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PAGE No
DATE : / 202

Parameter Estimation Assignment

Q1. normal distribution , mean $\rightarrow \theta_1$, variance $= \theta_2$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

joint density of $(x_1, x_2, x_3, \ldots x_n)$ is

$$L(\theta_1, \theta_2; x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x_i-\theta_1)^2}{2\theta_2}}$$

taking log on bothsides

$$\ln[L(\theta_1, \theta_2)] = \ln\left((2\pi\theta_2)^{-n/2} \cdot e^{-\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}}\right)$$

$$\ln[L(\theta_1, \theta_2)] = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\Sigma(x_i-\theta_1)^2$$

(θ_1) differentiate $\ln[L(\theta_1, \theta_2)]$ w.r.t to θ_1.

$$\frac{\partial \ln L}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1) = 0$$

$$\boxed{\theta_1 = \frac{\Sigma x_i}{n}}$$ ← Sample mean

(θ_2) differentiate $\ln[L(\theta_1, \theta_2)]$ w.r.t to θ_2

$$\frac{\partial \ln L}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2}\Sigma(x_i-\theta_1)^2 = 0.$$

$$\boxed{\theta_2 = \frac{1}{n}\Sigma(x_i-\theta_1)^2}$$ ← variance

$\therefore$ MLE of θ_1 ~~for~~ is $\overline{X}$.

$\therefore$ MLE of θ_2 is $Var(X)$.

PAGE No.
DATE: / 202

Q2 $B(m, \theta) \rightarrow$ binomial distribution

$m \rightarrow$ no. of ~~tri~~ trails $\qquad \theta = (0,1)$

$$f(x) = {}^{m}C_{x}\, p^{x} (1-p)^{m-x} \qquad \boxed{p = \theta}$$

Joint density,

$$L(\theta; x_1, x_2, \dots x_m) = \prod_{i=1}^{m} P(x_i | m, p)$$

$$L(\theta) = \prod_{i=1}^{n} \left({}^{n}C_{x_i} \cdot \theta^{x_i} \cdot (1-\theta)^{m-x_i} \right)$$

taking log on both sides

$$\ln(L(\theta)) = \sum_{i=1}^{n} \log({}^{n}C_{x_i}) + \sum_{i=1}^{n} x_i \log\theta + \sum_{i=1}^{n} (m-x_i) \log(1-\theta)$$

differentiate w.r.t θ.

$$\frac{\partial \ln(L)}{\partial \theta} = \frac{1}{\theta} \sum_{i=1}^{n} x_i + \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i)(-1) = 0$$

$$\frac{1}{\theta} \sum_{i=1}^{n} x_i = \frac{1}{1-\theta} \sum_{i=1}^{n} (m-x_i)$$

$$(1-\theta) \sum x_i = \theta \sum (m-x_i)$$

$$\sum x_i = \theta \left(\sum m \right)$$

$$\boxed{\theta = \frac{\sum x_i}{m}} \quad \leftarrow \text{mean}$$

M.L.E of θ for $B(m, \theta)$ is $\overline{X}$.