Technical University of Denmark

Computational Imaging and Spectroscopy

Scene analysis I: Spectrum representation and recovery

Thierry SOREZE
DTU July 2024

DTU Fotonik
Department of Photonics Engineering

Spectrum representation and spectrum recovery

Suppose we have a device with three colour sensors, whose spectral sensitivities are $C_k(\lambda)$, k = 1, 2, 3. The three sensor responses for a colour signal $S(\lambda)$ will be:

$$\begin{cases} R_1 = \sum_{\lambda} C_1(\lambda)S(\lambda) \\ R_2 = \sum_{\lambda} C_2(\lambda)S(\lambda) \\ R_3 = \sum_{\lambda} C_3(\lambda)S(\lambda) \end{cases}$$

Which can be written in matrix from as

$$\boldsymbol{r} = \mathbf{M}\boldsymbol{s}$$

Spectrum representation and spectrum recovery

Linear decomposition

We can use a linear decomposition to represent a spectra P discretized in n bins of width $\Delta\lambda$ and write:

$$P(\lambda) = \sum_{k=1}^{n} a_k b_k(\lambda)$$

Where a_k are weights and $b_k(\lambda)$ a set a of basis functions.

Several basis functions have been proposed in the literature. The most popular basis are given by **PCA**

Spectrum representation and spectrum recovery

Linear decomposition

Fourier

$$B = \sum_{i=0}^{D} a_i \cdot \sin(i\lambda\pi)$$

Polynomial

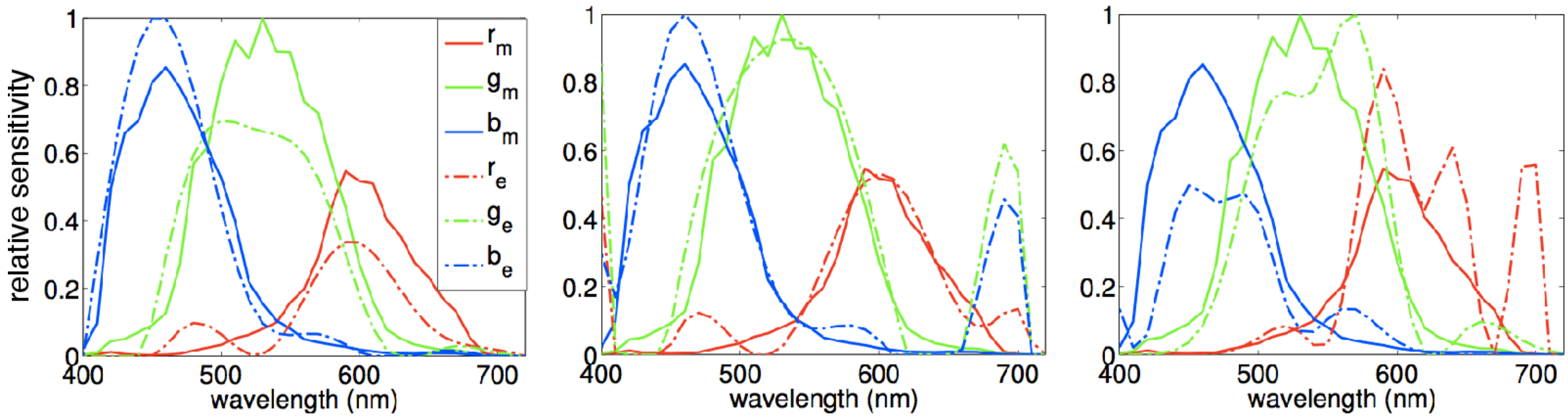
$$B = \sum_{i=0}^{D} a_i \cdot \lambda^i$$

Radial basis

$$B = \sum_{i=0}^{D} a_i \cdot \exp\left(-\frac{(\lambda - \mu_i)^2}{\sigma^2}\right)$$

