AN215A

RF Small Signal Design Using Two-Port Parameters

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INTRODUCTION

Design of the solid-state, small-signal RF amplifier using two-port parameters is a systematic, mathematical procedure, with an exact solution (free from approximation) available for the complete design problem. The only sources of error in the final design are parameter variations resulting from transistor parameter distributions and strays in the physical circuit. Parameter distributions result from limits in measurement and random variations among identically designed transistors.

The purpose of this paper is to provide, in a single working reference, the important relationships necessary for the complete solution of the RF small-signal design problem using two-port parameters.

The major portion of the report presents design equations in terms of admittance parameters. A section on design with scattering parameters is also included.

This paper is based on work by Linvill[1], Stern[2], and others. Those who may wish to consider the derivations of some of the expressions should refer to the bibliography.

This report assumes that the reader is familiar with the two-port parameter method of describing a linear active network. Several references are available on this subject.[1,2,6,8,11,12]

It has also been assumed that a suitable transistor or other active device for the task at hand has been selected, and that two-port parameters are available for the frequency and bias point which will be used. Device selection will not be covered as a separate topic in this report; rather, a thorough understanding of the material in the report should provide the designer with the tools he needs to select transistors for a particular small-signal application.

The equations given in the text of this report are applicable to the common-emitter, common-base, or common-collector configuration, if the applicable set of parameters (common-emitter, common-base, or common-collector parameters) is used. Equations for the conversion of the admittance or hybrid parameters of any configuration to either of the other two configurations of the same parameter set are given in the appendix.

While directed primarily toward circuit design with conventional bipolar transistors, two-port network theory has the advantage of being applicable to any linear active network (LAN). The same design approach and equations may therefore be used with field effect transistors[7,9], integrated circuits[10], or any other device which may be described as a linear active two-port network.

Finally, various parameter interrelationships and other data are given in the Appendix.

GENERAL DESIGN CONSIDERATIONS

Design of the RF small-signal tuned amplifier is usually based on a requirement for a specified power gain at a given frequency. Other design goals may include bandwidth, stability, input-output isolation, and low noise performance. After a basic circuit type is selected, the applicable design equations can be solved.

Circuits may be categorized according to feedback (neutralization, unilateralization, or no feedback), and matching at transistor terminals (circuit admittances either matched or mismatched to transistor input and output admittances). Each of these circuit categories will be discussed, including the applicable design equations and the considerations leading to the selection of a particular configuration.

STABILITY

A major factor in the overall design is the potential stability of the transistor. This may be determined by computing the Linvill stability factor[1] C using the following expression:†

$$C = \frac{|y_{12}\,y_{21}|}{2g_{11}\,g_{22} - R_e\,(y_{12}y_{21})} \qquad (1)$$

When C is less than 1, the transistor is unconditionally stable. When C is greater than 1, the transistor is potentially unstable.

The C factor is a test for stability under a hypothetical worst case condition; that is, with both input and output transistor terminals open circuited. With no external feedback, an unconditionally stable transistor will not oscillate with any combination of source and load. If a transistor is potentially unstable, certain source and load combinations will produce oscillations.

Although the C factor may be used to determine the potential stability of a transistor, the conditions of open circuited source and load which are assumed in the C factor test are not applicable to a practical amplifier. Consequently it is also desirable to compute the relative stability of actual amplifier circuits, and Stern[2] has defined a stability factor k for this purpose. The k factor is similar to the C factor except that it also takes into account finite source and load admittances connected to the transistor. The expression for k is:

$$k = \frac{2\,(g_{11} + G_s)\,(g_{22} + G_L)}{|y_{12}y_{21}| + R_e\,(y_{12}y_{21})} \qquad (2)$$

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†$R_e\,(y_{12}y_{21})$ = Real part of $(y_{12}y_{21})$

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If k is greater than one, the circuit will be stable. If k is less than one, the circuit will be potentially unstable and will very likely oscillate at some frequency.

Note that the C factor simply predicts potential stability of a transistor with an open circuited source and load, while the k factor provides a stability computation for a specific circuit.

Stability considerations will be discussed further in the descriptions of each basic circuit type to follow.

GENERAL DESIGN EQUATIONS

There are a number of design equations which are applicable to most types of amplifiers. These equations will be discussed first. Descriptions of specific amplifier types will then follow, and each will contain additional design equations applicable to that particular amplifier.

POWER GAIN

The general expression for power gain is:

$$G = \frac{|y_{21}|^2 \, R_e \, (Y_L)}{|Y_L + y_{22}|^2 \, R_e \left(y_{11} - \dfrac{y_{12} y_{21}}{y_{22} + Y_L} \right)} \tag{3}$$

Equation 3 applies to circuits with no external feedback. It can also be used with circuits which have external feedback if the composite y parameters of both the transistor and the feedback network are substituted for the transistor y parameters in the equation. The composite y parameters are determined by considering the transistor and the feedback network to be two "black boxes" in parallel:
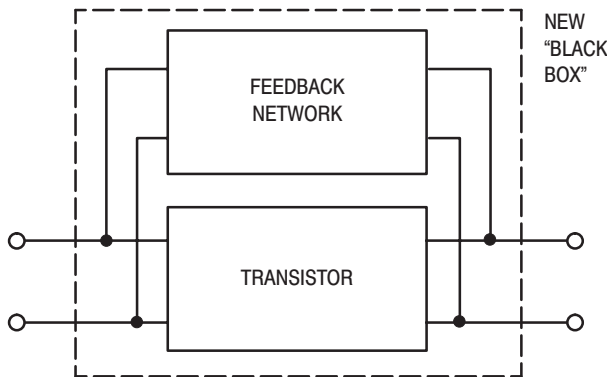
For example, the above combination of transistor and feedback network may be characterized as a single "black box" by the following equations:[†]

$$
\begin{aligned}
y_{11c} &= y_{11t} + y_{11f} \\
y_{12c} &= y_{12t} + y_{12f} \\
y_{21c} &= y_{21t} + y_{21f} \\
y_{22c} &= y_{22t} + y_{22f}
\end{aligned}
\tag{4}
$$

Where:

y_{11c}, y_{12c}, y_{21c}, y_{22c} are the composite y parameters of the parallel combination of transistor and feedback network.

y_{11t}, y_{12t}, y_{21t}, y_{22t} are the y parameters of the transistor.

y_{11f}, y_{12f}, y_{21f}, y_{22f} are the y parameters of the feedback network.

Note that, since this approach treats the transistor and feedback network combination as a single "black box" with y_{11c}, y_{12c}, y_{21c}, and y_{22c} as its y parameters, the composite y parameters may therefore be substituted in any of the design equations applicable to a linear, active two-port analysis.

