Activation functions in Neural Networks

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While building a neural network, one key decision is selecting the *Activation Function* for both the hidden layer and the output layer. Activation functions decide whether a neuron should be activated.

In this article, we will explore the role of activation functions in neural networks, their types, and their impact on the learning process.

Pre-requisite: *Neural Networks*, *Backpropagation*

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What is an Activation Function?

An activation function is a mathematical function applied to the output of a neuron. It introduces non-linearity into the model, allowing the network to learn and represent complex patterns in the data. Without this noncalculating the weighted sum of inputs and adding a bias term. This helps the model make complex decisions and predictions by introducing non-linearities to the output of each neuron.

Why is Non-Linearity Important in Neural Networks?

Neural networks consist of neurons that operate using **weights**, **biases**, and **activation functions**.

In the learning process, these weights and biases are updated based on the error produced at the output—a process known as **backpropagation**. Activation functions enable backpropagation by providing gradients that are essential for updating the weights and biases.

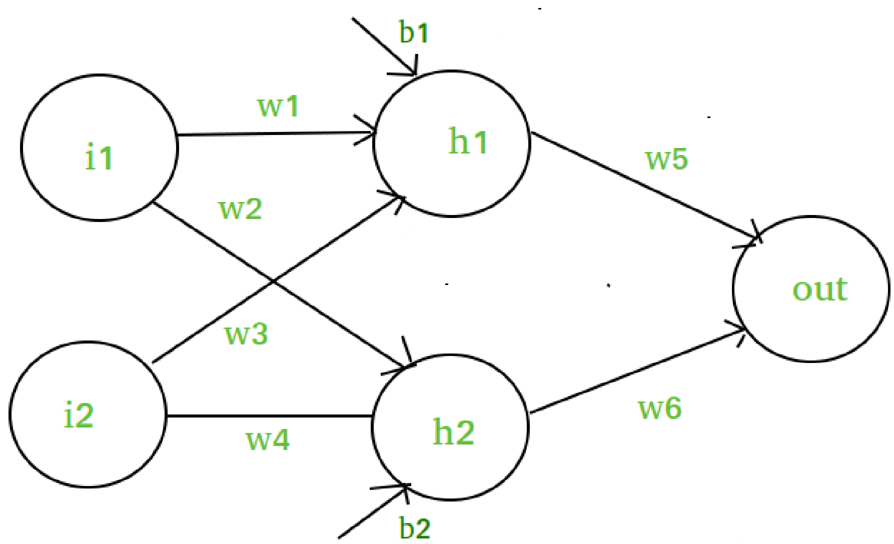
Without non-linearity, even deep networks would be limited to solving only simple, linearly separable problems. Activation functions empower neural networks to model highly complex data distributions and solve advanced deep learning tasks. Adding non-linear activation functions introduce flexibility and enable the network to learn more complex and abstract patterns from data.

Mathematical Proof of Need of Non-Linearity in Neural Networks

To illustrate the need for non-linearity in neural networks with a specific example, let's consider a network with two input nodes ($i_1$and i_2), a single hidden layer containing one neuron (h_1), and an output neuron (out). We will use w_1, w_2 as weights connecting the inputs to the hidden neuron, and w_5 as the weight connecting the hidden neuron to the output. We'll also include biases (b_1 for the hidden neuron and b_2 for the output neuron) to complete the model.

Network Structure

1. **Input Layer**: Two inputs i_1 and i_2.
2. **Hidden Layer**: One neuron h_1.
3. **Output Layer**: One output neuron.



Mathematical Model Without Non-linearity

Hidden Layer Calculation:

The input to the hidden neuron h1h_1h1 is calculated as a weighted sum of the inputs plus a bias:

$$z_{h_1} = w_1 i_1 + w_2 i_2 + b_1$$

Output Layer Calculation:

The output neuron is then a weighted sum of the hidden neuron's output plus a bias:

$$\text{output} = w_5 h_1 + b_2$$

If h_1 were directly the output of z_{h_1} (no activation function applied, i.e., $h_1 = z_{h_1}$), then substituting h_1 in the output equation yields:

$$\text{output} = w_5(w_1 i_1 + w_2 i_2 + b_1) + b_2$$

$$\text{output} = w_5 w_1 i_1 + w_5 w_2 i_2 + w_5 b_1 + b_2$$

This shows that the output neuron is still a linear combination of the inputs i_1 and i_2.

Thus, the entire network, despite having multiple layers and weights, effectively performs a linear transformation, equivalent to a single-layer perceptron.

Introducing Non-Linearity in Neural Network

To introduce non-linearity, let's use a non-linear activation function σ for the hidden neuron. A common choice is the ReLU function, defined as $\sigma(x) = \max(0, x)$.

