ENGR 599 Homework 3

Adrian Henle


```julia
using PlutoUI
```

Exercise 4.1.

Use the data in **Table 4.2** to confirm that the significant effect values in this design are really the same as those appearing in **Table 4.3**.

Table 4.2
Results of the 2^4 complete factorial design performed to study the catalytic action of Mo(VI)

Factors	(–)	(+)
1 $[H_2SO_4]$ (mol L^{-1})	0.16	0.32
2 [KI] (mol L^{-1})	0.015	0.030
3 $[H_2O_2]$ (mol L^{-1})	0.0020	0.0040
4 Time (s)	90	130

	Run	1	2	3	4	Response[a]
√	1	–	–	–	–	52
	2	+	–	–	–	61
	3	–	+	–	–	124
√	4	+	+	–	–	113
	5	–	–	+	–	85
√	6	+	–	+	–	66
√	7	–	+	+	–	185
	8	+	+	+	–	192
	9	–	–	–	+	98
√	10	+	–	–	+	86
√	11	–	+	–	+	201
	12	+	+	–	+	194
√	13	–	–	+	+	122
	14	+	–	+	+	139
	15	–	+	+	+	289
√	16	+	+	+	+	286

[a] Analytical signal intensity $\times$ 1000.

```
16×4 Matrix{Float64}:
 -1.0  -1.0  -1.0  -1.0
  1.0  -1.0  -1.0  -1.0
 -1.0   1.0  -1.0  -1.0
  1.0   1.0  -1.0  -1.0
 -1.0  -1.0   1.0  -1.0
  1.0  -1.0   1.0  -1.0
 -1.0   1.0   1.0  -1.0
  ⋮
 -1.0   1.0  -1.0   1.0
  1.0   1.0  -1.0   1.0
 -1.0  -1.0   1.0   1.0
  1.0  -1.0   1.0   1.0
 -1.0   1.0   1.0   1.0
  1.0   1.0   1.0   1.0
```

```
begin
    levels = zeros(16, 4)
    for i ∈ 1:4
        levels[:, i] =
        repeat([repeat([-1], 2^(i-1))..., repeat([1], 2^(i-1))...], Int(8/(2^(i-1))))
    end
    levels
end
```

```
responses =  [52, 61, 124, 113, 85, 66, 185, 192, 98, 86, 201, 194, 122, 139, 289, 286]
```

```
responses = vec([52 61 124 113 85 66 185 192 98 86 201 194 122 139 289 286])
```

Table 4.3
Analysis of the 2^4 factorial design to study the catalytic response of Mo(VI). The most significant values are italicized

Average = 143.31	**12** = −1.13
1 = −2.38	**13** = 2.88
2 = *109.38*	**14** = 1.13
3 = *54.38*	***23*** = *25.63*
4 = *67.13*	***24*** = *21.88*
	34 = 9.88
123 = 2.63	
124 = −2.63	**1234** = −8.88
134 = 5.38	
234 = 0.13	

```
factors_table4_3 =  [-2.38, 109.38, 54.38, 67.13]
```

```
factors_table4_3 = [-2.38, 109.38, 54.38, 67.13]
```

```
factors_calc =  [-2.375, 109.375, 54.375, 67.125]
```

```
factors_calc = [2 * sum(levels[:, i] .* responses) / length(responses) for i in 1:4]
```

```
true
```

```
all(isapprox.(factors_table4_3, factors_calc, rtol=0.0025))
```

The factors $\mathbf{1}$, $\mathbf{2}$, $\mathbf{3}$, and $\mathbf{4}$ in **Table 4.3** are identical to those calculated from **Table 4.2** to within 0.25%

Exercise 4.3.

How many experimental runs are there in a 2^(8–4) fractional factorial?

$$2^{8-4} = 16$$

Exercise 4.5.

All contrasts in **Table 4.5** represent the sum of two effects except $\mathbf{l_I}$, which estimates the overall average plus half of the $\mathbf{1234}$ interaction. Why half?

Table 4.5
Relations between the contrasts of the 2^{4-1} half-fraction and the effects of the 2^4 full factorial. **M** is the average of all the response values

Relations between columns of signs	Contrasts of the 2^{4-1} half-fraction in terms of the effects of 2^4 factorial
$\mathbf{1} = \mathbf{234}$	$l_1 = l_{234} \rightarrow \mathbf{1} + \mathbf{234}$
$\mathbf{2} = \mathbf{134}$	$l_2 = l_{134} \rightarrow \mathbf{2} + \mathbf{134}$
$\mathbf{3} = \mathbf{124}$	$l_3 = l_{124} \rightarrow \mathbf{3} + \mathbf{124}$
$\mathbf{4} = \mathbf{123}$	$l_4 = l_{123} \rightarrow \mathbf{4} + \mathbf{123}$
$\mathbf{12} = \mathbf{34}$	$l_{12} = l_{34} \rightarrow \mathbf{12} + \mathbf{34}$
$\mathbf{13} = \mathbf{24}$	$l_{13} = l_{24} \rightarrow \mathbf{13} + \mathbf{24}$
$\mathbf{14} = \mathbf{23}$	$l_{14} = l_{23} \rightarrow \mathbf{14} + \mathbf{23}$
$\mathbf{I} = \mathbf{1234}$	$l_I \rightarrow \mathbf{M} + \frac{1}{2}(\mathbf{1234})$

The values are contrasts between two halves of the set of 16 responses, except for $\mathbf{M}$, which is calculated on all 16; so, $\mathbf{1234}$ must be scaled.