The neutralized and unilateralized amplifiers are special cases of this general concept, and equations associated with those special cases will be given later.

Equation 3 provides a solution for power gain of the linear active network (transistor) only. Input and output networks are considered to be part of the source and load, respectively. Two important points should therefore be kept in mind:

1. Power gain computed from equation 3 will not take into account network losses. Input network loss reduces power delivered to the transistor. Power lost in the output network is computed as useful power output, since the load admittance Y_L is the combination of the output network and its load.

2. Power gain is independent of source admittance. An input mismatch results in less input power being delivered to the transistor. Accordingly, note that equation 3 does not contain the term Y_s.

The power gain of a transistor together with its associated input and output networks may be computed by measuring the input and output network losses, and subtracting them from the power gain computed with equation 3.

In some cases it may be desirable to include the effects of input matching in power gain computations. A convenient term is transducer gain G_T, defined as output power delivered to a load by the transistor, divided by the maximum input power available from the source.

The equation for transducer gain is:

$$G_T = \frac{4 \, R_e \, (Y_s) \, R_e \, (Y_L) \, |y_{21}|^2}{|(y_{11} + Y_s)(y_{22} + Y_L) - y_{12} y_{21}|^2} \tag{5}$$

In this equation, Y_L is the composite transistor load admittance–composed of both output network and its load, and Y_s is the composite transistor source admittance –composed of both input network and its source. Therefore, transducer gain includes the effects of the degree of admittance match at the transistor input terminals but does not take into account input and output network losses.

As in equation 3, the composite y parameters of a transistor feedback network combination may be substituted for the transistor y parameters when such a combination is used.

The Maximum Available Gain (MAG) is an often used transistor figure-of-merit. The MAG is the theoretical power gain of a transistor with its reverse transfer admittance y_{12} set equal to zero, and its source and load admittances conjugately matched to y_{12} and y_{22}, respectively.

If $y_{12} = 0$, the transistor exhibits an input admittance equal to y_{11} and an output admittance equal to y_{22}.[‡] The equation for MAG is, therefore, obtained by solving the general power gain expression, equation 3, with the conditions

$$
\begin{aligned}
y_{12} &= 0 \\
y_L &= y_{22}{}^* \\
\text{and } y_s &= y_{11}{}^*
\end{aligned}
$$

where * denotes conjugate

[†] Refer to Seshu and Balabanian, "Linear Network Analysis," John Wiley and Sons, 1959, P321

[‡] Obtained by solving the equations for transistor Y_{IN} and Y_{OUT} with Y_{12} equal to zero. These equations are given later in the report.

which yields:

$$MAG = \frac{|y_{21}|^2}{4\,R_e\,(y_{11})\,R_e\,(y_{22})} \qquad (6)$$

MAG is a figure of merit only, since it is physically impossible to reduce y_{12} to zero without changing the other parameters of the transistor. An external feedback network may be used to achieve a composite y_{12} of zero, but then the other composite parameters will also be modified according to the relationships given in the discussion of the composite transistor — feedback network "black box."

CASCADED LAN'S

Design calculations for cascaded LAN's may be performed by first computing composite two-port parameters as was done in the case of the parallel LAN's.

For the following cascaded LAN's
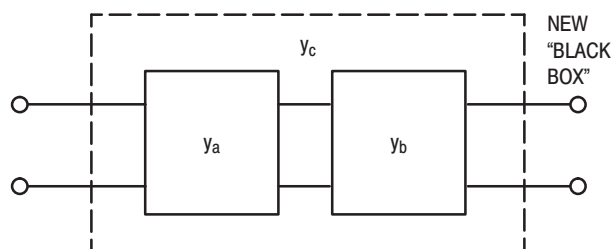
The composite y parameters are:

$$y_{11c} = y_{11a} - \frac{y_{12a}\,y_{21a}}{y_{22a} + y_{11b}}$$

$$y_{22c} = y_{22b} - \frac{y_{12b}\,y_{21b}}{y_{22a} + y_{11b}}$$

$$y_{21c} = \qquad - \frac{y_{21a}\,y_{21b}}{y_{22a} + y_{11b}} \qquad (7)$$

$$y_{12c} = \qquad - \frac{y_{12a}\,y_{12b}}{y_{22a} + y_{11b}}$$

where y_{11c}, y_{22c}, y_{21c}, y_{12c} are the composite y parameters of the cascaded LAN's.

TRANSISTOR INPUT AND OUTPUT ADMITTANCES

The expression for the input admittance of a transistor is:

$$Y_{IN} = y_{11} - \frac{y_{12}\,y_{21}}{y_{22} + Y_L} \qquad (8)$$

The expression for the output admittance of a transistor is:

$$Y_{OUT} = y_{22} - \frac{y_{12}\,y_{21}}{y_{11} + Y_s} \qquad (9)$$

When the feedback parameter y_{12} is not zero, Y_{IN} is dependent on load admittance and Y_{OUT} is dependent on source admittance.

AMPLIFIER STABILITY

One of the major considerations in RF amplifier design is stability. The stability of a final design can be assured by including stability computations and considering stability in all design decisions relating to feedback and transistor source and load admittances.

The potential stability of the transistor should first be computed using equation 1.

The various alternatives concerning input–output matching and neutralization–unilateralization will now be discussed for both the unconditionally stable transistor and the potentially unstable transistor.

THE UNCONDITIONALLY STABLE TRANSISTOR

When the Linvill stability factor of the transistor as determined by equation 1 is less than one, the transistor is unconditionally stable. Oscillations will not occur using any combination of source and load admittances without external feedback. Stability is therefore eliminated as a factor in the remainder of the design, and complete freedom is possible with regard to matching and neutralization to optimize the amplifier for other performance requirements.

AMPLIFIERS WITHOUT FEEDBACK

The amplifier with no feedback is a logical choice for the unconditionally stable transistor in many applications since it may offer the advantages of fewer components and a simple tuning procedure.

Source and load admittances may be selected for maximum gain and/or any number of other requirements. Power gain and transducer gain may be computed using equations 3 and 5, respectively; input and output admittances may be computed using equations 8 and 9, respectively.

The amplifier stability factor may be computed using equation 2. While amplifier stability was assured from the beginning by the use of an unconditionally stable transistor, the designer may still wish to perform this computation to provide some insight into danger of instability under adverse environmental conditions, source and load variations, etc.

G_{max}

G_{max}, the highest transducer gain possible without external feedback, forms a special case of the no feedback amplifier.

