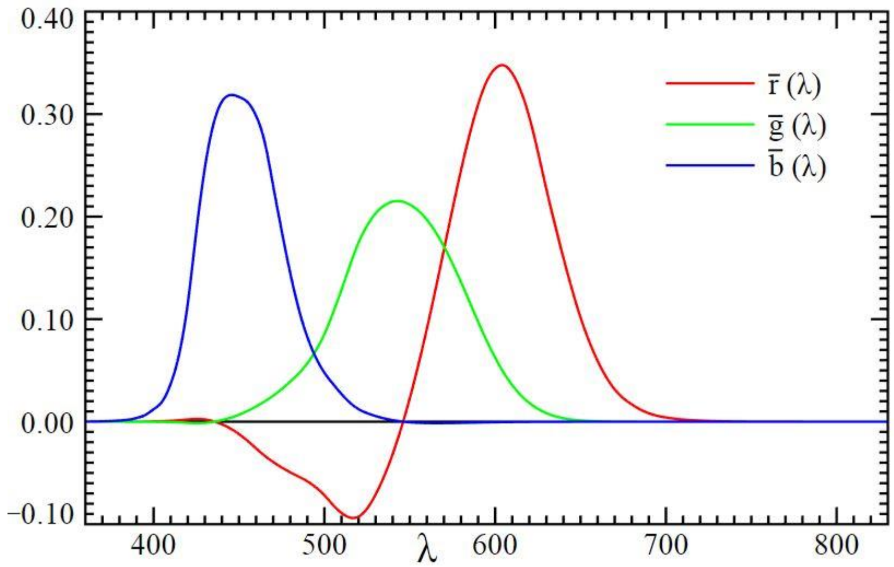
RGB system

$$[F_\lambda] = -R[R] + G[G] + B[B]$$
$$r = R/(R+G+B)$$
$$g = G/(R+G+B)$$
$$b = B/(R+G+B)$$



