Brief Reports

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Cross sections for single-tagged two-photon production of resonances

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Simple formulas are given for the production cross section of narrow resonances in single-tagged photon-photon collisions. Production of spin-zero and spin-one states is considered.

I. INTRODUCTION

In an earlier paper[1] I considered the production of spin-one particles in e^+e^--induced photon-photon collisions. Among the topics considered there was the production cross section evaluated in the equivalent-photon approximation.[2] That discussion was deficient in certain regards and those deficiencies are corrected here. More specifically, if there is a single tag, so the electron, say, is observed, but not the positron, the cross section contains an integral over the mass squared of the untagged photon $\int dQ_1^2/Q_1^2$ where the lower limit is $O(m_e^2)$ and the upper limit is $O(s)$, where s is the square of the center-of-mass energy of the e^+e^- collision. The factor $\int dQ_1^2/Q_1^2$ was replaced in Ref. 1 by $\ln s/4m_e^2$, which is correct only to leading-logarithmic factors.

Three corrections can be identified.[3] First, a more precise lower limit for the integration over Q_1^2 must be used. If the scattered electron has an energy $E'=(1-x)E$, where E is the initial beam energy, then for forward scattering, to leading order in m_e^2/E^2,

$$Q_{1\,\min}^2=m_e^2x^2/(1-x) \ . \tag{1.1}$$

Second, if the experiment in question antitags, that is, rejects, events in which Q_1^2 exceeds some value $Q_{1\,\max}^2$, the upper limit on the Q_1^2 integral is not s but $Q_{1\,\max}^2$. Third, for the production of a hadronic final state, there will be a form factor $F(Q^2)$ that will enter: $\int (dQ_1^2/Q_1^2)F^2(Q_1^2)$. If $Q_{1\,\max}^2$ is smaller than the characteristic cutoff of $F(Q_1^2)$, the influence of the form factor will be negligible. If, on the contrary, there is no antitagging, $F(Q^2)$ will play a decisive role. For a form factor $F(Q^2)=(1+Q^2/M_V^2)^{-1}$ we have

$$\int dQ^2F^2(Q^2)/Q^2\simeq\ln(M_V^2/m_e^2)$$

rather than $\ln(s/m_e^2)$. Thus, for production of exclusive hadronic states, the final result will always contain either $\ln(Q_{1\,\max}^2/m_e^2)$ or $\ln(M_V^2/m_e^2)$ and never $\ln(s/m_e^2)$.

Of particular interest is the production of narrow resonant states. In a single-tagging experiment the measured quantity is

$$d\sigma(e^+e^-\to e^+e^-R)/dQ_2^2 \ ,$$

where Q_2^2 is the negative of the four-momentum transfer squared of the tagged electron. In a high-statistics experiment, it is possible to compare the measured spectrum to one predicted from the partial width $\Gamma(R\to\gamma\gamma)$ and the appropriate form factors $F(Q^2)$. If the data are not so extensive it is necessary to assume a form for $F(Q^2)$ and compare

$$\int dQ^2 d\sigma(e^+e^-\to e^+e^-R)/dQ^2$$

to the observed data to deduce $\Gamma(R\to\gamma\gamma)$. A slight modification is necessary if R has $J=1$ since the $\Gamma(R\to\gamma\gamma)$ vanishes for real photons. Instead one has

$$\Gamma(R\to\gamma\gamma^*)\simeq(Q_2^2/M^2)\tilde\Gamma(R\to\gamma\gamma^*)$$

for small Q_2^2. It is then $\tilde\Gamma$ that is deduced from

$$\int dQ^2 d\sigma(e^+e^-\to e^+e^-R)/dQ^2 \ .$$

The plan of the paper is as follows. The kinematics of single-tagged cross sections are presented in Sec. II. The production of pseudoscalars is discussed in Sec. III and the production of spin-one states in Sec. IV. The double-tagged cross section is discussed briefly in Sec. V.