Spectrum representation and spectrum recovery

Linear decomposition

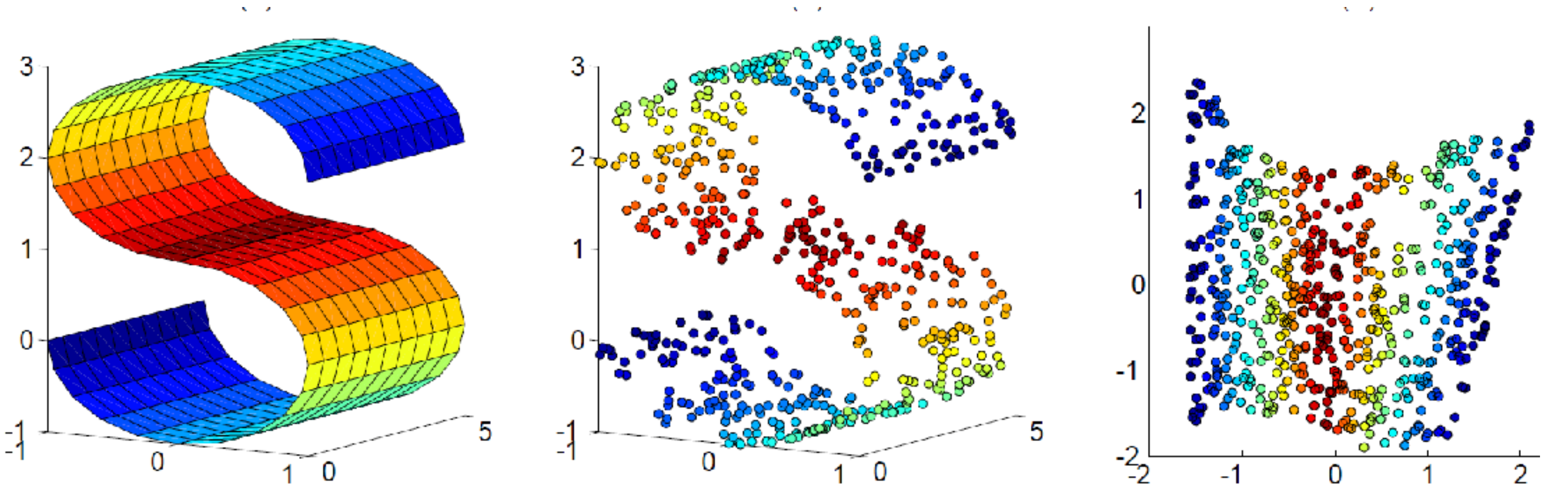


(a) Recovery by Fourier basis (b) Recovery by polynomial basis (c) Recovery by radial basis

Jiang et al.

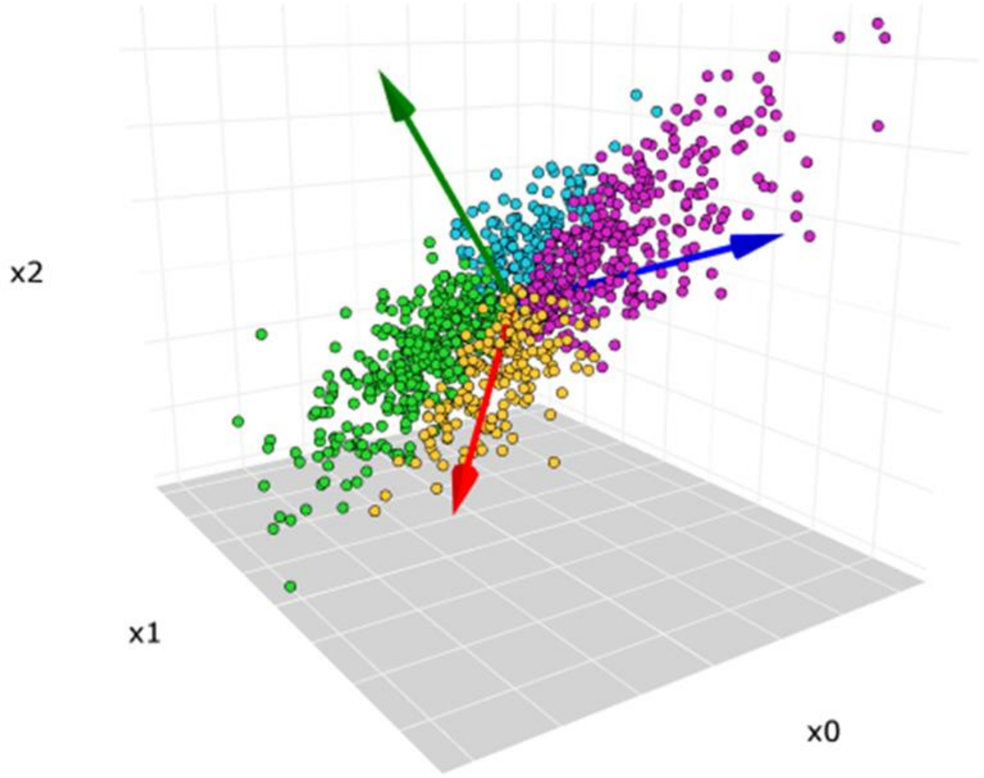
Spectrum representation and spectrum recovery