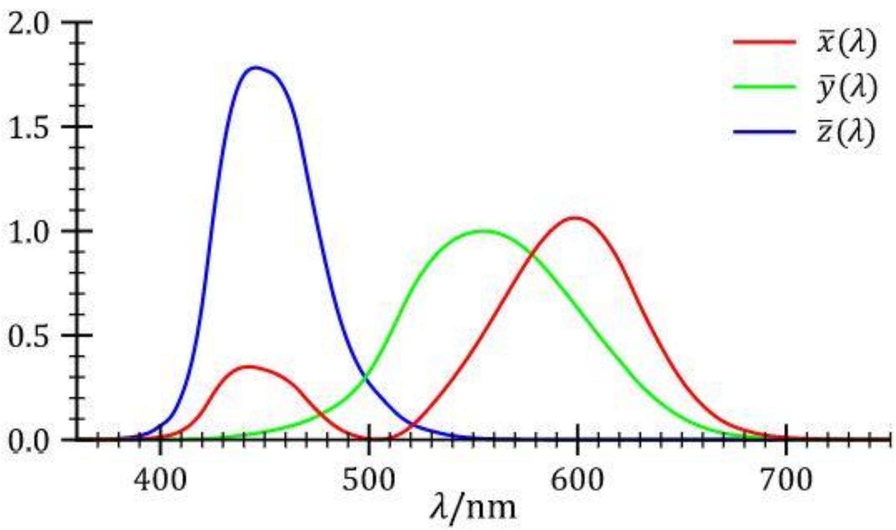
Colorimetry

CIE rgb matching functions



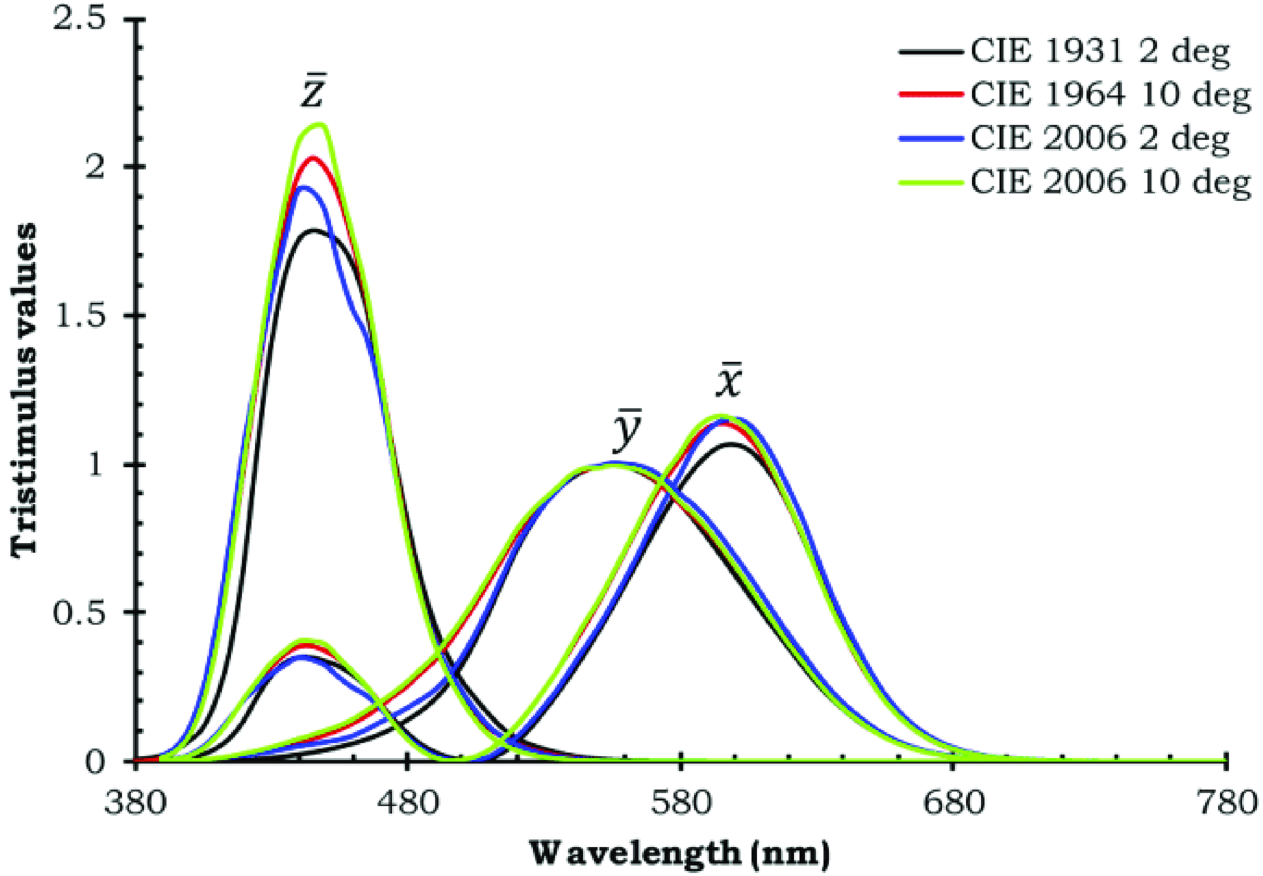
Colorimetry

CIE 1931 xyz matching functions



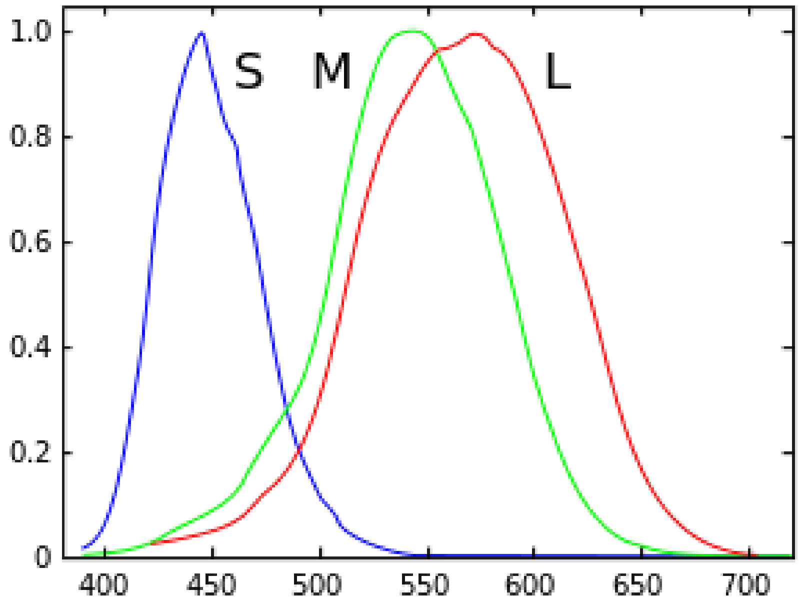
Colorimetry

Color matching functions



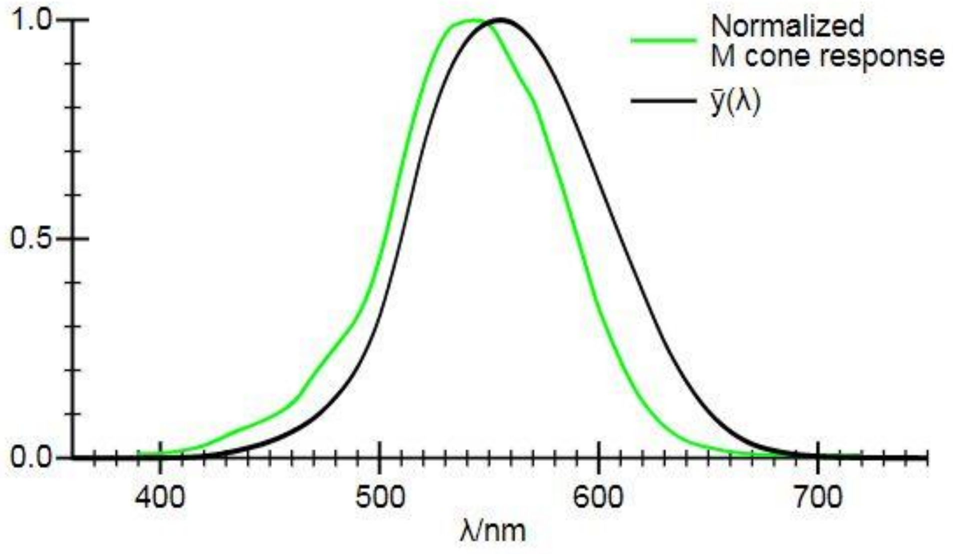
Colorimetry

LMS matching functions



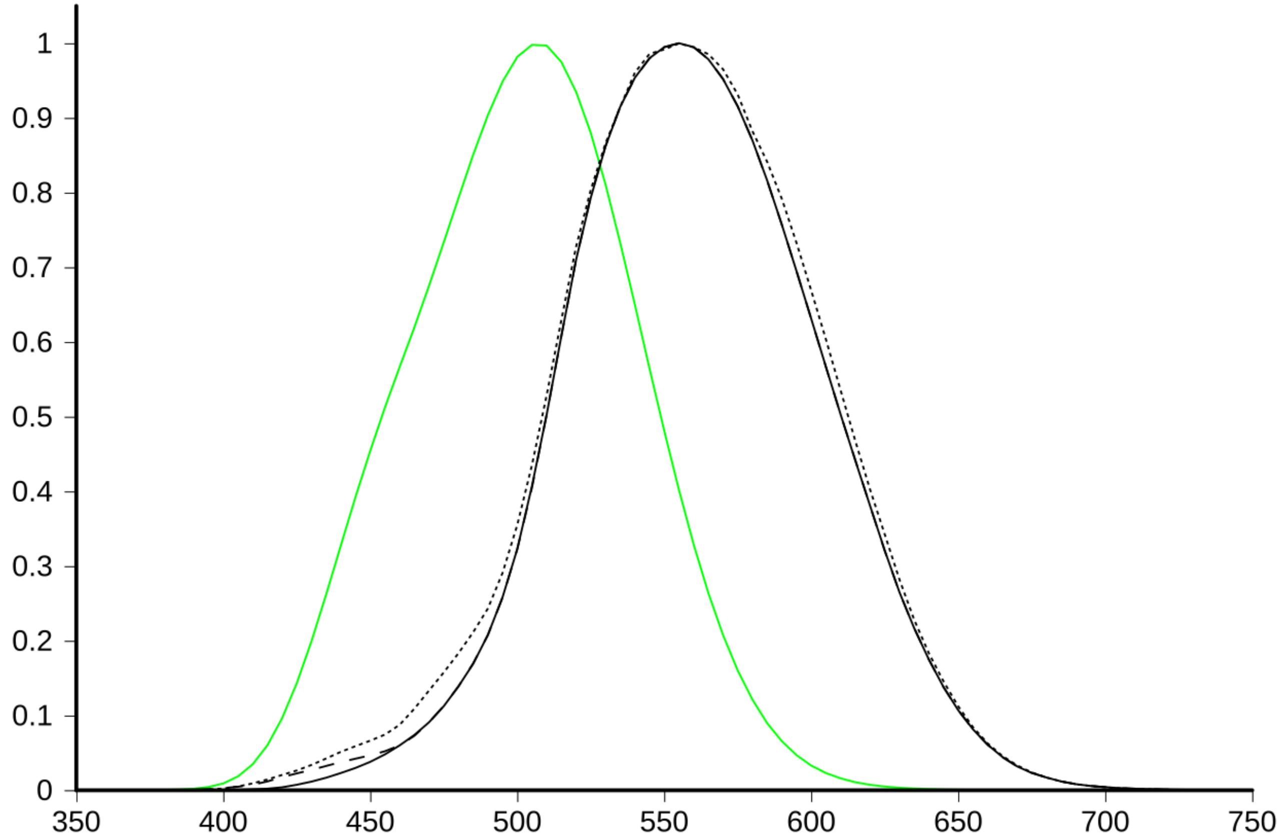
Colorimetry

$$V(\lambda)$$



Colorimetry

$$V(\lambda) and\, V'(\lambda)$$



Colorimetry

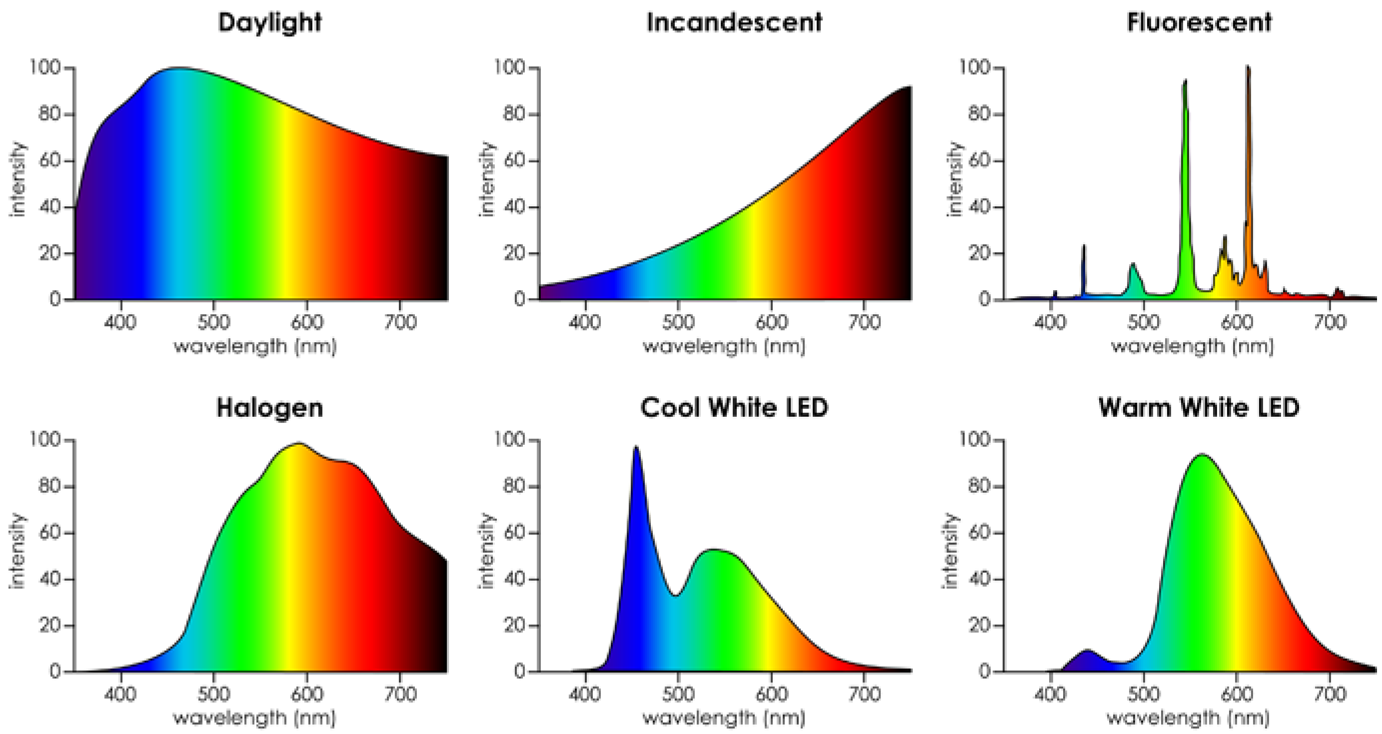
radiometric quantities		
Quantity	**Definition**	**Unit**
radiant energy	Q_e	J
radiant flux	$\Phi_e = dQ_e/dt$	W (J/s)
radiant intensity	$I_e = d\Phi_e/d\omega$	W/sr
irradiance	$E_e = d\Phi_e/dS$	W/m^2
radiant exitance	$M_e = d\Phi_e/dS$	W/m^2
radiance	$L_e = d\Phi_e/(dS \cdot \cos\theta \cdot d\omega)$	W/(sr · m^2)

photometric quantities		
Quantity	**Definition**	**Unit**
quantity of light	Q_v	lm · s
luminous flux	$\Phi_v = dQ_v/dt$	lm
luminous intensity	$I_v = d\Phi_v/d\omega$	lm/sr (cd)
illuminance	$E_v = d\Phi_v/dS$	lm/m^2 (lx)
luminous exitance	$M_v = d\Phi_v/dS$	lm/m^2
luminance	$L_v = d\Phi_v/(dS \cdot \cos\theta \cdot d\omega)$	lm/(sr · m^2)

(t: time, ω: solid angle, S: area, and θ: the angle between the normal of the plane element and the direction of observation)

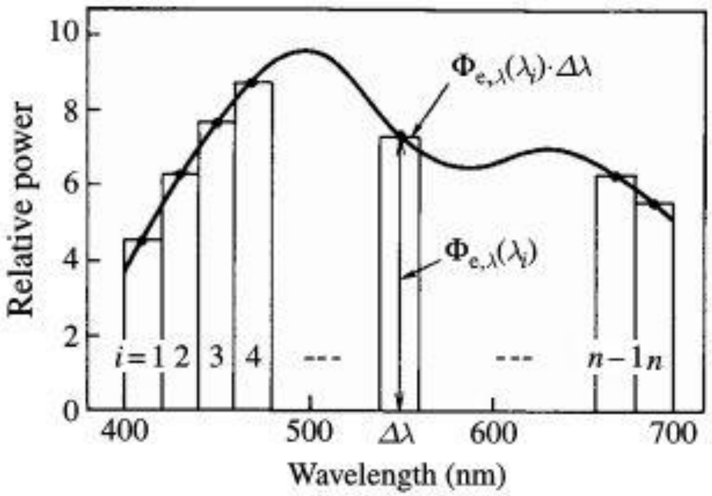
Colorimetry

Spectral Power Distribution (SPD)