Updated Hidden Layer Calculation:

$$h_1 = \sigma(z_{h_1}) = \sigma(w_1 i_1 + w_2 i_2 + b_1)$$

Output Layer Calculation with Non-linearity:

$$\text{output} = w_5 \sigma(w_1 i_1 + w_2 i_2 + b_1) + b_2$$

Effect of Non-linearity

The inclusion of the ReLU activation function \sigma allows h_1 to introduce a non-linear decision boundary in the input space. This non-linearity enables the network to learn more complex patterns that are not possible with a purely linear model, such as:

- Modeling functions that are not linearly separable.
- Increasing the capacity of the network to form multiple decision boundaries based on the combination of weights and biases.

Types of Activation Functions in Deep Learning

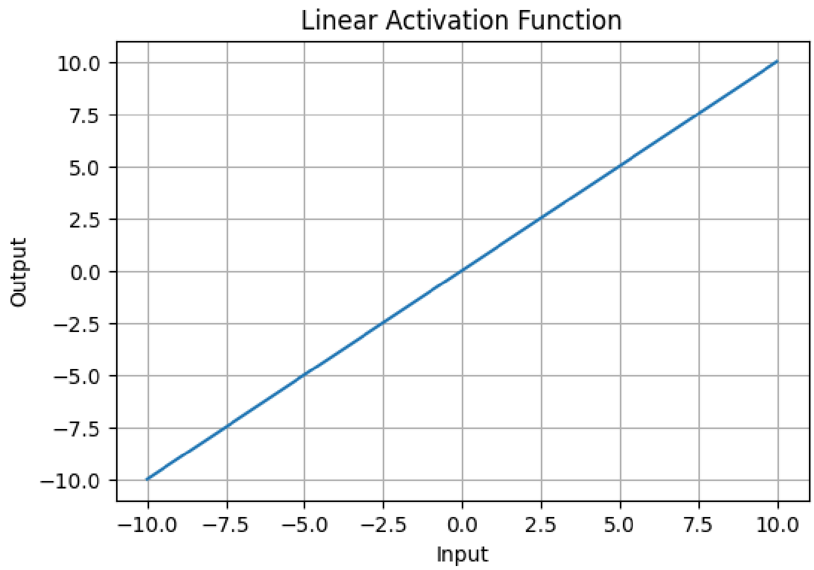
1. Linear Activation Function

Linear Activation Function resembles straight line define by y=x. No matter how many layers the neural network contains, if they all use linear activation functions, the output is a linear combination of the input.

- The range of the output spans from ($-\infty$ to $+\infty$).
- **Linear activation function** is used at just one place i.e. output layer.

- Using linear activation across all layers makes the network's ability to learn complex patterns limited.

Linear activation functions are useful for specific tasks but must be combined with non-linear functions to enhance the neural network's learning and predictive capabilities.



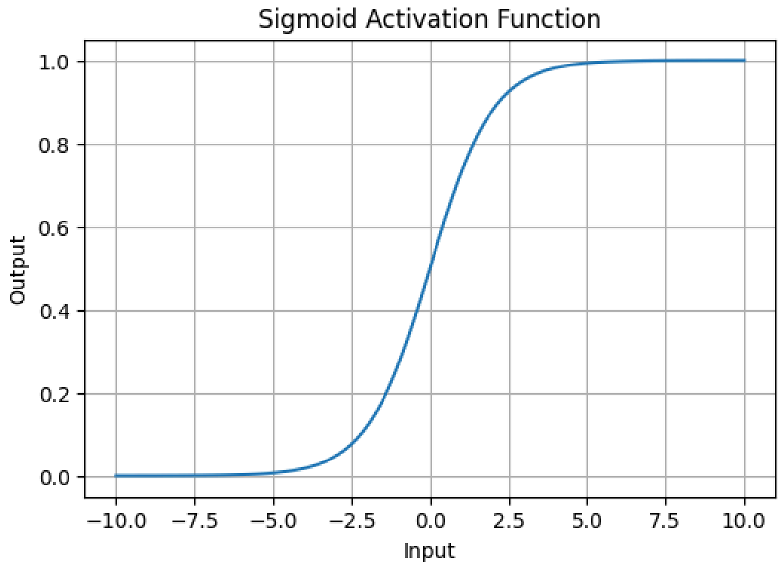
Linear Activation Function or Identity Function returns the input as the output

2. Non-Linear Activation Functions

1. Sigmoid Function

Sigmoid Activation Function is characterized by 'S' shape. It is mathematically defined as$A = \frac{1}{1+e^{-x}}$. This formula ensures a smooth and continuous output that is essential for gradient-based optimization methods.

- It allows neural networks to handle and model complex patterns that linear equations cannot.
- The output ranges between 0 and 1, hence useful for binary classification.
- The function exhibits a steep gradient when x values are between -2 and 2. This sensitivity means that small changes in input x can cause significant changes in output y, which is critical during the training process.



