

In general, we can show that inductive types are discrete and segal, as they offer no higher homotopical and simplicial data. We start with Unit.

define Unit
in section 2

Lemma 0.1. *Unit is discrete*

Proof. It suffices to show that the map induced by idtoarr between total spaces

$$\sum_{v:\text{Unit}} u = v \rightarrow \sum_{v:\text{Unit}} \text{hom}_{\text{Unit}}(u, v) \quad (1)$$

is an equivalence. The domain is contractible as it is a based path space. In the codomain, both u and v compute to \star , so it suffices to show $\text{hom}_{\text{Unit}}(\star, \star)$ is contractible. This is immediate as

$$\left\langle \Delta^1 \rightarrow \text{Unit} \Big|_{[\star, \star]}^{\partial \Delta^1} \right\rangle \quad (2)$$

has contractible fibers, thus the entire extension type is contractible by relative function extensionality. \square

We also expect the type of natural numbers to be discrete too.

define natural numbers

Lemma 0.2. *\mathbb{N} is discrete.*

Proof.

\square

The unit type corresponds to the category with one object and one morphism (the identity).