II. KINEMATICS OF SINGLE-TAGGED CROSS SECTIONS

A convenient starting point is given by Bonneau, Gourdin, and Martin.[4] We follow them with a few notational differences. The electron and positron initial momenta are k_1 and k_2 and the corresponding final momenta are k_1' and k_2'. The momentum transfers squared are $Q_1^2=-(k_1-k_1')^2=-q_1^2>0$ and $Q_2^2=-(k_2-k_2')^2=-q_2^2>0$. The virtual-photon energies in the e^+e^- c.m. are $x_1\sqrt{s}/2$ and $x_2\sqrt{s}/2$, where $s=(k_1+k_2)^2$ is the c.m. energy squared. The photon-photon invariant-mass squared is $W^2=(q_1+q_2)^2$. In the photon-photon c.m., the photon momentum is

$$K=\{[(W^2+Q_1^2+Q_2^2)^2-4Q_1^2Q_2^2]/4W^2\}^{1/2} . \quad (2.1)$$

From Eqs. (28) and (29) of Ref. 4, dropping terms involving azimuthal dependence or initial beam polarization,[5]

$$d\sigma=\frac{\alpha^2}{2\pi^4 s}\frac{KW}{Q_1^2Q_2^2}(\sigma_{LL}K_{LL}+\sigma_{LT}K_{LT}+\sigma_{TL}K_{TL}+\sigma_{TT}K_{TT})\frac{d^3k_1'}{E_1'}\frac{d^3k_2'}{E_2'} , \quad (2.2)$$

where $E_1'=k_{10}'$, $E_2'=k_{20}'$, and

$$K_{LL}=\left[2\left[\frac{k_1\cdot\bar{Q}_1}{KW}\right]^2-\frac{1}{2}\right]\left[2\left[\frac{k_2\cdot\bar{Q}_2}{KW}\right]^2-\frac{1}{2}\right] , \quad (2.3a)$$

$$K_{LT}=\left[2\left[\frac{k_1\cdot\bar{Q}_1}{KW}\right]^2-\frac{1}{2}\right]\left[2\left[\frac{k_2\cdot\bar{Q}_2}{KW}\right]^2+\frac{1}{2}-\frac{2m_e^2}{Q_2^2}\right] , \quad (2.3b)$$

$$K_{TL}=\left[2\left[\frac{k_1\cdot\bar{Q}_1}{KW}\right]^2+\frac{1}{2}-\frac{2m_e^2}{Q_1^2}\right]\left[2\left[\frac{k_2\cdot\bar{Q}_2}{KW}\right]^2-\frac{1}{2}\right] , \quad (2.3c)$$

$$K_{TT}=\left[2\left[\frac{k_1\cdot\bar{Q}_1}{KW}\right]^2+\frac{1}{2}-\frac{2m_e^2}{Q_1^2}\right]\times\left[2\left[\frac{k_2\cdot\bar{Q}_2}{KW}\right]^2+\frac{1}{2}-\frac{2m_e^2}{Q_2^2}\right] . \quad (2.3d)$$

Here

$$\bar{Q}_1=q_2-(q_1\cdot q_2/q_1^2)q_1,\quad \bar{Q}_2=q_1-(q_1\cdot q_2/q_2^2)q_2 , \quad (2.4)$$

so

$$k_1\cdot\bar{Q}_1=\tfrac{1}{4}[2(x_2s+Q_2^2)-(W^2+Q_1^2+Q_2^2)] , \quad (2.5a)$$

$$k_2\cdot\bar{Q}_2=\tfrac{1}{4}[2(x_1s+Q_1^2)-(W^2+Q_1^2+Q_2^2)] . \quad (2.5b)$$

Henceforth we write

$$W'^2=W^2+Q_1^2+Q_2^2 . \quad (2.6)$$

These results are particularly simple if $Q_1^2,Q_2^2 \ll W^2$ and $Q_1^2,Q_2^2 \gg m_e^2$. Then

$$K\approx\tfrac{1}{2}W \quad (2.7)$$

and

$$q_1\approx x_1k_1,\quad q_2\approx x_2k_2 , \quad (2.8)$$

so

$$W^2\simeq 2q_1\cdot q_2=x_1x_2s . \quad (2.9)$$

Then

$$k_1\cdot\bar{Q}_1=\tfrac{1}{4}(2x_2-x_1x_2)s, \text{ etc. } , \quad (2.10)$$

and

$$K_{LL}=[2(1-x_1)/x_1^2][2(1-x_2)/x_2^2] , \quad (2.11a)$$

$$K_{LT}=[2(1-x_1)/x_1^2]\{[(1-x_2)^2+1]/x_2^2\} , \quad (2.11b)$$

$$K_{TL}=\{[(1-x_1)^2+1]/x_1^2\}[2(1-x_2)/x_2^2] , \quad (2.11c)$$

$$K_{TT}=\{[(1-x_1)^2+1]/x_1^2\}\{[(1-x_2)^2+1]/x_2^2\} . \quad (2.11d)$$

This is useful for some of the double-tagging region.