Sigmoid or Logistic Activation Function Graph

2. Tanh Activation Function

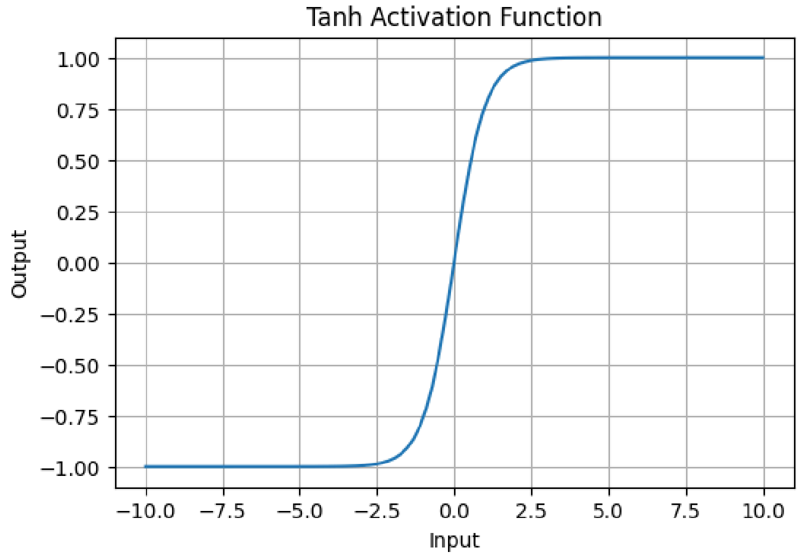
Tanh function or hyperbolic tangent function, is a shifted version of the sigmoid, allowing it to stretch across the y-axis. It is defined as:

$$f(x) = \tanh(x) = \frac{2}{1+e^{-2x}} - 1.$$

Alternatively, it can be expressed using the sigmoid function:

$$\tanh(x) = 2 \times \text{sigmoid}(2x) - 1$$

- **Value Range**: Outputs values from -1 to +1.
- **Non-linear**: Enables modeling of complex data patterns.
- **Use in Hidden Layers**: Commonly used in hidden layers due to its zero-centered output, facilitating easier learning for subsequent layers.

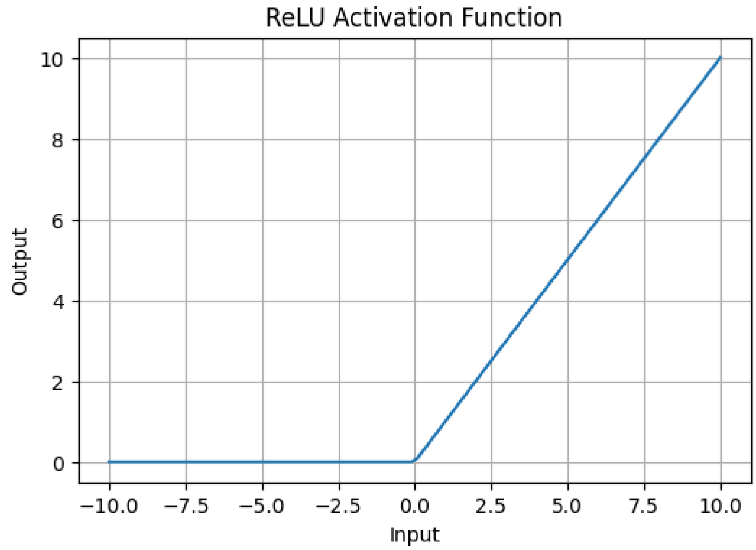


Tanh Activation Function

3. ReLU (Rectified Linear Unit) Function

ReLU activation is defined by $A(x) = \max(0, x)$, this means that if the input x is positive, ReLU returns x, if the input is negative, it returns 0.

- **Value Range**: $[0, \infty)$, meaning the function only outputs non-negative values.
- **Nature**: It is a **non-linear** activation function, allowing neural networks to learn complex patterns and making backpropagation more efficient.
- **Advantage over other Activation:** ReLU is less computationally expensive than tanh and sigmoid because it involves simpler mathematical operations. At a time only a few neurons are activated making the network sparse making it efficient and easy for computation.



