Maximum Likelihood and GMM

1. You plan to study the relationship between work and schooling. using the following parametric nonlinear model:

$$\Pr(work = 1|educ) = \frac{1}{1+\exp(-\beta_0 - \beta_1 educ)}$$

where *work* is a binary variable that takes value 1 if someone participates in the labor market and 0 otherwise, and *educ* represents the years of education and can take values between 6 and 20.

   (a) Write down the log likelihood function and derive the score.

   (b) Simulate a random $i.i.d.$ sample $\{work_i, educ_i\}$, $i = 1, \ldots, 1000$, where $educ_i \sim U(6, 20)$ and $\beta_0 = -2$, $\beta_1 = 0.2$ and

$$work_i = \begin{cases} 1 & \text{if } u_i < \Pr(work_i = 1|educ_i) \\ 0 & \text{otherwise} \end{cases}$$

   where $u_i \sim U(0, 1)$.

   (c) Estimate β using ML.

   (d) Explain why the estimator computed by the following Stata command

```
. gmm (work - {b0} - {b1} * educ}), instruments(educ)
```

   is not consistent.

   (e) Propose a consistent and asymptotically normal estimator of β other than the MLE.

   (f) Explain how you would implement your estimator in (b) in Stata.

   (g) Repeat (a)-(f), but now assuming that

$$\Pr(work = 1|educ) = \Phi(\beta_0 + \beta_1 educ)$$

   where $\Phi(\cdot)$ is the standard normal CDF.

   (h) Compute the Wald, LR and LM test for $H_0 : \beta_1 = 0$.

2. Consider the following special one parameter case of the gamma distribution,

$$f(y) = (y/\lambda^2)\exp(-y/\lambda)$$

where $y > 0$ and $\lambda > 0$.

(a) Show that $E[y_i] = 2\lambda$ and $V(y_i) = 2\lambda^2$.

Suppose that $\lambda_i = \exp(\beta_0 + \beta_1 x_{1i})/2 = \exp(x_i'\beta)/2$, then it follows from (a) that $E[y_i|x_i] = \exp(x_i'\beta)$ and $V[y_i|x_i] = \exp(x_i'\beta)^2/2$.

(b) Show that the moment conditions in (a) imply that

$$E[x_i\left((y_i - \exp(x_i'\beta))^2 - \exp(x_i'\beta)^2/2\right)] = 0$$

(c) Assume the structure above and $x_{1i} \sim U(0,1)$, $\beta_0 = 1$, $\beta_1 = 2$. Draw $N = 1000$ observations from this data generating process and estimate β using the moment condition in (b).

(d) Can you find another set of moment conditions other than those in (b)? Use these to estimate β.

(e) Which set of moment conditions you prefer? Tip: Compare the performance of the estimators in (c) and (d) using a Monte Carlo simulation (see -help simulate-).