Technical University of Denmark

Computational Imaging and Spectroscopy

Scene analysis II : Colour image acquisition

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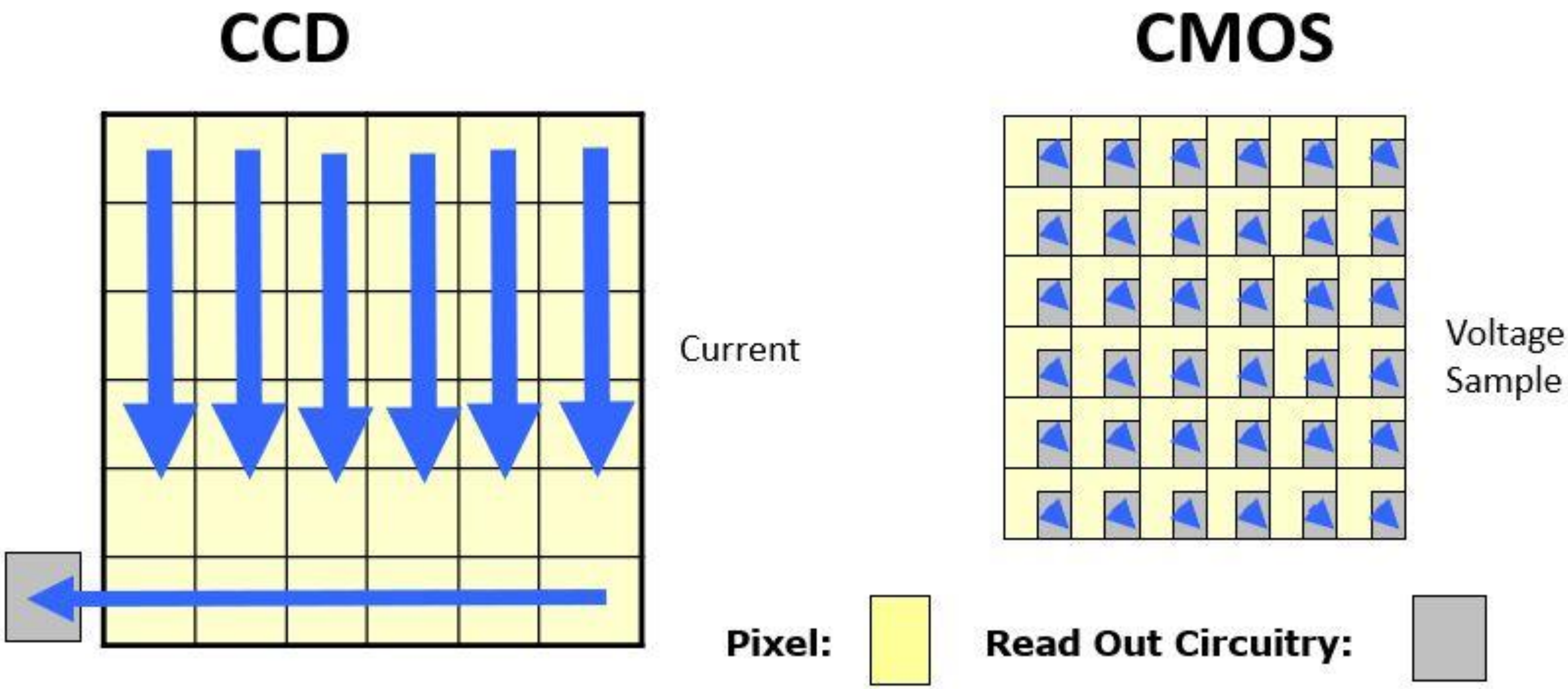
Colour image acquisition

Image sensor (CCD vs CMOS)



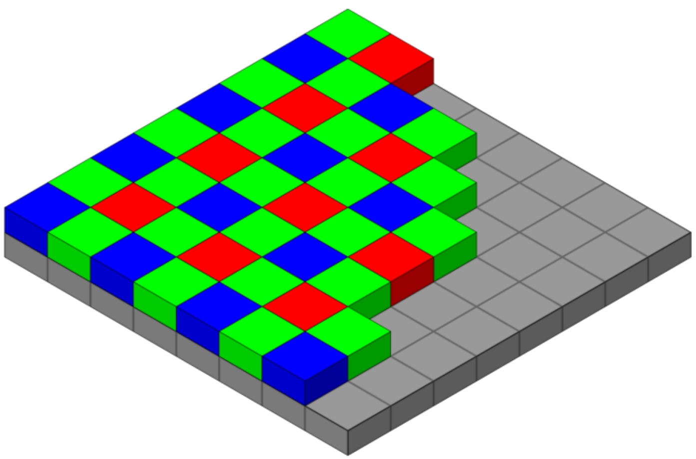
Colour image acquisition

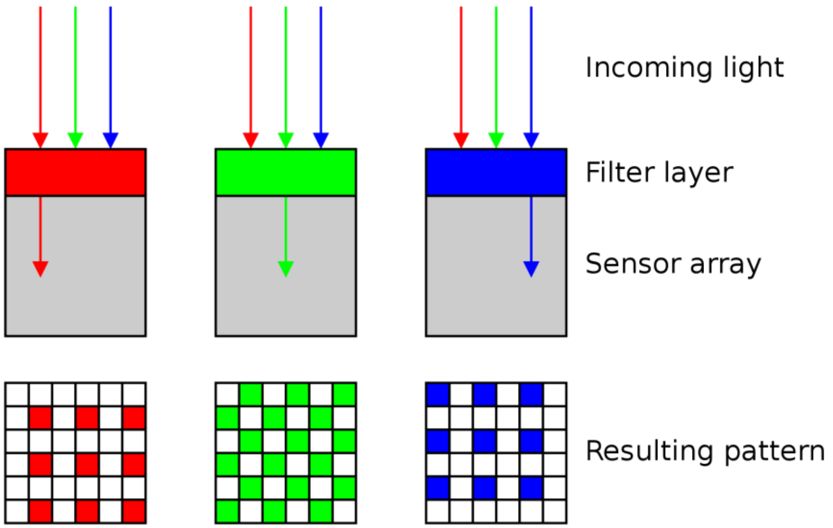
Image sensor

CCD	CMOS
High quality and low noise	Prone to noise
Expensive to produce	Cheaper to produce
Very high power consumption	Consume low power
High light sensitivity	Low light sensitivity
Very sensitive to blooming	Operate at higher speed