Colorimetry

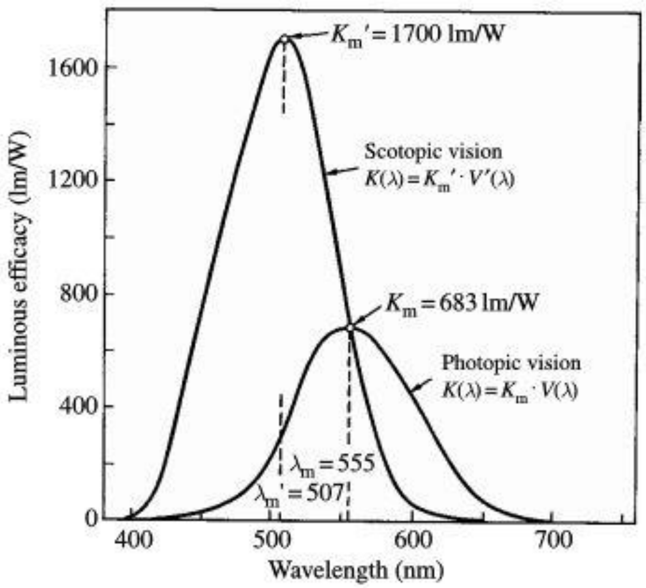
Radiant Flux



$$\Phi_e = \sum_{i=1}^{n} \Phi_{e,\lambda}(\lambda_i)\Delta\lambda$$

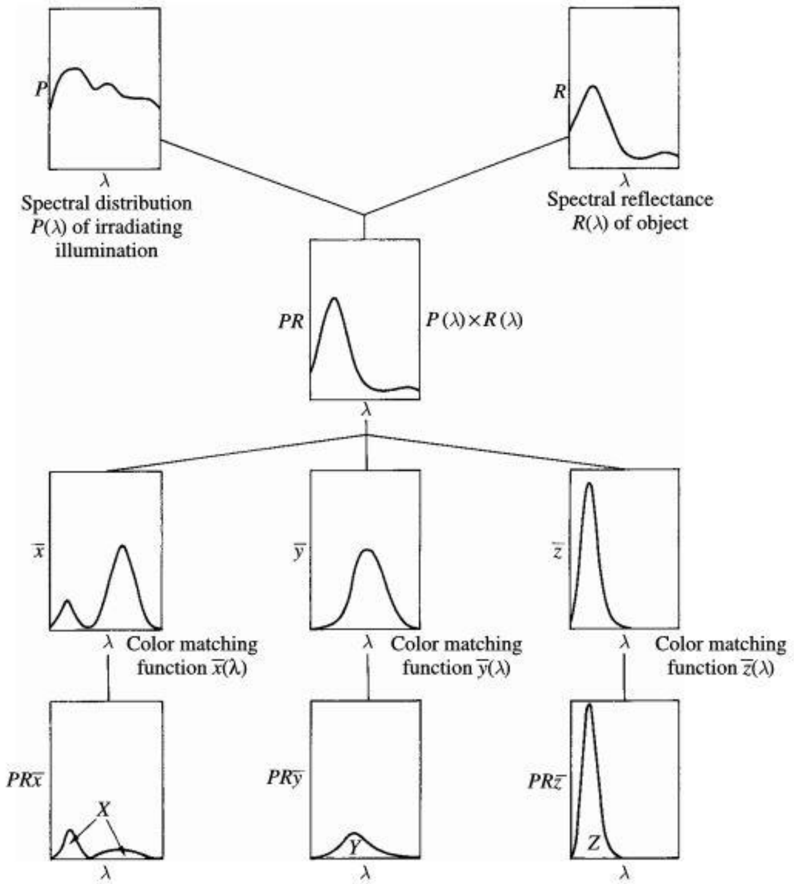
Colorimetry

Luminous Flux



$$\Phi_v = K_m \int_{vis} \Phi_{e,\lambda}(\lambda) V(\lambda) d\lambda$$

Colorimetry



Colorimetry

Tristimulus

Emissive case

$$X = \int_{vis} \Phi(\lambda)\,\overline{x}(\lambda)\,d\lambda$$

$$Y = \int_{vis} \Phi(\lambda)\,\overline{y}(\lambda)\,d\lambda$$

$$Z = \int_{vis} \Phi(\lambda)\,\overline{z}(\lambda)\,d\lambda$$

Colorimetry

Tristimulus

Reflective case

$$X = k\int_{vis} R(\lambda)P(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = k\int_{vis} R(\lambda)P(\lambda)\bar{y}(\lambda)d\lambda$$

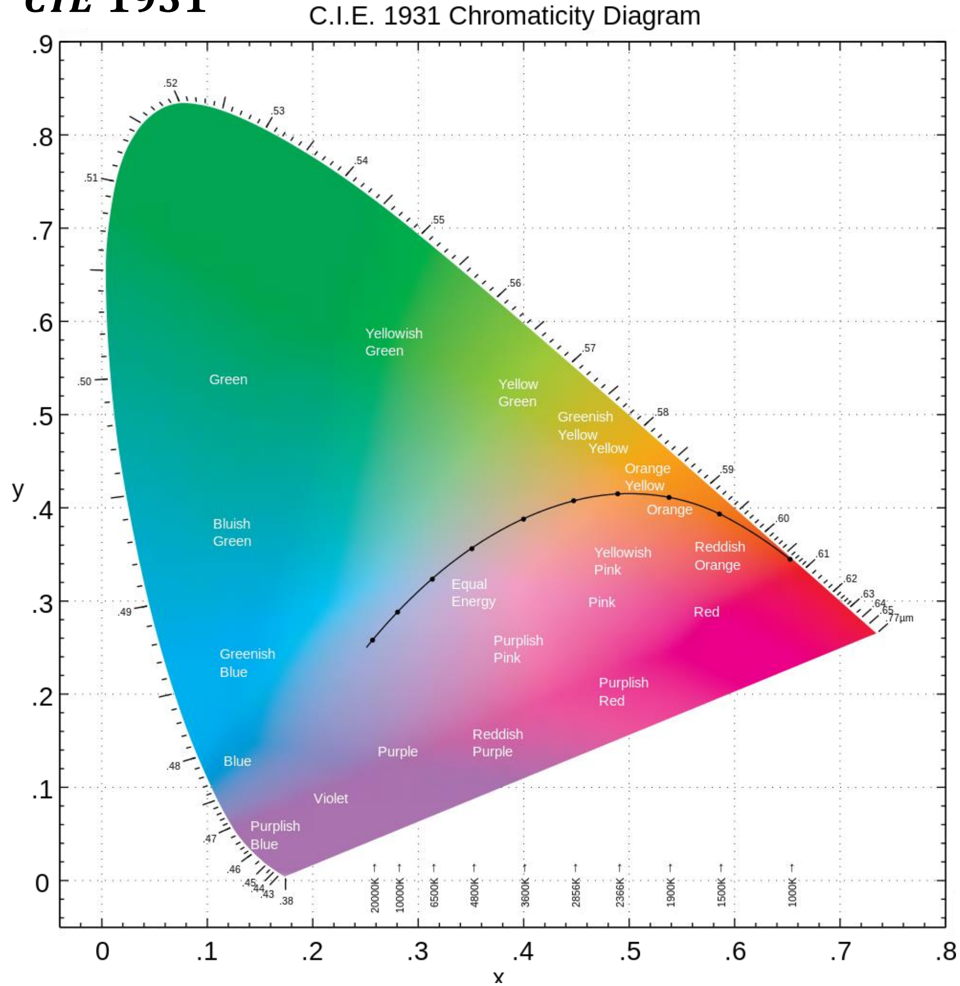
$$Z = k\int_{vis} R(\lambda)P(\lambda)\bar{z}(\lambda)d\lambda$$

$$k = 100 \Big/ \int_{vis} P(\lambda)\bar{y}(\lambda)d\lambda$$

Colorimetry

$$x = X/(X+Y+Z)$$
$$y = Y/(X+Y+Z)$$

$CIE\ 1931$



Colorimetry

$$CIE\ 1931\ (Planckian\ locus)$$

$$X_T = \int_0^\infty X(\lambda) M(\lambda, T)\, d\lambda$$

$$Y_T = \int_0^\infty Y(\lambda) M(\lambda, T)\, d\lambda$$

$$Z_T = \int_0^\infty Z(\lambda) M(\lambda, T)\, d\lambda$$

$$M(\lambda, T) = \frac{c_1}{\lambda^5} \frac{1}{\exp\left(\frac{c_2}{\lambda T}\right) - 1}$$

$c_1 = 2\pi hc^2$ is the first radiation constant

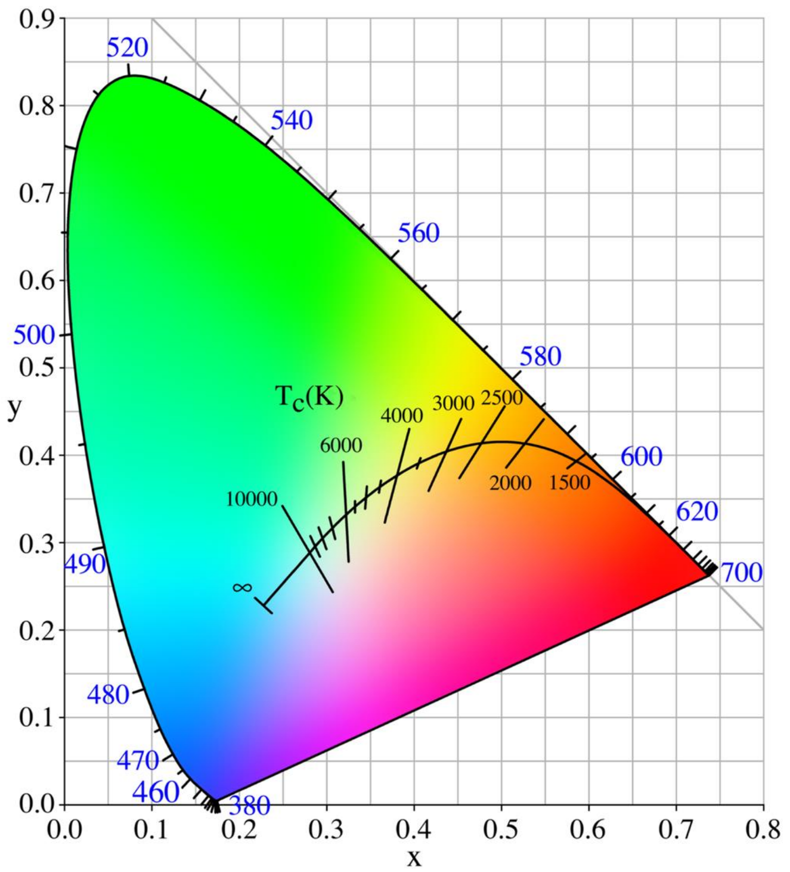
$c_2 = hc/k$ is the second radiation constant

T is the temperature of the black body

h is Planck's constant

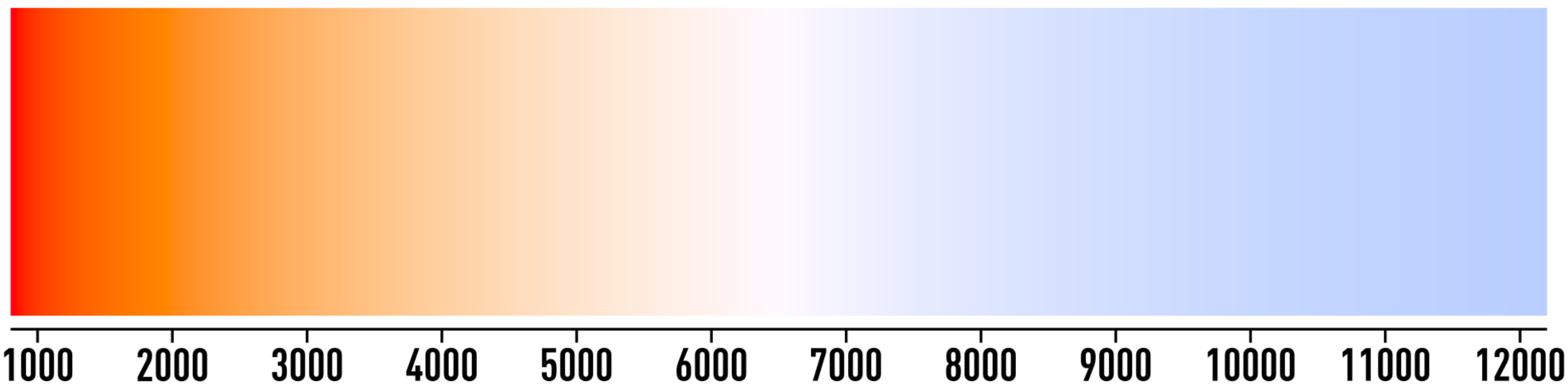
c is the speed of light

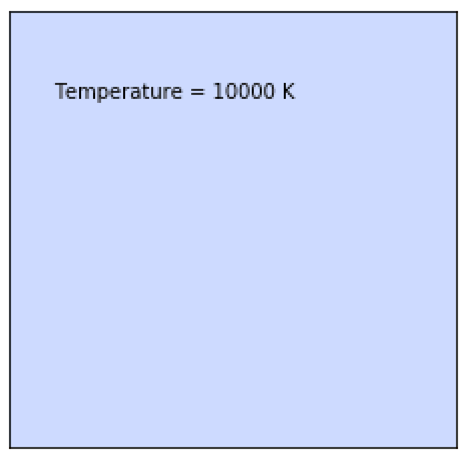
k is Boltzmann's constant



Colorimetry

Correlated color temperature (CCT)