Here, however, we are interested primarily in single tagging, i.e., where $Q_2^2\gtrsim 0.1$ GeV$^2 \gg m_e^2$, but where possibly $Q_1^2\approx m_e^2$. Now, in fact, we expect all cross sections to be damped by form factors in Q_1^2 and Q_1^2 with scales of roughly m_ρ^2. As a result the untagged photon will rarely have $Q_1^2>m_\rho^2$. If in addition we veto events with $Q_1^2>Q_{1\,\max}^2$ (antitagging) so $Q_1^2<Q_{1\,\max}^2 \ll W^2$, we can approximate

$$W'^2=W^2+Q_2^2,\quad KW=\tfrac{1}{2}W'^2 , \quad (2.12a)$$

$$k_1\cdot\bar{Q}_1=\tfrac{1}{4}[2(x_2s+Q_2^2)-W'^2] , \quad (2.12b)$$

$$k_2\cdot\bar{Q}_2=\tfrac{1}{4}(2x_1s-W'^2) . \quad (2.12c)$$

If we define

$$x_2'=x_2+Q_2^2/s , \quad (2.13)$$

we find

$$x_1x_2's=W'^2\equiv\zeta's \quad (2.14)$$

in place of (2.9). Thus

$$k_1\cdot\bar{Q}_1=\tfrac{1}{4}x_2'(2-x_1),\quad k_2\cdot\bar{Q}_2=\tfrac{1}{4}x_1(2-x_2') , \quad (2.15a)$$

$$KW=\tfrac{1}{2}W'^2 \quad (2.15b)$$

and

$$K_{LL}=\left[\frac{2(1-x_1)}{x_1^2}\right]\left[\frac{2(1-x_2')}{x_2'^2}\right] , \quad (2.16a)$$

$$K_{LT}=\left[\frac{2(1-x_1)}{x_1^2}\right]\left[\frac{(1-x_2')^2+1}{x_2'^2}\right] , \quad (2.16b)$$

$$K_{TL}=\left[\frac{(1-x_1)^2+1}{x_1^2}-\frac{2m_e^2}{Q_1^2}\right]\left[\frac{2(1-x_2')}{x_2'^2}\right] , \quad (2.16c)$$

$$K_{TT}=\left[\frac{(1-x_1)^2+1}{x_1^2}-\frac{2m_e^2}{Q_1^2}\right]\left[\frac{(1-x_2')^2+1}{x_2'^2}\right] . \quad (2.16d)$$

Using

$$d^3k_1'/E_1'=\pi\, dx_1dQ_1^2 , \quad (2.17)$$

Eq. (2.2) becomes

$$d\sigma=\frac{\alpha^2}{\pi^2}\frac{dQ_1^2}{Q_1^2}\frac{dQ_2^2}{Q_2^2}\frac{dx_1}{x_1}\frac{d\zeta'}{\zeta'}\left[\sigma_{LL}(1-x_1)(1-x_2')+\sigma_{LT}(1-x_1)\left[\frac{1+(1-x_2')^2}{2}\right]+\sigma_{TL}\left[\frac{1+(1-x_1)^2}{2}-\frac{m_e^2x_1^2}{Q_1^2}\right](1-x_2')+\sigma_{TT}\left[\frac{1+(1-x_1)^2}{2}-\frac{m_e^2x_1^2}{Q_1^2}\right]\left[\frac{1+(1-x_2')^2}{2}\right]\right] . \quad (2.18)$$

The minimum value of Q_1^2 is

$$Q_{1\,\min}^2 \approx m_e^2 x_1^2/(1-x_1) . \tag{2.19}$$

Integrating Q_1^2 from $Q_{1\,\min}^2$ to $Q_{1\,\max}^2 \gg m_e^2$ and ignoring the dependence of σ_{LL} and σ_{LT}, etc., on Q_1^2 and dropping for the moment σ_{LL} and σ_{LT},

$$d\sigma = \frac{\alpha^2}{\pi^2}\frac{dQ_2^2}{Q_2^2}\frac{dx_1}{x_1}\frac{d\zeta'}{\zeta'} \times \{\tfrac{1}{2}[1+(1-x_1)^2]\ln(Q_{1\,\max}^2/Q_{1\,\min}^2)-1+x_1\} \times \{\tfrac{1}{2}[1+(1-x_2')^2]\sigma_{TT}+(1-x_2')\sigma_{TL}\} . \tag{2.20}$$