Colour image acquisition

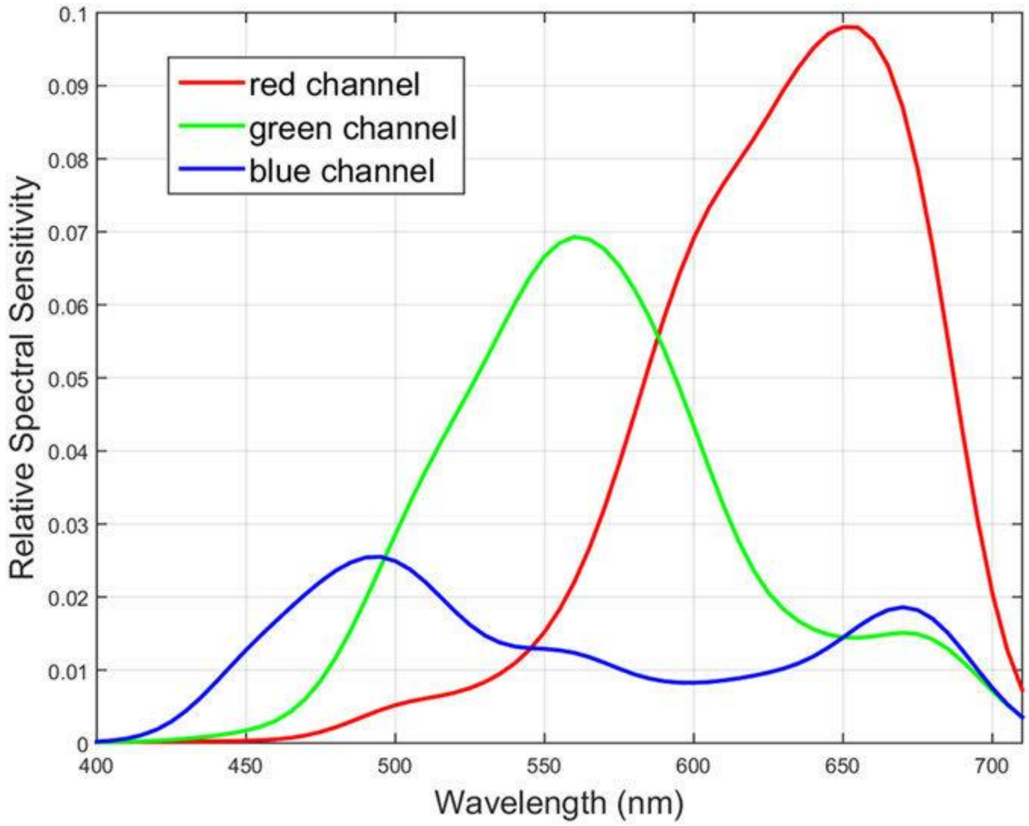
Image sensor (Colour Filter Array (CFA))





Colour image acquisition

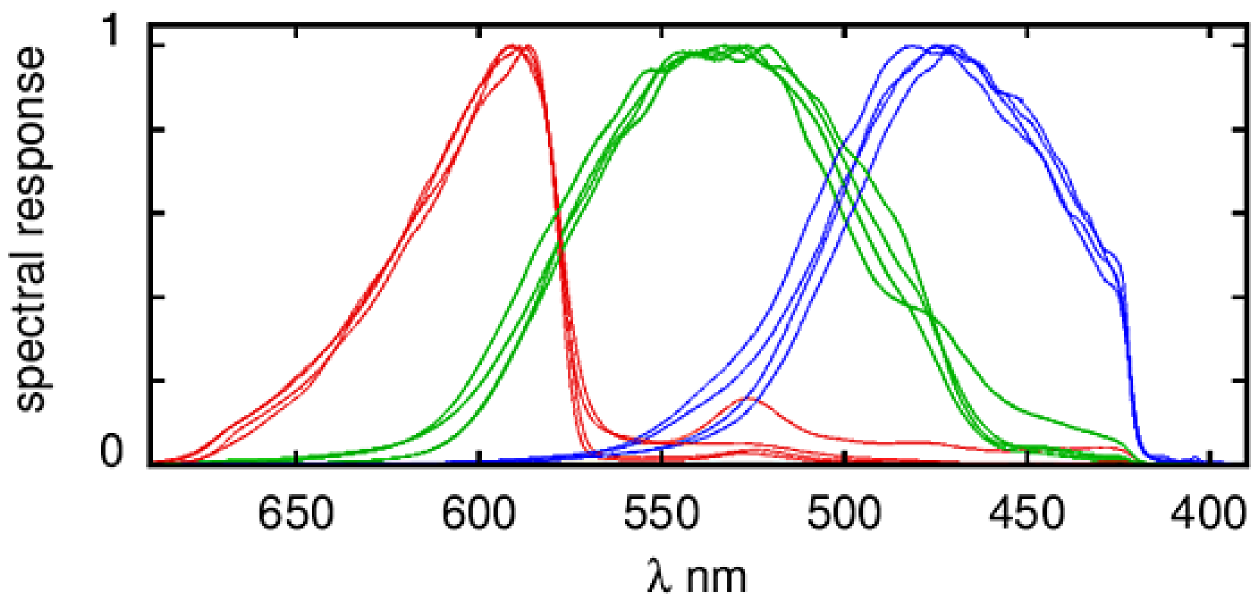
Image sensor (Colour Filter Array (CFA))



Spectral response of a Canon T3i APS-C CMOS image sensor with a Bayer pattern CFA

Colour image acquisition

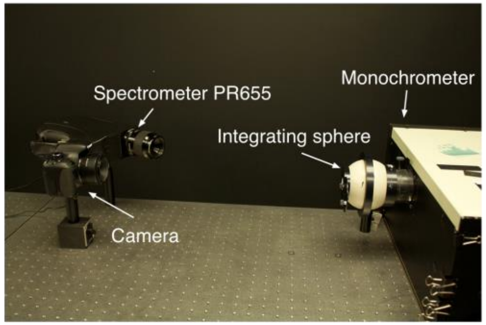
Image sensor spectral sensitivities (i.e.matching functions)