Colorimetry

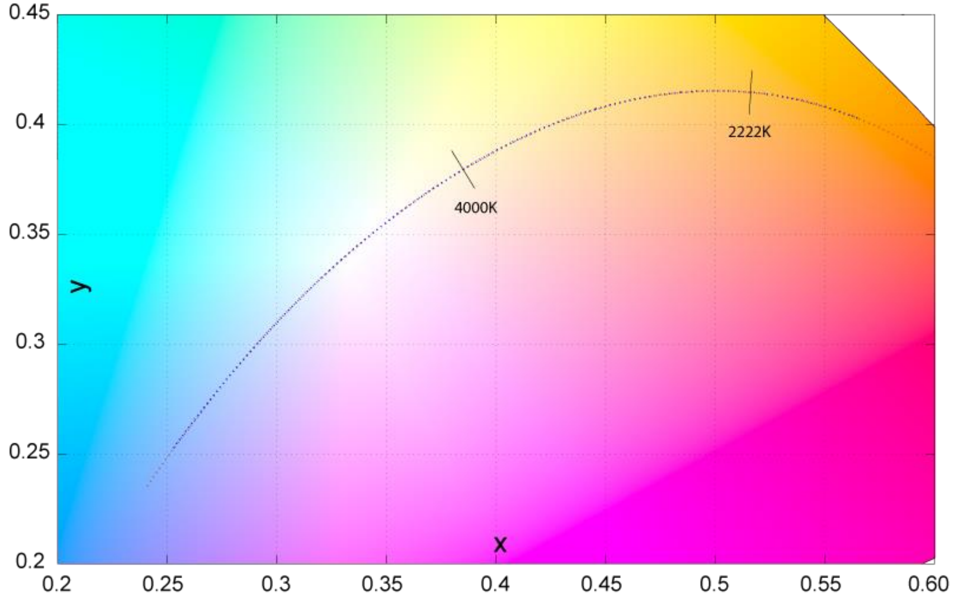
Approximation of CCT

$$T \approx 437n^3 + 3601n^2 + 6831n + 5517$$

$$n = (x - 0.3320)/(0.1858 - y)$$

Colorimetry

***CIE* 1931 (*Planckian locus*)**

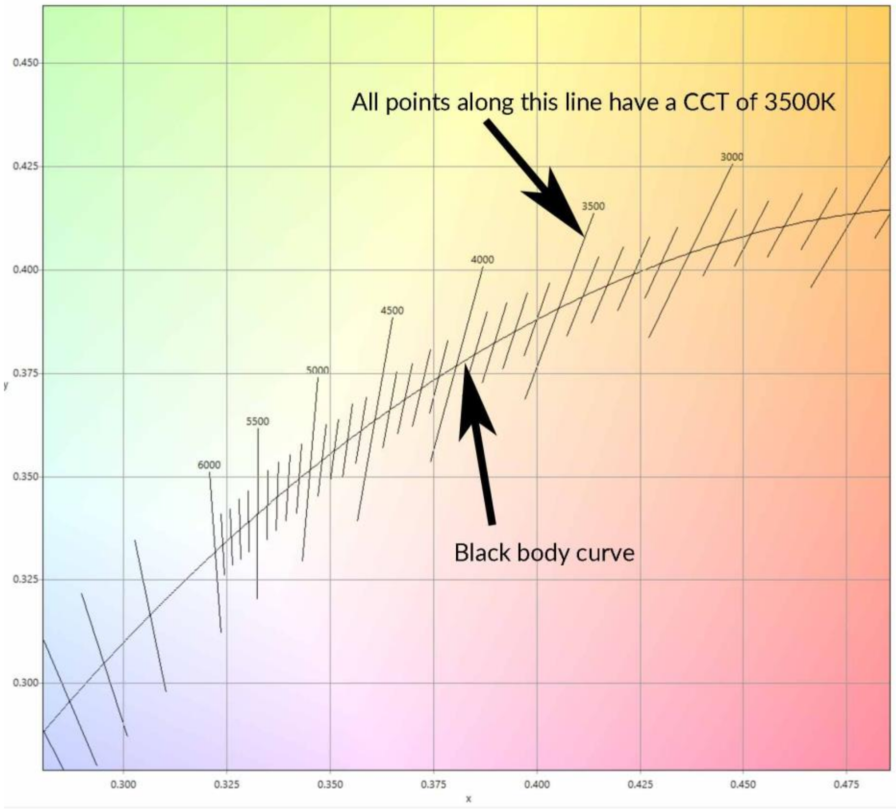


$$\bar{u}(T) = \frac{0.860117757 + 1.54118254 \times 10^{-4} T + 1.28641212 \times 10^{-7} T^2}{1 + 8.42420235 \times 10^{-4} T + 7.08145163 \times 10^{-7} T^2}$$

$$\bar{v}(T) = \frac{0.317398726 + 4.22806245 \times 10^{-5} T + 4.20481691 \times 10^{-8} T^2}{1 - 2.89741816 \times 10^{-5} T + 1.61456053 \times 10^{-7} T^2}$$

Colorimetry

$$CIE\ 1931\ (Duv)$$



Colorimetry

$CIE\ \mathbf{1931}$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{b_{21}} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \frac{1}{0.176{,}97} \begin{bmatrix} 0.490\,00 & 0.310\,00 & 0.200\,00 \\ 0.176\,97 & 0.812\,40 & 0.010\,63 \\ 0.000\,00 & 0.010\,00 & 0.990\,00 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.418\,47 & -0.158\,66 & -0.082\,835 \\ -0.091\,169 & 0.252\,43 & 0.015\,708 \\ 0.000\,920\,90 & -0.002\,549\,8 & 0.178\,60 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Colorimetry

sRGB

$$\begin{bmatrix} R_{\text{linear}} \\ G_{\text{linear}} \\ B_{\text{linear}} \end{bmatrix} = \begin{bmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix}$$

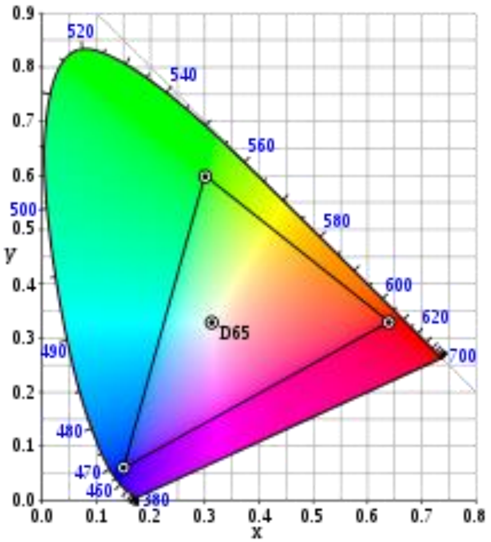
$$\gamma(u) = \begin{cases} 12.92u & u \le 0.0031308 \\ 1.055u^{1/2.4} - 0.055 & \text{otherwise} \end{cases}$$

- where u is R, G, or B.

$$\gamma^{-1}(u) = \begin{cases} u/12.92 & u \le 0.04045 \\ \left(\frac{u+0.055}{1.055}\right)^{2.4} & \text{otherwise} \end{cases}$$

- where u is R, G, or B.

$$\begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7152 & 0.0722 \\ 0.0193 & 0.1192 & 0.9504 \end{bmatrix} \begin{bmatrix} R_{\text{linear}} \\ G_{\text{linear}} \\ B_{\text{linear}} \end{bmatrix}$$

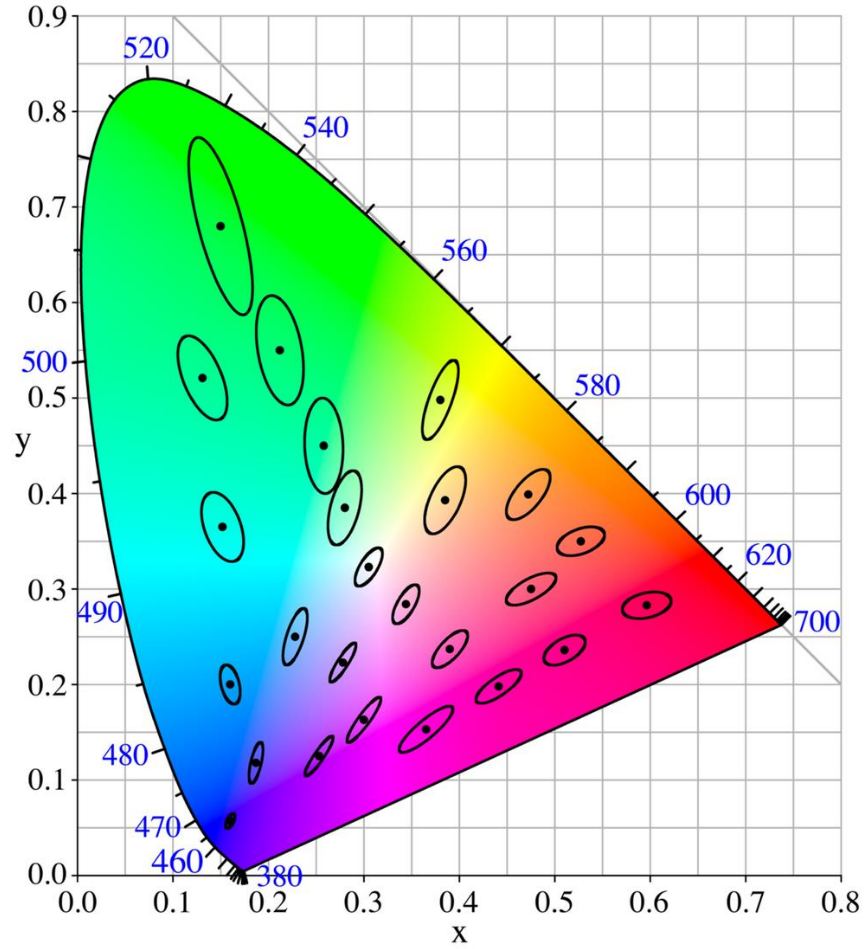


Colorimetry

❑ Ellipse's countour represent the just noticeable difference (JND)

