Problem 8.1

a) The implementation is in 1a.txt file.

b) The implementation is in 1b.txt file.

c) Base case:

Let rbn be the total number of nodes in a red black tree.

The first element added (root) has to be black according to property 2 (RooB property). (rbn=1)

If n=1 (rbn=2), the second element added as the left child is automatically colored to red. The NIL leaves should be black, according to property 3.

If the second element is black, there will be 1 black node between the second element's NIL nodes and the root, whereas there will be no black node between right NIL node of the root and the root, so it is violating property 5.

So, if n=1, the child node (second element) must be red.

Inductive Hypothesis:

Let rbn be the total number of red nodes in a red black tree. In a red black tree, there exists at least one red node where $1< n \leq$ rbn.

Inductive Step:

If we want to insert the element starting from rbn+1th node, there are two possibilities that the parent node will either be black or red.

If the parent node is black, the new added node must be red according to the base case.

If the parent node is red, there are some possible cases.

Case 1: If the uncle will also be red. Then, we have to recolor the grandparent, parent and uncle nodes as all paths must have same unique black nodes from the root (property 5), the NIL nodes must be black and the red node can only have black child nodes. Then, we should still remain the color of new child node to be red so that there will be at least one red node.

Case 2: After inserting child node, the parent and grandparent node need to have rotations. Then, recoloring of parent and grandparent is required. After that, the child node will still remain red, so we will still have at least one red node.

Case 3: After rotating the new node with the parent and then again with the grandparent node, the color of the parent will remain red after recoloring the grandparent and the child node. So, the tree will have at least one red node.

Problem 8.2

The implementation is in rbt.py file.

References

Bibeknam. (2019, February 25). Bibeknam/algorithmtutorprograms. Retrieved April 12, 2019, from https://github.com/Bibeknam/algorithmtutorprograms/blob/master/data-structures/red-black-trees/red_black_tree.py
(I used this website to get ideas for Problem 8.2)

Homework 5 – Red Black Trees. (2010, October 21). Retrieved April 12, 2019, from https://www.csee.umbc.edu/courses/undergraduate/341/fall10/homework/graded5.pdf
(I used this website to get ideas for Problem 8.1 c)