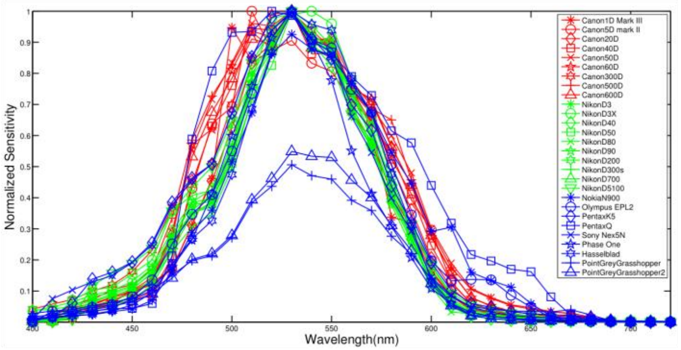
Response curves for CANON D800 D7100 D7200 D50

Colour image acquisition

Image sensor spectral sensitivities (i.e.matching functions)

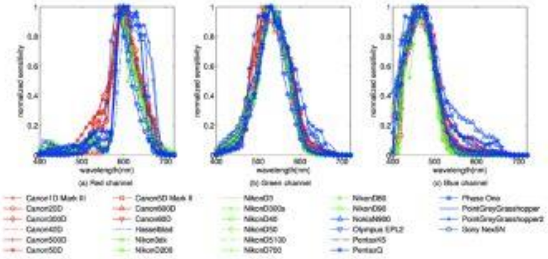


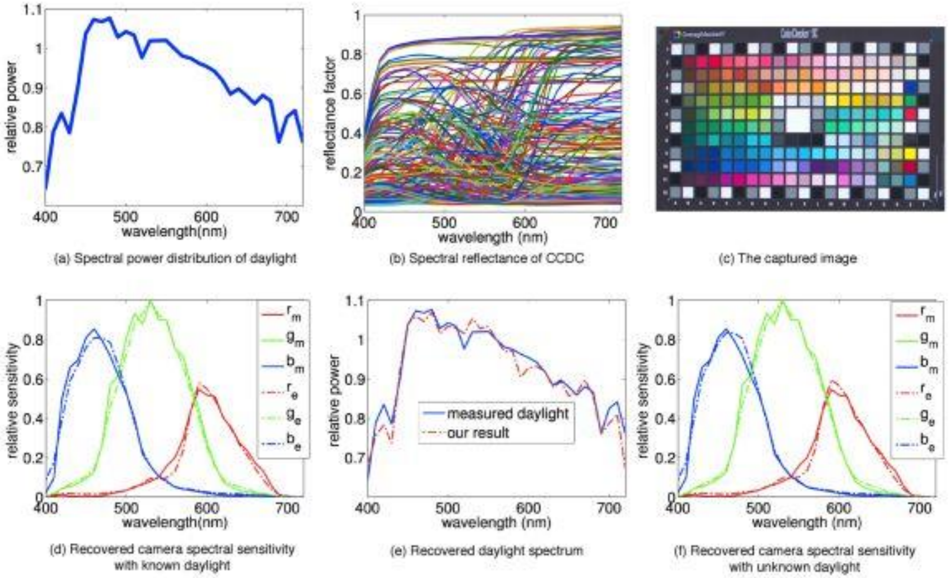
(a) Measurement Setup



(b) Camera Spectral Sensitivity (Green Channel)

Jiang et al.

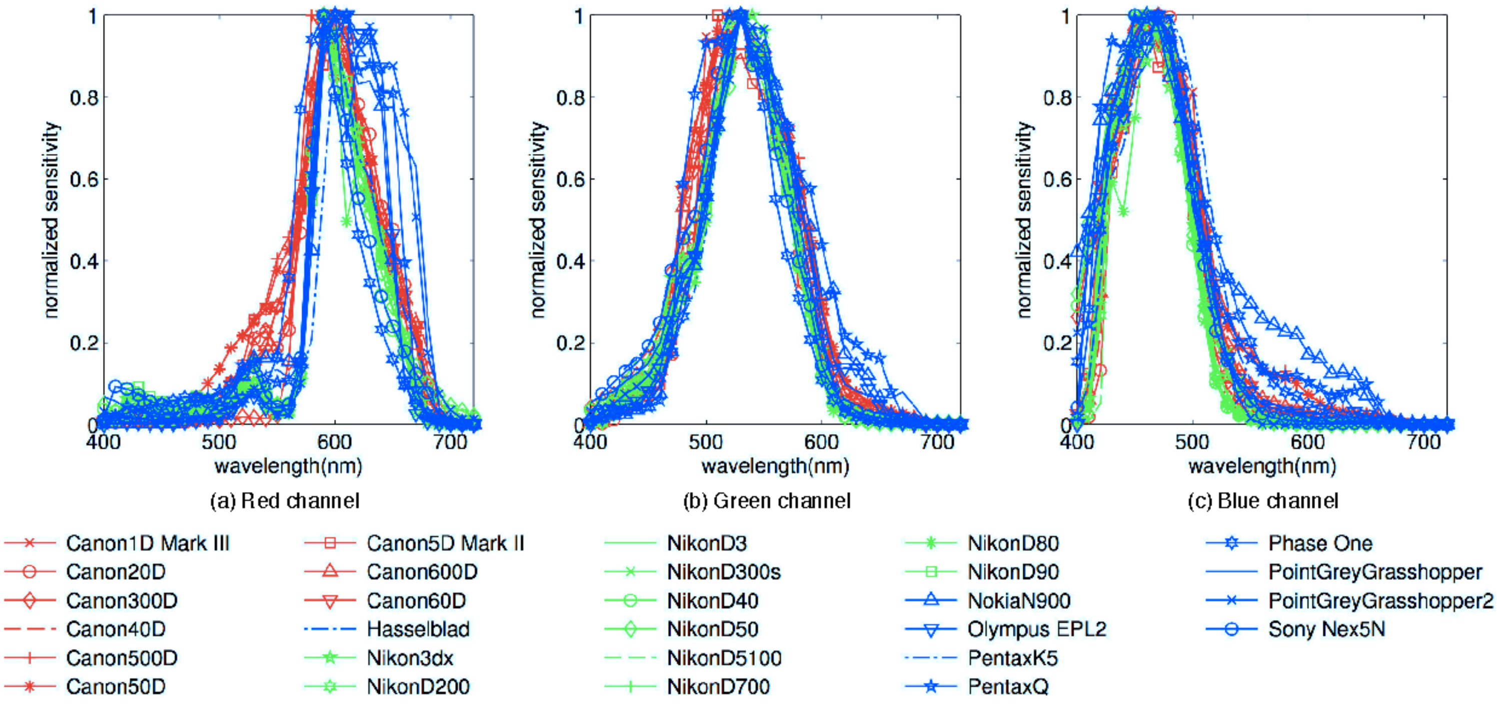




(a) Spectral power distribution of daylight
(b) Spectral reflectance of CCDC
(c) The captured image
(d) Recovered camera spectral sensitivity with known daylight
(e) Recovered daylight spectrum
(f) Recovered camera spectral sensitivity with unknown daylight