III. PRODUCTION OF PSEUDOSCALARS

Consider the production of a pseudoscalar resonance. The Breit-Wigner form for a narrow resonance is

$$\sigma_{TT} = (8\pi/K^2)\tfrac{1}{4}\Gamma_{R\to\gamma\gamma}\pi M\delta(W^2-M^2) . \tag{3.1}$$

To allow for the off-shell behavior we write

$$\sigma_{TT} = \frac{8\pi^2}{4K^2}\Gamma_{R\to\gamma\gamma}M\delta(W^2-M^2)F^2(Q_1^2)F^2(Q_2^2)\left[\frac{2K}{M}\right]^3 , \tag{3.2}$$

where the last factor reflects the p-wave nature of the decay. From Eqs. (2.20) and (3.2), and using $2K/M = \zeta' s/M^2$,

$$d\sigma = \frac{\alpha^2}{\pi^2}\frac{8\pi^2}{M^3}\Gamma_{R\to\gamma\gamma}\frac{dQ_2^2}{Q_2^2}F^2(Q_2^2)\frac{dx_1}{x_1} \times \left[\ln\frac{Q_{1\,\max}^2}{m_e^2}\frac{1-x_1}{x_1^2}\frac{1+(1-x_1)^2}{2}-(1-x_1)\right] \times \tfrac{1}{2}[1+(1-x_2')^2] , \tag{3.3}$$

where $x_1x_2' = (M^2+Q_2^2)/s = \zeta'$. From $x_1x_2'=\zeta'$ we see that x_1 is to be integrated from $x_1=\zeta'$ to 1. In practical circumstances $\zeta' \ll 1$ and we find

$$\int_{\zeta'}^1 \frac{dx}{x}\left[1-x+\frac{x^2}{2}\right]\left[1-\frac{\zeta'}{x}\right] \simeq \ln\frac{1}{\zeta'}-\frac{7}{4} , \tag{3.4a}$$

$$\int_{\zeta'}^1 \frac{dx}{x}\left[1-x+\frac{x^2}{2}\right]\left[1-\frac{\zeta'}{x}+\frac{\zeta'^2}{2x^2}\right] \simeq \ln\frac{1}{\zeta'}-\frac{3}{2} , \tag{3.4b}$$

$$\int_{\zeta'}^1 \frac{dx}{x}\left[1-x+\frac{x^2}{2}\right]\left[1-\frac{\zeta'}{x}\right]\ln\frac{1-x}{x^2} \simeq [\ln(1/\zeta')]^2-2\ln(1/\zeta')-\pi^2/6+\tfrac{7}{8} , \tag{3.4c}$$

$$\int_{\zeta'}^1 \frac{dx}{x}\left[1-x+\frac{x^2}{2}\right]\left[1-\frac{\zeta'}{x}+\frac{\zeta'^2}{2x^2}\right]\ln\frac{1-x}{x^2} \simeq [\ln(1/\zeta')]^2-\tfrac{3}{2}\ln(1/\zeta')-\pi^2/6+\tfrac{5}{8} , \tag{3.4d}$$

$$\int_{\zeta'}^1 \frac{dx}{x}(1-x)\left[1-\frac{\zeta'}{x}\right] \simeq \ln\frac{1}{\zeta'}-2 . \tag{3.4e}$$

Using the appropriate forms from Eq. (3.4) in Eq. (3.3),

$$d\sigma = \frac{\alpha^2}{\pi^2}\frac{8\pi^2}{M^3}\Gamma_{R\to\gamma\gamma}\frac{dQ_2^2}{Q_2^2}F^2(Q_2^2) \times \left[\left[\ln\frac{1}{\zeta'}-\frac{3}{2}\right]\ln\frac{Q_{1\,\max}^2}{m_e^2}+\left[\ln\frac{1}{\zeta'}\right]^2 -\frac{5}{2}\ln\frac{1}{\zeta'}+\frac{19}{8}-\frac{\pi^2}{6}\right] . \tag{3.5}$$

This can be integrated numerically over the region of acceptance. As an approximation we can set $\zeta'=\zeta$. As an example consider $\sqrt{s}=29$ GeV, $M=0.96$ GeV, $Q_{1\,\max}^2=0.1$ GeV^2, $\zeta=1.10\times10^{-3}$. Then the approximation gives

$$\sigma = (18.4\ \text{pb})[\Gamma\ (\text{keV})]\int\frac{dQ_2^2}{Q_2^2}F^2(Q_2^2) . \tag{3.6}$$

Of course in formulas (3.5) and (3.6) we have allowed only for tagging one side and if either the electron or positron is allowed as a tag, the result must be multiplied by 2.