The source and load admittances required to achieve G_{max} may be computed from the following:

$$G_s = \frac{1}{2\,R_e\,(y_{22})}\,\left\{[2\,R_e\,(y_{11})\,R_e\,(y_{22}) - R_e\,(y_{12}y_{21})]^2\right.$$
$$\left. - |y_{12}y_{21}|^2\right\}^{1/2} \qquad (10)$$

$$B_s = -\,Im(y_{11}) + \frac{Im(y_{21}y_{12})}{2\,R_e(y_{22})} \qquad (11)$$

$$G_L = \frac{1}{2\,R_e(y_{11})}\,\left\{[2\,R_e\,(y_{11})\,R_e\,(y_{22}) - R_e\,(y_{12}y_{21})]^2\right.$$
$$\left. - |y_{12}y_{21}|^2\right\}^{1/2} \qquad (12)$$

$$B_L = -\,Im(y_{22}) + \frac{Im(y_{21}y_{12})}{2\,R_e(y_{11})} \qquad (13)$$

Therefore, if the maximum possible power gain without feedback is desired for an amplifier, equations 10, 11 12, and 13 are used to compute Y_s and Y_L.

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The magnitude of G_{max} may be computed from the following expressions:

$$G_{max} = \frac{|y_{21}|^2}{2\,Re(y_{11})\,Re(y_{22}) - Re(y_{12}y_{21}) + \{[2\,Re(y_{11})\,Re(y_{22}) - Re(y_{12}y_{21})]^2 - |y_{12}y_{21}|^2\}^{1/2}} \quad (14)$$

Equations 10, 11, 12, and 13 can be obtained by differentiating equation 5 with respect to G_s, B_s, G_L, and B_L, and setting the four derivatives equal to zero. The G_s, B_s, G_L, and B_L thus computed can then be substituted in equation 5 to obtain the expression for G_{max}, equation 14.

THE LINVILL METHOD

The amplifier without feedback design problem may also be solved graphically using a technique developed by J. G. Linvill.[†] Linvill's technique is very useful for a certain class of problems. Since it is so fully discussed in many good references, we will not go into it further here. An advantage of the Linvill technique is that it provides a reasonably rapid graphic solution relating gain, bandwidth, and stability. A disadvantage is its scope of usefulness, since the standard Linvill solution applies only to an amplifier with no external feedback and the Y_s conjugately matched to the transistor input admittance, Y_{IN}.

THE UNILATERALIZED AMPLIFIER

Unilateralization consists of employing an external feedback network to achieve a composite y_{12} of zero.

While unilateralization is perhaps most often used to achieve stability with a potentially unstable transistor, other circuit considerations may also warrant the use of unilateralization with the unconditionally stable transistor. For example, the input-output isolation afforded by unilateralization may be desirable in a particular design.

Design equations for the unilateralized case are obtained by first computing the composite y parameters of the transistor — feedback network combination and then substituting the composite parameters in the general equations.

Referring to the discussion on composite y parameters and setting up the basic condition that y_{12c} must equal zero, the other composite y parameters can be computed. Assuming that a passive feedback network is being used, then

$$y_{11f} = y_{22f} = -y_{12f} = -y_{21f}.$$

and since $\quad y_{12c} = 0, \; y_{12t} + y_{12f} = 0$

then $\quad y_{12t} = -y_{12f},$

and $\quad y_{12t} = -y_{12f} = y_{11f} = y_{22f} = -y_{21f}$

Substituting the above results in equation 4 yields the following:

$$y_{11c} = y_{11t} + y_{12t}$$
$$y_{22c} = y_{22t} + y_{12t}$$
$$y_{12c} = y_{12t} - y_{12t} = 0$$
$$y_{21c} = y_{21t} = y_{12t}$$

Substituting these complete y parameters in equations 8, 9, 3, 7, and 5 respectively, yields equations 15, 16, 17, 18, and 19 respectively for the unilateralized case. Unilateralized input admittance

$$Y_{IN} = y_{11} + y_{12} \quad (15)$$

Unilateralized output admittance

$$Y_{OUT} = y_{22} + y_{12} \quad (16)$$

Unilateralized power gain, general expression:

$$G_{PU} = \frac{|y_{21} - y_{12}|^2\,Re(Y_L)}{|Y_L + y_{22} + y_{12}|^2\,Re(y_{11})} \quad (17)$$

Unilateralized power gain with Y_L conjugately matched to Y_{OUT}:

$$G_U = \frac{|y_{21} - y_{12}|^2}{4\,Re(y_{11} + y_{12})\,Re(y_{22} + y_{12})} \quad (18)$$

Unilateralized transducer gain:

$$G_{TU} = \frac{4\,Re(Y_s)\,Re(Y_L)|y_{21} - y_{12}|^2}{|(y_{11} + y_{12} + Y_s)(y_{22} + y_{12} + Y_L)|^2} \quad (19)$$

Note that equations 15, 16, 17, 18 and 19, are given entirely in terms of the transistor y parameters, not those of the feedback network or the composite.

Another benefit of unilateralization is input–output isolation. As can be seen in equations 15 and 16, Y_{IN} is completely independent of Y_L, and Y_{OUT} is similarly independent of Y_s. In a practical sense, this means that in a single or multi-stage amplifier using unilateralized stages, tuning of any one network will not affect tuning in other parts of the circuit. Thus, the troublesome task of having to re-peak an entire amplifier following a change in tuning at a single point can be eliminated.

NEUTRALIZATION

Neutralization consists of employing a feedback network to reduce y_{12} to some value other than zero. Neutralization is generally used for the same purposes as unilateralization, but provides something less than the ideal cancellation of the transistor feedback parameter which unilateralization achieves. A typical example of neutralization might be a feedback network which provides a composite b_{12} of zero while having only a negligible effect on the transistor g_{12}.

The equations for a particular neutralized case would be developed in the same manner as those for the unilateralized case. Since there are an infinite number of possibilities, no specific equations will be given here.

This completes the discussion of design with the unconditionally stable transistor. The potentially unstable transistor will now be considered.

THE POTENTIALLY UNSTABLE TRANSISTOR

When the Linvill stability factor of the transistor as determined by equation 1 is greater than one, the transistor is potentially unstable. Certain combinations of source and load admittances will cause oscillations if no feedback is used. In designing with the potentially unstable transistor, steps must be taken to insure that the amplifier will be stable.

Stability is usually achieved by one or both of two methods:

1. Using a feedback network which reduces the composite y_{12} to a value which insures stability.
2. Choosing a source and load admittance combination which provides stability.

[†]Application Note AN166 Motorola Semiconductor Products, Inc. Dept. TIC, 5005 E. McDowell Road, Phoenix, Arizona. See also reference 5 in the bibliography.

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A discussion of these basic methods is given below.