Colour image acquisition

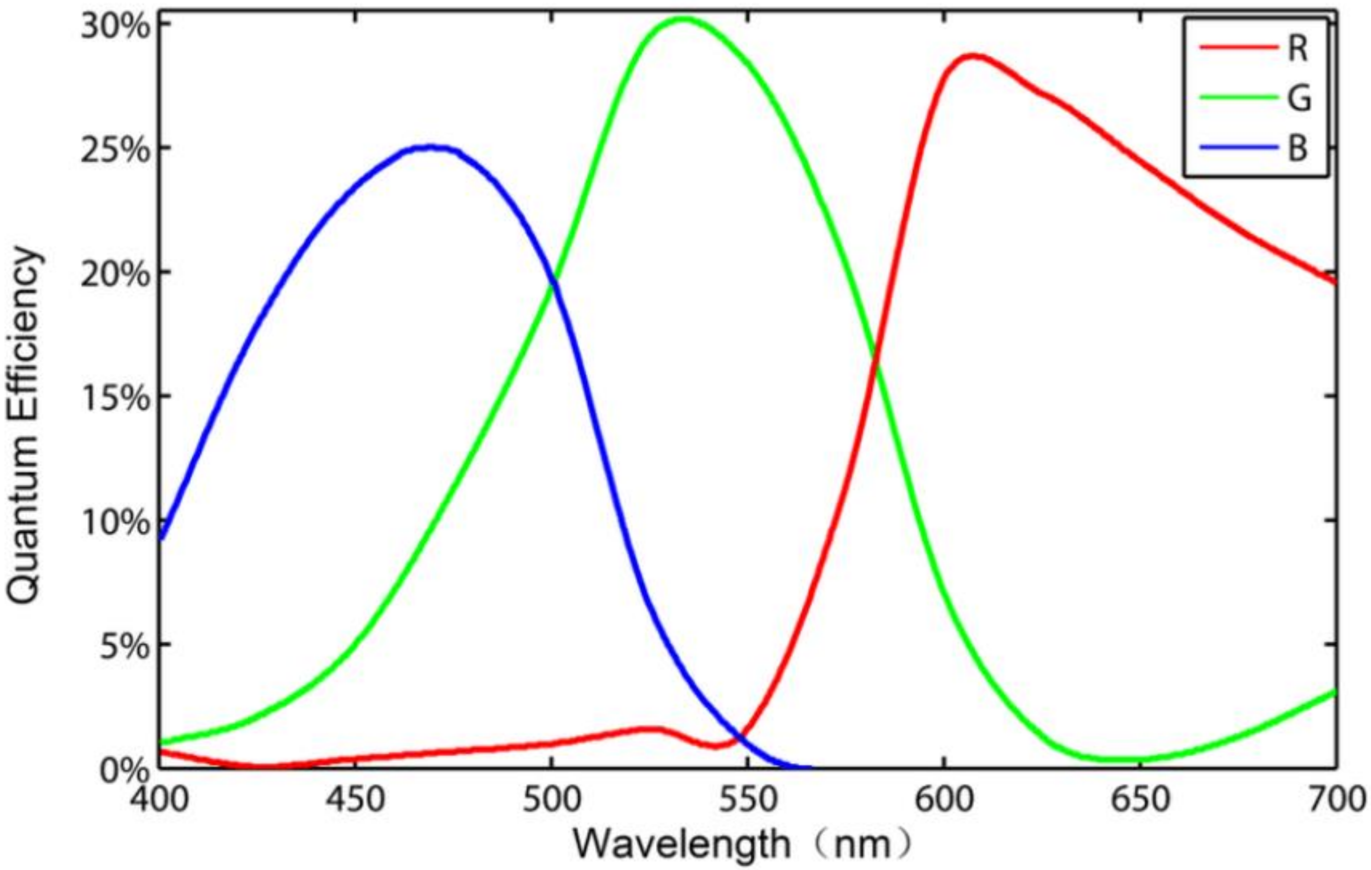
Image sensor spectral sensitivities (i.e.matching functions)



Jiang et al.

Colour image acquisition

Image sensor (Quantum efficiency)



Colour image acquisition

Dark current calibration

For a CCD sensor the output signal can be expressed as:

$$O = (PQ_e + D\ )t + R$$

Q_e *Quantum efficiency*
D thermal noise
P incident photon flux
R read noise
t exposure time

Colour image acquisition

Dark current calibration

Even if no light hit the sensor the thermal noise does not vanish, so in the dark we have:

$$O_D = Dt_0 + R.$$

In the case of high performance CCDs the read noise is negigible, we can then substract the dark output for the sensor outpout:

$$PQ_e t \approx O - \frac{t}{t_0} O_D$$

If the quantum efficiency is known the photon flux can be determined

$$P \approx \frac{1}{Q_e t}\left(O - \frac{t}{t_0} O_D\right)$$

Colour image acquisition

Dark current calibration

After calibration, for a narrow bandfilter the image intensity at a specific pixel x can be expressed as:

$$I(x,\lambda) = \beta_\lambda \left(O - \frac{t}{t_0} O_D \right) \approx \beta_\lambda P Q_e t$$

where βλ is a wavelength-dependent quantity that depends on the geometry of the sensor and the spectral transmission of the optical filter or grating used in the imager.