IV. PRODUCTION OF SPIN-ONE RESONANCES

For the cross section to produce spin-one resonances in photon-photon collisions we take[1]

$$\sigma_{TL} = \frac{4\pi}{K^2}\frac{3}{2}\left[\frac{2K}{M}\right]^3\left[\frac{Q_2^2}{M^2}\right]\tilde{\Gamma}\pi M \times \delta(W^2-M^2)F^2(Q_1^2)F^2(Q_2^2) , \tag{4.1}$$

$$\sigma_{TT} = \frac{4\pi}{K^2}\frac{3}{4}\left[\frac{2K}{M}\right]^3\left[\frac{Q_2^2}{M^2}\right]^2\tilde{\Gamma}\pi M \times \delta(W^2-M^2)F^2(Q_1^2)F^2(Q_2^2) , \tag{4.2}$$

which are approximate forms derived from the ansatz

$$\mathcal{M} = \mathcal{A}(Q_1^2,Q_2^2)\epsilon_{\alpha\beta\gamma\delta}(Q_1^2k_2^\alpha-Q_2^2k_1^\alpha)\xi^\beta\epsilon_1^\gamma\epsilon_2^\delta . \tag{4.3}$$

This form gives, as the partial widths to two virtual photons with energies ω_1 and ω_2 and virtualities Q_1^2 and Q_2^2,

$$\Gamma_{TL} = \frac{1}{12\pi}\frac{K}{M^2}\left[Q_2^2\omega_1-Q_1^2\omega_2+\frac{Q_1^2+Q_2^2}{\omega_2}K^2\right]^2 \times(\omega_2^2/Q_2^2)|\mathcal{A}(Q_1^2,Q_2^2)|^2 , \tag{4.4}$$

$$\Gamma_{TT} = \frac{1}{12\pi}\frac{K}{M^2}(Q_2^2\omega_1-Q_1^2\omega_2)^2|\mathcal{A}(Q_1^2,Q_2^2)|^2 , \tag{4.5}$$

where K is the c.m. photon momentum. In the limit $Q_1^2\to0$, $K=(M^2+Q_2^2)/(2M)=\omega_1$, $\omega_2=(M^2-Q_2^2)/(2M)$, and

$$\Gamma_{TL} \simeq (1/12\pi) K^3 Q_2^2 \, |\mathcal{A}(0,Q_2^2)|^2 , \quad (4.6)$$

$$\Gamma_{TT} \simeq (1/12\pi) K^3 (Q_2^2/M^2)^2 \, |\mathcal{A}(0,Q_2^2)|^2 M^2 . \quad (4.7)$$

This motivates the K and Q^2 dependence in Eqs. (4.1) and (4.2). Inserting Eq. (4.1) into Eq. (2.20) and approximating $Q_1^2 \approx 0$ where appropriate,

$$d\sigma_{TL} = \frac{\alpha^2}{\pi^2} \frac{dQ_1^2}{Q_1^2} \frac{dQ_2^2}{Q_2^2} \frac{dx_1}{x_1} \left[1 - x_1 + \frac{x_1^2}{2} - \frac{m_e^2 x_1^2}{Q_1^2} \right] \times \left[1 - \frac{\zeta'}{x_1} \right] \frac{24\pi^2}{M^3} \left[\frac{Q_2^2}{M^2} \right] \tilde{\Gamma} F^2(Q_2^2) . \quad (4.8)$$

Integrating over x_1 and Q_1^2,

$$d\sigma_{TL} = \frac{\alpha^2}{\pi^2} \frac{24\pi^2 \tilde{\Gamma}}{M^3} \left[\ln \frac{Q_{1\,\max}^2}{m_e^2} \left[\ln \frac{1}{\zeta'} - \frac{7}{4} \right] + \left[\ln \frac{1}{\zeta'} \right]^2 - 3 \ln \frac{1}{\zeta'} - \frac{\pi^2}{6} + \frac{23}{8} \right] \times (dQ_2^2/M^2) F^2(Q_2^2) . \quad (4.9)$$