USING FEEDBACK TO ACHIEVE STABILITY

Either unilateralization or neutralization may be used to achieve stability. If unilateralization is used, the transistor-feedback network combination will be unconditionally stable. This may be verified by computing the Linvill stability factor of the combination. Since $y_{12c} = 0$, the numerator in equation 1 would be zero.

With stability thus assured, the remainder of the design may then be done to satisfy other requirements placed on the amplifier. After unilateralization has converted the potentially unstable transistor to an unconditionally stable combination, all other aspects of the design are identical to the unilateralized case with the unconditionally stable transistor. Power gains and input and output admittances may be computed using equations 15 through 19.

If neutralization is used to achieve stability, the Linvill stability factor can be used to compute the potential stability of any transistor–neutralization network combination. Since in this case $y_{12c} \neq 0$, C will have a value other than zero.

After unconditional stability of the transistor-neutralization network combination has been achieved, the design may then be completed by treating the combination as an unconditionally stable transistor, and proceeding with the case of the unconditionally stable transistor in an amplifier without feedback. Power gains, input and output admittances, and the circuit stability factor may be computed by using the composite parameters of the combination in equations 2, 3, 5, 8, and 9.

STABILITY WITHOUT FEEDBACK

A stable design with the potentially unstable transistor is possible without external feedback by proper choice of source and load admittances. This can be seen by inspection of equation 2; G_S and/or G_L can be made large enough to yield a stable circuit regardless of the degree of potential instability of the transistor.

This suggests a relatively simple way to achieve a stable design with a potentially unstable transistor. A circuit stability factor k is selected, and equation 2 is used to arrive at values of G_S and G_L which will provide the desired k. In achieving a particular circuit stability factor, the designer may choose any of the following combinations of matching or mismatching of G_S and G_L to the transistor input and output conductances, respectively:

1. G_S matched and G_L mismatched
2. G_L matched and G_S mismatched
3. Both G_S and G_L mismatched

Often a decision on which combination to use will be dictated by other performance requirements or practical considerations.

Once G_S and G_L have been chosen, the remainder of the design may be completed using the relationships which apply to the amplifier without feedback. Power gain and input and output admittances may be computed using equations 3, 5, 8, and 9.

Although the above procedure may be adequate in many cases, a more systematic method of source and load admittance determination is desirable for designs which demand maximum power gain per degree of circuit stability.

Stern has analyzed this problem and developed equations for computing the conductance and susceptance of both Y_S and Y_L for maximum power gain for a particular circuit stability factor.[2,4] These equations are given here:

$$G_S = \sqrt{\frac{k\,[\,|y_{12}y_{21}| + R_e(y_{12}y_{21})]}{2}} \cdot \sqrt{\frac{g_{11}}{g_{22}}} - g_{11} \quad (20)$$

$$G_L = \sqrt{\frac{k\,[\,|y_{12}y_{21}| + R_e(y_{12}y_{21})]}{2}} \cdot \sqrt{\frac{g_{22}}{g_{11}}} - g_{22} \quad (21)$$

$$B_S = \frac{(G_S + g_{11})\,Z_o}{\sqrt{k\,[\,|y_{12}y_{21}| + R_e(y_{12}y_{21})]}} - b_{11} \quad (22)$$

$$B_L = \frac{(G_L + g_{22})\,Z_o}{\sqrt{k\,[\,|y_{12}y_{21}| + R_e(y_{12}y_{21})]}} - b_{22} \quad (23)$$

Where,

$$Z = \frac{(B_S+b_{11})(G_L+g_{22}) + (B_L+b_{22})\,k(L+M)/2\,(G_L+g_{22})}{\sqrt{k\,(L+M)}} \quad (24)$$

$$L = |y_{12}y_{21}| \quad (25)$$

$$M = R_e\,(y_{12}y_{21}) \quad (26)$$

Defining D as the denominator in equation 5 yields:

$$D = \frac{Z^4}{4} + \frac{[k(L+M) + 2M]\,Z^2}{2} - 2NZ\sqrt{k(L+M)} + A^2 + N^2 \quad (27)$$

where,

$$A = \frac{k(L+M)}{2} - M, \quad (28)$$

$$N = I_m(y_{12}y_{21}), \quad (29)$$

and,

Z_o = that real value of Z which results in the smallest minimum of D, found by setting,

$$\frac{dD}{dZ} = Z^3 + [k(L+M) + 2M]\,Z - 2N\sqrt{k(L+M)} \quad (30)$$

equal to zero.

Computation of Y_S and Y_L using equations 20 through 30 is a bit tedious to be done very frequently, and this may have discouraged wide usage of the complete Stern solution. However, examination of Stern's work suggests some interesting shortcuts:

(A) COMPUTATION OF G_S AND G_L ONLY, USING EQUATIONS 20 AND 21. If a value equal to $-b_{22}$ is then chosen for B_L, the resulting Y_L will be very close to the true Y_L for maximum gain. The transistor Y_{IN} can then be computed from Y_L using equation 8, and B_S can be set equal to $-I_m(Y_{IN})$.

Computation of B_S and B_L comprise by far the more complex portion of the Stern solution. This alternate method therefore permits the designer to closely approximate the exact Stern solution for Y_S and Y_L while avoiding that portion of the computations which are the most complex and time consuming. Further, the circuit can be designed with tuning adjustments for varying B_S and B_L, thereby creating the possibility of experimentally achieving the true B_S and B_L for

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maximum gain as accurately as if all the Stern equations had been solved.

(B) MISMATCHING G_s TO g_{11} AND G_L TO g_{22} BY AN EQUAL RATIO YIELDS A TRUE STERN SOLUTION FOR G_s AND G_L. This can be derived from equations 20 and 21, which lead to the following result:

$$\frac{G_L}{g_{22}} = \frac{G_s}{g_{11}} \qquad (31)$$

If a mismatch ratio, R, is defined as follows,

$$R = \frac{G_L}{g_{22}} = \frac{G_s}{g_{11}} \qquad (32)$$

then R may be computed for any particular circuit stability factor using the equation:

$$(1 + R)^2 = k \left[\frac{|y_{21}y_{12}| + R_e(y_{12}y_{21})}{2 \, g_{11} \, g_{22}} \right] \qquad (33)$$

Equation 33 was derived from equations 2 and 32. Having thus determined R, G_s and G_L can be quickly found using equation 32.

B_s and B_L can then be determined in the manner described above in alternate method (A).

This alternate method may be advantageous if source and load admittances and power gains for several different values of k are desired. Once the R for a particular k has been determined, the R for any other k may be quickly found from the equation

$$\frac{(1 + R_1)^2}{(1 + R_2)^2} = \frac{k_1}{k_2} \qquad (34)$$

where R_1 and R_2 are values of R corresponding to k_1 and k_2, respectively.