Colour image formation model

Image formation (BRDF)

Irradiance

$$E_i(\theta_i, \phi_i, \lambda) = L(\lambda) cos\theta_i d\omega_i$$

BRDF

$$f(\theta_i, \phi_i, \theta_o, \phi_o, \lambda) = \frac{E_o(\theta_i, \phi_i, \theta_o, \phi_o, \lambda)}{E_i(\theta_i, \phi_i, \lambda)}$$

Colour image formation model

Image formation (surface radiance)

Combining these expressions we have:

$$E_o(\theta_i, \phi_i, \theta_o, \phi_o, \lambda) = f(\theta_i, \phi_i, \theta_o, \phi_o, \lambda) L(\lambda) cos\theta_i d\omega_i$$

Assuming that the flux radiated from the surface is transmitted through the lens without any loss of energy, the spectral irradiance reaching the image plane is:

$$I_{im}(\lambda) = \frac{\pi}{4}\left(\frac{d}{z}\right)^2 \cos^4\alpha E_o(\theta_i, \pi, \theta_o, \phi_o, \lambda)$$

$$I_{im}(\lambda) = mf(\theta_i, \phi_i, \theta_o, \phi_o, \lambda) L(\lambda) cos\theta_i \cos^4\alpha d\omega_i$$

Colour image formation model

Image formation (surface radiance)

$$m = \frac{\pi}{4}\left(\frac{d}{z}\right)^2$$

Note:

d is the lens diameter, z is the distance between the lens and the image plane, and α is the angle between the optical axis of the camera and the line of sight from the surface patch to the centre of the lens.

Colour image formation model

Image formation (Colour response)

Let's denote C_c the spectral sensitivities of the camera, i.e. matching functions, we can express the color response

$$I_c = k_c \int_{vis} C_c(\lambda) I_{im}(\lambda) d\lambda$$

$$I_c = m k_c cos\theta_i \cos^4\alpha \times \int_{vis} C_c(\lambda) f(\theta_i, \phi_i, \theta_o, \phi_o, \lambda) L(\lambda) cos d\lambda$$

$$c \in [R, G, B]$$

κ_c corresponds to the color balance factor of the camera against a predetermined reference.

Colour image formation model

Image formation (Colour response)

The tristimulus value S_{ref} of the colour reference is given by

$$S_{ref_c} = mk_c d\omega_i \int_{vis} C_c(\lambda) L(\lambda) d\lambda$$

The sample is considered having a brdf equals to 1, and placed perpendicularly to the axis of the camera. By solving this equation for k_c we get

$$I_c = mk_c cos\theta_i \cos^4 \alpha d\omega_i \times \frac{\int_{vis} C_c(\lambda) f(\theta_i, \phi_i, \theta_o, \phi_o, \lambda) L(\lambda) cos d\lambda}{\int_{vis} C_c(\lambda) L(\lambda) d\lambda}$$

Colour image formation model

Image formation (Colour response)

This can be simplified to the following image formation equation, under some assumptions:

$$I_c = \frac{\int_{vis} C_c(\lambda)L(\lambda)d\lambda}{\int_{vis} L(\lambda)d\lambda}$$

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Colour image formation model

Image formation (Camera calibration)

Camera matrix estimation

$$\begin{bmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{bmatrix}_\lambda = M \begin{bmatrix} R \\ G \\ B \end{bmatrix}_\lambda$$

$$M = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \begin{bmatrix} R & G & B \end{bmatrix} \left(\begin{bmatrix} R \\ G \\ B \end{bmatrix} \begin{bmatrix} R & G & B \end{bmatrix} \right)^{-1}$$

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Colour image formation model

Image formation (Illuminant estimation and white balancing)

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Colour image formation models

Image formation (Illuminant estimation and white balancing)

Gray World

We assume that $\mathbf{R}_{avg} = \mathbf{B}_{avg} = \mathbf{G}_{avg}$

We calculate

$$\begin{cases} \mathrm{C}_{avg} = \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} I_c(x,y) \\ \hat{r} = \frac{G_{avg}}{R_{avg}}, \hat{b} = \frac{G_{avg}}{B_{avg}} \\ \hat{I}_r(x,y) = \hat{r} I_r(x,y), \hat{I}_b(x,y) = \hat{b} I_b(x,y) \end{cases}$$

The Green channel remains untouched

Colour image formation model

Image formation (Illuminant estimation and white balancing)

Retinex theory

We assume that $\mathbf{R}_{avg} = \mathbf{B}_{avg} = \mathbf{G}_{avg}$ and that the brightest pixels represent the ”white point”

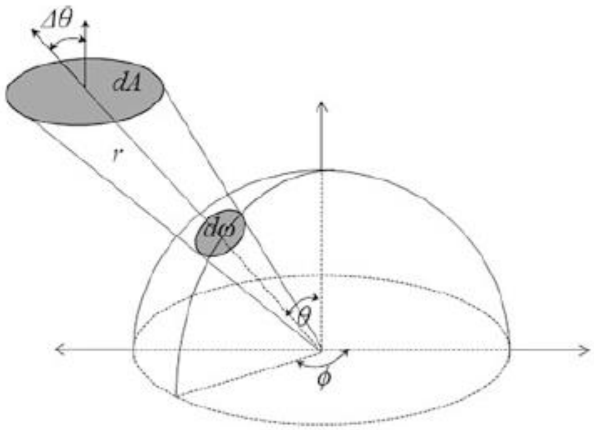
We calculate

$$\begin{cases} \mathrm{C}_{max} = \max_{x,y}\{I_c(x,y)\} \\ \hat{r} = \frac{G_{max}}{R_{max}}, \hat{b} = \frac{G_{max}}{B_{max}} \\ \hat{I}_r(x,y) = \hat{r} I_r(x,y), \hat{I}_b(x,y) = \hat{b} I_b(x,y) \end{cases}$$

The Green channel remains untouched

BRDF

Image formation (solid angle)

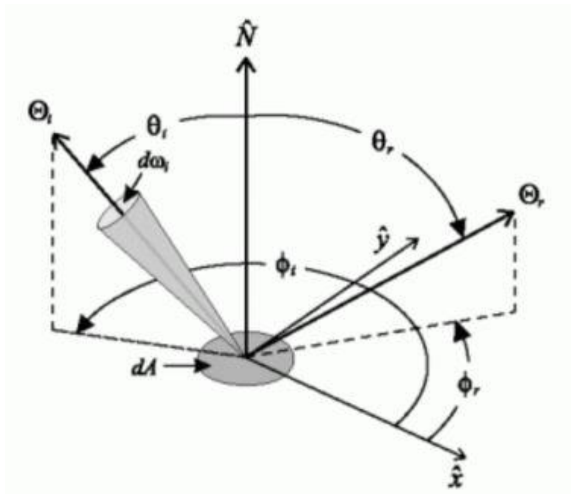


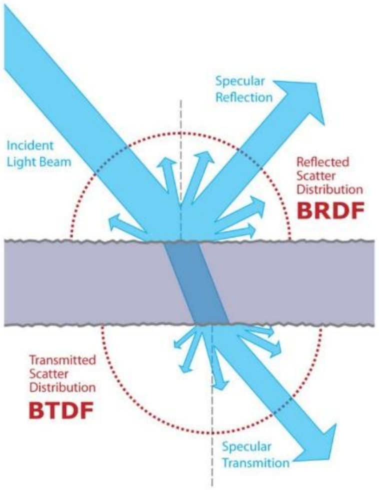
$$d\omega = \frac{dA\cos\Delta\theta}{r^2}$$

$$d\omega = sin\theta d\theta d\phi$$

BRDF

Image formation (BRDF)