Dimensionality reduction (PCA and NNMF)



Spectrum representation and spectrum recovery

Principal component analysis (PCA)



Spectrum representation and spectrum recovery

Non negative matrix factorization (NNMF)

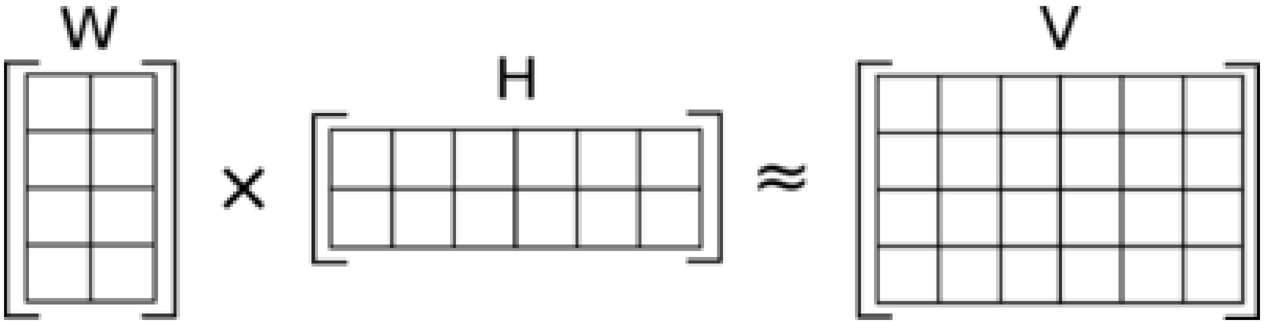
PCA has many applications:

Dimensionality reduction/compression
Orthogonalization (ex. Color space)
Classification
Face recognition
Etc.

https://en.wikipedia.org/wiki/Principal_component_analysis

Spectra representation and spectral recovery

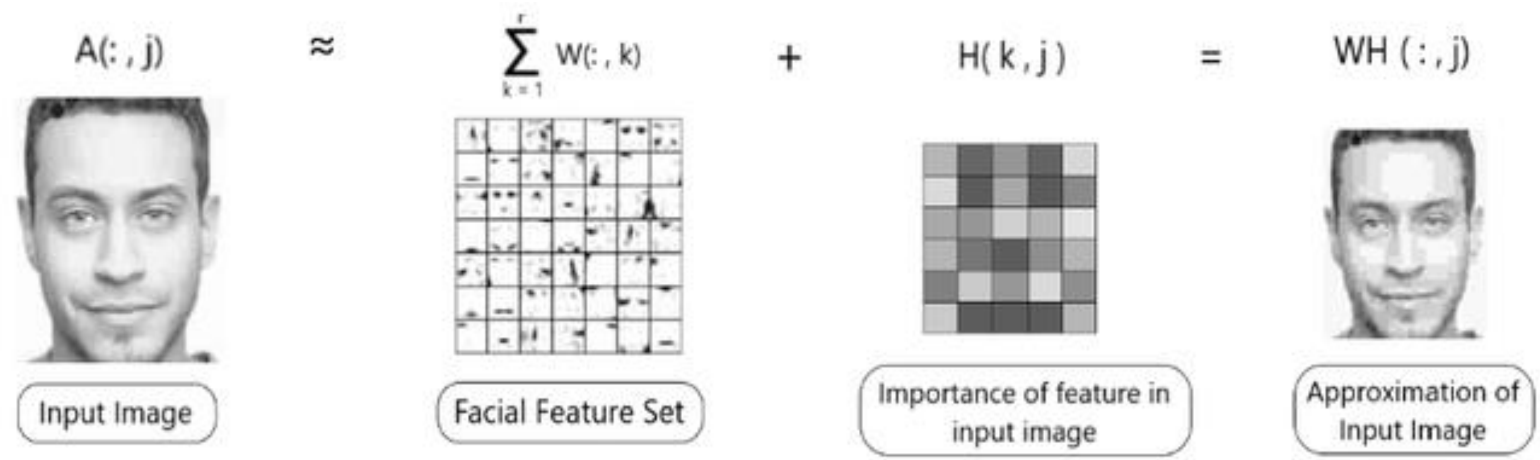
Non negative matrix factorization (NNMF)