❑ Used in **Standard Deviation Color Matching**

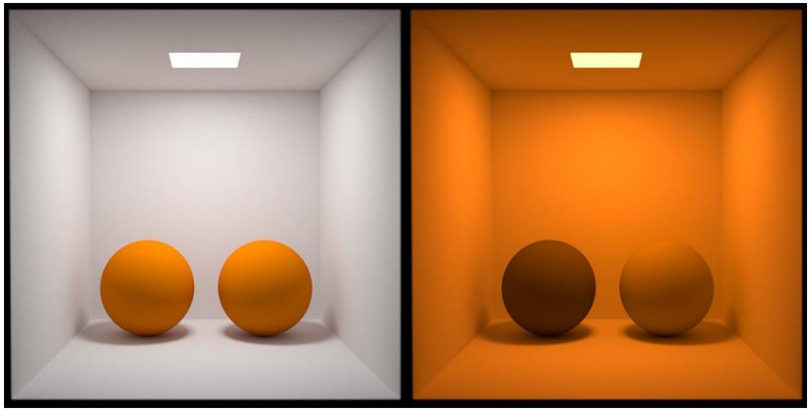
Mac Adam ellipses



Colorimetry

Metamerism

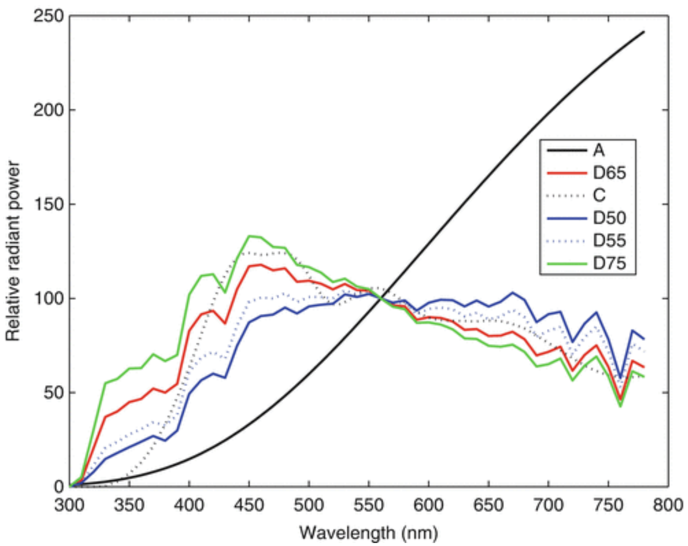
$$\int_{vis} R(\lambda)P(\lambda)\bar{x}(\lambda)d\lambda = \int_{vis} R'(\lambda)P(\lambda)\bar{x}(\lambda)d\lambda$$
$$\int_{vis} R(\lambda)P(\lambda)\bar{y}(\lambda)d\lambda = \int_{vis} R'(\lambda)P(\lambda)\bar{y}(\lambda)d\lambda$$
$$\int_{vis} R(\lambda)P(\lambda)\bar{z}(\lambda)d\lambda = \int_{vis} R'(\lambda)P(\lambda)\bar{z}(\lambda)d\lambda$$



Colorimetry

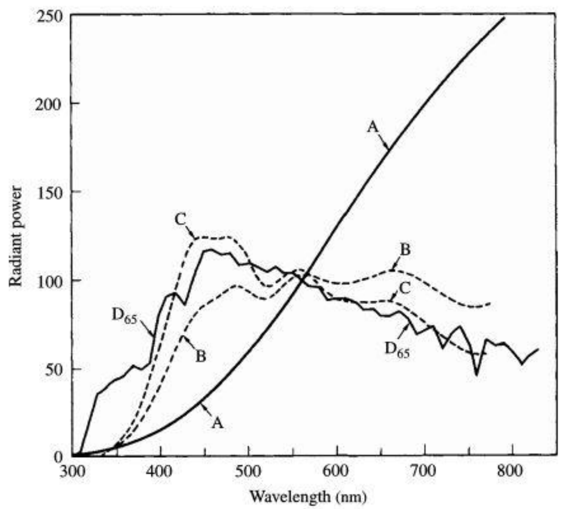
CIE standard Illuminants

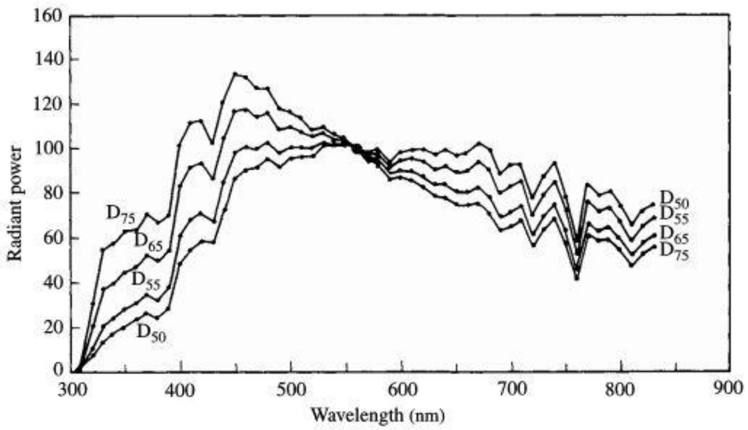
- **Illuminant A**
  - Incandescent following a black radiator law
- **Illuminant B et C**
  - Daylight simulator CCT 4874 K (B)/6774 K (C)
  - Reproducible with filters
- **Illuminant series D**
  - Daylight
  - Numerically reproducible
- **Illuminant E**
  - Equi-energy (x=y=1/3)
- **Illuminant series F**
  - Fluorescent
- **Illuminant series L**
  - LED (2017)

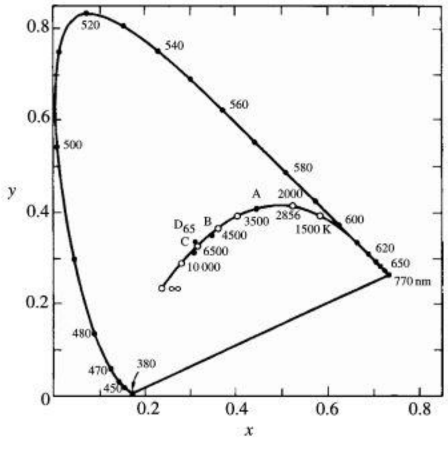


Colorimetry

CIE standard Illuminants



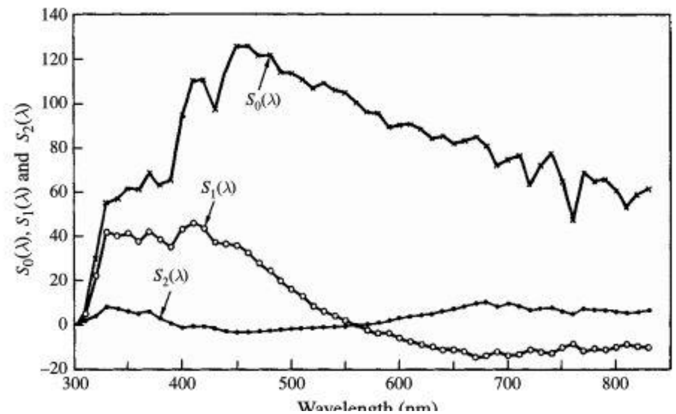


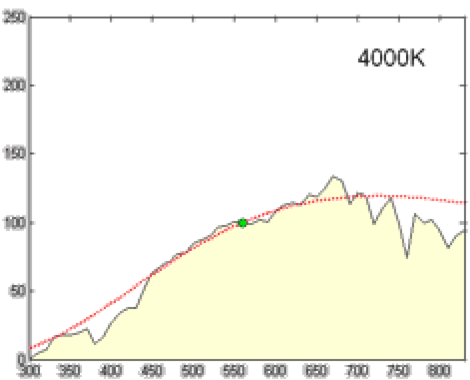


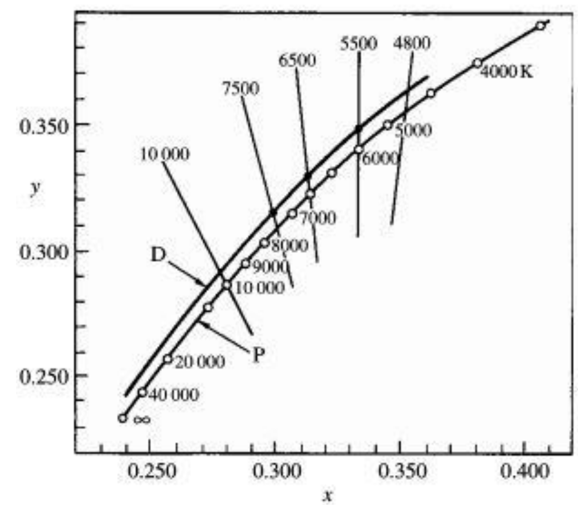
Colorimetry

CIE D Illuminants

$$y_D = -3.000x_D^2 + 2.870x_D - 0.275$$
$$M_1 = \left(-1.3515 - 1.7703x_D + 5.9114y_D\right) / \left(0.0241 + 0.2562x_D - 0.7341y_D\right)$$
$$M_2 = \left(0.0300 - 31.442x_D + 30.0717y_D\right) / \left(0.0241 + 0.2562x_D - 0.7341y_D\right)$$
$$S_D(\lambda) = S_0(\lambda) + M_1 S_1(\lambda) + M_2 S_2(\lambda)$$







Colorimetry

CIE standard Illuminants

$$4000K \le T_{cp} \le 7000K$$

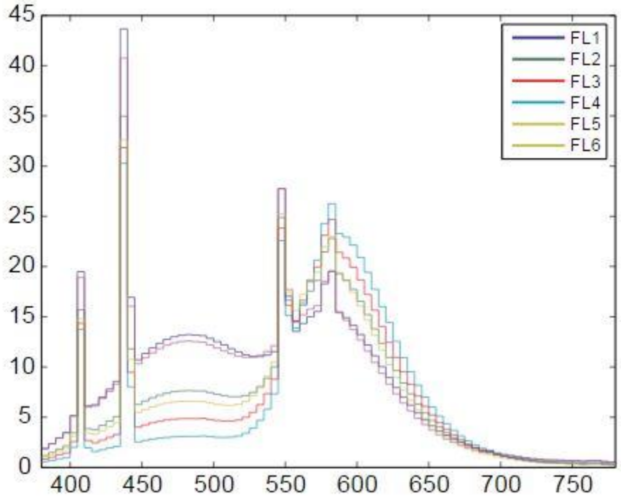
$$x_D = -4.6070 \times 10^9 \big/ T_{cp}^3 + 2.9678 \times 10^6 \big/ T_{cp}^2 + 0.09911 \times 10^3 \big/ T_{cp} + 0.244063$$

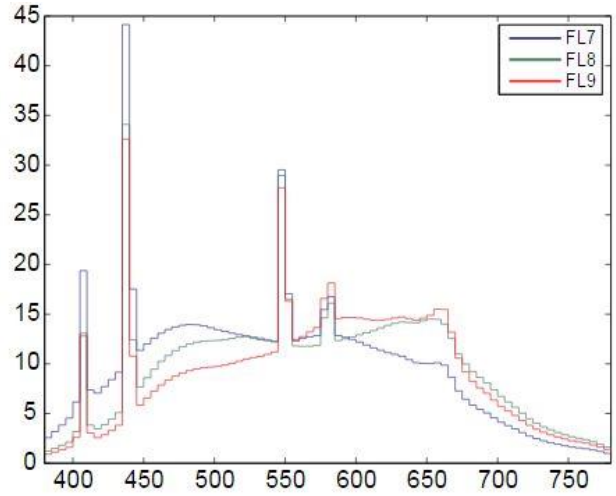
$$7000K \le T_{cp} \le 25000K$$

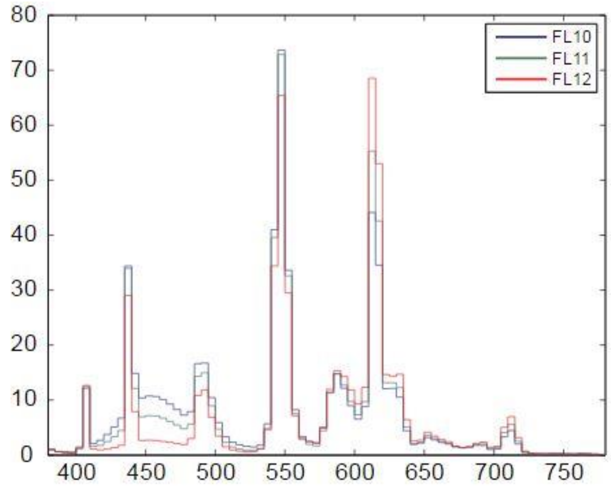
$$x_D = -2.0064 \times 10^9 \big/ T_{cp}^3 + 1.9081 \times 10^6 \big/ T_{cp}^2 + 0.24748 \times 10^3 \big/ T_{cp} + 0.237040$$