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BRDF

Image formation (BRDF)

Irradiance

$$E_i(\theta_i, \phi_i, \lambda) = L(\lambda) cos\theta_i d\omega_i$$

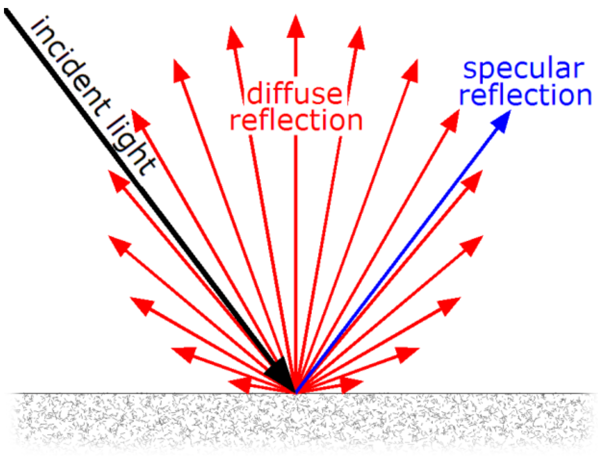
BRDF

$$f(\theta_i, \phi_i, \theta_o, \phi_o, \lambda) = \frac{E_o(\theta_i, \phi_i, \theta_o, \phi_o, \lambda)}{E_i(\theta_i, \phi_i, \lambda)}$$

Reflectance models

Reflectance models

- **Diffuse reflection**



$$f_r(\omega_i, \omega_o) = \frac{\rho}{\pi}$$

Reflectance models

- **Specular reflection**

$$\mathrm{R}\left(\omega_i, \vec{\mathbf{n}}\right) = 2\left(\vec{\mathbf{n}} \cdot \omega_i\right)\vec{\mathbf{n}} - \omega_i$$

- **Fresnel term (dielectrics)**

$$r_s = \frac{n_i \cos\left(\theta_i\right) - n_o \cos\left(\theta_o\right)}{n_i \cos\left(\theta_i\right) + n_o \cos\left(\theta_o\right)}$$

$$r_p = \frac{n_o \cos\left(\theta_i\right) - n_i \cos\left(\theta_o\right)}{n_i \cos\left(\theta_o\right) + n_o \cos\left(\theta_o\right)}$$

$$F = \frac{\left|r_p\right|^2 + \left|r_s\right|^2}{2}$$

Reflectance models

- **Fresnel term (conductors)**

$$r_s = \frac{n_i \cos(\theta_i) - (n_o - k_o)\cos(\theta_o)}{n_i \cos(\theta_i) + (n_i - k_i)\cos(\theta_o)}$$

$$r_p = \frac{(n_o - k_o)(\theta_i) - n_i \cos(\theta_o)}{(n_o - k_o)(\theta_i) + n_i \cos(\theta_o)}$$

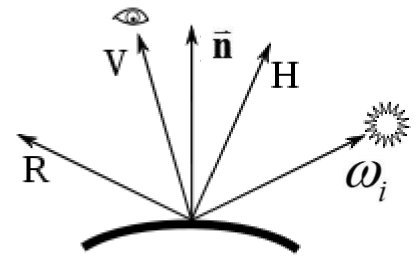
Reflectance models

- **BRDF (specular)**

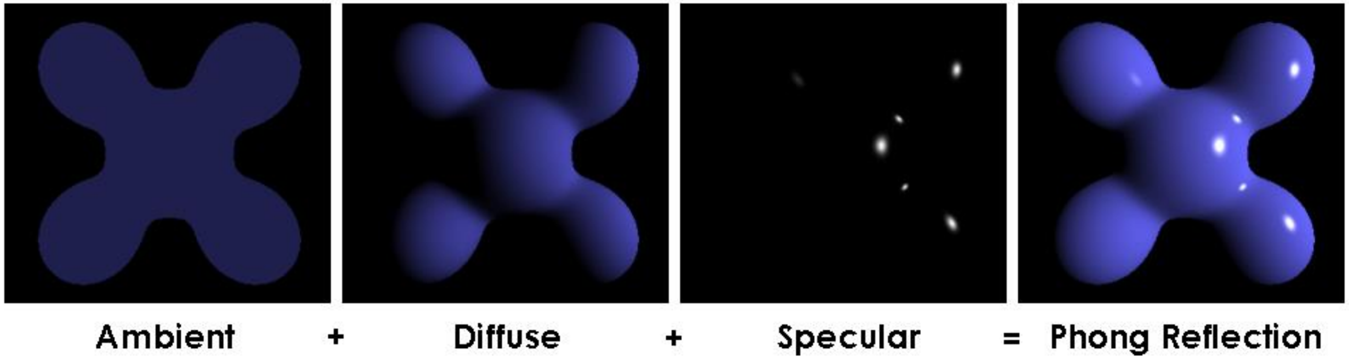
$$f_r\left(x,\omega_i,\omega_o\right)=\mathrm{F}\left(\omega_o\right)\frac{\delta\left(\omega_o-\mathrm{R}\left(\omega_i,\vec{\mathbf{n}}\right)\right)}{\left|\cos\left(\theta_i\right)\right|}$$

Reflectance models

- **BRDF (Phong)**

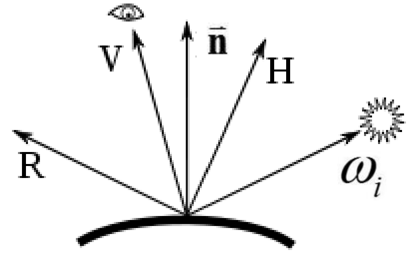


$$f_r\left(x,\omega_i,\omega_o\right)=k_a+k_s\frac{\left(\vec{\mathbf{V}}\cdot\vec{\mathbf{R}}\right)^n}{\vec{\mathbf{n}}\cdot\omega_i}+\frac{\rho}{\pi}$$