Exercise 4.7.

How would you combine the values of the contrasts to obtain the $\mathbf{1234}$ interaction effect? Do the corresponding calculations and compare the result with the value given in **Table 4.3**.

$$\mathbf{1234} = l_{1234} - l^*_{1234} = 2 * \mathbf{1} = -4.76$$

This is considerably lower than the value in table 4.3

Exercise 4.9.

In a half-fractional factorial design of resolution VI, the main effects are confounded with what other effects? And the two-factor interactions?

Main effects confound with 5^{th} order interactions, and binary interactions confound with 4^{th} order.

Exercise 4.11.

Construct a 2^{5-1} fractional design using $\mathbf{5} = \mathbf{124}$. Determine, for this fraction, the relations between the contrasts involving one or two factors and the effects calculated from a complete factorial. Can you imagine a situation in which this design would be preferable, instead of a fraction of maximum resolution?

Design matrix:

```
16×5 Matrix{Float64}:
 -1.0  -1.0  -1.0  -1.0  -1.0
  1.0  -1.0  -1.0  -1.0   1.0
 -1.0   1.0  -1.0  -1.0   1.0
  1.0   1.0  -1.0  -1.0  -1.0
 -1.0  -1.0   1.0  -1.0  -1.0
  1.0  -1.0   1.0  -1.0   1.0
 -1.0   1.0   1.0  -1.0   1.0
  ⋮
 -1.0   1.0  -1.0   1.0  -1.0
  1.0   1.0  -1.0   1.0   1.0
 -1.0  -1.0   1.0   1.0   1.0
  1.0  -1.0   1.0   1.0  -1.0
 -1.0   1.0   1.0   1.0  -1.0
  1.0   1.0   1.0   1.0   1.0
```

```
hcat(hcat([repeat([repeat([-1], 2^(i-1))..., repeat([1], 2^(i-1))...], Int(8/(2^(i-1)))) for i ∈ 1:4]...), [-1. +1. +1. -1. -1. +1. +1. -1. +1. -1. -1. +1. +1. -1. -1. +1.]')
```

Contrasts and effects:

$$l_1 = \mathbf{1} + \mathbf{245}$$

$$l_2 = \mathbf{2} + \mathbf{145}$$

$$l_3 = \mathbf{3} + \mathbf{12345}$$

$$l_4 = \mathbf{4} + \mathbf{125}$$

$$l_5 = \mathbf{3} + \mathbf{124}$$

$$l_{12} = \mathbf{12} + \mathbf{45}$$

$$l_{13} = \mathbf{13} + \mathbf{2345}$$

$$l_{14} = \mathbf{14} + \mathbf{25}$$

$$l_{15} = \mathbf{15} + \mathbf{24}$$

$$l_{23} = \mathbf{23} + \mathbf{1345}$$

$$l_{24} = \mathbf{15} + \mathbf{24}$$

$$l_{25} = \mathbf{25} + \mathbf{14}$$

$$l_{34} = \mathbf{34} + \mathbf{1235}$$

$$l_{35} = \mathbf{35} + \mathbf{1234}$$

$$l_{45} = \mathbf{12} + \mathbf{45}$$

Exercise 4.13.

Use the data of **Table 4.10** to calculate the value of the contrast corresponding to the main effect of the side of the tennis court.

Table 4.10 (excerpt):

Runs	1	2	3	4	5	6	7	% valid
1	−	−	−	+	+	+	−	56
2	+	−	−	−	−	+	+	66
3	−	+	−	−	+	−	+	51
4	+	+	−	+	−	−	−	52
5	−	−	+	+	−	−	+	54
6	+	−	+	−	+	−	−	70
7	−	+	+	−	−	+	−	42
8	+	+	+	+	+	+	+	64

Let vector $\mathbf{p}$ be the signs of column 5 (the court side factor):

```
p =  [1.0, -1.0, 1.0, -1.0, -1.0, 1.0, -1.0, 1.0]
```

```
p = [+1., -1., +1., -1., -1., +1., -1., +1.]
```

Let vector $\mathbf{y}$ be the percentage of valid serves for each run:

```
y =  [56.0, 66.0, 51.0, 52.0, 54.0, 70.0, 42.0, 64.0]
```

```
y = [56., 66., 51., 52., 54., 70., 42., 64.]
```

The contrast is:

$$l_5 = \frac{1}{4} \sum_i p_i y_i$$

```
6.75
```

```
0.25 * sum(p .* y)
```

Exercise 4.15.

Each run in **Tables 4.10** and **4.12** correspond to performing serves under the experimental conditions specified by the signs of the respective design matrices. Describe the experiment represented by run 4 in **Table 4.10**. In practice, what is the difference between this run and run 4 in **Table 4.12**?

Run 4 of **Table 4.10** is slice, high frequency, daytime, on concrete, from the right, shirt-on, medium racquet. Run 4 of **Table 4.12** has the same conditions, except the serve is from the left.

Exercise 4.17.

Use the generating relations given in **Table 4.15** and find the three-factor interactions confounded with factor 1.

The sign of 1 is negative, so the product of the 3 factors' signs must be negative.

Also,

$$\mathbf{I} = \mathbf{1248} = \mathbf{1358} = \mathbf{2368} = \mathbf{1237}$$

so we don't consider $\mathbf{368}$. That gives:

$$l_1 = \mathbf{1} + \mathbf{248} + \mathbf{358} + \mathbf{237} + \mathbf{346} + \mathbf{256} + \mathbf{678} + \mathbf{457}$$