Again, we can approximate this by setting $\zeta' = \zeta$. As an example, if $M = 1.4$ GeV, $Q_{1\,\max}^2 = 0.1$ GeV^2, $\sqrt{s} = 29$ GeV, $\zeta = 2.33 \times 10^{-3}$, then

$$\sigma_{TL} = (13.6 \text{ pb}) [\tilde{\Gamma} \text{ (keV)}] \int \frac{dQ_2^2}{M^2} F^2(Q_2^2) . \quad (4.10)$$

Similarly for the σ_{TT} portion,

$$d\sigma_{TT} = \frac{\alpha^2}{\pi^2} \frac{12\pi^2 \tilde{\Gamma}}{M^3} \left[\ln \frac{Q_{1\,\min}^2}{m_e^2} \left[\ln \frac{1}{\zeta'} - \frac{3}{2} \right] + \left[\ln \frac{1}{\zeta'} \right]^2 - \frac{5}{2} \ln \frac{1}{\zeta'} - \frac{\pi^2}{6} + \frac{19}{8} \right] \times (dQ_2^2/M^2)(Q_2^2/M^2) F^2(Q_2^2) . \quad (4.11)$$

For the same parameters used for Eq. (4.10) we obtain the approximation

$$\sigma_{TT} = (7.3 \text{ pb}) [\tilde{\Gamma} \text{ (keV)}] \int \frac{dQ_2^2}{M^2} \left[\frac{Q_2^2}{M^2} \right] F^2(Q_2^2) \quad (4.12)$$

if we replace ζ' by ζ. For the range 0.2 GeV$^2 < Q_2^2 < 1.2$ GeV^2, the approximations (4.10) and (4.12) are typically too large by 9% for $M \sim 1.4$ GeV.

V. DOUBLE-TAGGED CROSS SECTION

Even if it is intended to antitag, that is, reject events with $Q^2 > Q_{\max}^2$ on the side opposite the tagged lepton, this cannot be done perfectly, but only with some efficiency ϵ. Thus it is necessary to correct for the fraction $1-\epsilon$ of these events, which are incorrectly included with the events actually having $Q^2 < Q_{\max}^2$. Typically this is a small correction, so it suffices to do a very approximate calculation. From Eq. (4.10) we see that, for spin-one production,

$$\sigma(Q_1^2 > Q_{\max}^2) \approx \frac{\alpha^2}{\pi^2} \frac{24\pi^2 \tilde{\Gamma}}{M^2} \int_{Q_{\max}^2}^{\infty} \frac{dQ_1^2}{Q_1^2} F^2(Q_1^2) \int_{Q_-^2}^{Q_+^2} \frac{dQ_2^2}{M^2} F^2(Q_2^2) \int_{\zeta'}^{1} \frac{dx_1}{x_1} \left[1 - x_1 + \frac{x_1^2}{2} \right] \left[1 - \frac{\zeta'}{x_1} \right] \quad (5.1)$$

$$\approx \frac{\alpha^2}{\pi^2} \frac{24\pi^2 \tilde{\Gamma}}{M^3} \left[\ln \frac{m_\rho^2 + Q_{\max}^2}{Q_{\max}^2} - \frac{m_\rho^2}{m_\rho^2 + Q_{\max}^2} \right] \int_{Q_-^2}^{Q_+^2} \frac{dQ_2^2}{M^2} F^2(Q_2^2) \left[\ln \frac{1}{\zeta'} - \frac{7}{4} \right] . \quad (5.2)$$

For $M = 1.4$ GeV and $Q_{\max}^2 = 0.1$ GeV^2 this is approximately

$$\sigma(Q_1^2 > Q_{\max}^2) \approx (0.85 \text{ pb}) [\tilde{\Gamma} \text{ (keV)}] \int \frac{dQ_2^2}{M^2} F^2(Q_2^2) , \quad (5.3)$$

which is less than 7% of the single-tagged cross section.

VI. SUMMARY

The single-tagged cross section for photon-photon production of pseudoscalar and vector resonances can be expressed as simple integrals over Q^2 of the tagged photon. In typical circumstances corrections for events in the double-tag region are small.

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