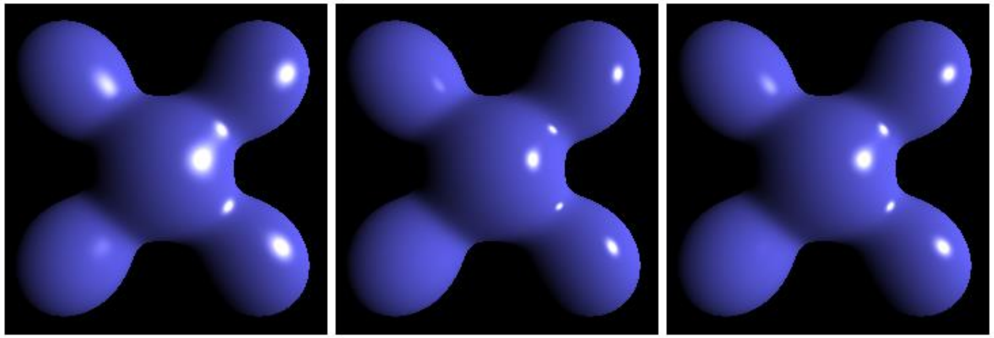


Reflectance models

- **BRDF (Blinn-Phong)**



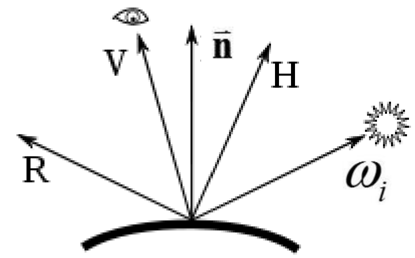
$$f_r(x,\omega_i,\omega_o) = k_s \frac{\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{H}}\right)^n}{\vec{\mathbf{n}}\cdot\omega_i} + \frac{\rho}{\pi}$$



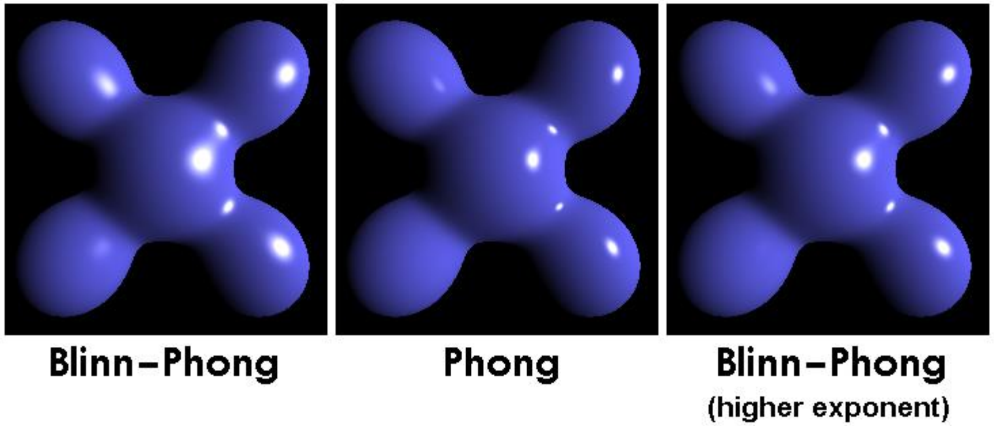
Blinn–Phong | Phong | Blinn–Phong (higher exponent)

Reflectance models

- **BRDF (Modified Blinn-Phong)**

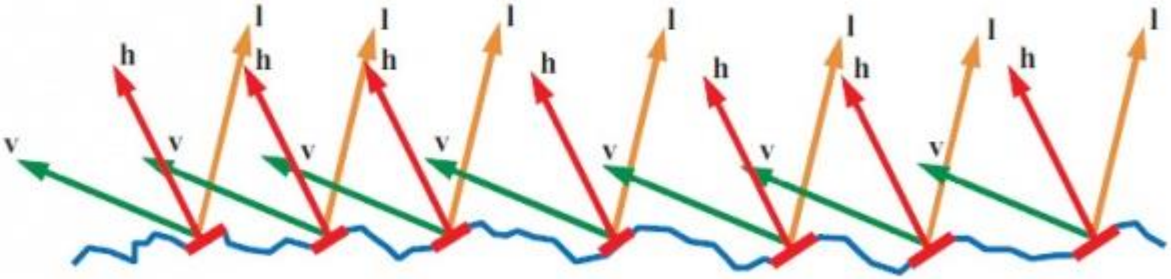


$$f_r\left(x,\omega_i,\omega_o\right)=k_s\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{H}}\right)^n+\frac{\rho}{\pi}$$



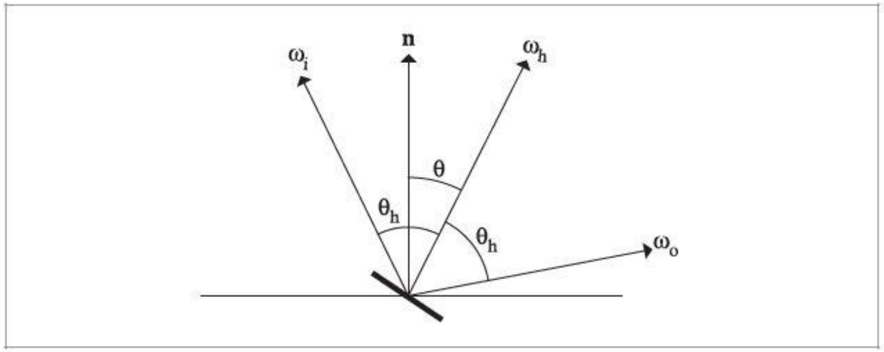
Reflectance models

- **Micro-facets theory**



Reflectance models

- **Micro-facets theory (generic model)**



$$f_r\left(x,\omega_i,\omega_o\right)=\frac{D\left(\omega_h\right)G\left(\omega_i,\omega_o\right)F_r\left(\omega_o\right)}{4\cos\left(\theta_i\right)\cos\left(\theta_o\right)}$$