https://en.wikipedia.org/wiki/Non-negative_matrix_factorization

Spectrum representation and spectrum recovery

Non negative matrix factorization (NNMF)



Spectrum representation and spectrum recovery

Principal component analysis (PCA)

Example 1 : camera calibration (Jiang et al.)

$$I_{k,x} = \int_{vis} C_k(\lambda) L(\lambda) R_x(\lambda) d\lambda. \quad k \in [R, G, B]$$

In matrix form we have

$$\mathbf{I_k} = \mathbf{c_k L R}$$

$$\mathbf{c_k} = \mathbf{i_k} \cdot (\mathbf{LR})^{+}$$

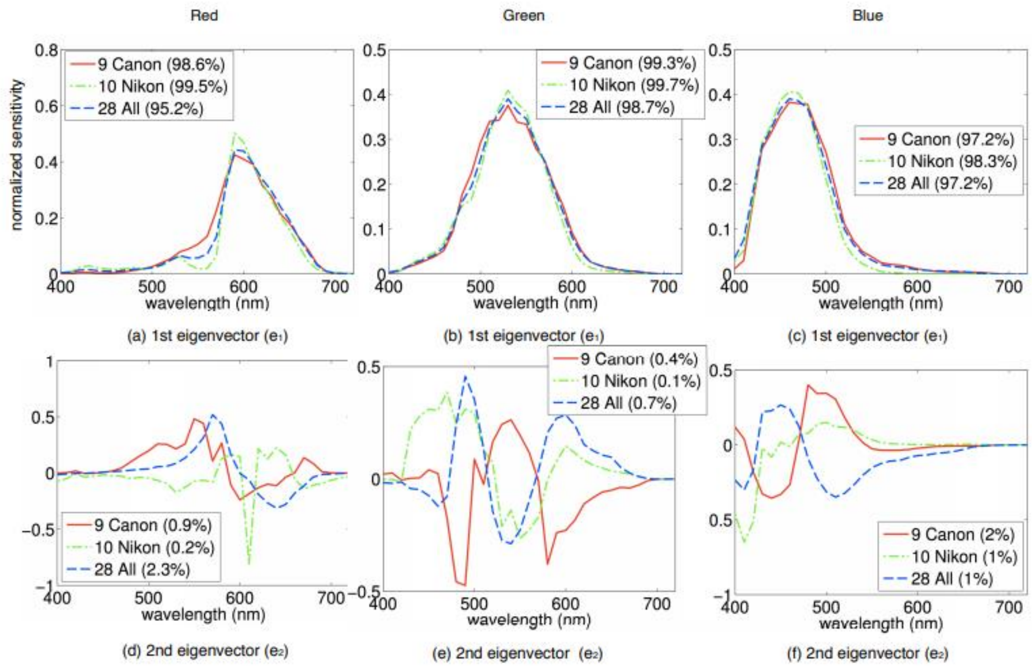
In practice direct inversion is not accurate for retrieving the camera sensitivities

Spectrum representation and spectrum recovery

Principal component analysis (PCA)

Example 1 : camera calibration (Jiang et al.)