Colorimetry

CIE Illuminants



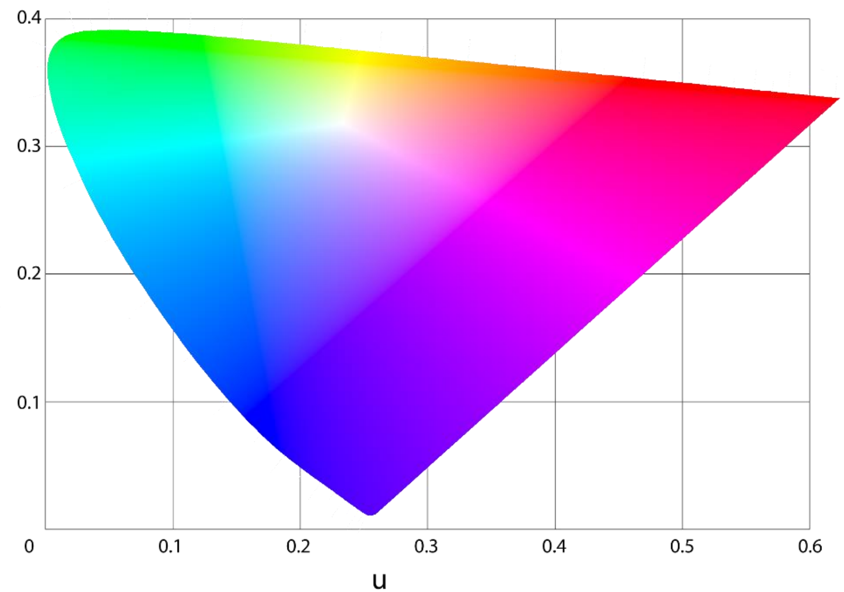


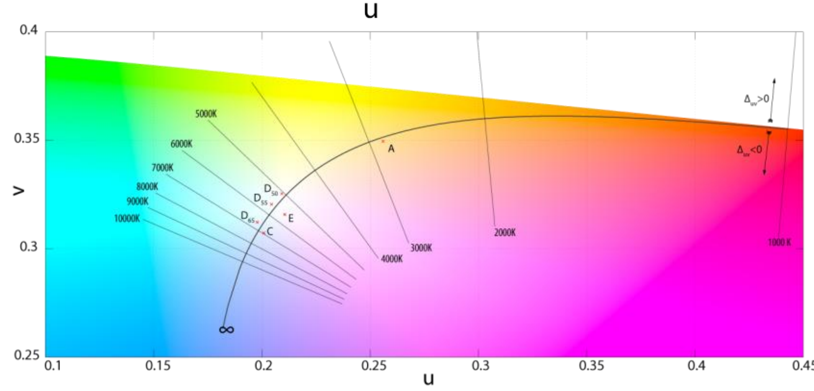


Colorimetry

uniform CIE 1960 space

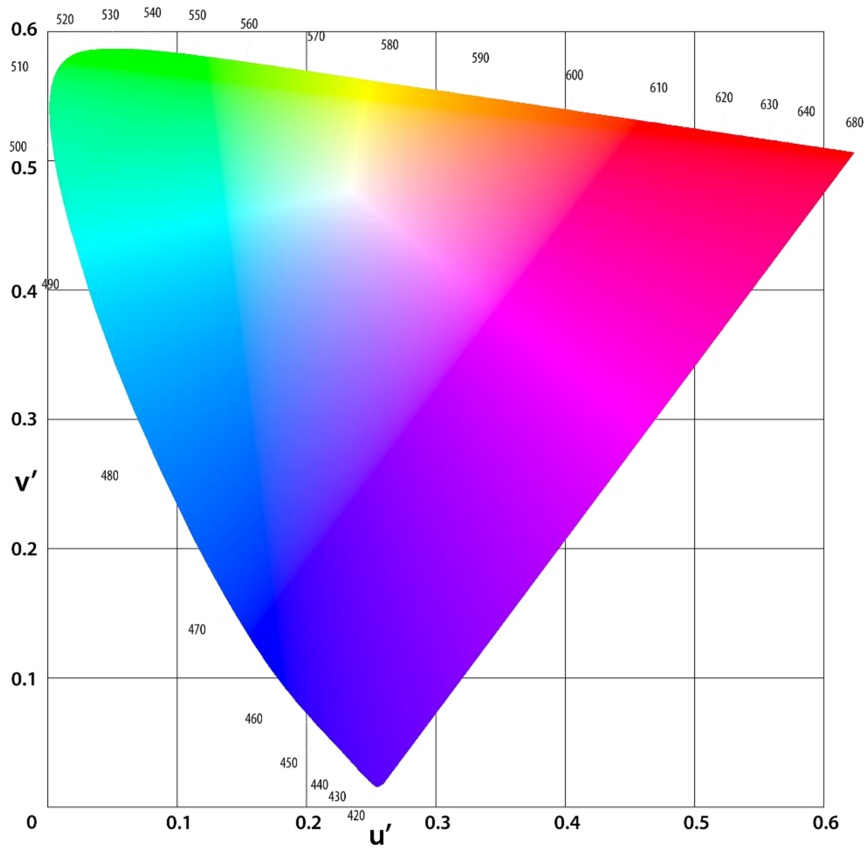
$$u = 4X/(X + 15Y + 3Z)$$
$$v = 6Y/(X + 15Y + 3Z)$$





Colorimetry

uniform CIE 1976 space



Colorimetry

CIE L*u*v*

$$u' = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v' = \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3}$$

$$L^* = \begin{cases} \left(\frac{29}{3}\right)^3 Y/Y_n, & Y/Y_n \le \left(\frac{6}{29}\right)^3 \\ 116\left(Y/Y_n\right)^{1/3} - 16, & Y/Y_n > \left(\frac{6}{29}\right)^3 \end{cases}$$

$$u^* = 13L^*\left(u' - u'_n\right)$$

$$v^* = 13L^*\left(u' - v'_n\right)$$

Colorimetry

CIE L*a*b*

$$L^* = 116 f\left(\frac{Y}{Y_n}\right) - 16$$

$$a^* = 500\left(f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right)$$

$$b^* = 200\left(f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right)$$

$$f(t) = \begin{cases} \sqrt[3]{t} & \text{if } t > \delta^3 \\ \dfrac{t}{3\delta^2} + \dfrac{4}{29} & \text{otherwise} \end{cases} \qquad \delta = \frac{6}{29}$$

Colorimetry

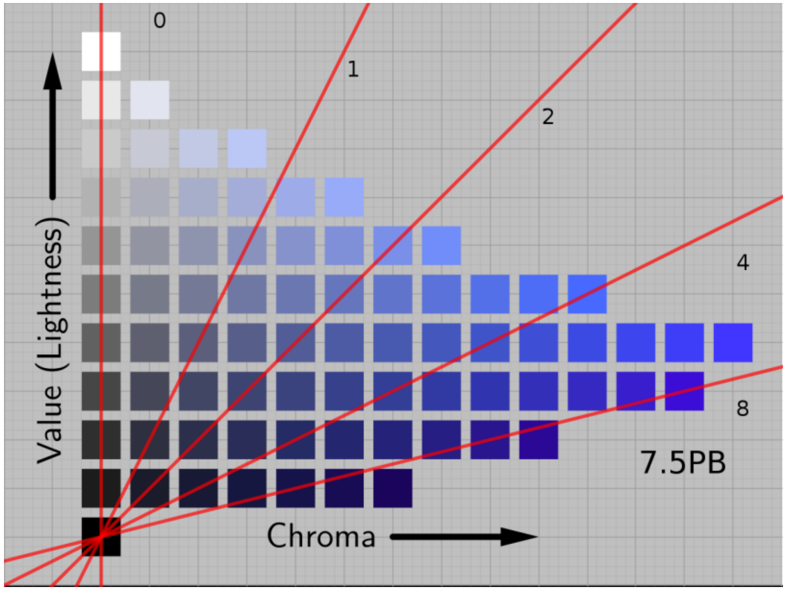
Color correlates

Absolute attributes

- Brightness
- Hue
- Colorfulness

Relative attributes

- Lightness
- Chroma
- Saturation



Colorimetry

$$\text{Lightness} = \frac{\text{Brightness}}{\text{Brightness of White}}$$

$$\text{Chroma} = \frac{\text{Colorfulness}}{\text{Brightness of White}}$$

$$\text{Saturation} = \frac{\text{Chroma}}{\text{Brightness}} = \frac{\dfrac{\text{Colorfunlness}}{\text{Brightness of White}}}{\dfrac{\text{Brightness}}{\text{Brightness of White}}} = \frac{\text{Chroma}}{\text{Lightness}}$$

Colorimetry

Color correlates

$$C_{ab} = \left\{ \left(a^*\right)^2 + \left(b^*\right)^2 \right\}^{1/2}$$

$$C_{uv} = \left\{ \left(u^*\right)^2 + \left(v^*\right)^2 \right\}^{1/2}$$

Chroma

$$S_{uv} = 13\left\{ \left(u^{'} - u_n^{'}\right)^2 + \left(v^{'} - v_n^{'}\right)^2 \right\}^{1/2}$$
$$= C_{uv}^* / L^*$$

Saturation

$$h_{ab} = \tan^{-1}\left(b^*/a^*\right)$$

$$h_{uv} = \tan^{-1}\left(v^*/u^*\right)$$

Hue angle

Colorimetry

$$\Delta E_{ab}^{*}=\sqrt{\left(L_{2}^{*}-L_{1}^{*}\right)^{2}+\left(a_{2}^{*}-a_{1}^{*}\right)^{2}+\left(b_{2}^{*}-b_{1}^{*}\right)^{2}}$$