(C) COMPUTER DESIGN. The complete Stern design problem may be programmed into a computer. Power gain, circuit stability factor, Y_s and Y_L can be obtained from the computer for any value of k. MAG, G_U, and the Linvill stability factor of the transistor may also be included in the program.

After employing either the complete Stern solution or an alternate method to obtain Y_s and Y_L for the potentially unstable transistor in an amplifier without feedback, power gains and input and output admittances may be obtained using equations 3, 5, 8, and 9.

SENSITIVITY

In all but the unilateralized amplifier, Y_{IN} is a function of load admittance. Thus Y_{IN} changes with output circuit tuning, and this can be troublesome. Consequently, it is sometimes desirable to compute the extent of variation of Y_{IN} with changes in Y_L. A term, sensitivity δ, has been defined to provide a measure of this characteristic, and is equal to percent change in Y_{IN} divided by percent change in Y_L. The equation for sensitivity is:

$$\delta = \left| \frac{Y_L}{y_{22} + Y_L} \right| \bullet \left| \frac{g_{11}}{y_{11}} \right| \bullet \frac{K}{\left| \frac{y_{22} + Y_L}{g_{22}} + \frac{g_{11}}{y_{11}} \, K \, e^{j\Theta} \right|} \qquad (35)$$

where,

$$K = \left| \frac{y_{21} \, y_{12}}{g_{11} \, g_{22}} \right|$$

$$\Theta = \arg\,(- y_{12}y_{21})^*$$

$$K \, e^{j\Theta} = K \,(\cos \Theta + j \sin \Theta)$$

A more complete discussion of sensitivity is given in reference 6.

DESIGN WITH SCATTERING PARAMETERS

Scattering, or s parameters have greatly increased in popularity since the late 1960's, largely due to the appearance of sophisticated new equipment for performing s parameter measurements.

A summary of s parameter design equations is given below.

Power gain:

$$G = \frac{|S_{21}|^2 \,(1 - |\Gamma_L|^2)}{(1 - |S_{11}|^2) + |\Gamma_L|^2 \,(\,|S_{22}|^2 - |_{\Delta S}|^2) - 2\, \text{Re}\,(\Gamma_L \, N)} \qquad (36)$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$N = S_{22} - D \, S^*_{11}$$

Transducer gain:

$$G_T = \frac{|S_{21}|^2 \,(1 - |\Gamma_S|^2)\,(1 - |\Gamma_L|^2)}{|(1 - |S_{11}|\Gamma_S)\,(1 - S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_L\Gamma_S|^2} \qquad (37)$$

Input reflection coefficient:

$$S'_{11} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \qquad (38)$$

Output reflection coefficient:

$$S'_{22} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \qquad (39)$$

Linvill stability factor:

$$C = K^{-1}$$

$$K = \frac{1 + |\Delta S|^2 - |S_{11}|^2 - |S_{22}|^2}{2 \,|S_{12}S_{21}|} \qquad (40)$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

Equation 40 which gives K, the reciprocal of C, is presented in this form because it is the s parameter stability expression most often seen in the literature. K in equation 40 must not be confused with Stern stability factor k given in equation 2.

Maximum unneutralized transducer gain, unconditionally stable LAN:

$$G_{max} = \left| \frac{S_{21}}{S_{12}} \,(K \pm \sqrt{K^2 - 1}) \right| \qquad (41)$$

$$K = C^{-1}$$

C = Linvill Stability Factor

Source and load reflection coefficients for a conjugate match of the unconditionally stable LAN in an amplifier without feedback:

$$\Gamma_{mS} = M^* \left[\frac{B_1 \pm \sqrt{B_1{}^2 - 4 \,|M|^2}}{2 \,|M|^2} \right] \qquad (42)$$

$$\Gamma_{mL} = N^* \left[\frac{B_2 \pm \sqrt{B_2{}^2 - 4 \; |N|^2}}{2|N|^2} \right] \qquad (43)$$

Where $B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta S|^2$

$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta S|^2$

$M = S_{11} - (\Delta S)(S_{22}{}^*)$

$N = S_{22} - (\Delta S)(S_{11}{}^*)$

A more comprehensive treatment of amplifier design with s parameters is given in references 8, 11, and 12.

One cautionary note is in order.

Several papers have been published on the subject of simplifying the s parameter design procedure by making the assumption that the reverse transfer parameter, s_{12}, is equal to zero. This procedure totally ignores the entire problem of amplifier stability.

Modern high gain solid-state RF devices will readily oscillate under a wide variety of circuit conditions. Stability problems are encountered even with extremely low feedback devices such as Linear IC's and dual gate MOSFET's. Therefore, amplifier design calculations which do not include device and circuit feedback are only an approximation which will yield either an inaccurate solution or possibly even an oscillator when the design is tested in the laboratory. Reference 13 provides more detail on the shortcomings of this procedure, including an amplifier design example which did turn out to be an oscillator.

SUMMARY OF DESIGN PROCEDURE

A summary of the amplifier design procedure using two-port parameters is given below.

1. Determine the potential instability of the active device.
2. If the device is not unconditionally stable, decide on a course of action to insure circuit stability.
3. Determine whether or not feedback is to be used.
4. Determine source and load admittances.
5. Design appropriate networks to provide the desired source and load admittances.

Stability (Steps 1 and 2 above)

A stability computation for the worst case conditions of open circuit source and load is provided by Linvill's stability factor C. If the C factor indicates unconditional stability, no combination of passive terminations can cause oscillations.

Stability calculations should include the total feedback of the amplifier. In the case of extremely low feedback devices such as dual gate MOSFET's and Linear IC's, external circuit feedback often eclipses the internal device feedback. In such a case, the designer should measure the external circuit feedback and include it in the design calculations. To accomplish this, see the earlier section of this note on the composite parameters of two-port LAN's in parallel.

If the device is unconditionally stable, the design may proceed to fulfill other objectives without fear of oscillations. If the device is potentially unstable, steps must be taken to prevent oscillations in the final design. Stability is achieved by proper selection of source and load admittances, by the use of feedback, or both.

Feedback (Step 3)

Feedback may be employed in the tuned high frequency amplifier to achieve stability, input-output isolation, or to alter the gain and terminal admittances of the active device. A decision to employ feedback would be based on whether or not its use was the optimum way to accomplish one of the foregoing objectives in a particular application.

If feedback is employed, the device parameters may be modified to include the feedback network in accordance with standard two-port network theory. The remainder of the design may then proceed by treating the transistor–feedback network combination as a single, new two-port linear active network.