They measured the spectral sensitivities of 28 cameras and performed PCA on each channel individually on this dataset to derived the space of spectral sensitivities of cameras.



Jiang et al.

Spectrum representation and spectrum recovery

Principal component analysis (PCA)

Example 1 : camera calibration (Jiang et al.)

In their experiments the data were explained by the first two components of **PCA** at 97%. We can write, under a known illuminant:

$$\mathbf{c_k} = \mathbf{i_k}(\mathbf{E_k}\mathbf{L}\mathbf{R})^{+}\mathbf{E_k}$$

Spectrum representation and spectrum recovery

Principal component analysis (PCA)

Example 2 : spectral recovery

We seek to recover a spectrum $S(\lambda\ )$ for a triplet of XYZ. As seen we can express this spectra with a linear decomposition:

$$s = \sum_{k=1}^{n} a_k b_k$$

We can truncate this summation to the first three terms and write:

$$s \approx a_1 b_1 + a_2 b + a_3 b_3 + \mu$$

μ is the average of the data

Spectrum representation and spectrum recovery

Principal component analysis (PCA)

Example 2 : spectral recovery

We can express the XYZ triplet by using the following expression

$$c \approx \begin{bmatrix} \mathbf{X}^{\mathbf{T}} \\ \mathbf{Y}^{\mathbf{T}} \\ \mathbf{Z}^{\mathbf{T}} \end{bmatrix} \tilde{s} = \begin{bmatrix} \mathbf{X}^{\mathbf{T}} \\ \mathbf{Y}^{\mathbf{T}} \\ \mathbf{Z}^{\mathbf{T}} \end{bmatrix} [\mathbf{b_1 b_2 b_3}] \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + c_\mu = \mathbf{M} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + c_\mu$$

We can therefore approximate the weights:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \approx \mathbf{M}^{-1}(c - c_\mu) = \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

Spectrum representation and spectrum recovery

Principal component analysis (PCA)

Example 2 : spectral recovery

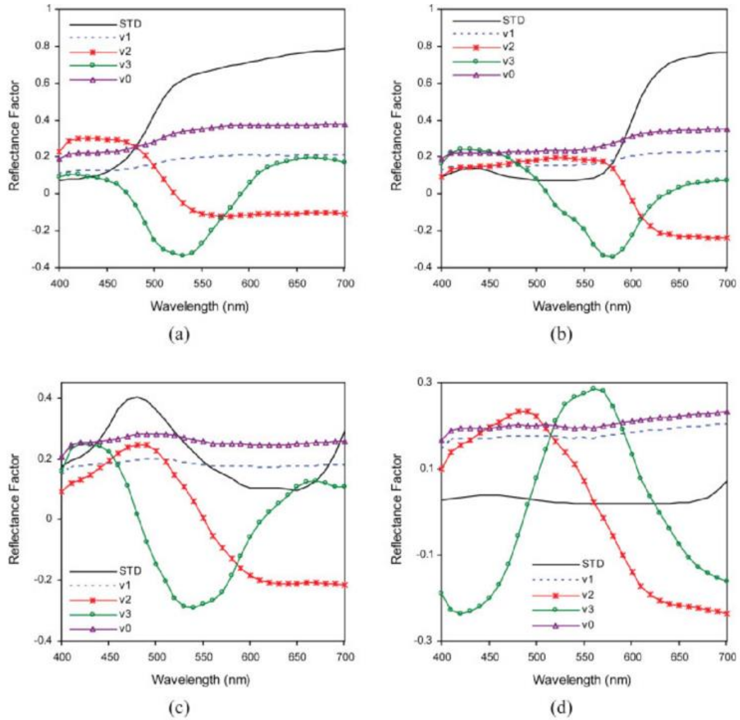
The reconstructed spectrum s is given by