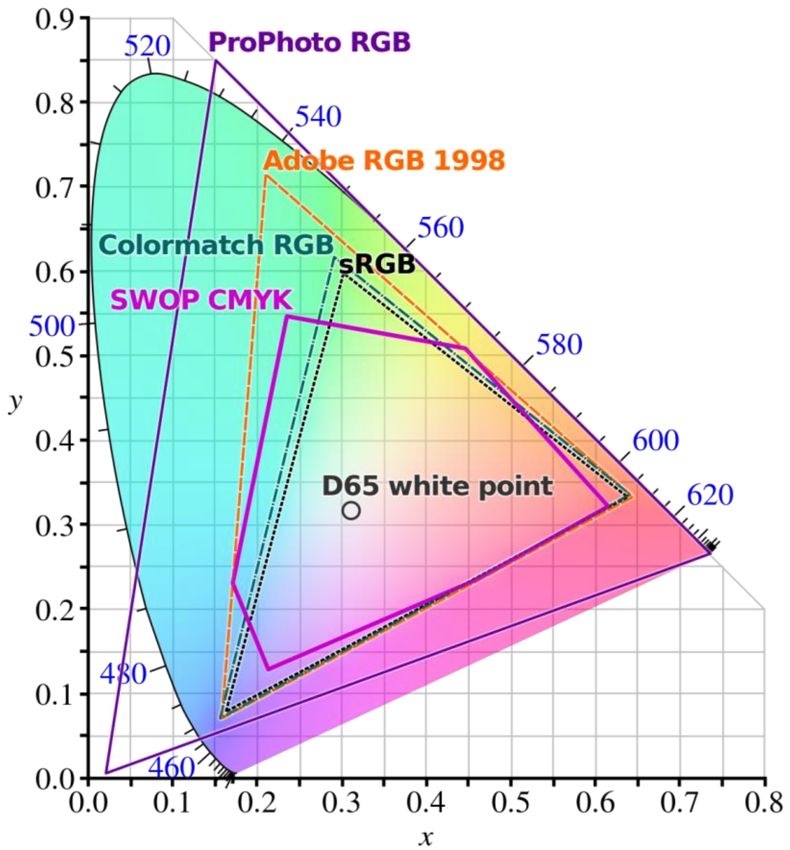
$$\Delta E_{94}^{*}=\sqrt{\left(\frac{\Delta L^{*}}{k_{L}S_{L}}\right)^{2}+\left(\frac{\Delta C_{ab}^{*}}{k_{C}S_{C}}\right)^{2}+\left(\frac{\Delta H_{ab}^{*}}{k_{H}S_{H}}\right)^{2}}$$

$$\Delta E_{00}^{*}=\sqrt{\left(\frac{\Delta L^{'}}{k_{L}S_{L}}\right)^{2}+\left(\frac{\Delta C^{'}}{k_{C}S_{C}}\right)^{2}+\left(\frac{\Delta H^{'}}{k_{H}S_{H}}\right)^{2}+R_{T}\frac{\Delta C^{'}}{k_{C}S_{C}}\frac{\Delta H^{'}}{k_{H}S_{H}}}$$

https://en.wikipedia.org/wiki/Color_difference

TONE and COLOR REPRODUCTION

Color Spaces

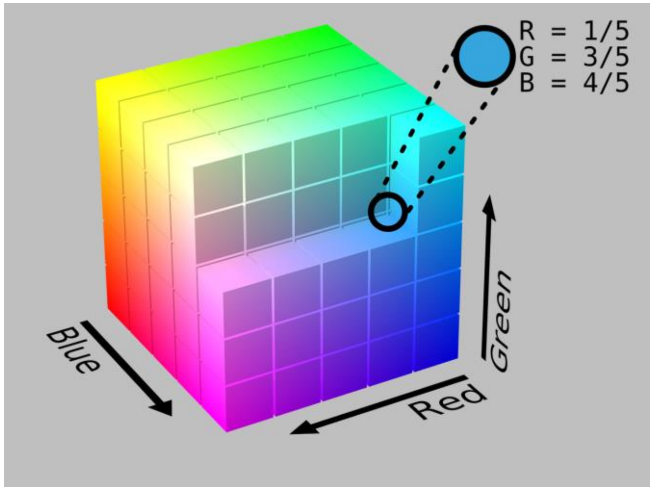


Beside the CIE 1931 color space there exist several other color spaces

Development of a new Color space can be motivated by:

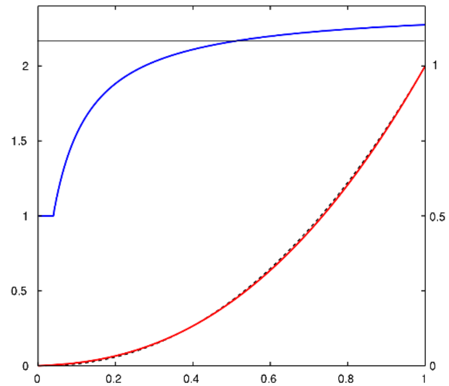
- Physical realizability
- Efficient encoding
- Perceptual uniformity
- Intuitive color specification

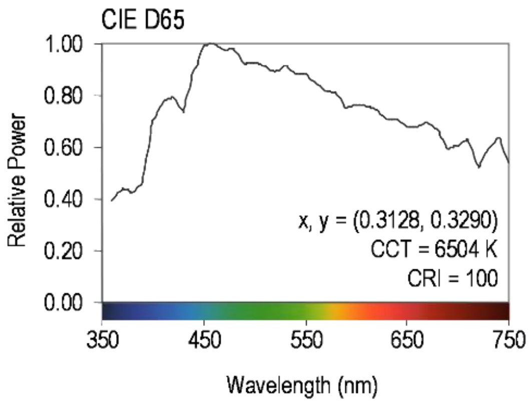
(RGB) Color Spaces



A RGB color space is characterize by a color set that can be reproduction by a mix of red, green and blue monochromatic lights.

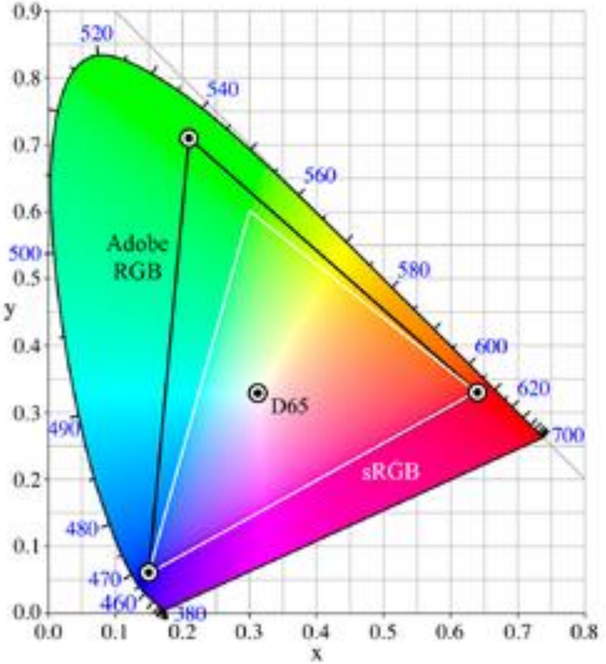
Its specification also requires a white point chromaticity, i.e. D65 for instance, and a gamma correction curve





(RGB) Color Spaces

- **ADOBE 1998 RGB**



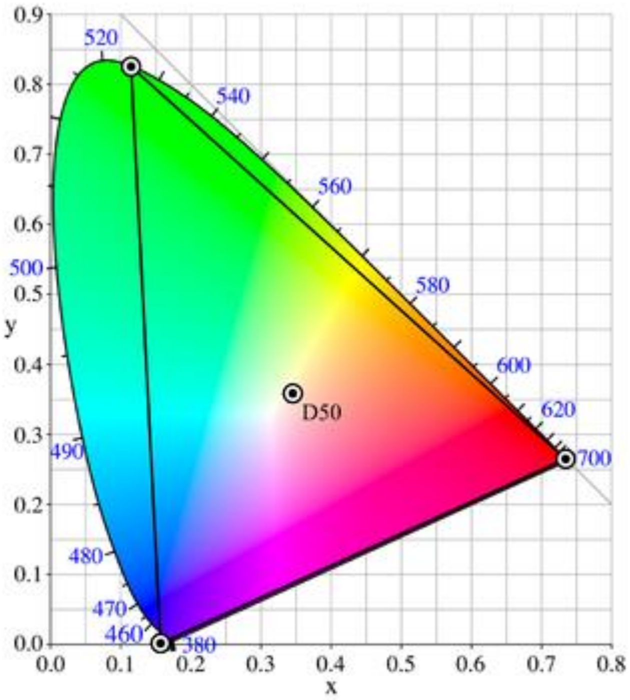
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 2.04159 & -0.56501 & -0.34473 \\ -0.96924 & 1.87597 & 0.04156 \\ 0.01344 & -0.11836 & 1.01517 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.57667 & 0.18556 & 0.18823 \\ 0.29734 & 0.62736 & 0.07529 \\ 0.02703 & -0.07069 & 0.99134 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

White point: D65; $\gamma = 0.45$

(RGB) Color Spaces

- **Wide gamut (ADOBE)**



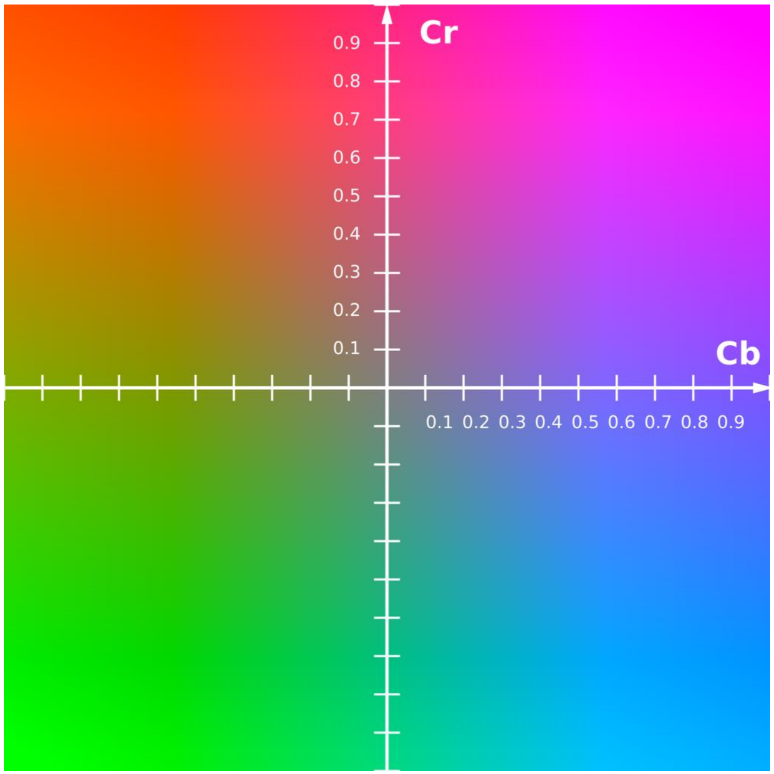
$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.465 & -0.1845 & -0.2734 \\ -0.5228 & 1.4479 & 0.0681 \\ 0.0346 & -0.0958 & 1.2875 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.7164 & 0.1010 & 0.1468 \\ 0.2587 & 0.7247 & 0.0166 \\ 0.0000 & -0.0512 & 0.7740 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