Source and Load Admittances (Step 4)

Source and load admittance determination is dependent upon gain and stability considerations, together with practical circuit limitations.

If the device is either unconditionally stable itself or has been made stable with feedback, stability need not be a major factor in the determination of source and load. If the device is potentially unstable and feedback is not employed, then a source and load which will guarantee a certain degree of circuit stability must be used. Also, it is a good idea to check the circuit stability factor during this step even when an unconditionally stable device is used.

Finally, practical limitations in matching networks and components may also play an important part of source and load admittance determination.

Network Design (Step 5)

The final step consists of network synthesis to achieve the desired source and load admittances computed in step 4.

Sometimes, it will be difficult to achieve a desired source and load due to tuning range limitations, excess network losses, component limitations, etc. In such cases, the source and load admittances will be a compromise between desired performance and practical limitations.

SUMMARY

The small signal amplifier performance of a transistor is completely described by two-port admittance parameters. Based on these parameters, equations for computing the stability, gain, and optimum source and load admittances for the unilateralized, neutralized, and no-feedback amplifier cases have been discussed.

The unconditionally stable transistor will not oscillate with any combination of source and load admittances, and circuits using a stable transistor may be optimized for other performance requirements without fear of oscillations.

The potentially unstable transistor requires that steps be taken to guarantee a stable design. Stability is usually achieved by unilateralization, neutralization, or selection of source and load admittances which result in a stable amplifier.

Unilateralization and neutralization reduce the composite reverse transfer admittance. They may be used to achieve stability, input–output isolation, or both.

Maximum power gain per degree of circuit stability without feedback may be achieved using Stern's equations.

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The degree of input–output isolation is described by the term sensitivity, which makes it possible to compute changes in input admittance for any change in load admittance.

The theory and design equations in this report are applicable to any linear active device which may be characterized as a two-port network. Therefore, the term "transistor" used herein refers generally to all such devices, including FETs and integrated circuits.

BIBLIOGRAPHY

1. "Transistors and Active Circuits," by Linvill and Gibbons, McGraw-Hill, 1961.
2. "Stability and Power Gain of Tuned Transistor Amplifiers," by Arthur P. Stern, Proc. IRE, March, 1957.
3. "Using Linvill Techniques for R. F. Amplifiers," Motorola Semiconductor Products, Inc., Application Note 166.
4. "High-Gain, High-Frequency Amplifiers," by Peter M. Norris, Electro-Technology, January, 1966.
5. "Linvill Technique Speeds High Frequency Amplifier Design," by John Lauchner and Marvin Silverstein, Electronic Design, April 12, 1966.
6. "The Design of Alignable Transistor Amplifiers," by J. F. Gibbons, Stanford University Technical Report No. 106, May 7, 1956.
7. "Field Effect Transistor RF Amplifier Design Techniques," Motorola Semiconductor Products, Inc., Application Note 423.
8. "Circuit Design and Characterization of Transistors by Means of Three-Part Scattering Parameters," by George E. Bodway, The Microwave Journal, May, 1968.
9. "Small-Signal RF Design with Dual-Gate MOSFET's," Motorola Semiconductor Products, Inc. Application Note 478A.
10. "A High Gain Integrated Circuit RF-IF Amplifier with Wide Range AGC," Motorola Semiconductor Products, Inc. Application Note 513.
11. "S Parameter Design," Hewlett-Packard Company, Palo Alto, California, Application Note 154.
12. "S Parameters," Hewlett-Packard Company, Palo Alto, California, Application Note 95.
13. "Staying Stable with S Parameters," by Roy Hejhall, Motorola Monitor, Vol. 7, No. 2.

GLOSSARY

C = Linvill's stability factor
k = Stern's stability factor
G_s = Real part of the source admittance
G_L = Real part of the load admittance
B_s = Imaginary part of the source admittance
B_L = Imaginary part of the load admittance
g_{11} = Real part of y_{11}
g_{22} = Real part of y_{22}
G = Generalized power gain
Y_L = Complex load admittance
Y_s = Complex source admittance
G_T = Transducer gain
MAG = Maximum available gain
* = Conjugate
Y_{IN} = Input admittance
Y_{OUT} = Output admittance
G_{max} = Maximum gain without feedback
G_U = Unilateralized gain

G_{TU} = Unilateralized transducer gain
δ = Sensitivity
s'_{11} = Input reflection coefficient
s'_{22} = Output reflection coefficient
Γ_L = Load reflection coefficient
Γ_S = Source reflection coefficient
K = Scattering parameter stability factor

APPENDIX I

A. Conversions among parameter types for y, z, h, and g parameters.

h to y

$$y_{11} = \frac{1}{h_{11}} \qquad y_{12} = \frac{-h_{12}}{h_{11}} \qquad y_{21} = \frac{h_{21}}{h_{11}} \qquad y_{22} = \frac{\Delta h}{h_{11}}$$

where $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

y to h

$$h_{11} = \frac{1}{y_{11}} \qquad h_{12} = \frac{-y_{12}}{y_{11}} \qquad h_{21} = \frac{y_{21}}{y_{11}} \qquad h_{22} = \frac{\Delta y}{y_{11}}$$

where $\Delta y = y_{11} y_{22} - y_{12} y_{21}$

h to z

$$z_{11} = \frac{\Delta h}{h_{22}} \qquad z_{12} = \frac{h_{12}}{h_{22}} \qquad z_{21} = \frac{-h_{21}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}$$

z to h

$$h_{11} = \frac{\Delta z}{z_{22}} \qquad h_{12} = \frac{z_{12}}{z_{22}} \qquad h_{21} = \frac{-z_{21}}{z_{22}} \qquad h_{22} = \frac{1}{z_{22}}$$

where $\Delta z = z_{11} z_{22} - z_{12} z_{21}$

h to g

$$g_{11} = \frac{h_{22}}{\Delta h} \qquad g_{12} = \frac{-h_{12}}{\Delta h} \qquad g_{21} = \frac{-h_{21}}{\Delta h} \qquad g_{22} = \frac{h_{11}}{\Delta h}$$

where $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

g to h

$$h_{11} = \frac{g_{22}}{\Delta g} \qquad h_{12} = \frac{-g_{12}}{\Delta g} \qquad h_{21} = \frac{-g_{21}}{\Delta g} \qquad h_{22} = \frac{g_{11}}{\Delta g}$$

where $\Delta g = g_{11} g_{22} - g_{12} g_{21}$

z to y

$$y_{11} = \frac{z_{22}}{\Delta z} \qquad y_{12} = \frac{-z_{12}}{\Delta z} \qquad y_{21} = \frac{-z_{21}}{\Delta z} \qquad y_{22} = \frac{z_{11}}{\Delta z}$$