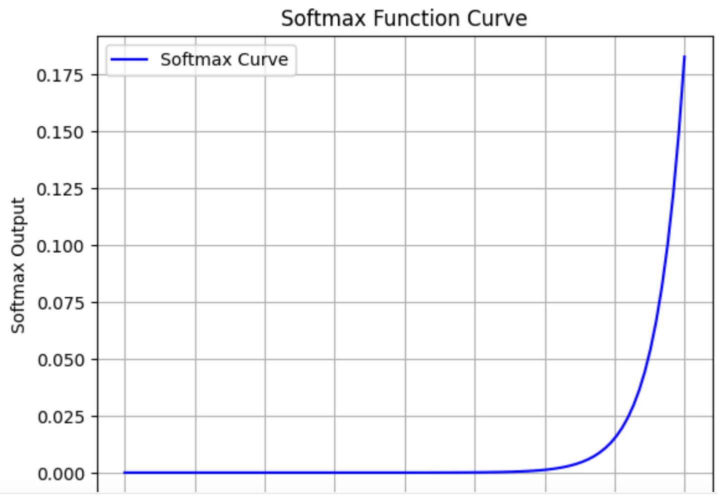
ReLU Activation Function

3. Exponential Linear Units

1. Softmax Function

[Softmax function] is designed to handle multi-class classification problems. It transforms raw output scores from a neural network into probabilities. It works by squashing the output values of each class into the range of 0 to 1, while ensuring that the sum of all probabilities equals 1.

- Softmax is a **non-linear** activation function.
- The Softmax function ensures that each class is assigned a probability, helping to identify which class the input belongs to.

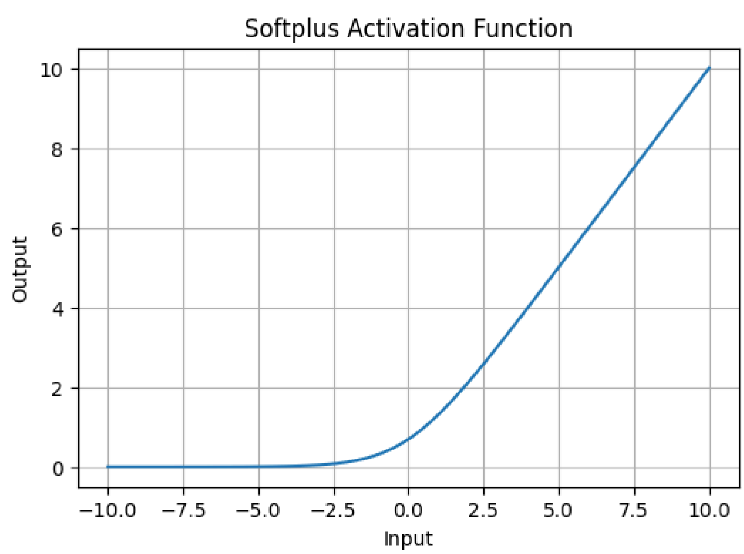


Softmax Activation Function

2. SoftPlus Function

Softplus function is defined mathematically as: $A(x) = \log(1 + e^x)$. This equation ensures that the output is always positive and differentiable at all points, which is an advantage over the traditional ReLU function.

- **Nature**: The Softplus function is **non-linear**.
- **Range**: The function outputs values in the range $(0, \infty)$, similar to ReLU, but without the hard zero threshold that ReLU has.
- **Smoothness**: Softplus is a smooth, continuous function, meaning it avoids the sharp discontinuities of ReLU, which can sometimes lead to problems during optimization.



Softplus Activation Function

Impact of Activation Functions on Model Performance

The choice of activation function has a direct impact on the performance of a neural network in several ways:

1. **Convergence Speed:** Functions like **ReLU** allow faster training by avoiding the vanishing gradient problem, while **Sigmoid** and **Tanh** can slow down convergence in deep networks.
2. **Gradient Flow:** Activation functions like **ReLU** ensure better gradient flow, helping deeper layers learn effectively. In contrast, **Sigmoid** can lead to small gradients, hindering learning in deep layers.
3. **Model Complexity:** Activation functions like **Softmax** allow the model to handle complex multi-class problems, whereas simpler functions like **ReLU** or **Leaky ReLU** are used for basic layers.

Conclusion

Activation functions are the backbone of neural networks, enabling them to capture non-linear relationships in data. From classic functions like Sigmoid and Tanh to modern variants like ReLU and Swish, each has its place in different types of neural networks. The key is to understand their behavior and choose the right one based on your model's needs.

Frequently Asked Questions : Activation Functions

What is the activation function?

> *An activation function determines the output of a neuron in a neural network by adding non-linearity, enabling the network to learn complex patterns from the data.*

What is ReLU and softmax?

- ***ReLU*** *outputs the input directly if it's positive, or zero otherwise, and is used in hidden layers to speed up training.*
- ***Softmax*** *is used in the output layer for multi-class classification, converting raw outputs into probabilities for each class.*

What is the ReLU activation function?

ReLU is activation function that helps avoid vanishing gradients and computationally efficient in deep learning.

What is the difference between ReLU and TANH?

ReLU *outputs positive values directly and zero for negatives, while* ***Tanh*** *maps inputs between -1 and 1. Tanh is zero-centered but suffers from vanishing gradients, unlike ReLU which does not for positive values.*