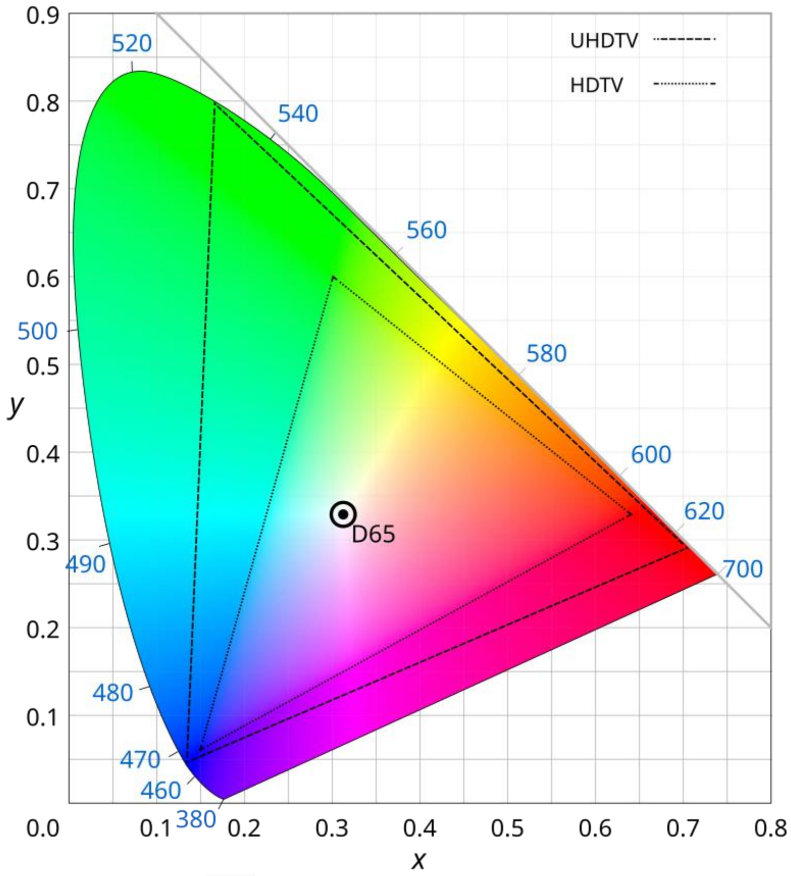
White point: D50; $\gamma = 0.45$

Optimized color space for digital video

- **YCbCr**



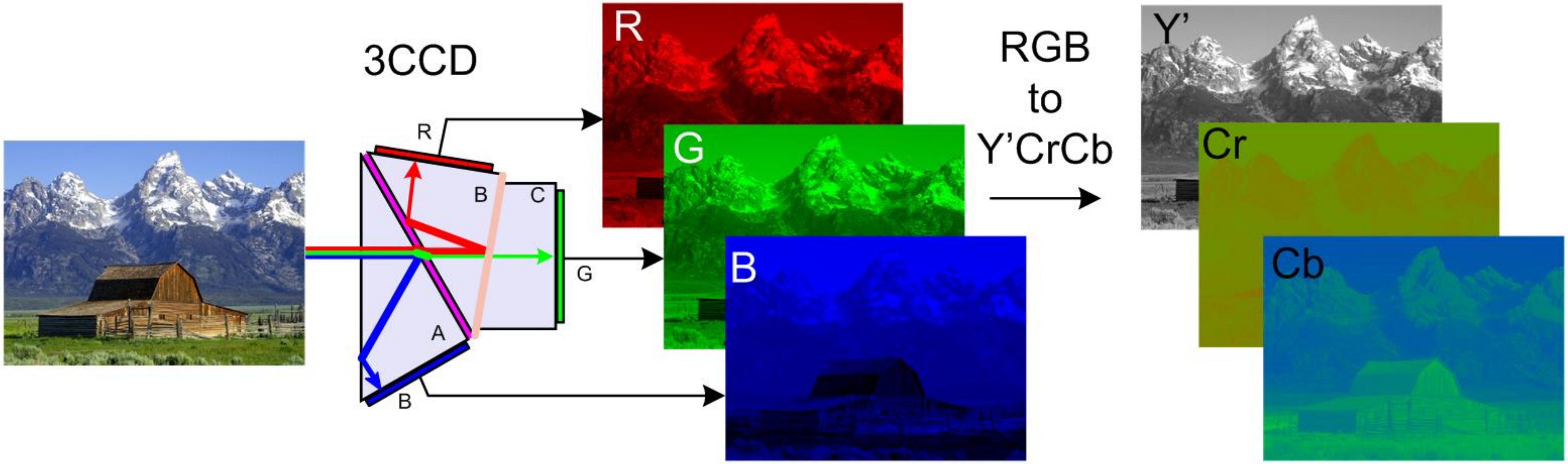
The CbCr plane at constant luma Y′=0.5



Rec. 709 vs Rec. 2020

Optimized color space for digital video

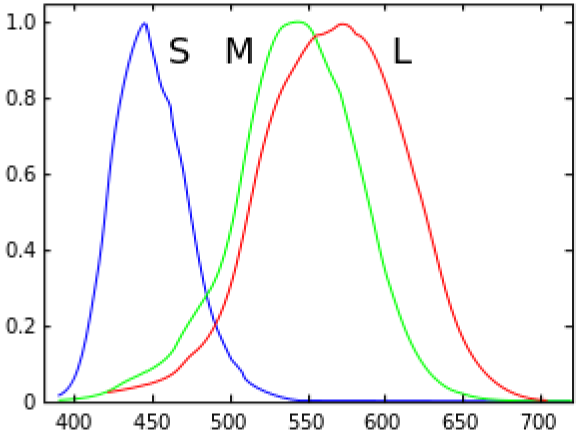
- **YCbCr**



RGB to YCbCr conversion

(HVS derived) Color Spaces

- **LMS color space**



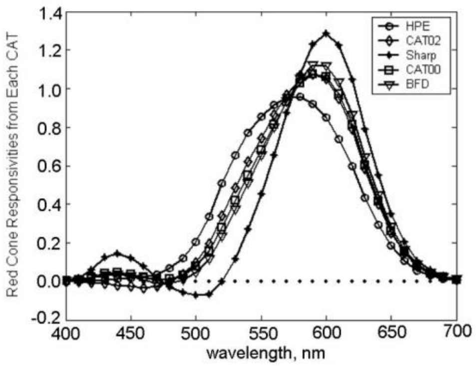
$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.3897 & 0.6890 & -0.0787 \\ -0.2298 & 1.1834 & 0.0464 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

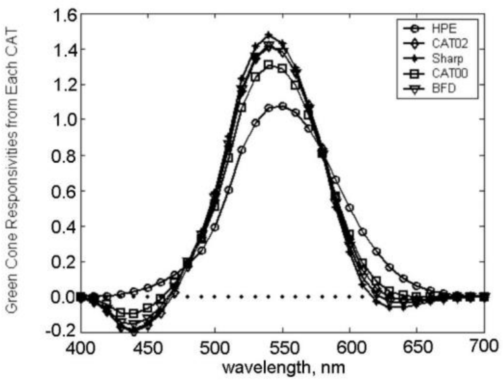
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.9102 & -0.1121 & 0.2019 \\ 0.3710 & 0.6291 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

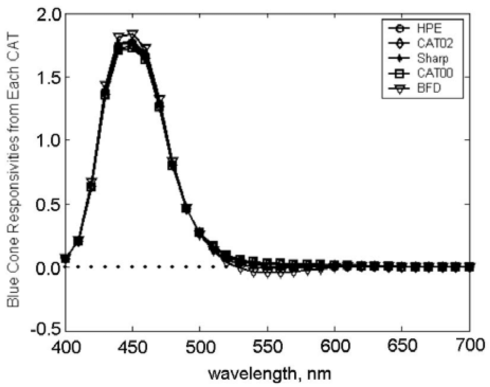
(HVS derived) Color Spaces

- **Sharpened cones responses**

Chromatic Adaptation Transform models use sharpened cones responses in order to cope with the overlap of the M and L cones responses







Color Opponent Spaces

- **IPT**

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.4002 & 0.7075 & -0.0807 \\ -0.2280 & 1.1500 & 0.0612 \\ 0.0000 & 0.0000 & 0.9184 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$L' = \begin{cases} L^{0.43} & \text{if } L \geq 0 \\ -(-L)^{0.43} & \text{if } L < 0 \end{cases}$$

$$M' = \begin{cases} M^{0.43} & \text{if } M \geq 0 \\ -(-M)^{0.43} & \text{if } M < 0 \end{cases}$$

$$S' = \begin{cases} S^{0.43} & \text{if } S \geq 0 \\ -(-S)^{0.43} & \text{if } S < 0 \end{cases}$$

$$\begin{bmatrix} I \\ P \\ T \end{bmatrix} = \begin{bmatrix} 0.4000 & 0.4000 & -0.2000 \\ -4.4550 & -4.8510 & 0.3960 \\ 0.8056 & 0.3572 & -1.1628 \end{bmatrix} \begin{bmatrix} L' \\ M' \\ S' \end{bmatrix}$$

Color Opponent Spaces

- **IPT**

$$\begin{bmatrix} L' \\ M' \\ S' \end{bmatrix} = \begin{bmatrix} 1.8502 & -1.1383 & -0.2384 \\ 0.3668 & 0.6439 & -0.0107 \\ 0.0000 & 0.0000 & 1.0889 \end{bmatrix} \begin{bmatrix} I \\ P \\ T \end{bmatrix}$$

$$L = \begin{cases} L'^{1/0.43} & \text{if } L' \geq 0 \\ -\left(-L'\right)^{1/0.43} & \text{if } L' < 0 \end{cases}$$

$$M = \begin{cases} M'^{1/0.43} & \text{if } M' \geq 0 \\ -\left(-M'\right)^{1/0.43} & \text{if } M' < 0 \end{cases}$$

$$S = \begin{cases} S'^{1/0.43} & \text{if } S' \geq 0 \\ -\left(-S\right)'^{1/0.43} & \text{if } S' < 0 \end{cases}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1.0000 & 0.0976 & 0.2052 \\ 1.0000 & -1.1139 & 0.1332 \\ 1.0000 & 0.0326 & -0.6769 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

Chromatic Adaptation Transform

Chromatic Adaptation

$$D = D_1^{-1} D_2 = \begin{bmatrix} L_2/L_1 & & \\ & M_2/M_1 & \\ & & S_2/S_1 \end{bmatrix}$$

Chromatic Adaptation Transform

- **CIE CAT02**

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \mathbf{M}_{\text{CAT02}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\mathbf{M}_{\text{CAT02}} = \begin{bmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6974 & 0.0061 \\ 0.0030 & -0.0136 & 0.9834 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M}^{-1}{}_{\text{CAT02}} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

$$\mathbf{M}^{-1}{}_{\text{CAT02}} = \begin{bmatrix} 1.0961 & -0.2789 & 0.1827 \\ 0.4544 & 0.4739 & 0.0721 \\ 0.0096 & -0.0057 & 1.0153 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \mathbf{M}^{-1}{}_{\text{CAT02}} \begin{bmatrix} \frac{R_{w,2}}{R_{w,1}} & 0 & 0 \\ 0 & \frac{G_{w,2}}{G_{w,1}} & 0 \\ 0 & 0 & \frac{B_{w,2}}{B_{w,1}} \end{bmatrix} \mathbf{M}_{\text{CAT02}} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix}$$

Application 1

- **Calculate 1931 coordinates of two Munsell samples under D65 and 1931 standard observers**

- **Calculate their color difference (Delta LAB)**