where $\Delta z = z_{11} z_{22} - z_{12} z_{21}$

y to z

$$z_{11} = \frac{y_{22}}{\Delta y} \qquad z_{12} = \frac{-y_{12}}{\Delta y} \qquad z_{21} = \frac{-y_{21}}{\Delta y} \qquad z_{22} = \frac{y_{11}}{\Delta y}$$

where $\Delta y = y_{11} y_{22} - y_{12} y_{21}$

z to g

$$g_{11} = \frac{1}{z_{11}} \qquad g_{12} = \frac{-z_{12}}{z_{11}} \qquad g_{21} = \frac{z_{21}}{z_{11}} \qquad g_{22} = \frac{\Delta z}{z_{11}}$$

where $\Delta z = z_{11} z_{22} - z_{12} z_{21}$

g to z

$$z_{11} = \frac{1}{g_{11}} \qquad z_{12} = \frac{-g_{12}}{g_{11}} \qquad z_{21} = \frac{g_{21}}{g_{11}} \qquad z_{22} = \frac{\Delta g}{g_{11}}$$

where $\Delta g = g_{11} g_{22} - g_{12} g_{21}$

g to y

$$y_{11} = \frac{\Delta g}{g_{22}} \qquad y_{12} = \frac{g_{12}}{g_{22}} \qquad y_{21} = \frac{-g_{21}}{g_{22}} \qquad y_{22} = \frac{1}{g_{22}}$$

where $\Delta g = g_{11} g_{22} - g_{12} g_{21}$

y to g

$$g_{11} = \frac{\Delta y}{y_{22}} \qquad g_{12} = \frac{y_{12}}{y_{22}} \qquad g_{21} = \frac{-y_{21}}{y_{22}} \qquad g_{22} = \frac{1}{y_{22}}$$

where $\Delta y = y_{11} y_{22} - y_{12} y_{21}$

B. Conversions among common emitter, common base, and common collector parameters of the same type for y, and h parameters.

Common emitter y parameters in terms of common base and common collector y parameters.

$$y_{11e} = y_{11b} + y_{12b} + y_{21b} + y_{22b} = y_{11c}$$

$$y_{12e} = -(y_{12b} + y_{22b}) = -(y_{11c} + y_{12c})$$

$$y_{21e} = -(y_{21b} + y_{22b}) = -(y_{11c} + y_{21c})$$

$$y_{22e} = y_{22b} = y_{11c} + y_{12c} + y_{21c} = y_{22c}$$

Common base y parameters in terms of common emitter and common collector y parameters.

$$y_{11b} = y_{11e} + y_{12e} + y_{21e} + y_{22e} = y_{22c}$$

$$y_{12b} = -(y_{12e} + y_{22e}) = -(y_{21c} + y_{22c})$$

$$y_{21b} = -(y_{21e} + y_{22e}) = -(y_{12c} + y_{22c})$$

$$y_{22b} = y_{22e} + y_{11c} + y_{12c} + y_{21c} + y_{22c}$$

Common collector y parameters in terms of common emitter and common base y parameters.

$$y_{11c} = y_{11e} = y_{11b} + y_{12b} + y_{21b} + y_{22b}$$

$$y_{12c} = -(y_{11e} + y_{12e}) = -(y_{11b} + y_{21b})$$

$$y_{21c} = -(y_{11e} + y_{21e}) = -(y_{11b} + y_{12b})$$

$$y_{22c} = y_{11e} + y_{12e} + y_{21e} + y_{22e} = y_{11b}$$

Common emitter h parameters in terms of common base and common collector h parameters.

$$h_{11e} = \frac{h_{11b}}{(1 + h_{21b})(1 - h_{12b}) + h_{22b} h_{11b}} \approx \frac{h_{11b}}{1 + h_{21b}} = h_{11c}$$

$$h_{12e} = \frac{h_{11b}h_{22b} - h_{12b}(1+h_{21b})}{(1 + h_{21b})(1-h_{12b}) + h_{22b}h_{11b}} \approx \frac{h_{11b}h_{22b}}{1+h_{21b}} - h_{12b} = 1 - h_{12c}$$

$$h_{21e} = \frac{-h_{21b}(1-h_{12b}) - h_{22b}h_{11b}}{(1+h_{21b})(1-h_{12b})+h_{22b}h_{11b}} \approx \frac{-h_{21b}}{1 + h_{21b}} = -(1+h_{21c})$$

$$h_{22e} = \frac{h_{22b}}{(1 + h_{21b})(1 - h_{12b}) + h_{22b} h_{11b}} \approx \frac{h_{22b}}{1 + h_{21b}} = h_{22c}$$

Common has h parameters in terms of common emitter and common collector h parameters.

$$h_{11b} = \frac{h_{11e}}{(1 + h_{21e})(1 - h_{12e}) + h_{11e} h_{22e}} \approx \frac{h_{11e}}{1 + h_{21e}}$$

$$= \frac{h_{11c}}{h_{11c} h_{22c} - h_{21c} h_{12c}} \approx \frac{-h_{11c}}{h_{21c}}$$

$$h_{12b} = \frac{h_{11e}h_{22e} - h_{12e}(1+h_{21e})}{(1+h_{21e})(1-h_{12e})+h_{11e}h_{22e}} \approx \frac{h_{11e}h_{22e}}{1+h_{21e}} - h_{12e}$$

$$= \frac{h_{21c}(1-h_{12c})+h_{11c}h_{22c}}{h_{11c}h_{22c} - h_{21c} h_{12c}} \approx (h_{12c}-1) - \frac{h_{11c}h_{22c}}{h_{21c}}$$

$$h_{21b} = \frac{-h_{21e}(1-h_{12e})-h_{11e}h_{22e}}{(1+h_{21e})(1-h_{12e})+h_{11e}h_{22e}} \approx \frac{-h_{21e}}{1+h_{21e}}$$

$$= \frac{h_{12c}(1+h_{21c})-h_{11c}h_{22c}}{h_{11c}h_{22c}-h_{21c}h_{12c}} \approx \frac{-(1+h_{21c})}{h_{21c}}$$

$$h_{22b} = \frac{h_{22e}}{(1+h_{21e})(1-h_{12e})+h_{11e}h_{22e}} \approx \frac{h_{22e}}{1+h_{21e}}$$

$$= \frac{h_{22c}}{h_{11c}h_{22c}-h_{21c}h_{12c}} \approx \frac{h_{22c}}{h_{21c}}$$

Common collector h parameters in terms of common base and common emitter h parameters.

$$h_{11c} = \frac{h_{11b}}{(1+h_{21b})(1-h_{12b})+h_{22b}h_{11b}} \approx \frac{h_{11b}}{1+h_{21b}} = h_{11e}$$

$$h_{12c} = \frac{1+h_{21b}}{(1+h_{21b})(1-h_{12b})+h_{22b}h_{11b}} \approx 1 = 1 - h_{12e}$$

$$h_{21c} = \frac{h_{12b}-1}{(1+h_{21b})(1-h_{12b})+h_{22b}h_{11b}} \approx \frac{-1}{1+h_{21b}} = -(1+h_{21e})$$

$$h_{22c} = \frac{h_{22b}}{(1 + h_{21b})(1 - h_{12b}) + h_{22b} h_{11b}} \approx \frac{h_{22b}}{1 + h_{21b}} = h_{22e}$$

Expressions for voltage gain, current gain, input impedance, and output impedance in terms of y, z, h, and g parameters.