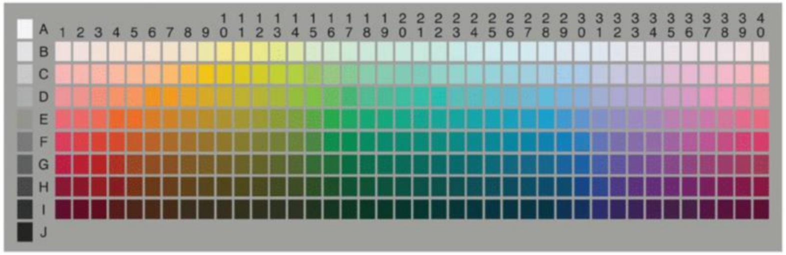
$$s \approx \hat{a}_1 b_1 + \hat{a}_2 b_2 + \hat{a}_3 b_3 + \mu$$

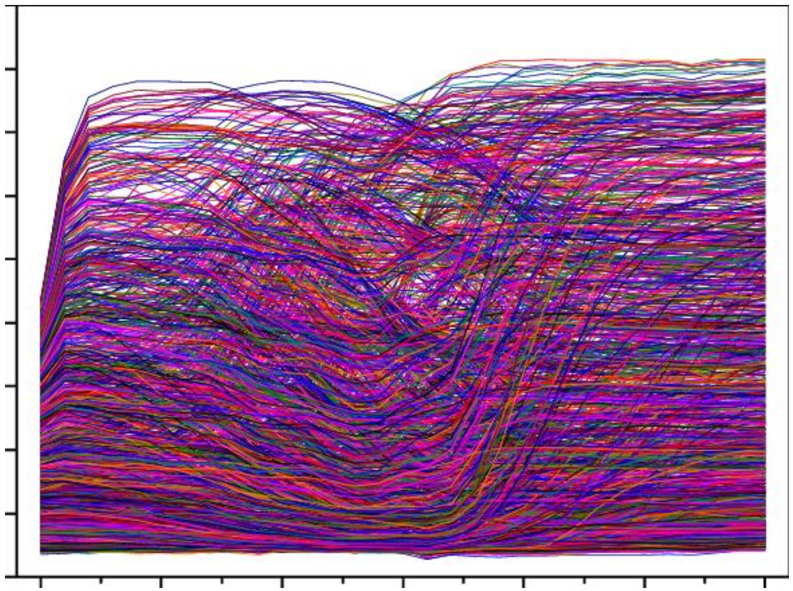
Spectrum representation and spectrum recovery

Assignment

Objective: Spectrum reconstruction of samples from the Munsell dataset







Spectrum representation and spectrum recovery

Assignment

Objective: Spectral reconstruction of samples from the Munsell dataset

Tasks:

- ❑Split the dataset at random (80/20): Training and test sets
- ❑Perform **PCA** on the training dataset to reconstruct spectra of few Munsell samples from the test datset

- ❑Compute error metrics (**MRAE**, **RMSE**, **GFC**) using 5 folds cross validation
- ❑Convert reconstructed spectra to XYZ
- ❑Compute color difference (delta Lab)

Spectrum representation and spectrum recovery

Assignment

Objective: Spectral reconstruction of samples from the Munsell dataset

Improvements:

- ❑Cluster the dataset with k-means
- ❑Perform reconstruction model per cluster
- ❑Compare results with non clustered model

https://en.wikipedia.org/wiki/K-means_clustering

Spectrum representation and spectrum recovery

Assignment

Objective: Spectral reconstruction of samples from the Munsell dataset

https://scikit-learn.org/stable/modules/generated/sklearn.cluster.KMeans.html

https://scikit-learn.org/stable/auto_examples/cluster/plot_kmeans_silhouette_analysis.html

https://scikit-learn.org/stable/modules/generated/sklearn.decomposition.PCA.html

https://scikit-learn.org/stable/modules/generated/sklearn.decomposition.NMF.html

https://scikit-learn.org/stable/modules/cross_validation.html

Spectrum representation and spectrum recovery

Assignment

Objective: Spectral reconstruction of samples from the Munsell dataset

Goodness of fit coefficient

$$GFC = \frac{|\sum_\lambda r(\lambda)\hat{r}(\lambda)|}{\sqrt{\sum_\lambda |r(\lambda)|^2}\sqrt{\sum_\lambda |\hat{r}(\lambda)|^2}}$$