Reflectance models

- **Micro-facets theory (Distribution)**

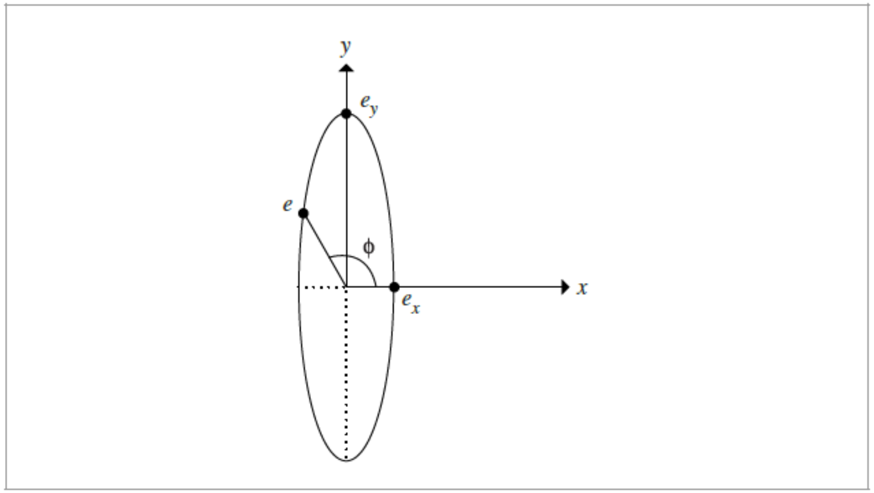
$$D_{Gaussian}\left(\omega_h\right)=\frac{1}{\sqrt{2\pi m}}\exp\left(-\frac{\arccos\left(\vec{\mathbf{n}}\cdot\omega_h\right)^2}{2m^2}\right)$$

$$D_{Beckamnn}\left(\omega_h\right)=\frac{\exp\left(-\tan^2\left(\arccos\left(\vec{\mathbf{n}}\cdot\omega_h\right)\right)/m^2\right)}{\pi m^2\cos^4\left(\arccos\left(\vec{\mathbf{n}}\cdot\omega_h\right)\right)}$$

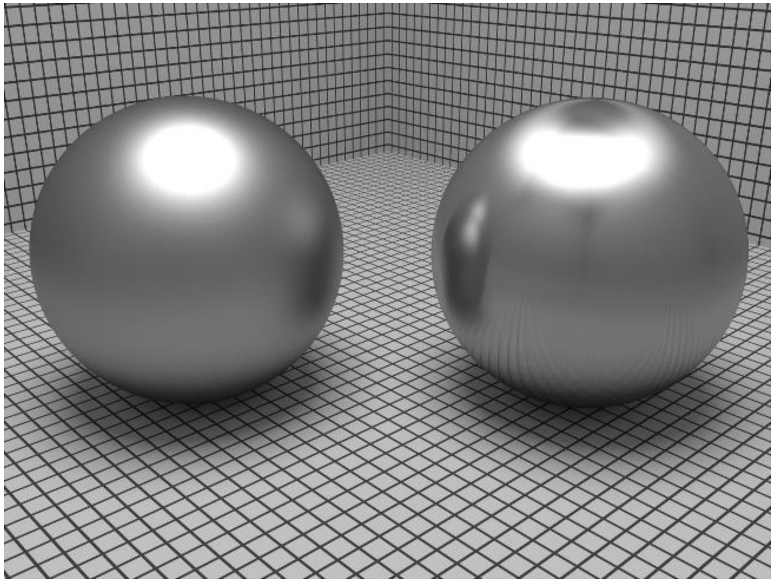
$$D_{anisotropic}\left(\omega_h\right)=\frac{\sqrt{\left(e_x+2\right)\left(e_y+2\right)}}{2\pi}\left(\omega_h\cdot\vec{\mathbf{n}}\right)^{e_x\cos^2(\phi)+e_y\sin^2(\phi)}$$

Reflectance models

- **Micro-facets theory (Distribution)**



Geometry of the anisotropic distribution



Isotropic vs anisotropic microfacets highlights

Reflectance models

- **Micro-facets theory (Geometric term)**

$$G_{CT}\left(\omega_o, \omega_i\right) = \min\left(1, \min\left(\frac{2\left(\vec{\mathbf{n}}\cdot\omega_h\right)\left(\vec{\mathbf{n}}\cdot\omega_o\right)}{\omega_o\cdot\omega_h}, \frac{2\left(\vec{\mathbf{n}}\cdot\omega_h\right)\left(\vec{\mathbf{n}}\cdot\omega_i\right)}{\omega_o\cdot\omega_h}\right)\right)$$

Reflectance models

- **Micro-facets theory (Schlick's approximation)**

$$F(\theta) = R_0 + (1 - R_0)(1 - \cos(\theta))^5$$

$$R_0 = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

$$\cos(\theta) = \vec{\mathbf{n}} \cdot \omega_i$$

Reflectance models

- **Micro-facets theory (GGX)**

$$D_{GGX}\left(\omega_h\right) = \frac{m^2}{\pi\left(\left(\vec{\mathbf{n}}\cdot\omega_h\right)^2\left(m^2-1\right)+1\right)^2}$$

$$G_{Smith}\left(\omega_o,\omega_i\right) = \frac{2\left(\vec{\mathbf{n}}\cdot\omega_i\right)\left(\vec{\mathbf{n}}\cdot\omega_o\right)}{\left(\vec{\mathbf{n}}\cdot\omega_o\right)\sqrt{m^2+\left(1-m^2\right)\left(\vec{\mathbf{n}}\cdot\omega_i\right)^2}+\left(\vec{\mathbf{n}}\cdot\omega_i\right)\sqrt{m^2+\left(1-m^2\right)\left(\vec{\mathbf{n}}\cdot\omega_o\right)^2}}$$

Reflectance models

- **Micro-facets theory (GGX)**

Colour image formation model

Image formation (Illuminant estimation and white balancing)

Application 1

Color balance the blue_cast.jpg image using both methods
Compare your results and conclude

Camera CANON EOS 550D

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Colour image formation model

Image formation (Illuminant estimation and white balancing)

Application 2 (WB_girl images)

Use CAT02 to perform the white balance (Output White is D65)

1 Estimate the white point (Retinex) for each channel
2 Calculate the adaptation matrix (diagonal)
3 Calculate (M^-1sRGB x M^-1CAT02) x adatM x (MCAT02 x MsRGB)
4 Try with arbitrary input White points

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