Voltage Gain

$$A_V = \frac{z_{21}Z_L}{\Delta z + z_{11}Z_L} = \frac{-y_{21}}{y_{22}+Y_L} = \frac{-h_{21}Z_L}{h_{11}+\Delta h Z_L} = \frac{g_{21}Z_L}{g_{22}+Z_L}$$

$$= \frac{S_{21}(1 + \Gamma_L)}{(1-S_{22}\Gamma_L)(1+S_{11}')}$$

Current Gain

$$A_I = \frac{-z_{21}}{z_{22}+Z_L} = \frac{-y_{21}Y_L}{\Delta y + y_{11}Y_L} = \frac{h_{21}Y_L}{h_{22}+Y_L} = \frac{-g_{21}}{\Delta g + g_{11}Z_L}$$

Input Impedance

$$Z_{IN} = \frac{\Delta z + z_{11}Z_L}{z_{22}+Z_L} = \frac{y_{22}+Y_L}{\Delta y + y_{11}Y_L} = \frac{\Delta h + h_{11}Y_L}{h_{22}+Y_L} = \frac{g_{22}+Z_L}{\Delta g + g_{11}Z_L}$$

Output Impedance

$$Z_{OUT} = \frac{\Delta z + z_{22}Z_s}{z_{11}+Z_s} = \frac{y_{11}+Y_s}{\Delta y + y_{22}Y_s} = \frac{h_{11}+Z_s}{\Delta h + h_{22}Z_s} = \frac{\Delta g + g_{22}Y_s}{g_{11}+Y_s}$$

Conversion between y parameters and s (scattering) parameters:

$$s_{11} = \frac{(1-y_{11})(1+y_{22}) + y_{12}y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \quad \dagger$$

$$s_{12} = \frac{-2y_{12}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \quad \dagger$$

$$s_{21} = \frac{-2y_{21}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \quad \dagger$$

$$s_{22} = \frac{(1+y_{11})(1-y_{22}) + y_{21}y_{12}}{(1+y_{11})(1+y_{22}) - y_{12}y_{21}} \quad \dagger$$

$$y_{11} = \left[\frac{(1+s_{22})(1-s_{11}) + s_{12}s_{21}}{(1+s_{11})(1+s_{22}) - s_{12}s_{21}} \right] \frac{1}{Z_o}$$

$$y_{12} = \left[\frac{-2s_{12}}{(1+s_{11})(1+s_{22}) - s_{12}s_{21}} \right] \frac{1}{Z_o}$$

$$y_{21} = \left[\frac{-2s_{21}}{(1+s_{11})(1+s_{22}) - s_{12}s_{21}} \right] \frac{1}{Z_o}$$

$$y_{22} = \left[\frac{(1+s_{11})(1-s_{22}) + s_{12}s_{21}}{(1+s_{22})(1+s_{11}) - s_{12}s_{21}} \right] \frac{1}{Z_o}$$

Where Z_o = the characteristic impedance of the transmission lines used in the scattering parameter system, usually 50 ohms.

Conversion between h parameters and s parameters:

$$s_{11} = \frac{(h_{11}-1)(h_{22}+1) - h_{12}h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \quad \ddagger$$

$$s_{12} = \frac{2h_{12}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \quad \ddagger$$

$$s_{21} = \frac{-2h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \quad \ddagger$$

$$s_{22} = \frac{(1+h_{11})(1-h_{22}) + h_{12}h_{21}}{(h_{11}+1)(h_{22}+1) - h_{12}h_{21}} \quad \ddagger$$

$$h_{11} = \left[\frac{(1+s_{11})(1+s_{22}) - s_{12}s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}} \right] Z_o$$

$$h_{12} = \frac{2s_{12}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$$

$$h_{21} = \frac{-2s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}}$$

$$h_{22} = \left[\frac{(1-s_{22})(1-s_{11}) - s_{12}s_{21}}{(1-s_{11})(1+s_{22}) + s_{12}s_{21}} \right] \frac{1}{Z_o}$$

$\dagger$ In converting from y to s parameters, the y parameters must first be multiplied by Z_o and then substituted in the equations for conversion to s parameters.

$\ddagger$ In converting from h to s parameters, the h parameters must first be normalized to Z_o in the following manner and then substituted in the equations for conversion to s parameters:

Parameter	To Normalize
h_{11}	divide by Z_o
h_{12}	use as is
h_{21}	use as is
h_{22}	multiply by Z_o

Conversion between z parameters and s parameters:

$$Z_{11} = \left[\frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \right] Z_o$$

$$Z_{12} = \left[\frac{2S_{12}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \right] Z_o$$

$$Z_{21} = \left[\frac{2S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \right] Z_o$$

$$Z_{22} = \left[\frac{(1+S_{22})(1-S_{11}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \right] Z_o$$

$$S_{11} = \frac{(Z_{11}-1)(Z_{22}+1) - Z_{12}Z_{21}}{(Z_{11}+1)(Z_{22}+1) - Z_{12}Z_{21}} \quad \dagger\dagger\dagger$$

$$S_{12} = \frac{2Z_{12}}{(Z_{11}+1)(Z_{22}+1) - Z_{12}Z_{21}} \quad \dagger\dagger\dagger$$

$$S_{21} = \frac{2Z_{21}}{(Z_{11}+1)(Z_{22}+1) - Z_{12}Z_{21}} \quad \dagger\dagger\dagger$$

$$S_{22} = \frac{(Z_{11}+1)(Z_{22}-1) - Z_{12}Z_{21}}{(Z_{11}+1)(Z_{22}+1) - Z_{12}Z_{21}} \quad \dagger\dagger\dagger$$

$\dagger\dagger\dagger$ In converting from z to s parameters, the z parameters must first be divided by Z_o, and then substituted in the equations for conversion to s parameters.

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